



<b>Title</b>	Allocation of Wind Capacity Subject to Long Term Voltage Stability Constraints
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<b>Publication date</b>	2016-05
<b>Publication information</b>	Bakhtvar, Mostafa, and Andrew Keane. "Allocation of Wind Capacity Subject to Long Term Voltage Stability Constraints." IEEE, May 2016. <a href="https://doi.org/10.1109/TPWRS.2015.2454852">https://doi.org/10.1109/TPWRS.2015.2454852</a> .
<b>Publisher</b>	IEEE
<b>Item record/more information</b>	<a href="http://hdl.handle.net/10197/7986">http://hdl.handle.net/10197/7986</a>
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<b>Publisher's version (DOI)</b>	10.1109/TPWRS.2015.2454852

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# Allocation of wind capacity subject to long term voltage stability constraints

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**Abstract**—Increasing wind capacity integration results in displacement of active power from conventional generators and a reduction in reactive power sources available. As such voltage stability may become a concern in certain periods for power system operation particularly in weaker areas of the network. Thus, it is of importance to consider the AC constraints for optimal wind generation planning (long term) in order to decrease the possibility of a wind capacity allocation that requires costly remedies from the power system operation perspective (short term). In this work a procedure is proposed for wind capacity allocation with the aim of benefiting from the potential of an optimal wind capacity allocation for enhancing the voltage stability margin. The procedure is based on a multi operating conditions voltage stability constrained optimal power flow. The wind capacity target is set and the loadability margin is tracked. The results will show the applicability of the proposed procedure and will emphasize the effects of the pattern of wind capacity allocation on the loadability margin. This will result in a wind capacity allocation that enhances the minimum loadability margin among the possible future operating conditions considered for planning. The procedure uses the Maximin concept for this purpose.

**Index Terms**—Voltage stability; Optimal power flow; Wind; Capacity allocation; VSCOPF; Voltage control; Sigmoid; Tap changer;

## NOMENCLATURE

$O$	Set of normal operating points ( $o \in O$ )
$S$	Set of stressed operation conditions ( $s \in S$ )
$N$	Set of operating conditions ( $n \in N$ )
$B$	Set of buses
$L$	Set of branches
$T$	Set of transformers ( $T \subseteq L$ )
$G$	Set of generators
$W$	Set of wind farms ( $W \subseteq G$ )
$D$	Set of demands
$V_b$	Voltage at bus ( $b \in B$ )
$V^{max}, V^{min}$	Secure operation voltage limit
$I_l$	Current flow through branch ( $l \in L$ )
$I_l^{rated}$	Current limit of branch
$R_t$	Off nominal ratio of transformer ( $t \in T$ )
$R_t^{max}, R_t^{min}$	Off nominal ratio limit of transformer

This work was conducted in the Electricity Research Centre, University College Dublin, Ireland, which is supported by the Commission for Energy Regulation, Bord Gáis Energy, Bord na Móna Energy, Cylon Controls, EirGrid, Electric Ireland, EPRI, ESB International, ESB Networks, Gaelectric, Intel, SSE Renewables, and UTRC. The work of M. Bakhtvar was supported by the Science Foundation Ireland under Grant Number SFI/09/SRC/E1780.

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$P_g, Q_g$	Active and reactive power dispatch of generator ( $g \in G$ )
$P_g^{max}, P_g^{min}$ $Q_g^{max}, Q_g^{min}$	Active power dispatch limit of generator Reactive power dispatch limit of generator
$\omega_g$	Status of generator (on: 1, off: 0)
$C_w$	Capacity of wind farm ( $w \in W$ )
$C_w^{max}, C_w^{min}$	Capacity limit for wind farm
$C_{target}$	Wind capacity target
$C_{tolerance}$	Tolerance for the wind capacity target
$f_w$	generation factor of wind farm
$Q_w$	Reactive dispatch of wind farm
$\mu_{max,w}, \mu_{min,w}$	Limit of ratio between reactive power output and capacity of wind farm
$\lambda$	Maximum allowed instantaneous wind penetration
$P_d, Q_d$	Active and reactive power consumption of demand ( $d \in D$ )
$\alpha_d^p, \alpha_d^i, \alpha_d^z$	Percentage of constant power, constant current and constant impedance component of demand
$f_d$	demand factor
$\delta_d$	Load increment factor of demand
$\kappa$	Required load increment percentage

## I. INTRODUCTION

THE trend of wind capacity integration in power systems indicates a gradual move towards high penetrations of wind realized in terms of both energy production and instantaneous generation [1], [2]. Implications of such a move in liberalized energy markets include displacement of active power generated by conventional generation units, accompanied by a reduction of reactive power sources available. Uncommitted units cannot provide reactive power to the network for regulation purposes and as a consequence, a shift in reactive power production from these units to the remaining online sources is anticipated. Firstly, such alterations may force the available reactive power sources to their limits [3]. Secondly, the alternative reactive power source may not necessarily be as close to the demand as in the case prior to the integration of wind capacity. Furthermore, with the installation of wind capacity new points of active power injection are introduced to the power system changing both the power flow and voltage profile across the network particularly in weaker areas. A loading reduction in the network due to stability limits may lead to costly remedies such as network reinforcement and load shedding [4]. Thus it is of importance to consider the AC constraints for an optimal wind generation planning (long

term) in order to decrease the possibility of a wind capacity allocation that requires costly remedies from the power system operation perspective (short term).

#### A. Literature review

Optimal power flow (OPF) is a powerful framework for power system planning and operation. Inclusion of voltage stability constraints in the AC OPF has been proposed in [5]–[9]. The authors of [5] propose a security constrained OPF which considers both static and voltage stability constraints. Bender’s decomposition is used to decompose the preventive and corrective actions which are based on the four cases describing the network normal and stressed base and post contingency cases. In [6] a reactive reserve based OPF is presented. The contingencies violating the voltage stability margin constraint are identified with the aid of a modified continuation power flow. Accordingly, the minimum reactive power reserve required to meet the voltage stability margin constraint is evaluated and fed to an OPF module that optimizes the preventive actions required. In [7], [9] two sets of variables (TSV) are utilized to include both normal and critical operating points in the voltage stability constrained OPF. The loadability margin is used to link these sets of variables together. Reference [8] considers a voltage stability constrained redispatching of the generators with a scalar loadability margin to minimize the cost of redispatching considering both load and generation ramp up and ramp down costs.

Allocation of optimal wind capacity in power systems with an indication of unit commitment has been addressed in several works using both linear and non-linear OPF [10]–[13]. The authors of [10] propose a method that sequentially increases the wind energy penetration target while updating the unit commitment and wind capacity allocation pattern formulated as a linear problem. In [11] a DC OPF based procedure is proposed for optimal wind capacity allocation. A piecewise transmission cost function is also incorporated to reflect the investment associated with remoteness of wind rich areas. Reference [12] coordinates the generation expansion problem with the transmission expansion problem by applying benders decomposition to each of these problems.

