



Title	Structural Analysis of Bridges with Time-Variant Modulus of Elasticity under Moving Loads
Authors(s)	Aied, Hussein, González, Arturo
Publication date	2012
Publication information	Aied, Hussein, and Arturo González. "Structural Analysis of Bridges with Time-Variant Modulus of Elasticity under Moving Loads." Taylor and Francis, 2012. https://doi.org/10.1201/b12352-407 .
Conference details	6th International Conference on Bridge Maintenance, Safety and Management (IABMAS), Villa Erba, Lake Como, Italy, 8-12 July, 2012
Publisher	Taylor and Francis
Item record/more information	http://hdl.handle.net/10197/6436
Publisher's version (DOI)	10.1201/b12352-407

Downloaded 2026-05-02 01:12:30

The UCD community has made this article openly available. Please share how this access benefits you. Your story matters! (@ucd_oa)



© Some rights reserved. For more information

Structural analysis of bridges with time-variant modulus of elasticity under moving loads

H. Aied, A. Gonzalez

University College Dublin, Dublin, Ireland

ABSTRACT: A simply supported bridge model is used to investigate the effect of a strain rate dependent modulus of elasticity on the dynamic response of the structure to a moving load. The bridge is modelled as a one-dimensional discretized finite element beam and the moving load is represented by a point force. A constant modulus of elasticity is traditionally employed when simulating the dynamic response of structures under moving loads. In this paper, a time-variant modulus is used to calculate strains and displacements and compare them to the traditional approach for different speeds and bridge spans. The time-variant modulus is obtained from the strain rate of the structure which is used in turn to update the strain at each point in time. The results show significant changes in the modulus and in the resulting load effect as load magnitude and speed increase.

1 INTRODUCTION

Most investigations on the response of bridges to moving loads take into consideration a constant modulus of elasticity (E_c) for each section of the structure, i.e., the influence of strain rate on the modulus is neglected. However, mechanical properties of the structure can vary depending on the type of load and how the load is applied. Past research based on the testing of concrete samples under dynamic loads indicates that the modulus of elasticity (known as time-variant modulus (E_d) in this context) tends to increase with increasing strain rate (Bischoff & Perry, 1991, Shkolnik, 2008). Below a specific lower strain rate limit, the structural material will react with a constant modulus of elasticity to the load, and above an upper strain rate limit, the modulus of elasticity will not increase any further (i.e. due to impact or explosive loading (Bischoff & Perry, 1991)). In between both limits, the time-variant modulus will vary with strain rate. As a result, if a section within a bridge had a time-variant strain rate that reached sufficiently high values, then the time-variant modulus would also vary in time, and it will affect other parameters (e.g., stiffness matrix) and the resulting load effects at each time step. This investigation focuses on assessing the influence of allowing for this time-variant modulus when calculating the overall response of a bridge to a moving load. In this paper, the strain calculated using a constant modulus is

denoted as normal strain, while the strain calculated using a time-variant modulus is called modified strain.

2 SIMULATION MODEL AND VARIATION OF MODULUS OF ELASTICITY WITH STRAIN RATE

A one-dimensional finite element discretized beam representing a simply supported bridge that allows for a time-variant modulus depending on the strain rate is modelled using MATLAB (MathWorks, 2010). The moving load is modelled as a constant point force. Unless stated otherwise, the bridge under investigation is 10m long, 16m wide and 0.65m deep (cross-sectional area of 10.4m^2) with a density of 2400 kg/m^3 and static modulus of $35 \times 10^9\text{N/m}^2$. The latter serves as a reference value when calculating the time-variant modulus at each point in time. These properties result in a first natural frequency of the beam of 11.25Hz, and a damping ratio of 2% is adopted, which is a typical value for small to medium sized bridges. Complex damping modelling is typically ignored in bridges because it is relatively small and has no significant impact on the outcome. Therefore, a linear damping model is adopted here which it is typically considered acceptable for simulation purposes. The

global damping matrix $[C_g]$ is expressed as a linear combination of global mass $[M_g]$ and stiffness $[K_g]$ matrices (i.e. Rayleigh damping) (Yang et al., 2004):

