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Sources of uncertainty in the seismic design of submerged free-standing racks

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Abstract

Free-standing racks are 5 m tall structures that store spent fuel removed from the nuclear power reactor on the depths of a spent fuel pool. Rack units are arranged on the floor of this 12 meters deep pool separated by only a few centimeters. Their response to an earthquake event is a troubling safety issue as they are in submerged and free-standing conditions. Such a seismic analysis deals with a highly nonlinear behavior, a transient dynamic response and a fluid-structure interaction problem. To overcome these difficulties in a cost-effective manner, the current analysis methodology implements the hydrodynamic mass concept in commercial finite element analysis software. However, some dispersion of results still exists in the application of this ad-hoc methodology. This paper reviews the seven major sources of uncertainty inherent to the current analysis methodology together with the main challenges of the seismic analysis.

Keywords: uncertainty; seismic analysis; free-standing; racks; fluid-structure interaction; nonlinear; transient analysis; hydrodynamic mass;

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1. Introduction to the rack context

Spent fuel storage racks are tailored stainless steel structures made up of an array of storage cells. They are used in the first step of the waste management process, during the wet storage of the radioactive spent fuel within the spent fuel pool. Rack units rest free-standing on the floor of a 12 meters depth pool and separated by only a few centimeters to fit within the pool dimensions (Fig. 1). Recent high density racks include neutron-absorbing materials to reduce the critical distance that allow putting the fuel assemblies closer and maximizing the storage capacity of each rack.

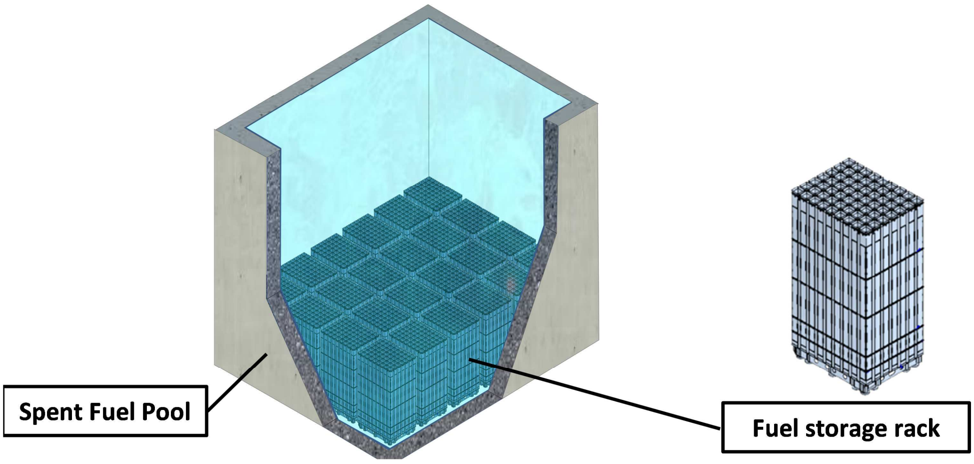

Fig. 1. Location of free-standing racks within the fuel spent fuel pool

When racks are subjected to an earthquake, they experience displacements and forces on supports that determine their final arrangement and design. Nuclear regulatory authorities classify racks as seismic category I and perform safety reviews according to the OT position paper [1], the U.S. NRC Standard Review Plan NUREG-0800 [2] and the ASME code. A complementary compilation of the few basic guidelines have been published by Ashar and DeGrassi [3] and DeGrassi [4].

The seismic analysis deals with a fluid-structure interaction problem, a very highly nonlinear behavior and a transient dynamic response. These challenges have been overcome thanks to an ad-hoc methodology, but some uncertainties remain and cause scatter in the results. Real safety margins are extremely difficult to predict, so the nuclear regulatory authorities request further studies as part of the approval and licensing process.

This paper summarizes the state of the art and improves the understanding of the underwater seismic response of storage racks, by identifying and discussing the sources of uncertainty inherent to the current analysis methodology.

