Abstract

The problem of coating steel by passing it through a bath of molten alloy is considered. The thickness of the coating is determined by the operating conditions of the process, including the speed of travel, temperature of the sheeting and the air pressure and shear from the air knives that operate to strip off excess liquid alloy. During the process, on wider sheets and those with thicker coatings, there sometimes appear defects - known as bananas. These appear at the edges and curve upwards. These regions are almost completely devoid of any coating and hence are not suitable for sale. The group revisited previous work before considering several processes that may be relevant to the problem. These included a two-dimensional model allowing variation in air pressure across the sheet, a study of the backflow generated by the air knives and the Marangoni (surface tension) effect near the edge of the sheet.
1 Introduction and problem description

The process of “continuous hot-dipped galvanisation” is used to coat metal sheeting to protect it from corrosion. The steel surface is heated to high temperature and then is passed through a bath of molten alloy. After passing through the bath it is pulled upwards past air knives that are calibrated to strip off an appropriate amount of the alloy, which falls back into the bath. The strip with the remaining coating travels upward where it is cooled by air and then water before proceeding to the next phase of the industrial process.

At some point during this process, usually when the coating is thicker or the sheet is wider, small diamond shaped regions appear that have no coating. These only appear near the edges and seem to grow upwards and inwards, causing the formation of bare patches, called bananas. These patches are a major flaw in the coating and hence make this part of the sheet unsuitable for distribution.

The problem to be considered by the study group was to attempt to find the reason for the formation and evolution of the bananas.

- What are the physical phenomena responsible for the Banana problem?
- What is the solution to prevent the Bananas from occurring?
  - Proposed solution should work at all line speeds (40 – 125 m/min).
  - Proposed solution should work within the capabilities of the industrial plant.
  - Fluid flow, temperature distribution in the zinc pot, as well as temperature distribution of the steel strip entering the zinc pot seem to be important.

Building on earlier work, the group considered several aspects of the process including the effects of lateral pressure variation (edge-effects) on the coating thickness, the backflow beneath the air knives and the effects of temperature variation on surface tension near the edge of the sheet (Marangoni effect).

There has been a good deal of important work on the coating process, such as [16, 19, 17, 7, 8]. Tuck (1983) developed a two-dimensional, almost-unidirectional fluid flow model that has proven to be quite robust in describing the flow conditions across most of the sheet. More recently, the problem
was considered at a study group in Australia [7, 8]. They considered the stability of any deformations to the coating and found it to be marginally stable (disturbances neither grow nor decay), but that depending on flow conditions the perturbations could steepen at either the front or back and hence break and form droplets in both directions.

Most of this work was predicated on there being very little variation across the sheet, and is therefore most relevant away from the edges. The resulting “simplified” equation is a first-order partial differential equation, the solution of which turns out to be rather delicate depending on the upstream flux condition. However, in the problem under consideration at this study group, there is a need to consider this variation and in particular to consider edge effects.

The magnitudes of the different terms including surface tension, shear, heat transfer, air pressure and gravity, were estimated from typical parameter values to determine their relative importance to the setting up of the process. Surface tension was found to have only a small effect, but it is possible that in the edge region it may be much more significant. The air pressure and shear are the dominant terms in determining the final layer thickness across the sheet.

In this particular case, the likely indicators of the problem seem to be generated near the edge of the sheet. The group considered the following possible causes;

- Instabilities on the edge of the sheet beneath the air knife (in the back flow) propagating upward through the air knife leading to a dry or very thin coating near the edge. These occur during a weaker back flow that is consistent with thicker coatings.

- Wider sheets have a higher temperature gradient and higher temperature near the edge than narrower ones. This gradient in temperature across (and also upward as it cools) may lead to surface tension pulling the coating apart, leaving “dry” regions.

- The pressure and shear due to the air knife are more erratic due to edge effects. The result is that the coating near the edge is slightly thicker than further inward. The effect of this is that flow upward is slightly slower in the edge region (due to lower air shear from the air knives). This may explain the upward curve of the bananas, but may also lead to viscous shear in the region very close to the edge of the sheet.
In what follows we will consider each of these aspects of the flow to test their potential to be the cause. The report finishes with some discussion and suggestions for future work.