$$[C_g] = \mathbf{a}_0 [M_g] + \mathbf{a}_1 [K_g] \quad (1)$$

where a_0 and a_1 are the Rayleigh coefficients of first and second modes which can be determined using :

$$\begin{Bmatrix} \mathbf{a}_0 \\ \mathbf{a}_1 \end{Bmatrix} = \frac{2\xi}{\omega_1 + \omega_2} \begin{Bmatrix} \omega_1 \omega_2 \\ \mathbf{1} \end{Bmatrix} \quad (2)$$

where ξ is the damping ratio for the two modes which are assumed to have the same damping ratio, and ω_1 and ω_2 are the circular frequencies of vibration of first and second modes respectively.

The finite element beam is discretized into 100 elements and strain is calculated every 0.001s. Once the displacements have been obtained from integrating the equation of motion of the beam, the strain is calculated based on Equation (3) (Rowley, 2007, Przemieniecki, 1985):

$$\begin{aligned} \varepsilon(x_e) = & \frac{-y}{j_e^3} [(6l_e - 12(x_e))u_i + (4l_e^2 - 6l_e(x_e))\theta_i \\ & + (-6l_e + 12(x_e))u_{i+1} + (2l_e^2 - 6l_e(x_e))\theta_{i+1}] \end{aligned} \quad (3)$$

where l_e is the length of each discretized beam element, y is the distance to neutral axis of the section, x_e is the point along the element and $(u_i, \theta_i, u_{i+1}, \theta_{i+1})$ are the degrees of freedom on the nodes i and $i+1$ of the beam element. Then, the strain rate ($\dot{\varepsilon}$) is obtained from Equation (4):

$$\dot{\varepsilon} = (\varepsilon(t) - \varepsilon(t - \Delta t)) / \Delta t \quad (4)$$

where Δt is the time step (seconds), $\varepsilon(t)$ is the strain at the instant t , and $\varepsilon(t - \Delta t)$ is the strain at the previous instant $t - \Delta t$.

Here, the time-variant modulus is calculated based on the CEB-FIP Model Code formula given by Equation (5) (Rowe & Rene, 1991):

$$E_d / E_c = (\dot{\varepsilon} / \dot{\varepsilon}_0)^{0.026} \quad (5)$$

where E_d is the time-variant dynamic modulus of elasticity, E_c is the constant modulus of elasticity calculated from the specified characteristic strength of normal weight concrete (E_c assumed to be $35 \times 10^9 \text{ N/m}^2$ in this paper). $\dot{\varepsilon}_0$ is the static strain rate taken as $3 \times 10^{-6} \text{ s}^{-1}$ and $3 \times 10^{-5} \text{ s}^{-1}$ for tension and compression regions of the cross-section

respectively. Above these values, the time-variant modulus will differ from the constant modulus. Figure 1 illustrates the variation of the ratio of E_d to E_c for different strain rates in tension and compression when applying Equation (5).

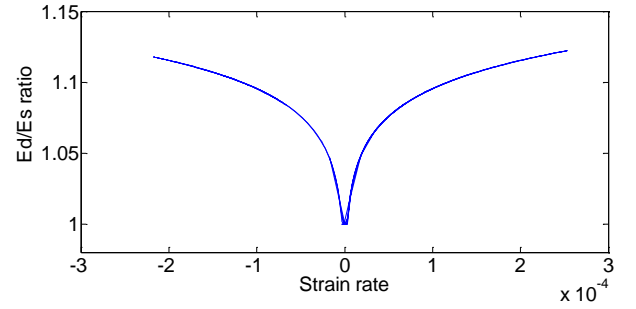


Figure 1: Ratio of dynamic modulus of elasticity to static modulus of elasticity versus strain rate

Figure 2 shows the variation of strain rate and time-variant dynamic modulus (E_d) from the time the load crosses the first element of the beam for the case of 100 kN crossing a 10 m span at 25 m/s. The strain is lower in the compression region than in the tension region. This is due to the use of a larger value of static strain rate ($\dot{\varepsilon}_0$) in the compression region as specified by (Rowe & Rene, 1991) resulting into a smaller value of time-variant modulus (Equation (5)). Therefore, Figure 2 and the rest of the figures throughout the paper, show results for the bottom (tension) strip of the cross-section. It can be seen for the example in Figure 2 that the time-variant modulus increases for high changes in strain rate with a maximum increase with respect to the constant modulus of about 9%. A detailed discussion on how strain rate and dynamic modulus of elasticity varies with magnitude and speed of a moving load can be found in Aied & González (2011).