#### B. Contribution

It is reported in the literature that the voltage stability margin may be improved by optimal allocation of the generation capacity in distribution systems [14] as well as transmission systems [9]. However, to the best knowledge of the authors none of the approaches that consider unit commitment, take into account the potential of an optimal wind capacity allocation for voltage stability enhancement. In this paper, a multi operating conditions VSCOPF is presented. A procedure for wind capacity allocation with an indication of unit commitment is proposed that aims to enhance the voltage stability margin through optimal sizing and siting of wind capacity. Unit commitment is important as it constraints the amount of active and reactive power capacity online. Allocation of wind capacity can alter the unit commitment in the system. Thus, for each instance of power system operation the level

of conventional active and reactive power capacity online may change. Reactive power, supports the voltage in the network. A reduction in the available reactive power in an instance of power system operation may result in a reduction of voltage stability margin in that instance. However, the trend for this reduction is not linear due to the fact that the reduction in the available reactive power capacity occurs in steps (a unit turns off and its reactive power capacity is not available anymore). In addition, the proximity of the source and sink of reactive power may change significantly with the change in the unit commitment; this affects the voltage support especially in weak areas in the network. Injection of active power in these areas may help improving the voltage profile and subsequently the loadability margin. Also, the proximity of the active power to loads improves the loadability margin. The procedure in this paper optimizes capacity allocation regarding to these factors. The methodology provides insight into the optimal wind capacity allocation (from the voltage stability perspective) to the system operator in order to prioritize granting network access. Introduction of the wind capacity target allows for management of the problem scale. The wind capacity allocation to be found by the proposed procedure minimizes the negative effect of wind capacity integration on loadability margin of the network and in the best case will enhance it. The proposed procedure may also be adapted for capacity allocation problems with similar structure such as photovoltaic generation.

#### C. Paper organization

This paper is structured as follows: section II formulates the OPF problem. The procedure for wind capacity allocation and its implementation are explained in Section III. Section IV describes the test network. Results of applying the proposed methodology on the test network are presented and discussed in section V. Finally Section VI concludes this paper.

## II. OPF FORMULATION

The concept of optimization allows for modeling the AC power flow as a non-linear optimization problem. The result of the optimization problem is a numerically stable operating point of the network subject to a number of constraints in such a way that an objective is achieved. Here the power network is modeled as a graph in which the vertices and edges represent the buses and branches in the network, respectively. The ACOPF is optimized by applying Kirchoff laws to the vertices and edges of the graph as well as constraints that require variables of the network to remain within the technical and physical boundaries.

#### A. multi operating conditions VSCOPF

Power system planning is a complex task that requires investigation of several operation conditions (e.g. demand level, line outage, unit commitment) to ensure that the proposed changes in the network do not result in violation of any of the limits. A multi operating conditions AC OPF has an extra dimension compared to the ordinary AC OPF.

Steady state voltage stability assessment may be integrated in the OPF by the means of loadability margin. This is achieved by including a further set of variables in the OPF. The new set of variables represent the state of the system at a stressed point. A stressed operating point is the system steady state after being subject to a gradual load increase at a load serving bus (or any combination of a number of load serving buses) while it still respects the technical and physical limits (e.g. voltage) of the network. Therefore, the OPF will involve two operating points of the network, normal and stressed. These two operating points are linked with a load increase factor. The maximum achievable load increase factor is equivalent to the loadability margin. This is shown in the PV curve in Fig.1. However, it should be noted that the OPF does not produce a complete PV curve but the two aforementioned numerically stable operating points. Furthermore, the PV curve may widen or shrink due to changes in the system. The explained approach is extended to include multiple operation conditions and multiple stressed operating points. In the new problem, for every operating condition there is a normal operating point and a number of stressed operating points that represent the system state after being subject to a load increase at a load serving bus (or a number of load serving buses) in the network. Fig.2 illustrates the structure of this problem; this can be defined as:

$$\begin{aligned} & \text{Max/Min } f(x_o^n, x_s^n) \\ & \text{s.t. } h(x_o^n, x_s^n) = 0 \\ & g^{\min}(x_o^n, x_s^n) \leq g(x_o^n, x_s^n) \leq g^{\max}(x_o^n, x_s^n) \end{aligned} \quad (1)$$

Where  $n, o, s, x, f(\cdot), h(\cdot), g(\cdot), h^{\max}(\cdot)$  and  $h^{\min}(\cdot)$

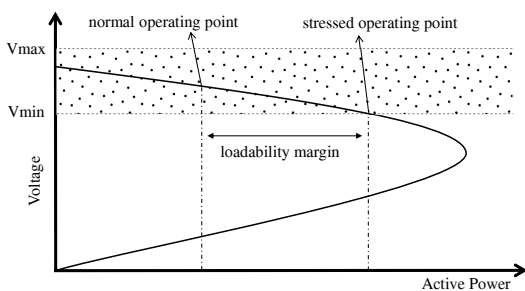


Fig. 1. VSCOPF problem

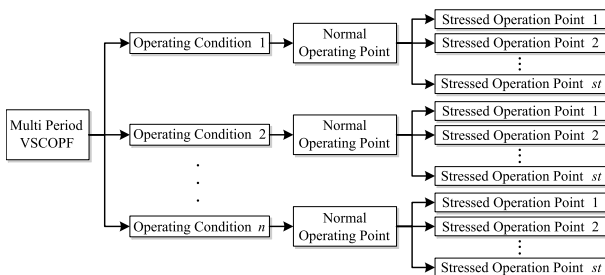


Fig. 2. multi operating conditions VSCOPF problem

represent the operating condition, normal operating point, stressed operating point, vector of variables, objective, equality constraints, inequality constraints, upper and lower bound of

inequality constraints functions, respectively. The objective function and voltage control constraints are defined as follows. Further constraints may be found in appendix A.

### B. Objective Function

Various objective functions may be used with the non-linear optimization problem of wind capacity allocation e.g. total wind capacity maximization and cost minimization are two common objective functions considered in the literature. Here the objective function is voltage stability maximization. This will result in a wind capacity allocation that maximizes the minimum loadability margin in the network across the study operating conditions and is denoted as:

$$\text{Max } (\kappa) \quad (2)$$

### C. Voltage Control in the OPF

Automatic voltage regulation is applied on several components in the network; this limits a variable corresponding to the controlled component (e.g. reactive power generation in synchronous generators) in order to achieve a desired voltage at a designated bus. Such behavior involves an inherently discrete function where decision on whether increasing or decreasing the controlled variable is dependent on the voltage at the designated bus. In [15] the complementary constraints are used to produce this discrete function. Here a tuned sigmoid function is used to get a continuous approximation of such behavior.

$$\Gamma(V) = \frac{2}{\pi} \arctan\left\{\frac{\pi}{2} \rho (V - V_{sp})\right\} \quad (3)$$

Where  $V_{sp}$  and  $\rho$  are the voltage set point and a parameter for tuning sensitivity of the control function respectively. Application of sigmoid function for voltage control has also been reported in [16]. Fig.3 plots the sigmoid function for various sensitivities. A significantly large  $\rho$  provides an almost step behavior in the sigmoid function, however, this causes the slope to tend to infinity at voltages close to the set point which is not desired in non-linear optimization. Thus it is essential to tune  $\rho$  such that while the problem does not become infeasible, an acceptable approximation of the discrete behavior is achieved. The required constraints for voltage control by generators and tap changers are as follows.