2 A two-dimensional approximation

In the earlier work, the mathematical model for the coating process [19, 7, 8] assumed that the flow was mainly vertical and varied little across the sheet. While for most of the sheet this is a good approximation, it is clear that near the edges this no longer holds. Adopting the coordinate scheme shown in Figure 1 and assuming the flow to be two-dimensional, incompressible, laminar and unsteady, we arrive at the following one-dimensional dynamic equation,

$$h_t + \left( h + \frac{1}{2} h^2 G(x) - \frac{1}{3} h^3 (S + P'(x) - Ch_{xxx}) \right)_x = 0 \quad (1)$$

where $h(x,t)$ is the thickness of the coating, $G(x)$ is the shear on the surface and $S$ is the Stokes number. $P(x)$ and $G(x)$ are the pressure and shear generated by the air knives. A full derivation of this equation can be found in [19, 7, 8].

This equation has been non-dimensionalized using $x = L\tilde{x}$, $z = \epsilon L\tilde{z}$, $u = U\tilde{u}$, $w = \epsilon U\tilde{w}$, $t = (L/U)\tilde{t}$, $p = (\mu U/\epsilon^2 L)\tilde{p}$, $h = \epsilon L h_0$, $\tau_0(x) = (\mu U/\epsilon L)G(x)$ and $p_0(x) = (\mu U/\epsilon^2 L)P(x)$, where $\rho$ denotes density, $\mu$ denotes dynamic viscosity, subscripts are used to indicate differentiation and $g$ is the acceleration due to gravity. Here bars denote non-dimensional variables, $h_0$ denotes a typical value of $h$, $L$ denotes a typical length over which the air knife is active, and $\epsilon = h_0/L << 1$.

$$S = \frac{\epsilon^2 \rho g L^2}{\mu U}, \quad C = \frac{\epsilon^3 \gamma}{\mu U}$$

($S$ is the Stokes number and $C$ is $\epsilon^3$ times the Capillary number $Ca$) and, according to the usual “thin layer” assumptions, terms multiplied by $\epsilon^2 Re = h_0^2 \mu U / L^3 \sim 5 \times 10^{-5}$ (see values in Appendix 1) have been ignored.

It is the steady version of this equation that can be used to determine the shape and thickness of the coating [19, 8]. However, our interest here is to consider a simple model of the variable impact of pressure and shear variations from the air knife close to the edge of the sheet. Surface tension will be neglected by setting $C \sim 10^{-6}$ equal to zero.
Suppose now that a coating process is established and running in a steady state, with the coating varying smoothly. Thus \( h = h_0(x) \) where \( h_0 \) is determined by

\[
h_0 + \frac{h_0^2}{2} G(x) - \frac{h_0^3}{3} (S + P'(x)) = Q_c
\]

and \( Q_c \) is the minimum of the maximum fluxes as determined by [19, 7, 8].

Wupperman Staal use baffles along the side of the sheet in an attempt to mitigate the edge effects. An acute angled baffle was used originally (covering slightly the edge of the sheet), but this was replaced by an obtuse angled baffle (angled away from the sheet edge), and this improved the behaviour in this region. However, to minimize edge effects it would seem the best strategy is to make the air flow in this region as close as possible to two-dimensional, much as on airline wing tips or the “skirts” on Formula 1 racing cars. Therefore, it would seem the best shape for baffles might be perpendicular to the sheet, thus forcing the flow to retain its two dimensional structure as much as possible. This should reduce the variation in pressure across the sheet as the edge is neared. However, it will not change the temperature effect.

It is of interest to examine a few cases in which a steady solution is
disturbed in some way to see what will happen to the disturbance. However, by solving for different pressure and shear at different $y$ locations we can approximate the behaviour of a horizontal dip that has formed on the sheet at the air knife. At the edge of the sheet the pressure is lower due to edge-effects of the air knife, and this increases as we move inward.

Equation (1), for the evolution of the surface height of the coating, can be arranged into the more convenient form,

$$h_t + (1 + hG(x) - h^2(S + P'(x))) h_x = \frac{1}{3} h^3 P''(x) - \frac{1}{2} h^2 G'(x). \tag{2}$$

In the usual way, the characteristic traces can be written as

$$\frac{dt}{1} = \frac{dx}{1 + hG(x) - h^2(S + P'(x))} = \frac{dh}{\frac{1}{3} h^3 P''(x) - \frac{1}{2} h^2 G'(x)}$$

and since the equations are autonomous, we have two first-order ordinary differential equations for $h$ and $x$, i.e.

$$\frac{dx}{dt} = 1 + hG(x) - h^2(S + P'(x)), \quad x(0) = x_0 \tag{3}$$

$$\frac{dh}{dt} = \frac{1}{3} h^3 P''(x) - \frac{1}{2} h^2 G'(x), \quad h(0) = h_0 \tag{4}$$

where $x_0$ and $h_0$ are the initial values of $x$ and $h$ on the surface. These two differential equations can be solved using fourth-order Runge-Kutta integration along each characteristic starting at different values of $x$.