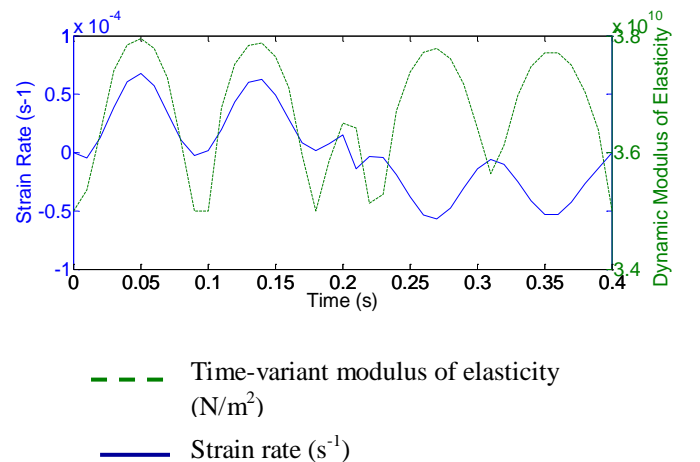


Figure 2: Time-variant modulus with strain rate versus time at mid-span

While Figure 2 has been obtained using a time step of 0.01s to simplify computational time, the rest of the graphs in the paper have been obtained using a time step of 0.001s.

The dynamic response of the modelled beam depends on the variation of displacement and strain with time. The cross-section is divided into a series of strips 0.0325 m high, and the time-variant modulus is calculated for each strip and for each elementary beam at each point in time. Since the one-dimensional beam model only allows for a single modulus and a single inertia to be entered for each elementary beam, a modified width (b_i) is calculated for each strip i to ensure a stiffness equivalent to the one provided by the use of a time-variant modulus. The modified width is obtained using the following equation:

$$b_i = b_0 \left(\frac{E_{di}}{E_c} \right) \quad (6)$$

where b_0 is the original width (16m), E_c is the constant modulus, and E_{di} is the time-variant modulus for each strip i . The modified section is then used to calculate the entire moment of inertia (I_z) of the cross-section that will be input into the stiffness matrix. For each strip, the inertia I_{zi} about its own axis is given by:

$$I_{zi} = \frac{b_i * d^3}{12} \quad (7)$$

where d is the height of each strip i (0.0325m for the model used in this paper). The values of I_z are obtained at each time step for the mid-section of each elementary beam. Applying the parallel-axis theorem and adding together the contribution of each strip, the inertia of the entire section is given by:

$$I_z = \sum_{i=1}^N (I_{zi} + A_i (d_{yi})^2) \quad (8)$$

where d_{yi} is the distance between the centre of the strip i to the neutral axis of the entire cross-section (m), A_i is the area of the strip ($b_i * d$), and N is the total number of strips. Figure 3 illustrates how the width of the model is modified to ensure an equivalent dynamic stiffness at each time step.

The variation of stiffness of the mid-span beam element as the load moves across the modelled bridge is shown in Figure 4 where $I_z * E_c$ is plotted versus time (I_z obtained using Equation (8) is the modified inertia necessary to ensure the correct dynamic stiffness, i.e., the variation of moment of inertia is proportional to the variation of time-variant modulus).