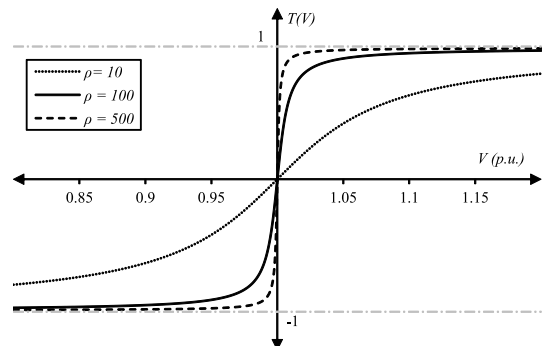


Fig. 3. The sigmoid function at various sensitivities for  $V_{sp} = 1p.u.$

1) *Generators:*

$$\beta_1 \frac{Q^{max} + Q^{min} - 2Q}{Q^{max} - Q^{min}} \leq \Gamma(V) \quad (4)$$

$$\beta_2 \frac{Q^{max} + Q^{min} - 2Q}{Q^{max} - Q^{min}} \geq \Gamma(V) \quad (5)$$

Where  $\beta_1$  and  $\beta_2$  are two auxiliary variables limited by the continuous interval  $[0 \ 1]$ . Taking into account the fact that reactive power output of the generator is limited by  $Q^{max}$  and  $Q^{min}$  then as a result of the applied constraints the generator's reactive power output will ideally end to one of the states below:

$$\text{voltage control} \implies \begin{cases} Q \approx Q^{max} & V \leq V_{sp} \\ Q^{min} \leq Q \leq Q^{max} & V = V_{sp} \\ Q \approx Q^{min} & V \geq V_{sp} \end{cases}$$

2) *Tap changers:* The voltage dead band in tap changers requires alteration of the behavior in (3) this is achieved by

$$M(V, \Delta) = \frac{\Gamma(V + \sigma) + \Gamma(V - \sigma)}{2} + \Delta \quad (6)$$

Where  $\pm\sigma$  is the dead band specified for the tap changer.  $\Delta$  is an auxiliary variable for correction of the altered sigmoid function at points where the voltage is within the dead band.  $\Delta$  is limited by  $\Delta^{max}$  and  $\Delta^{min}$ ; these are defined as

$$\Delta^{min} = \frac{\Gamma(V_{sp} - 2\sigma + \varepsilon) + \Gamma(V_{sp} + \varepsilon)}{2} \leq \Delta \quad (7)$$

$$\Delta^{max} = \frac{\Gamma(V_{sp} + 2\sigma - \varepsilon) + \Gamma(V_{sp} - \varepsilon)}{2} \geq \Delta \quad (8)$$

Here  $\varepsilon$  is a tuning parameter with small value ( $\varepsilon \ll \sigma$ ). With tap changers, the voltage control constraints also depend on the side at which the voltage is being controlled. Assuming that the tap changer controls the transformer turns ratio at the primary side (starting bus of the branch) the respective constraints are

- Voltage control applied to the primary side (starting bus of the branch):

$$\beta_1(R - 1) \geq \frac{(1 - R^{max})M(V, \Delta)}{1 + \Delta^{min}} \quad (9)$$

$$\beta_2(R - 1) \leq \frac{(R^{min} - 1)M(V, \Delta)}{1 - \Delta^{max}} \quad (10)$$

- Voltage control applied to the secondary side (ending bus of the branch):

$$\beta_1(R - 1) \leq \frac{(R^{max} - 1)M(V, \Delta)}{1 + \Delta^{min}} \quad (11)$$

$$\beta_2(R - 1) \geq \frac{(1 - R^{min})M(V, \Delta)}{1 - \Delta^{max}} \quad (12)$$

The accuracy of the proposed voltage control method is dependent on the value of  $\rho$ . In order to find a suitable value for  $\rho$ , a large enough number of random operating points may be simulated at different levels of  $\rho$ . The error in the voltage control implementation may be monitored as well as the number of infeasibility reports. A suitable value for  $\rho$  can be chosen accordingly

### III. WIND CAPACITY ALLOCATION PROCEDURE

The proposed procedure for wind capacity allocation is shown in Fig.4.

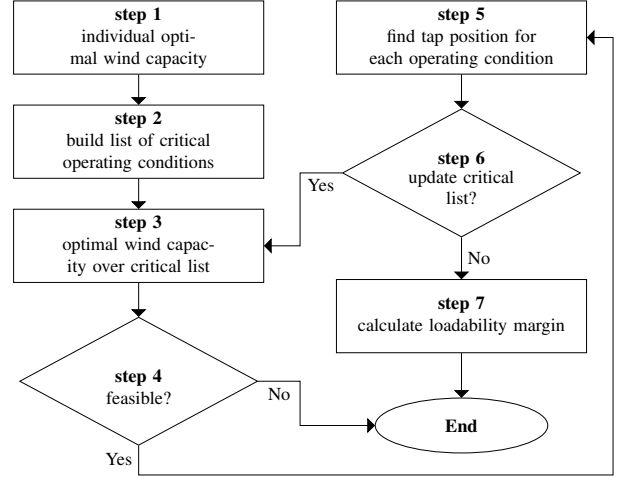


Fig. 4. Wind capacity allocation procedure

- Step 1: An ideal optimal wind capacity allocation is performed in this step. For every set of normal and stressed operating points of every wind-demand operating condition a wind capacity allocation is found individually (multiple separate executions). The objective function is to maximize loadability margin. This is an unrealistic case in which the wind capacity allocation and generator voltage set points are not common among the operating conditions and the transformers are allowed to tap-freely. The optimization problem consists of
  - constraints: (4), (5) and (13)-(26) (from Appendix A)
  - objective function: (2)
  - voltage dependent control variables:  $Q$  in conventional generators
  - decision variables:  $C_w$ ,  $P_g$ ,  $R$ ,  $\kappa$ ,  $\delta_d$  and  $V_{sp}$  in voltage controlling generators

The wind capacity allocation found for one set of normal and stressed operating points in this step may not be feasible for all other set of normal and stressed operating points. The reason for carrying this process is to provide data for step 2 to reduce the number of operating conditions that are to be solved at once in step 3.

- Step 2: In order to form the initial critical list, a subset of the operating points is selected. Two approaches can be used to select the operating points for the critical list
  - Select all operating points that result in loadability margin lower than a specific value
  - Select a specific number of operating points with the lowest loadability margins

Thus, the standard for choosing operating points is based on the comparison of the best achievable loadability margins for the analyzed operating points. Regardless of which of the two approaches is used, the operating points chosen are the ones that lead to the lowest loadability margins ( $\delta_d$ ) in Step 1. The initial critical list is formed with these operating points.

- Step 3: The operating points in the critical list are passed to the multi operating condition VSCOPF. Within the multi operating condition VSCOPF, the generator output variables are defined individually for each operating point. The variables that are common across all the operating points and connect them to each other are the voltage set points of the voltage controlling devices,  $V_{sp}$ , capacity of wind farms,  $C_w$ , and the minimum required loadability margin,  $\kappa$ . Further, each stressed operating point is linked to its respective normal operating point by the loadability margin,  $\delta_d$ , i.e. the third terms in (25) and (26). As defined by equations (23) and (25) the wind farm capacity is translated to the output of the wind farm by being multiplied to the generation factor of the wind farm at each operating condition,  $f_w$ . This parameter is based on the wind strength at each candidate allocation bus at each operating condition. The Maximin technique is applied using the constraints and objective function defined by equations (22) and (2), respectively. These constraints require the loadability margins that link each pair of normal and stressed operating points to be greater than (or equal to) the minimum required loadability margin while the objective function is maximization of the minimum required loadability margin. The benefit of using this method is that the loadability margin is maximized for the worst case. As the wind capacity at candidate buses are defined as variables that are common among all the operating points, the wind capacity is allocated by the multi-operating condition VSCOPF in such way that the minimum loadability margin is maximized. It should be noted that the total wind capacity to be allocated in this step is constrained by equation (20). The optimization problem is summarized as

- constraints: (4), (5), (7) - (12) and (13)-(26) (from Appendix A)
- objective function: (2)
- voltage dependent control variables:  $R$  in tap-changing transformers and  $Q$  in voltage controlling generators
- decision variables:  $C_w$ ,  $P_g$ ,  $\kappa$ ,  $\delta_d$  and  $V_{sp}$  in voltage controlling devices

The tap changing transformers are set to control voltage. The voltage set point for tap changing transformers is an optimization decision as well as the voltage set point of the voltage controlling generators. Only passing the operating points in the critical list to the multi-operating condition VSCOPF allows the scale of the problem to be reduced; this is reasonable due to the fact that not all of the sets of normal and stressed operating points impose a limit on the wind capacity allocation problem.