Starting with an initial condition of the steady solution for the pressure at each $y$-location and then perturbing slightly with the introduction of a dip, the evolution of the disturbance can be followed very accurately using a routine such as ODE45 in Matlab or lsode in Octave.

### 2.1 Results

Figure 2(a) shows a surface plot of the shape of the disturbance after some time where the flow is rotated so it is from right to left. Note that the surface is higher at $y = 0$ and decreases as $y$ increases due to the increased effect of the air knife. It is clear that the dip is travelling more slowly at the outer edge of the sheet ($y = 0$) where the coating is thicker. The ridge at the right is the backflow beneath the air knife. The pressure effect is not linear and so the dip curves. Near $y = 0$ the dip is about to break backwards into itself due to the different speeds at different heights.
Figure 2: The coating thickness showing an introduced dip progressing up the sheet due to differential pressure near the edge. The large ridge at $x = 0$ is due to the air knife. Other quantities; $P(x, y) = P_{\text{max}}(y)e^{-x^2}$, $S = 0.03$ and $G(x, y) = G_{\text{max}}(y)[x(1 + x^4/4)^{-1}]$ and $P_{\text{max}}(y) = G_{\text{max}}(y) = 0.01 + 0.19y^{3/2}$. 
Figure 2(b) is the contours at the same time. The compact contours near \( x = 0 \) are due to the drop off of the sheet thickness as it passes under the air knife. The contours to the right of this ridge are showing that the back flow is thinner where the coating is thicker. The ridges near \( x = 10 \) are the dip propagating forward. Again the curvature of the ridge is clear. The plot is limited in time by the breaking of the ridge near the edge and this has had an effect on the contours. The front of the dip steepens and breaks back into itself, and this can not easily be represented using contours.

Using the pressure distribution \( P_{max}(y) = 0.01 + 0.19y^{3/2} \) it is clear that the curvature is the wrong way to be responsible for the evolution of the shape of the bananas. In order to get the upward curve displayed by the bananas, a different pressure distribution would be required. The initial calculations would seem to suggest that this upward curvature of the bananas is not due to the differential speed of propagation. It would also seem that the velocity shear is not of sufficient magnitude to cause any kind of instability.

Once the dip has formed, however, it is clear that its front will break backwards onto itself, and this effect can be seen at the top of the bananas as drips. As the location moves away from the edge of the sheet, the pressure and shear terms level off, suggesting that the velocity of the flaws will also level off. This would seem to rule out the shear at the edge of the sheet from being the cause of the formation or shape of the bananas.

These computations do not take into account the temperature variation of the sheet near the edge, but it would seem unlikely that this would change the shape of the results in any significant manner.

3 Wetting, dewetting and surface tension

The effect of temperature variations on the surface tension and its potential to cause flows within the coating need to be considered. Although surface tension plays no significant role in the set up of the steady state, conditions near the edge of the sheet may be sufficiently subtle to allow it to come into play. Here we analyse the magnitude of this effect to see if it may be significant. There are three questions that need to be answered.

Firstly, is it possible for the surface tension to pull apart a dip with sufficient force to dewet the surface? If so, then this may be the origin of the banana. Secondly, once a ‘dry’ region has formed are surface tension forces sufficient to pull it apart into the shapes seen? The final question relates to the shape of the bananas. Can this effect cause such a dry spot to
propagate upward faster than the sheet is moving, thus creating the upward curve seen on the sample sheet. An analysis of the heat transfer between the steel sheet and the coating has shown that both will very quickly reach the temperature of the steel sheet after it exits the pre-heating phase [7, 8]. It is therefore likely that any such effects must be due to the lateral temperature gradient across or up the sheet. Therefore, we need to consider the lateral and lengthwise temperature gradients and the so-called Marangoni effect.

Dewetting is the spontaneous withdrawal of liquid film from a 'hostile' surface. Solid-state dewetting generally occurs at defects but it may also be influenced by the substrate structure/roughness, the substrate morphology, the elastic properties, the grain boundaries in the thin film or even the film geometry. It is driven by the surface free-energy minimization and kinetically mediated by the hydrodynamics of the liquid [4]. A simple eutectic type phase diagram of the $Fe - Zn$ alloy system [10] indicates that liquid zinc will not adhere to a steel surface and so an intermediate coating layer of aluminium is used to prepare the steel for hot-dipped galvanisation. The zinc attaches more readily to this layer due to the presence of intermetallic phases in the $Al - Fe$ system [14].