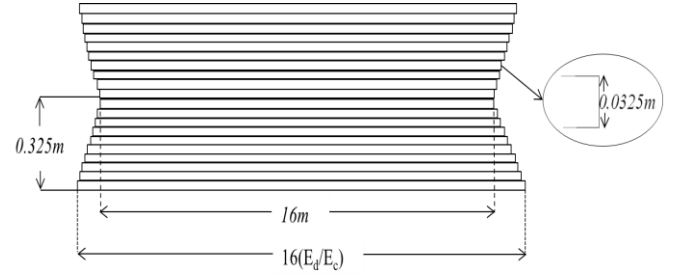


Figure 3: Modified cross-section of beam element to calculate the moment of inertia at each point in time

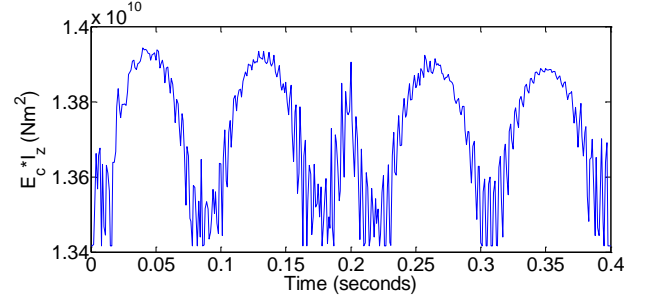


Figure 4: Stiffness ($E_c * I_z$) versus time

The global stiffness matrix of the bridge changes with time due to the variation in time-variant modulus. The global stiffness matrix is the result of assembling elementary stiffness matrices and it depends on the degrees of freedom and number of elements of the model. The modified elementary stiffness matrix can be calculated as:

$$[K'_e] = [K_e] \times \left(\frac{I_z}{I} \right) \quad (9)$$

where $[K'_e]$ is the modified stiffness matrix of the elementary beam, $[K_e]$ is the elementary stiffness matrix due to a constant modulus E_c , I is the moment of inertia based on the original cross-section and I_z is the moment of inertia due to time-variant modulus (Equation (8)).

3 TOTAL RESPONSE USING A VARYING MODULUS OF ELASTICITY

3.1 Total strain and displacement at the mid-span section

The strain derived from using the time-variant stiffness matrix described in Section 2 gives the results shown in Figure 5. The strain obtained using a constant modulus of elasticity (35×10^9 N/m²) is slightly higher than the strain calculated using a time-variant modulus. The largest difference between the strain and modified strain is about 5% and it occurs when the load is between the mid-span

and quarter span (i.e., about 0.15s after entering the bridge). For this example, maximum normal and modified strain take place when the load is located at mid-span, and the difference between both maximum strains is small.

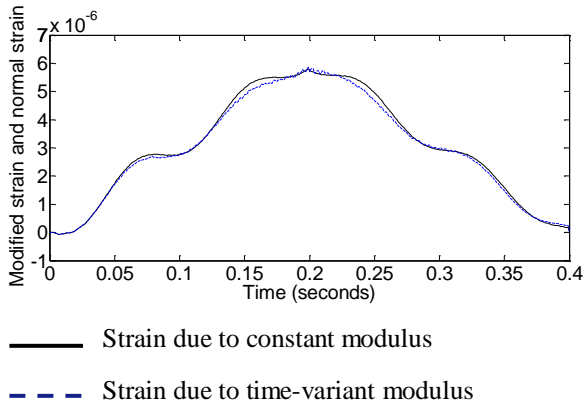


Figure 5: Modified strain and normal strain versus time at mid-span of the beam

The result of ignoring or including damping on the modified strain is shown in Figure 6. Both undamped and damped results are unfiltered, but the introduction of damping in the calculation gives a smoother graph through the integration process.