2. Challenges of the seismic analysis

The seismic design of a free-standing rack faces three complex physical phenomena as described by Soler and Singh [5]:

- *Fluid-Structure Interaction* (FSI). The underwater conditions determine the dynamic response of the racks and fuel assemblies. Hydrodynamic forces arise at the wet boundary when the system of racks undergoes transient motion. These inertial fluid forces couple racks units, which otherwise were independent.
- *Highly nonlinear behavior*. The seismic load causes large displacements (e.g. sliding, rocking, twisting and turning), which involve energy dissipative effects such as friction and damping. In addition, this combination of displacements may cause impacts among racks or between the fuel assemblies and their storage cell. These singularities lead to nonlinearities to the point that Moudrik et al [6] show that the first mode frequency of the rack decreases when the excitation level increases.
- *Transient dynamic response*. The response of the rack system cannot be obtained using modal superposition due to the inherent nonlinearities. Hence, the dynamic analysis can only be performed through a time-history simulation with direct integration of the equations of motion in the three orthogonal directions. This method requires iterative step-by-step algorithms that are computationally very expensive. The entire seismic event is discretized into numerous small time steps where displacements and rotations are calculated through the basic equation of motion:

  $$[M]\{\ddot{u}(t)\}+[C]\{\dot{u}(t)\}+[K]\{u(t)\}=\{F(t)\} \tag{1}$$

  where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the structural system respectively and $\{u(t)\}$, $\{\dot{u}(t)\}$ and $\{\ddot{u}(t)\}$ are vectors containing the translational and rotational degrees of freedom of the structure and their respective velocities and accelerations. $\{F(t)\}$ is a vector containing the time-varying forces applied at each nodal degree of freedom.

3. Current analysis methodology

In their licensing efforts, industrial designers combine computer-aided Finite Element (FE) analysis with the hydrodynamic mass concept to perform this seismic analysis in a cost-effective manner. This scheme, described by Zhao et al [7, 8], generally accepts the following conservative assumptions:

- The earthquake is characterized by large accelerations during short intervals of time, so water velocities remain limited. Thus, Dong [9] states that fluid damping could be neglected.
- Relative rack displacements remain small compared to nominal water gaps. Hence, fluid coupling forces can be calculated once at the initial arrangement and kept constant for the entire analysis duration.
- Structural damping is consistent with 5% of the critical linear viscous damping.
- Fuel assemblies rattle in phase within the storage cells. Their dynamic effect can be represented through a single assembly with the combined mass at the corresponding centroid.
- Water is assumed inviscid and incompressible. The pool flow satisfies Laplace's equation ($\nabla^2\Phi=0$).

4. Sources of uncertainty inherent to the current analysis methodology

Following the thoughts of Bathe [10], the uncertainty existing in the racks seismic analysis can be attributed to seven different sources [11]: 'Accuracy of input data', 'conceptual and mathematical modeling', 'finite element methods', 'large displacement analysis in 3D', 'contact and friction conditions', 'hydrodynamic mass approach to fluid-structure interaction' and 'numerical integration of the dynamic equation'. These sources are reviewed next.

4.1. Accuracy of input data

Accurate input data are rarely, if ever, available. There is always a degree of uncertainty associated to the variables defining the scenario. Any type of measurement contains deviations from reality due to sampling, modelling and instrument errors. Queval et al [12] and Champonier et al [13] show the importance of this issue in rack seismic analysis where critical input data including friction coefficients, Rayleigh damping coefficients, fuel loading distribution, clearance between racks, acceleration time-history, etc. are stochastic variables. Furthermore, the radioactive ambiance on site prevents measurement campaigns that would contribute to reduce the uncertainty associated to the real conditions.

For instance, the experimental work by Rabinowicz [14] demonstrates that the values of the friction coefficient for stainless steel-to-stainless steel contact in underwater conditions may lie in a wide range bounded by values from 0.2 to 0.8. The seismic loading is even more uncertain, i.e. a given design response spectra can lead to a boundless number of ground acceleration-time stories. In order to address the latter, the designer follows the Power Spectral Method to create a set of synthetic histories that meet the criteria of statistical independence and envelope the target design response spectra and a target Power Spectra Density function.

The impact of these stochastic variables in the racks behavior cannot be known a priori. A probabilistic analysis would be worth to accurately assess the bounds of the response and to gather a better understanding on how uncertainties propagate from the input data to the output results when real conditions differ from those reflected by the inputs.

4.2. Conceptual and mathematical modeling

The modelling process (Fig. 2) is the biased transformation of the reality under study into a manageable simplification. The usefulness of the simplification depends on its capability to predict the real behaviour but not upon its completeness or complexity.