Under equilibrium conditions, the wettability of a homogeneous, smooth flat rigid surface by a liquid is defined by the Young equation

$$\gamma_{SV} = \gamma_{LS} + \gamma_{LV} \cos \theta$$  \hspace{1cm} (5)

where $\gamma_{SV}$ is the solid-vapour energy, $\gamma_{LV}$ is the liquid-vapour-energy (surface tension), $\gamma_{LS}$ is the liquid-solid energy and $\theta$ is the contact angle. If $\theta > \pi/2$ then the surface is not wettable, otherwise the surface is wettable. For simplicity, we define $\gamma = \gamma_{LV} \cos \theta$. The special case of complete wetting is achieved for $\theta = 0$ and thus equation (5) becomes

$$\gamma = \gamma_{SV} - \gamma_{LS}$$ \hspace{1cm} (6)

where for this case $\gamma = \gamma_{LV}$.

Technological processes in which solid and liquid surfaces come into contact represent nonequilibrium situations and the wetting of solids by liquids is connected to physical chemistry (wettability), statistical physics (pinning of the contact line, wetting transitions, etc.), long-range forces (van der Waals, double layers), and fluid dynamics (external forces). The magnitude of the effect of each factor on the process depends on its operating parameters.

Dealing with processes under nonequilibrium conditions, we introduce a solid/vapour interfacial energy, $\gamma_{SO}$, which is larger than $\gamma_{SV}$ i.e. $\gamma_{SO} >$
\[ \gamma_{LS} + \gamma \]. The difference, \( S \), defined as

\[ S = \gamma_{SO} - \gamma_{LS} - \gamma \]  

is called the spreading coefficient. Physically, \( \gamma_{SO} \) is associated with a ‘dry’ solid surface, while \( \gamma_{SV} \) is related to a ‘moist’ surface. A negative value of the spreading coefficient, \( S < 0 \) indicates a partial wetting.

In liquid/solid systems that are not subjected to external forces, how the dewetting is initiated by the nucleation of a hole is not yet clear. Once nucleated, an increase in its surface area will result in the reduction of the free energy of the system and the hole will grow at a constant velocity, leading to a spontaneous dewetting of a partially wetting substrate [5]. The effect of external forces on the dewetting process has been found experimentally. Indeed, the flux of gas may induce the formation of a dry spot on a liquid film and its extension with time. The velocity of dewetting depends on the external forcing, i.e. the gas flux, the spinning and the Marangoni forces, but a singular contribution of each factor is difficult to determine [2].

### 3.1 Thermo-capillary instability

In thin films it is sometimes seen that surface tension non-uniformity drives the flow from smaller to higher surface tension regions. This is for example the case of the well-known tears of wine, that appear on the side of a glass because the alcohol evaporates more quickly than the water resulting in a surface tension imbalance which drives the wine from the bottom to the top, while the drops fall down due to gravity.

In the present problem, patterns of zinc very similar to tears are observable after the formation of the bananas, see Figure 3. This would suggest that non-uniformity in surface tension together with high vapour pressure of liquid zinc could take place here too. However, it is likely in this scenario that it is the temperature gradient causing surface tension non-uniformity, according to the so-called Marangoni effect. Indeed, a temperature increase leads to a decrease in surface tension, whereby the flow is driven from regions of smaller surface tension (higher temperature) to regions of higher surface tension (smaller temperature). It is possible that in this problem this may be the driver of the banana growth and even the de-wetting of the edge. Certainly, just before the zinc pot, the sheet does not have a uniform temperature since the edge is 30 – 40°C hotter than the center (absolute temperature is around 490°C in the center).