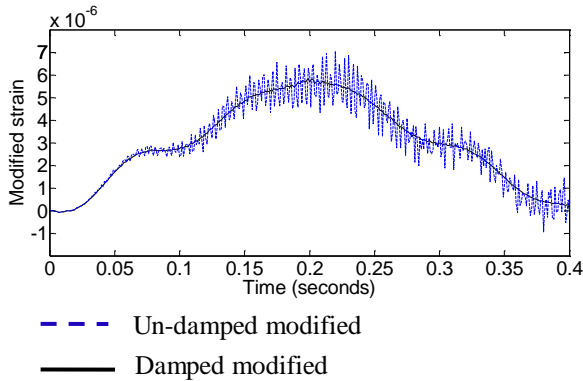


Figure 6: Comparison of damped and un-damped system in the calculation of modified strain

The displacement that results from the consideration of a time-variant modulus is shown in Figure 7 again for a load of 100kN travelling at 25m/s. The pattern of modified displacement is similar to that of the strain, and a smaller value of displacement can be appreciated when using a time-variant modulus compared to the constant modulus due to variations of the stiffness matrix with time.

The severity of the change is strongly influenced by bridge type and the load applied although some other factors are also important such as cross-sectional area and slab thickness. These properties also have a pronounced effect on the constant modulus and frequency of vibration.

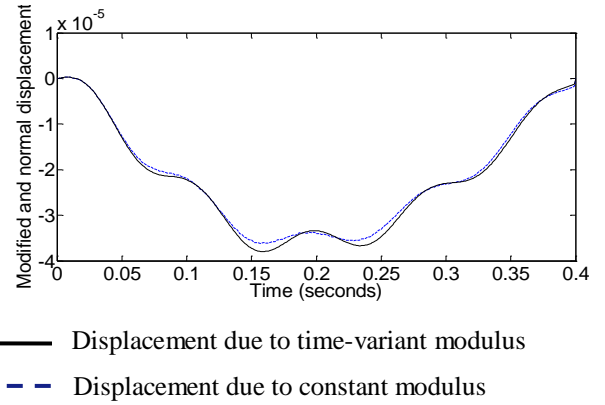


Figure 7: Modified and normal displacement at mid-span

3.2 Total strain response for different longitudinal sections and depths

The response of the bridge model is illustrated for other two longitudinal locations in Figure 8. While at the quarter-span section (25th element) the maximum difference between the modified strain and normal strain is 6%, at the three-quarter span section (75th element) the difference is 5%. The differences between the modified and normal strains are comparable to those found at mid-span (Figure 5).

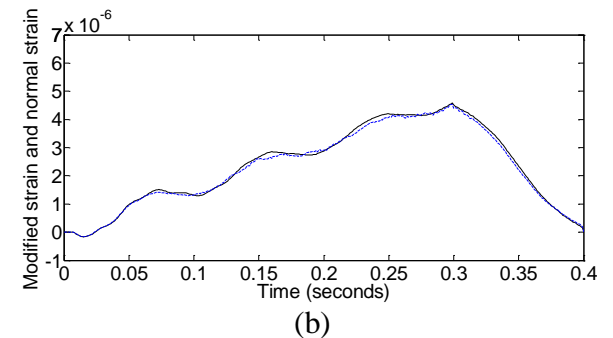
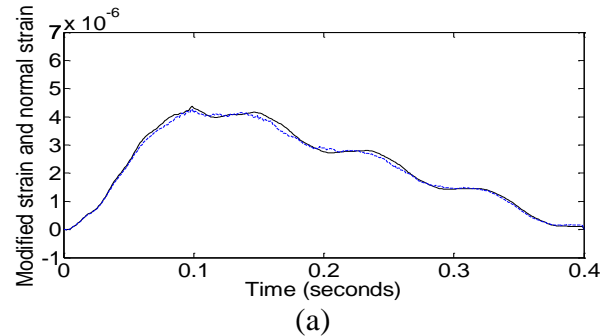


Figure 8: Modified strain and normal strain versus time at (a) quarter span and (b) three-quarter span

The variation of strain through the depth of the cross-section is shown in Figure 9 where normal and modified strains are illustrated for strips at 0.1625m and 0.325m from the neutral axis. Differences

between normal and modified strains for different depths take place in similar locations that are related to the location of the load, although these differences are of a smaller magnitude for the strips closer to the neutral axis.