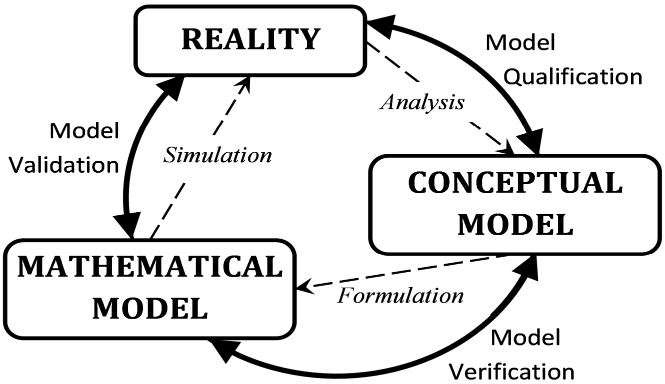

Fig. 2. Modeling process. Adapted from Oberkampf et al [15] based on the work by Schlesinger [16]

Conceptual models are imperfect abstractions influenced by our personal belief about the governing elements and dominant factors. Their only purpose is to answer the analysis questions so physical details can be filtered out in areas of low interest if irrelevant to the analysis process.

Mathematical models are the consequent translation into the language of mathematics. They set the constitutive relationships to approach the structural behaviour through continuum mechanics theory. They generally include the classical theory of materials and beams as well as the complete specification of the initial conditions, boundary conditions and auxiliary conditions defining the Partial Differential Equations (PDEs) of the system.

Consequently, the features of the model are influenced by the analysis features. For instance, Hinderks et al [17] employ two significantly different models in the rack analysis. Fig. 3(a) shows an example of a very detailed structural model aimed to derive the dynamic properties and to check local instabilities and final static stresses. The seismic model of Fig. 3(b) only target dynamic loads and displacements, so it incorporates simplifications to reduce the computational effort of transient calculations. It must be noted that the most basic transient analysis of a 2-rack pool can last for 3 hours in an Intel core i3 processor.

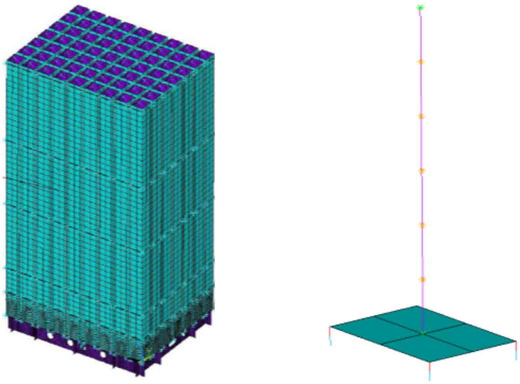
(a) (b)

Fig. 3. (a) Detailed structural model; (b) Simplified seismic model

This simplified seismic model represents the rack as a 3D wired model. The ensemble of storage cells is represented through a unique vertical beam, the fuel assemblies are grouped in a collinear vertical beam, while the base plate and rack feet are shaped as rigid bodies. This model is compatible with the motion of free-standing racks and maintains their principal dynamic properties. It is able to simulate the rattling of the fuel assemblies inside the storage cells and the eventual lift-offs with subsequent impacts of support pedestals with the pool liner.

However, there are some remaining uncertainties that can affect the rack motion. Among others, the manner by which the masses are lumped to nodes, the replication of the higher vibration modes, the surface distribution of the hydrodynamic coupling force, the stiffness of the grouped fuel assembly, the inertial contribution of the water mass between the storage cells, the application of the load input, etc. Being aware of the advantages and disadvantages, accuracy and range of applicability of each model is key for an accurate design and assessment. For example, in order to avoid acceleration discontinuities, Lee et al [18] recommend the use of the acceleration-time history rather than the displacement-time history as a seismic input.

4.3. Finite Element Methods

Building an appropriate model is only the first step of the analysis process. Models may be too complex to find an analytical solution. They often involve multiple degrees of freedom coupled through differential equations in the time-space domain. Hence, the continuous model is generally discretized into elements of simple geometry that can be addressed through computational analysis. This physical concept is the so-called physical FE method. The finite elements are then assembled according to the original model by coupling displacements and rotations of concatenated nodes. When the level of discretization and the connexions are appropriate, the FE model is considered to be a reasonable approximation of the continuous model.

FE methods influence the analysis of the racks system. The height of each rack is discretised in several nodes where the system connexions are applied (Fig. 4). Lumped masses, hydrodynamic forces and fuel contacts are established at those levels. The discretisation error decreases with the number of considered levels, but the computational time increases. Beyond a certain level of discretization, a finer mesh may not necessarily result in greater accuracy.