- Step 4: If the problem in step 3 is infeasible then the process stops otherwise it continues to the next step. An infeasibility in this step may arise due to line congestion or unacceptable voltage. This problem may be solved by altering the generator status vector by adding constraints to the unit commitment. This has been addressed in [17] and is out of the scope of this paper. Further, it may

result in an increase in the operational costs. We treat an infeasibility in this step as an actual infeasibility and assume that no further constraints are allowed in the unit commitment.

- Step 5: An optimization problem is run for each operating condition. The problem includes the normal and all of the stressed operating points corresponding to the operating condition. The wind capacity and voltage set points in this step are treated as parameters; these are fixed to values found in step 3. The objective function of the optimization problem similarly is maximization of the minimum loadability margin throughout the network. For all operating conditions the loadability margin has to be greater than or equal to the minimum loadability margin found in step 3 (i.e.  $\kappa_{step5} \geq \kappa_{step3}$ ). There are two reasons for carrying this step
  - Finding transformer tap-position: In order to find the loadability margin for each set of normal and stressed operating points we should consider that the tap-position for the voltage controlling transformers need to be common among all operating points corresponding to each operating condition. However, due to the dead-band used in the transformer voltage control function there might be more than one feasible solution. Moreover, while one combination may be optimal for one stressed operating point it might not be in favor for other stressed operating points corresponding to an operating condition. Thus, it is necessary to find the optimum tap-positions for each operating condition.
  - Check the validity of the solution found in step 3: The wind capacity allocation in step 3 is only based on the sets of operating points in the critical list. Thus, it is necessary to ensure that the solution found is valid for all other operating points under study.

The optimization problem consists of

- constraints: (4), (5), (7) - (12) and (13)-(26) (from Appendix A)
- objective function: (2)
- voltage dependent control variables:  $R$  in tap-changing transformers and  $Q$  in voltage controlling generators
- decision variables:  $P_g$ ,  $\delta_d$  and  $\kappa$

If an operating condition is found to be infeasible, it is added to the critical list. It should be noted that the strategies for controlling tap position of transformers may be different from one system operator to another. Accordingly, other approaches for deriving tap-position of transformers may also be adopted in this step.

- Step 6: If more items have been added to the critical list then return to step 3; this will consider the fact that addition of further constraints may change the optimum allocation of wind capacity.
- Step 7: The loadability margin for each set of normal and stressed operating point is found using an optimization problem. The generator voltage set points and wind capacity are treated as parameters equal to the values

found in step 3. Transformer tap position are also treated as parameters equal to the values found in step 5. The optimization problem consists of

- constraints: (4), (5), (13)-(18) and (22)-(26) (from Appendix A)
- objective function: (2)
- control variables:  $Q$  in conventional generators
- decision variables:  $P_g$ ,  $\kappa$  and  $\delta_d$

This step is optional. It is carried once the final solution has been found in step 1 to 6. It differs from step 1 in that the wind capacity and generator voltage set points are common parameters throughout all operating points in this step. Also transformers are not allowed to tap-freely.

The adjustment of the voltage set point of voltage controlling devices may be used as a preventive action. In order to consider this fact and also avoid constraining the capacity allocation by a parameter that may be used as a preventive control, the voltage set point was treated as a decision variable in the proposed procedure. This will decide on voltage set-point in favor of maximization of the minimum loadability margin. It should be noted that this value is common across all of the operating points. Accordingly, the set-points are fixed in steps 5 and 7 at the value found in step 3. Thus the voltage set points for one capacity target might be different to another. However, other schemes may also be used.

Constraint (18) is a function of the operating conditions and the capacity target. At low capacity targets it will hold for all of the operating conditions. However, at large enough capacity targets, it becomes constraining for part or all of the operating conditions and may drive the proposed procedure to infeasibility; this indicates that with the current parameters it is not possible to allocate wind in such way that the total allocated wind capacity equals to the specific capacity target studied.

It should be noted that the multiple separate executions in step 1 and step 7 are not simulating the actual continuous operation of the power system. These separate executions are carried to find the loadability margins at each of the considered possible future operating condition. The loadability margin is the metric used to assess the security of the system from the long term small disturbance voltage stability perspective. It doesn't mean that there will necessarily be a load increase at a bus or a number of buses in the network when it comes to the operation of the power system. Thus, the reason for using non-sequential operating conditions is that the proposed procedure is intended for planning purposes. Ramp up/down and reserve requirements may be included in the unit commitment; these are reflected in the generator status vector  $\omega_j$ . It is emphasized that accounting for the dynamic and transient voltage stability is not in the scope of this paper.

The presented algorithm allocates wind capacity using a wind capacity target (constraint in (20)) and a respective generator status vector; this vector is produced by running unit commitment according to the wind capacity target. The wind capacity allocation may be found for a sufficient number of targets separately using this algorithm. The result provides

valuable information with regard to the optimum wind capacity allocation such that the negative effects on steady state voltage stability are minimized and it is improved in the best case.

It is essential to consider that due to the non-linear and non-convex nature of the power flow and voltage control equations, multiple local optima may exist for the problem. Therefore, the solution found with the presented algorithm is not necessarily the globally optimum point.

#### IV. TEST NETWORK

The three area IEEE RTS 96 [18] and the IEEE 118 bus [19] networks are used in this study. The candidate buses for wind capacity allocation are listed in Table I. Three different wind profiles were assigned to the candidate buses (one per area). The number of possible future operating conditions is infinite. However, the planning decisions may be based on sufficient number of operating conditions that represent the realistic possible future power system operating conditions. It should be noted that the result found is valid for these specific operating conditions. The combination of 8 demand (30-100%) and  $10^3$  wind (10-100% in each area) levels were employed to form 8000 ( $= 8 \times 10 \times 10 \times 10$ ) wind-demand operating conditions. The wind and demand profiles used correspond to the Irish power network and were obtained from [20]. Inspection of the data showed that only 2566 of aforementioned operating conditions are probable to occur thus we considered these operating conditions for the demonstration purpose. However, the proposed procedure is not limited to this number and further operating conditions and wind and demand profiles may be employed with the cost of increasing the problem size and run time. The wind and demand profiles are also used for unit commitment. The allowed voltage set point range of 0.95 – 1.05 *p.u.* was applied to conventional generators in the network with tuning parameter,  $\rho = 100$ . The tap changing transformers were given an allowed off nominal ratio control range of 0.85 – 1.15 *p.u.* at the primary side with tuning parameters,  $\rho = 300$  and  $\varepsilon = 0.001$  and an allowed voltage set point range of 0.95 – 1.05 *p.u.* with 2% dead band. The maximum instantaneous wind penetration is set to 60%. It is worth mentioning that wind capacity is modeled as a virtual generator at the candidate buses. At each operating condition, the output of the wind turbine is a fraction of its capacity. Thus  $P_w^n = C_w f_w^n$ .  $f_w^n$  is the generation factor of the wind farm. It is a number in the [0 1] interval. This parameter is based on the wind strength at each candidate bus for wind capacity allocation at each operating condition. A unity power factor is preferred by wind operators [21] and also required by some grid codes e.g. the Nordic grid code [22]. For the purpose of this paper i.e. focusing on the pattern of wind capacity allocation, it was assumed that wind farms will inject power at unity power factor. This is achieved by letting  $\mu_{max,w} = \mu_{min,w} = 0$ . This will require  $Q_w = 0$ . Any other power factor (including optimal power factor) or voltage control may also be applied instead. It should be noted that 200 MW is the limit imposed on the size of wind farm that may be allocated at any of the candidate buses. This limit may

also be set in accordance with the wind resources available at each location in the network and (or) other technical and economical constraints. The tolerance for wind capacity target was set to 100 MW. The load increment factor,  $\delta_d$ , is assumed to be applied to both  $P_d$  and  $Q_d$  thus resulting in a constant  $P - Q$  ratio logic. This is an accepted approaches in the literature. However, any other approach such as a voltage dependent  $Q_d$  may be introduced to represent motor loads. It is worth mentioning that the number of stressed operating points for each operating condition is equal to 51 and 91 for the IEEE RTS 69 and IEEE 118 bus study cases, respectively, i.e. the number of load serving buses. Any other combination of load serving buses may be used which in fact may result in a reduction of the problem size. A summary of the optimization parameters is given in Table III. COOPR package [23] is