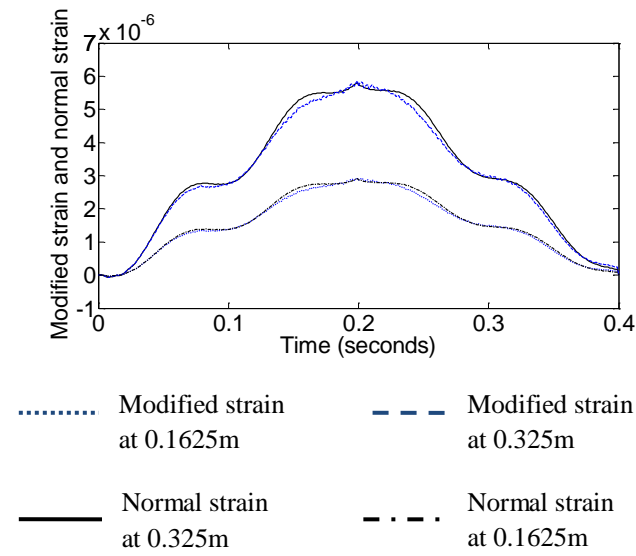


Figure 9: Strain at two different distances from the neutral axis for the beam mid-span cross-section

3.3 Total response at mid-span for varying speeds

Varying the speed of the load as it crosses the beam impacts the shape, magnitude and ultimately the difference between normal and modified strain. In Figure 10, the effect of speed of the moving load is investigated using different speeds ranging from 10m/s to 35m/s. Maximum modified and normal strains develop when the load is located at the mid-span point for all speeds and largest differences exist between both at this load location, except for 25m/s (Figure 5) where both maximum strain values are similar in magnitude. At speeds of 15m/s and 20m/s the difference between the maximum strain values for load located at mid-span is close to 3.2% and at a speed of 30m/s, the difference is 2.8%. The highest difference between the modified and normal strain is 3.5% at a speed of 35m/s. At 25m/s there is negligible difference between normal and modified strain when the load is at mid-span, however larger differences manifest for locations of the load between mid-span and quarter-span (Figure 5).

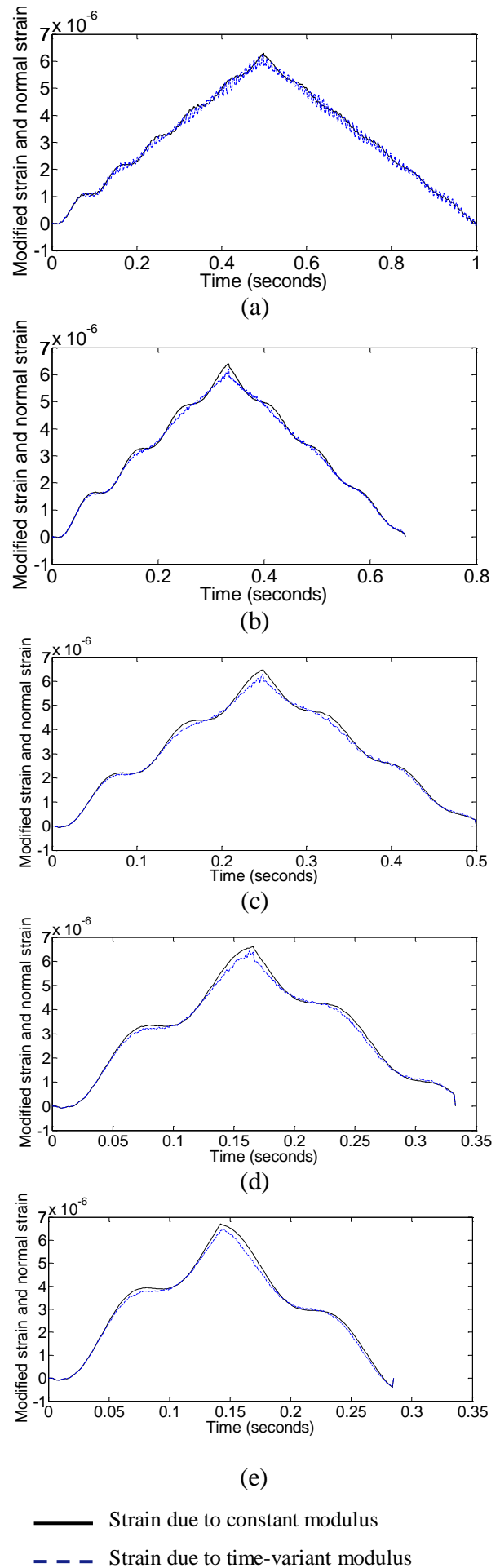


Figure 10: Modified and normal strain at (a) 10m/s, (b) 15m/s, (c) 20m/s, and (d) 30m/s (e) 35m/s