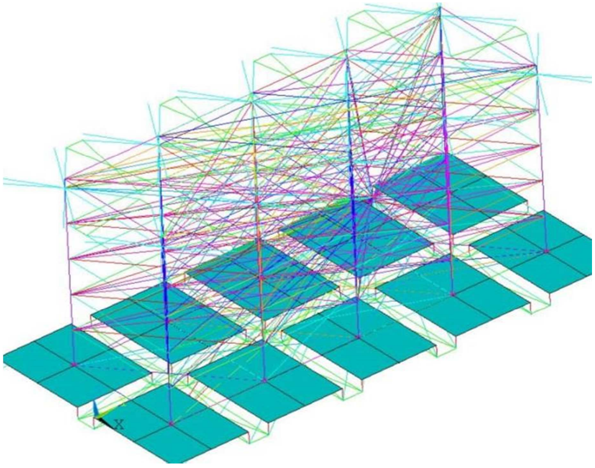

Fig. 4. Finite element model of a 10-racks pool with their multiple connexions for seismic assesment purpose

FE methods also deal with the stability of the numerical integration algorithm, the approximations of mathematical singularities, the consistency of the discrete equations with the PDEs and the differences in zones of influence between continuum and discrete systems. Irrespective of the accuracy in the numerical calculations, the resulting convergence and error characteristics of the FE equations can be determined through the analysis of the ordinary and PDEs created during the Taylor series expansions.

4.4. Large displacement analysis in 3D

The analysis of large displacements becomes an intricate task when the successive finite effects are not commutable. Unless the equations of motion are simultaneously solved in the three dimensions, nonlinear displacements will not be accurately predicted. This issue poses a great challenge in the mathematical computation and leads to incremental equilibrium formulations.

Different formulations have been developed to connect the computational mesh to the deforming continuum. Bathe et al [19, 20] point out their differences regarding the reference configuration that is used for tracking the boundary conditions and the linearization of the incremental equations of motion.

In the Lagrangian algorithms, the initial configuration is used as reference so each individual node of the mesh follows the associated material particle during motion. It is mainly used in structural mechanics since it allows an easy tracking of the boundary conditions and facilitates the treatment of materials with history-dependent constitutive relations. However, these algorithms may lead to constant re-meshing operations. These issues and the PDEs solving on the moving reference may become computationally too expensive in three dimensions. In the Eulerian algorithms, the reference corresponds to the last calculated configuration. After each time step, the computational mesh is fixed on a simple Cartesian grid. Hence, the Eulerian formulation turns the geometric problem into a PDEs problem adapted to deal with the discontinuities and nonlinearities typical of large deformations and topological changes. Finally, the Arbitrary Lagrangian-Eulerian (ALE) algorithms combine the best features of the two previous formulations: equations are related to a fixed coordinate system during a load increment and updated immediately after. It allows to handle greater distortions of the continuum than would be allowed by a purely Lagrangian method, with more resolution than is afforded by a pure Eulerian approach.

Free-standing racks are called to undergo large displacements namely sliding, rocking, twisting and turning, so the algorithm has to deal with these large displacements and the mobile boundaries. However, since racks only experience minor structural deformations and the water volume is replaced by the hydrodynamic mass concept, the Lagrange formulation is the most suitable to monitor these displacements.

4.5. Contact and friction conditions

Contact takes place when two independent surfaces touch each other. A contact is a singular discontinuity which represents a “changing-status” form of nonlinearity largely affecting and ill-conditioning the stiffness matrix of the system. Hu [21] states that the convergence difficulties derived from these sudden changes in acceleration, velocity and stress fields significantly increase the computational cost. Furthermore, contact events add several uncertainties to the analysis including the prediction of the point of contact, the orientation of the contact normal, the stiffness of the deformable surfaces and the associated friction effects.

Contacts represent the main source of nonlinearities in racks seismic analysis as they bring discontinuities in both forces and displacements. During the transient analysis, fuel assemblies rattle within the storage cell meanwhile vertical contacts appear and disappear at the rack's support due to its rocking behaviour. Moreover, lateral rack-to-rack and rack-to-pool impacts can happen. However, neither the real contact surfaces nor their orientation vector cannot be accurately represented in the wired model of racks. Only node-to-node linear contact elements are used in the potential contact locations. For instance, within physical gaps as fuel assemblies-storage cell, rack face-to-rack face, pedestal-to-pool liner and rack-to-pool wall. These contacts are generally simulated by means of compression-only spring elements with gap capability which provide for opening and closing interfaces without transmission of tensile normal forces.