TABLE I  
WIND CAPACITY ALLOCATION CANDIDATE BUSES

Test Network	Candidate bus
IEEE RTS 96	104, 105, 112, 119, 120, 124
	203, 204, 205, 206, 210, 217, 220
	309, 312, 317, 319, 324, 325
IEEE 118 bus	7, 9, 14, 24, 71, 117
	37, 44, 48, 57, 58, 68, 97
	84, 86, 93, 104, 107, 108, 109

employed for modeling the optimization problems; These are solved using KNITRO non-linear solver. Plexos [24] is used to model and solve the unit commitment problem by XPRESS MP solver [25]. Time performance of optimization problems is highly dependent on the size and complexity of the problem as well as the starting point. For the IEEE RTS 96 network, finding each individual allocation in step 1 and each loadability margin in step 7 took on average 0.13 seconds (solver time). The time required in step 3 depends on the number of items in the critical list. For lists with 179 and 943 items, it took approximately 1 and 5 hours, respectively, to find the optimal solution. Finding tap position for each operating condition in step 5 took approximately 38 seconds.

## V. RESULTS AND DISCUSSION

### A. IEEE RTS 96

The presented algorithm was run to allocate wind capacity for 7 study cases. The wind capacity target in these cases is listed in Table II. The first case,  $C_{target} = 0$  GW, is a base case with no wind capacity. Conventional generator voltage set point and transformer tap position is found for this case using the presented algorithm in a similar fashion as the rest of the studied cases except for the wind capacity target tolerance which was set to zero for this specific case i.e.  $C_{tolerance} = 0$  MW. Fig.5 shows the minimum loadability margin achieved throughout the system in all of the operating conditions at each of the studied cases. It can be seen that initially with the allocation of 0.5 GW wind capacity, the minimum loadability margin is increased by 1% compared to the base case. However, this is not sustained with further addition of wind capacity as more conventional generation

TABLE II  
IEEE RTS 96, WIND CAPACITY TARGETS FOR THE STUDY CASES,  $C_{target}$  (GW)

0	0.5	1	1.5	1.75	2	2.37	2.5
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needs to be turned off to facilitate the higher power injection by wind capacity in cases with  $C_{target} = 1$  and further on. Interestingly the network experiences a steep fall in the minimum loadability margin value going from  $C_{target} = 2$  GW to  $C_{target} = 2.5$  GW. Fig.6 explains this behavior. For  $C_{target} = 1.75, 2,$  and  $2.5$  GW, this figure shows how many times, how much more (or less) reactive power injection capacity is online compared to the base case; this illustrates how addition of wind capacity affects the conventional reactive capacity online. Comparing for  $C_{target} = 2.5$  and  $C_{target} = 1.75$  in Fig.6, it can be noted that the number of the operating conditions with 1.04 GVar less conventional reactive power capacity online has approximately doubled with the 750 MW increase in the  $C_{target}$ . Similar behavior is seen for values less than -1.04 GVar. It should be noted that less reactive power injection capacity online results in weaker voltage support and thus a drop in the minimum loadability margin. Fig.7

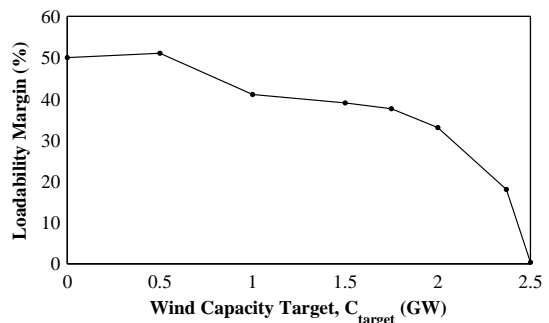


Fig. 5. IEEE RTS 96, minimum loadability margin versus wind capacity targets

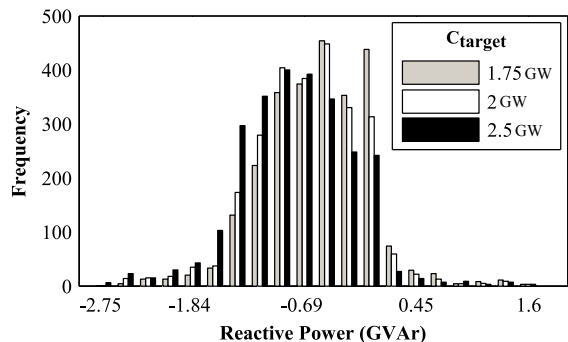


Fig. 6. IEEE RTS 96, frequency of the conventional reactive power injection capacity online difference from base case for cases with  $C_{target} = 1.75, 2$  and  $2.5$  GW

shows the share of wind capacity allocated to each area for the studied wind capacity targets. While the percentage of wind capacity allocated to area 1 in the network is approximately constant for cases with  $C_{target} = 1$  and  $1.5$  GW but it

starts rising for the next studied cases. It is evident that the optimal pattern of wind capacity allocation does not remain constant throughout the studied cases. Fig.8 shows this aspect in detail by illustrating the wind capacity allocated to each candidate bus. It is noted that buses 119, 120, 217 and 220 are not favorable for wind capacity allocation while part of the total wind capacity is allocated at buses 102, 105, 206 and 309 in all of the study cases. Further, buses 206, 312 and 324 have accommodated the maximum allowed wind capacity in cases with  $C_{target} = 1.5$  GW and onwards. This information may help the system operator to grant network access for wind farms on a phased basis. Fig.9 illustrates