The displacement is affected by the speed of the load similarly to strain. The relationship between dynamic amplification and speed consists of a series of peaks and valleys, i.e., critical speeds causing higher amplification due to constructive interference of the oscillatory inertial forces of the bridge with the static response (i.e., reaching a maximum at the same point) and other speeds causing destructive interference and reducing this amplification (González et al., 2010). For the speeds under investigation, maximum displacement tends to increase with speed of the load except for 25m/s. The amplitude, location and number of dynamic peaks is different for each speed and they are the cause of constructive or destructive interference that lead to larger or smaller maximum strains. At 10m/s, the peaks are small but at higher speeds the peaks are more evident producing the maximum displacements. The difference between normal and modified displacement tends to be highest close to 0.15s and 0.25s for a speed of 25m/s (Figure 7). For other speeds (Figure 11), largest differences in displacements occur when the load is located near the mid-span point.

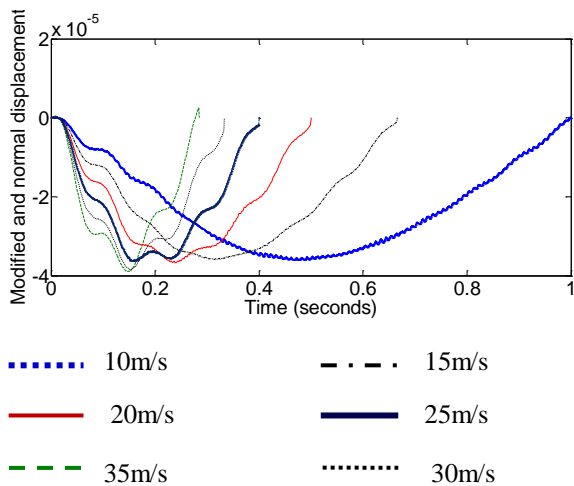


Figure 11: Modified (using time-variant modulus) displacement at mid-span versus time for different speeds

3.4 Total response for a number of bridge spans

The influence of a time-variant modulus on the strain response is investigated for a number of span lengths in this section. Figure 12 shows the strain due to a 100kN load travelling at 25m/s for 20m and 30m spans. Depths of the cross-section of 1m and 1.32m are assumed for the 20m and 30m bridge spans respectively. The dynamic peaks reach a maximum when the load is at mid-span for both spans.