Several contact formulations have been developed to ensure compatibility at the contact surface. Fig. 5(a) represents the Normal Lagrange method which enforces no penetration. The contact status is therefore a binary step function (i.e. either open or closed) arising a discontinuity that makes difficult to achieve convergence. In order to overcome this concern, designers generally use the penalty-based method of Fig. 5(b) to avoid the step change and facilitate convergence. It allows some slight penetration leading to a perfectly elastic impact where the reaction force ranges from zero just before the impact to near infinite when contact occurs. The stiffness coefficients have been studied by Oh and Ryu [22] for the fuel assemblies and by Stabel et al [23] for the pedestal. No energy dissipation is generally enforced but its conservatism has not been showed yet.

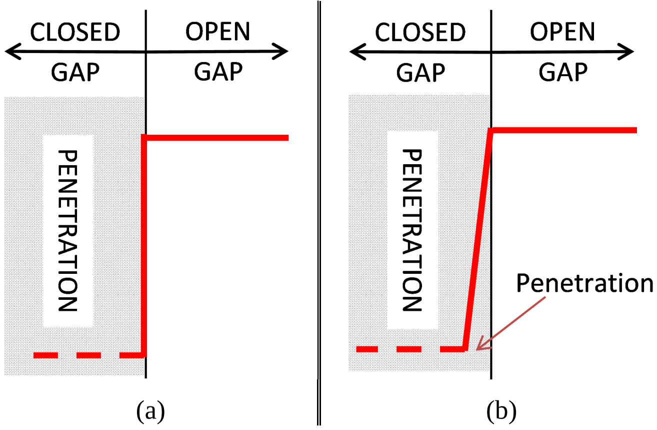

Fig. 5. Representation of two possible contact formulations in ANSYS [24]: (a) Normal Lagrange Method, (b) Penalty-based Method

After the contact event, the two solids are not stuck and may slide relative to each other along the interface. A frictional force opposite to the movement appears at the point of contact. The frictional force is often approached through a basic Coulomb model ignoring the difference between static and dynamic friction. Frictional elements behave as stiff springs until the force reaches a limiting value equal to the specified friction coefficient times the normal force. Low values of the friction coefficients at the interface pedestal-to-pool liner lead to a preponderating sliding motion whereas high values push to a rocking behaviour. Hence, a conservative value of friction cannot be prescribed a priori and separate analyses are usually performed to consider a maximum sliding case and a maximum rocking case respectively.

4.6. Hydrodynamic mass approach to Fluid-Structure Interaction (FSI)

FSI is a key factor in the dynamic analysis of a submerged structure. In addition to the buoyancy vertical force, fluid forces appear at the wet boundary when the fluid volume undergoes transient acceleration. These forces are derived from inertial effects of the fluid self-mass. During an earthquake event, both fuel assemblies and racks units vibrate inside the water volume. Therefore, hydrodynamic forces are established between the fuel assemblies and the storage cells, between the faces of the racks and the pool walls, and between the racks themselves.

These hydrodynamic forces are directly proportional to the exposure area and inversely proportional to the clearance, so their importance is given by the geometry and arrangement of the racks. In general, the area of a rack face is in the order of 10 m^2 and they are separated by relatively small water gaps (in the order of 50 mm). Consequently, the relative accelerations between rack units produces a large fluid pressure within the water gaps which somewhat couples the dynamic behaviour of the whole rack system. As a result, the natural frequencies of the system differ from those of a single rack.

This FSI could be approached through full 3D turbulent Navier-Stokes equations, but computational time would often make them prohibitive. Therefore, industrial designers assume some cost-effective simplifications in water behaviour, ignoring fluid damping and sloshing effects, and resort to the hydrodynamic mass concept which treats the water as a virtual extension of the structure. This approach replaces the external inertial effect of the surrounding water volume by an added mass changing the effective mass of the system as detailed by Chung and Chen [25]. Thus, Equation (1) becomes for the rack structure:

$$\lfloor M + m_{hydro} \rfloor \{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{F(t)\} \tag{2}$$

where the added matrix [m_{hydro}] is referred to as hydrodynamic mass matrix and represents the mass added to each rack itself (diagonal terms) and the coupling effect among the system elements (off-diagonal terms). This matrix affects each element individually by adding an equivalent mass to each element.