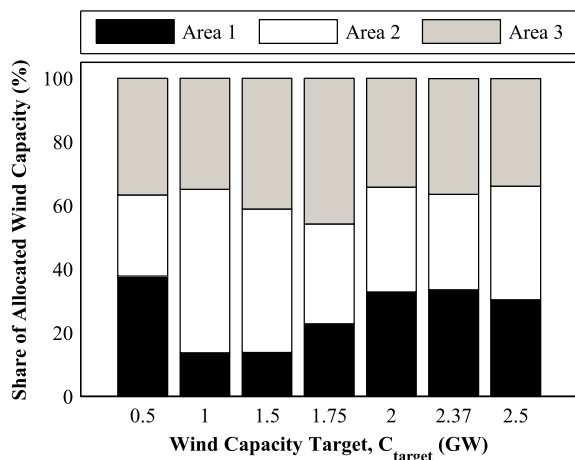


Fig. 7. IEEE RTS 96, share of wind capacity per area for different cases

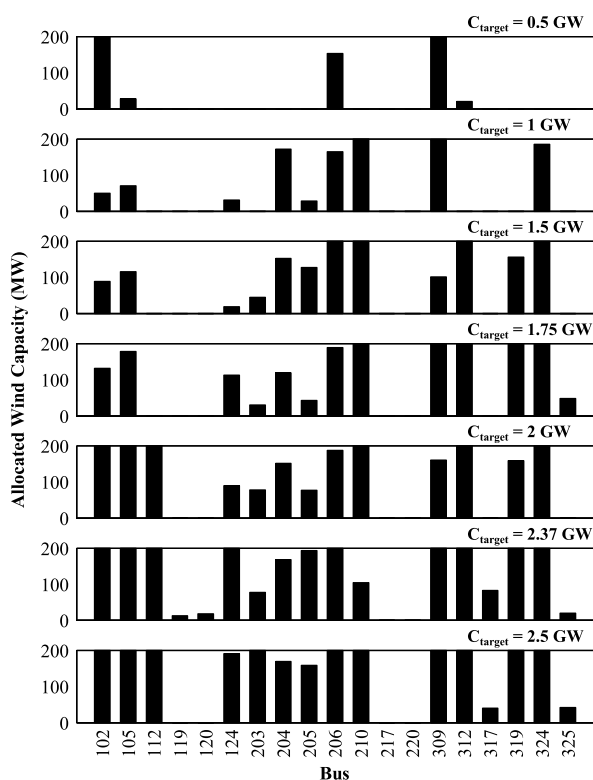


Fig. 8. IEEE RTS 96, wind capacity allocation at each candidate bus for every study wind capacity target

the cumulative probability of the minimum loadability margin throughout the system in all operating conditions for study cases with  $C_{target} = 0, 0.5, 1.75$  and  $2.5$  GW. It is seen that the probability of having a minimum loadability margin less than 100% increases with allocation of wind capacity (shift in the respective trends towards right). ! The generator

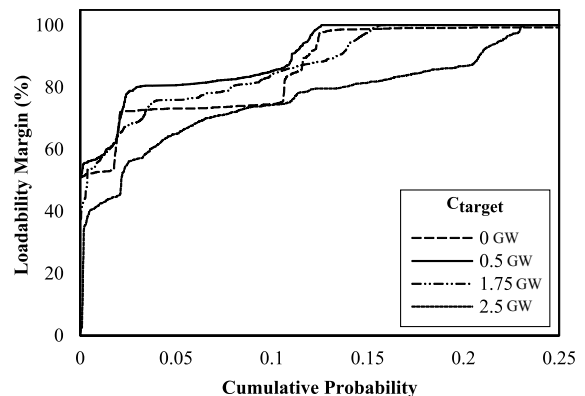


Fig. 9. IEEE RTS 96, cumulative probability of the minimum loadability margin throughout the system in all operating conditions

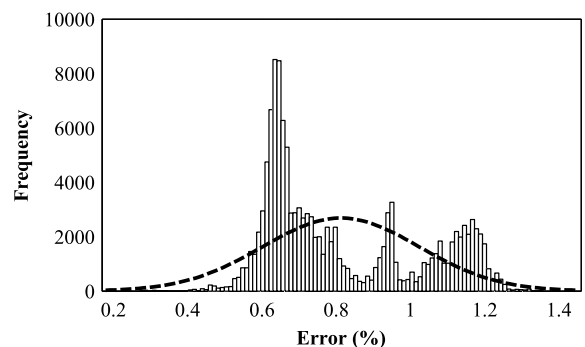


Fig. 10. IEEE RTS 96, frequency of error in voltage with respect to the voltage set point at bus 121

at bus 121 was set to control voltage at 0.965 pu. Fig.10 illustrates the frequency of error in voltage at this bus for 130,866 operating points. It can be noted that the presented continuous formulation of the discrete voltage control function in the optimal power flow results in less than 1% error for 75% of the operating points and less than 1.35% error for all of the operating points. Thus, a satisfactory performance has been achieved with the voltage control implementation presented.

### B. IEEE 118 Bus

The presented algorithm was run to allocate 2GW of wind capacity in the IEEE 118 bus network. Fig.11 shows the allocated wind capacity at each bus. The wind capacity allocation with the proposed procedure has resulted in allocating the wind close to the locations with major loads. It is also seen that area 2 of the network is preferred by the procedure for accommodating the wind capacity. Fig.12 illustrates the cumulative probability of the minimum loadability margin across each area separately and the whole system in all operating conditions. The probability of having a minimum

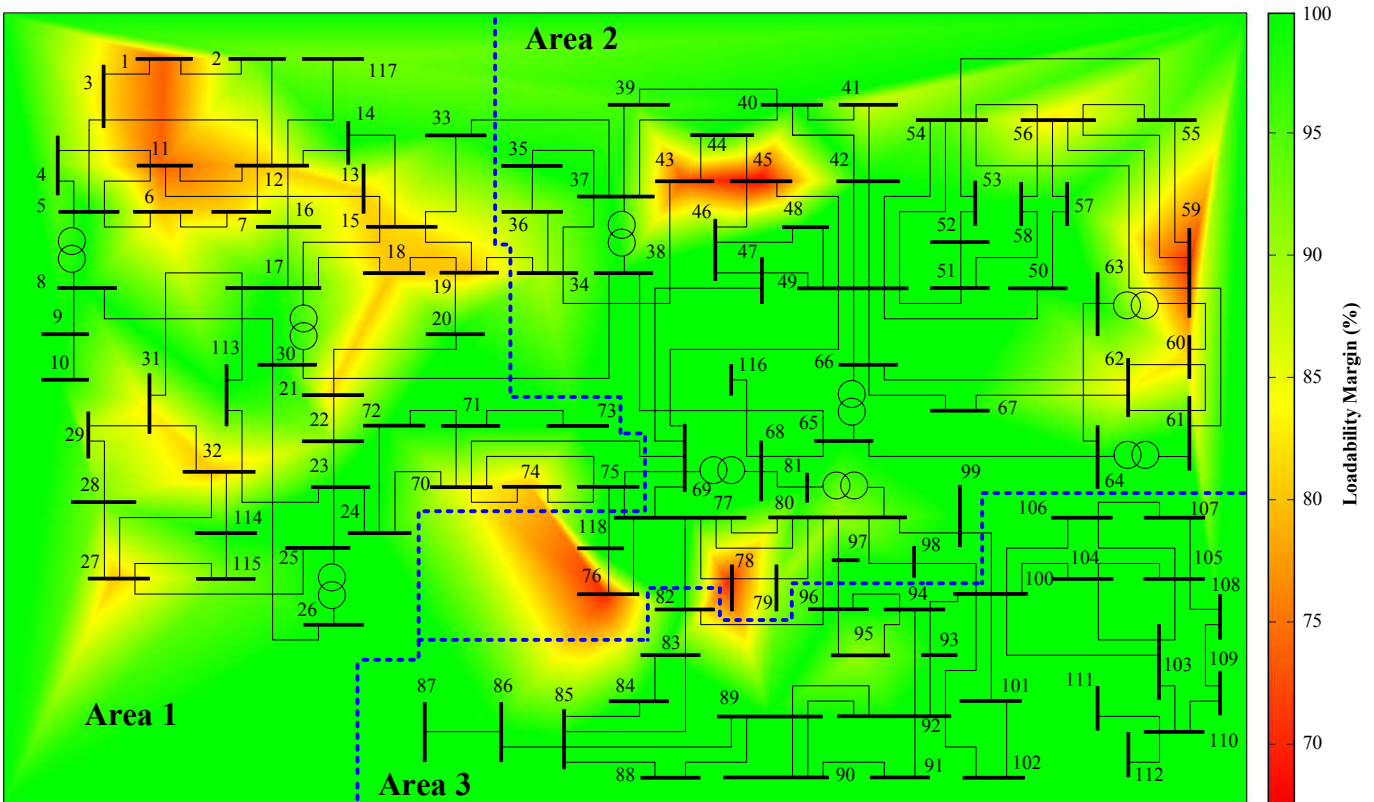


Fig. 13. IEEE 118 bus, minimum loadability margin at each node of the IEEE 118 bus network