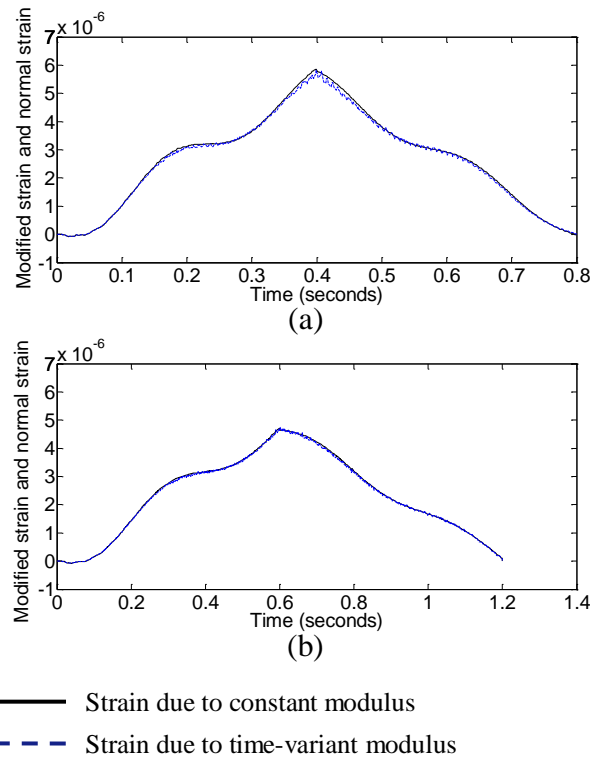


Figure 12: Modified and normal strain for spans (a) 20m and (b) 30m

From Figure 12(a), the 20m long bridge model produces small differences between modified and normal strains compared to that at 10m (Figure 5). For the 30m long bridge (Figure 12(b)), the same magnitude of the load induces a smaller strain than in the 20m span due to the larger stiffness of the section for the longer span, and the differences between normal and modified strains are even smaller.

4 SUMMARY AND CONCLUSIONS

This paper has shown that the stiffness of a bridge section may vary under a moving force once the magnitude and speed of the load is sufficiently high. To the authors' best knowledge, this is the first publication in the literature that investigates the effect of a time-variant stiffness in the total response of a simply supported structure under a moving load.

The CEB-FIP model code formula has been used to an estimation of the time-variant modulus (which depends on strain rate) at each time step. The moment of inertia is modified to provide an equivalent stiffness and the elementary and global stiffness matrix are updated accordingly at each point in time. Differences can be noticed when comparing strain results using a constant modulus or a time-variant modulus as suggested in the paper, i.e., the structural response is stiffer with smaller displacements/strains than when using a constant modulus. This phenomenon is more pronounced the

higher the load and the higher the speed. Most of the results have been focused on the mid-span location where strains are typically largest. Other locations have been analysed and analogous results have been obtained regarding the percentage decrease in strain.

The results shown here have been based on a moving point force theoretical model. Some authors (Michaltsos, 2002) suggest that two axles/forces have more significant and important impact on the variation of displacement and strain due to varying speed. Hence, the investigation of the effect of more complex forms of loading, including heavy high-speed trains that could lead to significant strain rates, needs to receive attention. Therefore, introducing reinforcement within the model might have different impacts on the response. The latter needs further investigation and experimental evaluation as the behaviour of the structure may vary unexpectedly for different materials and structural configurations.

REFERENCES

- Aied, H. & González, A. 2011. The Effect of Strain Rate on the Total Response of a Structure subject to a Moving Load. *In*: B.H.V. TOPPING, Y. T. (ed.) *Civi*. Stirlingshire, UK: Civil-Comp Press.
- Bischoff, P. H. & Perry, S. H. 1991. Compressive behavior of concrete at high strain rates. *Materials and Structures*, 24, 425-450.
- González, A., OBrien, E. J., Cantero, D., LI, Y., Dowling, J. & Žnidarič, A. 2010. Critical speed for the dynamics of truck events on bridges with a smooth road surface. *Journal of Sound and Vibration*, 329, 2127-2146.
- MathWorks 2010. MATLAB R2010b. 7.11.0.584 ed. U.S.: MathWorks.
- Michaltsos, G. T. 2002. Dynamic behaviour of a single-span beam subjected to loads moving with variable speeds. *Journal of Sound and Vibration*, 258, 359-372.
- Przemieniecki, J. S. 1985. *Theory of matrix structural analysis*, New York, Dover.
- Rowe, R. E. & Rene, W. 1991. Bulletin D'Information No. 203 CEB-FIP Model Code 1990- Final Draft *In*: (CEB), C. E.-I. D. B. (ed.) *Comite Euro-International du Beton (CEB)*.
- Rowley, C. W. 2007. *Moving force identification of axle forces on bridges*. Doctor of Philosophy, University College Dublin.
- Shkolnik, I. E. 2008. Influence of high strain rates on stress-strain relationship, strength and elastic modulus of concrete. *Cement & Concrete Composites*, 30, 1000-1012.
- Yang, Y. B., Wu, Y. S. & Yau, J. D. 2004. *Vehicle-bridge interaction dynamics: with applications to high-speed railways*, Singapore [u.a.], World Scientific.