The evaluation of the hydrodynamic masses is not a trivial task. Analytical formulations have only been found for simple configurations on the assumption of a potential fluid flow. For instance, the work by Fritz [26] in two coaxial cylinders in 2D, Yang and Moran [27] in two coaxial hexagonal cylinders, Soler and Singh [28] in two coaxial squares in 2D, and Ren and Stabel [29] in two coaxial rectangles in 3D. More recent Computational Fluid Dynamics (CFD) studies by Stabel and Ren [30] validate these formulations when amplitudes of oscillations about the concentric position remain small relative to the fluid gap.

Even if these formulations have been traditionally applied to the racks system, their more complex arrangement has been subject of specific studies carried on by Moreira and Antunes [31, 32, 33]. However, designers prefer numerical approaches based on the Boundary Element Method (BEM) where boundary conditions are given by the acceleration of the in-water structures. Everstine and Henderson [34] directly solves the pressure field of a potential flow through the Helmholtz equation whereas Rangette [35] solves it for a Laplacian flow. Champonier et al [36] apply a thermal analogy with the steady-state heat conduction phenomena.

4.7. Transient analysis and numerical integration of the equation of motion

Dynamic analyses involve inertial and damping forces and require the resolution of differential equations involving time-dependant variables. When simple approaches are not adequate, it is necessary to implement numerical methods to integrate the equation of motion. These direct integration methods performs iterative calculations to solve the equation of motion with finite precision at a series of discrete points in space and time which are small relative to the loading duration. In general, they are not very sensitive to round-off errors, but truncation errors are committed during the series expansions. Hence, the accuracy of the solution depends, to some extent, on the effectiveness of the numerical method.

The transient analysis is the major issue of the racks seismic analysis. Rayleigh damping coefficients should be treated with caution. Stiffness-proportional damping should not eliminate important high-frequency modes whereas mass-proportional damping is not justified since it causes external forces reducing the base shear for seismic loading.

In addition, their nonlinear behavior rules out the modal superposition method and forces the direct integration of the equation of motion. Multiple step-by-step algorithms are available in the literature, but the single-step, implicit, unconditional stable methods are the best suited due to the rough accelerations of racks. This category encompasses algorithms developed in the last 50 years as Newmark [37], Wilson et al [38, 39], Wood and Bossak [40], alpha-method of Hilbert, Hughes and Taylor [41] and the generalized alpha-method of Chung and Hulbert [42]. However, only the most common algorithms are implemented in the commercial software. They allow to introduce artificial numerical damping through controlling external parameters to dissipate energy of the spurious high frequency oscillations. It enhances the numerical robustness since the numerical noise and overshoot effects are reduced. This algorithmic damping property is function of the relationship between the time step and the natural period and ensures adequate dissipation when affects the higher nodes without impacting the lower modes.

For linear systems, these time integration schemes have been thoroughly studied. Andujar et al [43] compare their features and state that the higher accuracy of the simulation, the lower becomes the stability field. In general, it was shown that the convergence, the stability and the accuracy directly depend on the time step parameter. However, in geometrically high nonlinear problems as in the case of racks, the behaviour needs to be carefully examined. Belytschko and Schoeberle [44] prove that dynamic simulations may become unstable even when using methods that are unconditionally stable for linear structures. Rapholder and Wunderlich [44] state that nonlinear simulations are stable only when there is no additional energy blow up due to numerical characteristics. Hence, Krenk [46] recommends specific studies to check the error in the conservation of energy and find the best settings. Examples of adaptive time stepping procedures with error propagation estimators for nonlinear systems have been proposed by Xie and Steven [47, 48] and Chang [49].

5. Conclusions and future work

The usefulness of any assessment depends on the accuracy and reliability of its output. The propagation of uncertainties plays a predominant role in the transitory analysis at hand, because the outputs at the end of a calculus step are taken as initial conditions for the following step. The uncertainty accumulated all over these stages may affect the final outputs in an unpredictable manner. This paper has reviewed seven sources of uncertainty that the practitioner needs to be aware of when carrying out a seismic analysis of spent fuel storage racks. Further steps seek to evaluate the outlined sources of uncertainty and to provide error estimates and an error bound via a probabilistic analysis and an experimental validation with data from a novel 2-rack physical model testing campaign.

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