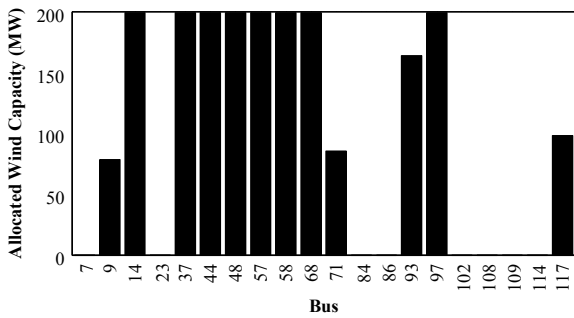


Fig. 11. IEEE 118 bus, wind capacity allocation at each candidate bus in IEEE 118 bus network for every study wind capacity target

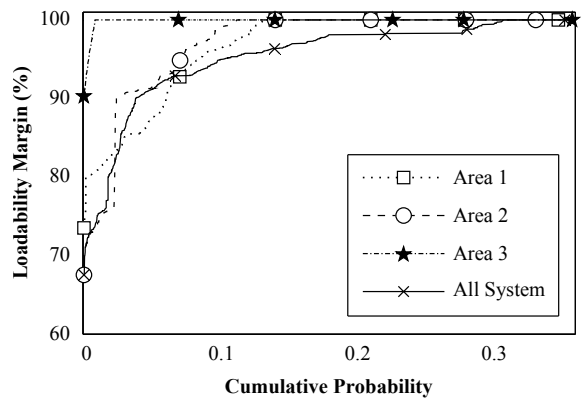


Fig. 12. IEEE 118 bus, cumulative probability of the minimum loadability margin for the IEEE 118 bus system in all operating conditions

loadability margin lower than 100% in area 3 is approximately 1%; this number is 11% and 13% for area 1 and area 2, respectively. This number is relatively larger from the all system perspective i.e. 31%, due to the fact that the location at which minimum loadability margin occurs is dependent on the operating condition. Fig.13 is a heat map overlaid with network single line diagram. It shows the minimum loadability margin across the network in all operating condition. It can be seen that buses 1, 11, 45, 43, 59, 76 and 78 are locations with the lowest minimum loadability margin. Also, there is only one bus, 95, that has a minimum loadability margin less than 100% in area 3. Most of the wind capacity was allocated to locations close to these buses.

## VI. CONCLUSION

A VSCOPF considering changes in the unit commitment is proposed in this paper which provides optimal wind capacity allocation for enhancing voltage stability margin. The changes occur in unit commitment as a result of wind capacity allocation by the wind capacity target. Further, the problem scale is managed by using the wind capacity target and feeding the respective generator status vector to a multi operating conditions VSCOPF tool for wind capacity allocation. The implementation of voltage control for both generators and transformers allowed the consideration of reactive power and transformer tap positions as voltage dependent variables rather

than unrestrained decision variables. This procedure provides valuable information for the system operator with regard to wind capacity allocation and the network voltage stability. The proposed procedure was applied to a 73 bus test system for several wind capacity targets and the larger 118 bus network for one capacity target. It was shown that considering the change in the unit commitment alters the optimal wind capacity allocation. It was also seen that the trend of the minimum loadability margin is not linear with respect to the wind capacity target. It may experience a sharp fall with increasing allocation of wind capacity; this shows the importance of the presented method in order to reduce the possibility of an investment in wind capacity which requires costly remedies from the power system operation perspective.

#### APPENDIX A

$$V^{min} \leq V_b^{n,o}, V_b^{n,s} \leq V^{max} \quad (13)$$

$$I_l^{n,o}, I_l^{n,s} \leq I_l^{rated} \quad (14)$$

$$P_g^{min} \leq P_g^{n,o}, P_g^{n,s} \leq P_g^{max} \quad (15)$$

$$Q_g^{min} \leq Q_g^{n,o}, Q_g^{n,s} \leq Q_g^{max} \quad (16)$$

$$\mu_{min,w}^n C_w \leq Q_w^{n,o}, Q_w^{n,s} \leq \mu_{max,w}^n C_w \quad (17)$$

$$\sum_{g=1}^G P_g^{n,o} \omega_g^n + \sum_{w=1}^W f_w^n C_w \geq \lambda \sum_{w=1}^W f_w^n C_w \quad (18)$$

$$C_w^{min} \leq C_w \leq C_w^{max} \quad (19)$$

$$C_{target} - C_{tolerance} \leq \sum_{w=1}^W C_w \leq C_{target} + C_{tolerance} \quad (20)$$

$$R_t^{min} \leq R_t \leq R_t^{max} \quad (21)$$

$$\begin{cases} \delta_d^{n,s} \geq \kappa & d \text{ corresponding to } s \\ \delta_d^{n,s} = 0 & \text{otherwise} \end{cases} \quad (22)$$

$$\begin{aligned} & \sum_{g=1}^G (P_g^{n,o} \omega_g^n)_b + \sum_{l=1}^L (P_l^{n,o})_b + \sum_{w=1}^W (f_w^n C_w)_b = \\ & \sum_{d=1}^D \{(\alpha_d^p + \alpha_d^i V_b^{n,o} + \alpha_d^z V_b^{n,o^2}) f_d^n P_d\}_b \end{aligned} \quad (23)$$

$$\begin{aligned} & \sum_{g=1}^G (Q_g^{n,o} \omega_g^n)_b + \sum_{l=1}^L (Q_l^{n,o})_b + \sum_{w=1}^W (Q_w^{n,o})_b = \\ & \sum_{d=1}^D \{(\alpha_d^p + \alpha_d^i V_b^{n,o} + \alpha_d^z V_b^{n,o^2}) f_d^n Q_d\}_b \end{aligned} \quad (24)$$

$$\begin{aligned} & \sum_{g=1}^g (P_g^{n,s} \omega_g^n)_b + \sum_{l=1}^L (P_l^{n,s})_b + \sum_{w=1}^W (f_w^n C_w)_b = \\ & \sum_{d=1}^D \{(\alpha_d^p + \alpha_d^i V_b^{n,s} + \alpha_d^z V_b^{n,s^2}) f_d^n P_d\}_b + \\ & \sum_{d=1}^D \{(\alpha_d^p + \alpha_d^i V_b^{n,s} + \alpha_d^z V_b^{n,s^2}) \delta_d^{n,s} f_d^n P_d\}_b \end{aligned} \quad (25)$$

$$\begin{aligned} & \sum_{g=1}^G (Q_g^{n,s} \omega_g^n)_b + \sum_{l=1}^L (Q_l^{n,s})_b + \sum_{w=1}^W (Q_w^{n,s})_b = \\ & \sum_{d=1}^D \{(\alpha_d^p + \alpha_d^i V_b^{n,s} + \alpha_d^z V_b^{n,s^2}) f_d^n Q_d\}_b + \\ & \sum_{d=1}^D \{(\alpha_d^p + \alpha_d^i V_b^{n,s} + \alpha_d^z V_b^{n,s^2}) \delta_d^{n,s} f_d^n Q_d\}_b \end{aligned} \quad (26)$$

Equation (13) reflects the voltage range defined by the grid code for power system operation. Thermal limit of branches is considered by (14). Equations (15) and (16) require the generators to be within the specified limits, respectively. Reactive power output of wind farms is bounded with the inequality constraint in (17);  $\mu_{max}$  and  $\mu_{min}$  may be extracted from the reactive capability diagram of any type of wind farm in each operating condition. With (18) the instantaneous wind generation penetration is maintained lower than a target value,  $\lambda$ . This constraint is set due to technical limits in the network such as inertia. Equation (19) allows for limiting the allocation of wind capacity based on physical and technical limitations in each candidate location for wind capacity allocation. Equation (20) limit the total wind capacity allocated by the wind capacity target  $\pm$  a tolerance value. Equation (21) maintains the tap ratio of the transformers within the available range. The constraint in (22) requires the load increase to be higher than the minimum loadability margin if the load is assigned for increase in the stressed operating point. Otherwise the load increase factor is required to be maintained at zero. Equality constraints in (23)-(26) are standard power flow equations; these require the power injected to a bus to be equal to the power drawn from that bus. The extra term in (25) and (26) applies the load increase factor to the corresponding load serving bus/buses in every stressed operating point.

#### APPENDIX B

TABLE III  
OPTIMIZATION PARAMETERS

Parameter	Value	Parameter	Value
$\lambda$	0.6	$\sigma_t$	0.02
$\rho_g$	100	$\alpha^p$	0.34
$\rho_t$	300	$\alpha^i$	0.33
$\epsilon$	0.001	$\alpha^z$	0.33
$\mu_{max,w}$	0	$C_w^{max}$	200
$\mu_{min,w}$	0	$C_w^{min}$	0
$C_{tolerance}$	100	-	-

#### ACKNOWLEDGMENT

The authors would like to thank A. Soroudi and P. Cuffe for their insightful advice.

#### REFERENCES

- [1] Eirgrid. (2009) Facilitation of renewables. [Online]. Available: <http://www.eirgrid.com/renewables/facilitationofrenewables/>

- [2] T. Ackermann, G. Ancell, L. D. Borup, P. B. Eriksen, B. Ernst, F. Groome, M. Lange, C. Mohrlen, A. G. Orths, J. O'Sullivan, and M. De-la Torre, "Where the wind blows," *Power and Energy Magazine, IEEE*, vol. 7, no. 6, pp. 65–75, 2009.
- [3] J. O'Sullivan, "Maximizing renewable generation on the power system of ireland and northern ireland," in *Wind Power in Power Systems*, 2nd ed., T. Ackermann, Ed. John Wiley & Sons, Ltd, 2012, pp. 623–647.
- [4] J. D. McCalley, V. Vittal, H. Wan, Y. Dai, and N. Abi-Samra, "Voltage risk assessment," in *Power Engineering Society Summer Meeting, 1999. IEEE*, vol. 1, 1999, pp. 179–184 vol.1.
- [5] E. Vaahedi, Y. Mansour, C. Fuchs, S. Granville, M. D. L. Latore, and H. Hamadanizadeh, "Dynamic security constrained optimal power flow/VAr planning," *Power Systems, IEEE Transactions on*, vol. 16, no. 1, pp. 38–43, 2001.
- [6] H. Song, B. Lee, S.-h. Kwon, and V. Ajjarapu, "Reactive reserve-based contingency constrained optimal power flow (RCCOPF) for enhancement of voltage stability margins," *Power Systems, IEEE Transactions on*, vol. 18, no. 4, pp. 1538–1546, 2003.
- [7] W. Zhang, F. Li, and L. M. Tolbert, "Voltage stability constrained optimal power flow (VSCOPF) with two sets of variables (TSV) for reactive power planning," in *Transmission and Distribution Conference and Exposition, 2008. IEEE*, 2008, pp. 1–6.
- [8] R. Zarate-Minano, A. J. Conejo, and F. Milano, "OPF-based security redispatching including FACTS devices," *Generation, Transmission Distribution, IET*, vol. 2, no. 6, pp. 821–833, 2008.
- [9] M. Bakhtvar and A. Keane, "Optimal allocation of wind generation subject to voltage stability constraints," in *Innovative Smart Grid Technologies Europe (ISGT EUROPE), 2013 4th IEEE/PES*, Oct. 2013, pp. 1–5.
- [10] D. J. Burke and M. J. O'Malley, "Maximizing Firm Wind Connection to Security Constrained Transmission Networks," *Power Systems, IEEE Transactions on*, vol. 25, no. 2, pp. 749–759, 2010.
- [11] M. Nick, G. H. Riahy, S. H. Hosseinian, and F. Fallahi, "Wind power optimal capacity allocation to remote areas taking into account transmission connection requirements," *Renewable Power Generation, IET*, vol. 5, no. 5, pp. 347–355, 2011.
- [12] Y. Gu, J. D. McCalley, and M. Ni, "Coordinating Large-Scale Wind Integration and Transmission Planning," *Sustainable Energy, IEEE Transactions on*, vol. 3, no. 4, pp. 652–659, 2012.
- [13] M. Zhao, Z. Chen, and F. Blaabjerg, "Probabilistic capacity of a grid connected wind farm based on optimization method," *Renewable Energy*, vol. 31, no. 13, pp. 2171–2187, 2006.
- [14] R. S. Al Abri, E. F. El-Saadany, and Y. M. Atwa, "Optimal Placement and Sizing Method to Improve the Voltage Stability Margin in a Distribution System Using Distributed Generation," *Power Systems, IEEE Transactions on*, vol. 28, no. 1, pp. 326–334, 2013.
- [15] R. Avalos, C. Canizares, F. Milano, and A. Conejo, "Equivalency of continuation and optimization methods to determine saddle-node and limit-induced bifurcations in power systems," *Circuits and Systems I: Regular Papers, IEEE Transactions on*, vol. 56, no. 1, pp. 210–223, Jan 2009.
- [16] P. N. Vovos, A. E. Kiprakis, A. R. Wallace, and G. P. Harrison, "Centralized and Distributed Voltage Control: Impact on Distributed Generation Penetration," *Power Systems, IEEE Transactions on*, vol. 22, no. 1, pp. 476–483, Feb. 2007.
- [17] Y. Fu, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with ac constraints\*," *Power Systems, IEEE Transactions on*, vol. 20, no. 3, pp. 1538–1550, Aug 2005.
- [18] C. Grigg *et al.*, "The IEEE Reliability Test System-1996. A report prepared by the Reliability Test System Task Force of the Application of Probability Methods Subcommittee," *Power Systems, IEEE Transactions on*, vol. 14, no. 3, pp. 1010–1020, 1999.
- [19] Z. Li and M. Shahidehpour, "Security-constrained unit commitment for simultaneous clearing of energy and ancillary services markets," *Power Systems, IEEE Transactions on*, vol. 20, no. 2, pp. 1079–1088, May 2005.
- [20] Eirgrid Plc. [Online]. Available: <http://www.eirgrid.com/>
- [21] D. Opila, A. Zeynu, and I. Hiskens, "Wind farm reactive support and voltage control," in *Bulk Power System Dynamics and Control (iREP) - VIII (iREP), 2010 iREP Symposium*, Aug 2010, pp. 1–10.
- [22] ENTSOE. [Online]. Available: <http://www.entsoe.eu/>
- [23] A COmmon Optimization Python Repository. [Online]. Available: <https://software.sandia.gov/coopr>
- [24] PLEXOS for Power systems. [Online]. Available: [www.energyexemplar.com](http://www.energyexemplar.com)
- [25] FICO Xpress Optimiser. [Online]. Available: <http://www.fico.com>



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