In order to determine whether the banana formation is triggered by the Marangoni effect, we need to consider the temperature gradient along the
Figure 3: Tears of wine (left) and similar zinc structures after the formation of bananas (right).

sheet and its effect on the surface tension. The relevant parameter in this case is the Marangoni number along the $y$ direction (span-wise). Following [13] this is given by

$$Ma = -\frac{d\gamma}{dT} \frac{dT}{dy} \frac{h^2 \rho c_p}{\mu k}, \quad (8)$$

where $T$ is temperature, $\gamma$ surface tension, $h$ film thickness, $\mu$ is dynamic viscosity and $\alpha = k/(\rho c_p)$ is thermal diffusivity (with $k$ thermal conductivity, $\rho$ density and $c_p$ specific heat), see appendix A for details. In this case $dT/dy \approx 25^\circ/m$ is the temperature gradient along the span-wise direction of the plate and is considered constant, whereas $d\gamma/dT \approx -0.17 N/m/\circ C$ is the variation of surface tension with temperature for zinc in this temperature range. When temperature changes across the sheet, the viscosity of the zinc also changes according to

$$\log_{10} (\frac{\mu}{\mu_0}) = -a_1 + \frac{a_2}{T}, \quad (9)$$

where $\mu_0 = 1 mPa s$ and $a_1 = 0.3291$ and $a_2 = 631.12 K$ are coefficients valid for zinc in the temperature range 695 – 1100 K, (422 – 827$^\circ C$) see [1]. Expression (9) is to be used to express the viscosity variation with temperature when computing the Marangoni number in (8).

It is clear from (8) that:

1. for wider sheets (and constant zinc thickness $h$) $Ma$ is greater because $dT/dy$ increases, in line with the observations by the company
2. for thicker zinc layers (and constant sheet width) $Ma$ is greater because $h$ increases, in line with the observations by the company. Noteworthy is that the Marangoni effect (and thus the Marangoni number) is generally stronger for thinner liquid films, but this is not the case here because the temperature gradient plays across the $y$ (span-wise) direction rather than the $z$ direction (cross-stream).

Figures 4 and 5 show the Marangoni number with respect to the sheet width and the zinc thickness, respectively. As anticipated, we note that an increase in the width of the metal sheet leads to an increase in the Marangoni number and increasing the zinc thickness also results in an increase in the Marangoni number. These dependences would explain the banana growth occurring with both larger plates and thicker zinc layers.

However, it must be noted that the Marangoni number is smaller than the Reynolds number, given by $Re = \frac{LY\rho}{\mu} \approx 10$. A more accurate analysis of magnitude is therefore needed. In addition, given previous work [6, 7] it seems unlikely that there is a temperature gradient through the zinc layer, although the air jet could cause a sudden cooling of the outer surface. This requires a detailed analysis of all the phenomena playing in the problem.
Figure 5: Marangoni number versus thickness of the zinc layer. Note that doubling the thickness more than doubles $Ma$.

4 Flow beneath the air knife

Based on observations from footage provided by Wupperman Steel, a snapshot of which is shown in Figure 3(a), we attempt to investigate the flow regime beneath the air knife as a possible origin of the defects observed further downstream of this region. The salient features of the footage are as follows:

- A film of excess molten alloy cascades downward into the bath whose edges appear to neck in as the film falls under gravity
- This backflow of fluid competes against the fluid drawn up from the bath by the metal sheet
- The flow is unsteady, and banana-like features can be observed throughout the footage.

A sophisticated model should incorporate all of the features described above, however it was decided that the study group should focus on one aspect of this regime in isolation since it is unclear which of these processes is dominant. In that spirit, we concentrate our efforts on the downward cascade of fluid from the air knife. For simplicity we assume that the steel sheet is stationary and no fluid is being drawn up from the bath. To imitate the
cascade of fluid from the air knife we consider a controlled flux of fluid of known thickness flowing downward from a boundary $y = 0$ as shown in Figure 6(b). Finally we assume that the flow beneath the air knife has reached a steady state configuration. We note that, in principle, it should be relatively straightforward to incorporate the effects of a moving sheet, but coupling the backflow of fluid to the flow of fluid drawn up by the sheet is likely to be more complex. We also stress that the assumption of steady state flow is a drastic simplification in our model, since the footage appears to show that the flow is in fact unsteady and subject to perturbations. However, the origin of these perturbations is not clear and so we make no attempt to incorporate such phenomena here.

Figure 6: (a) A still obtained from footage provided by Wupperman Steel of the flow regime below the air knife. The air knife strips off excess liquid alloy which causes a flux of fluid downwards into the bath in the direction of the black arrows. The red arrow demarcates the inward necking of the falling film, while the blue arrow indicates the direction in which the steel sheet is being pulled. It appears from the footage that the bananas may originate as a result of these two opposing effects in the region highlighted by the black dashed box. (b) A schematic of the flow regime under consideration. A constant known flux of fluid is supplied from $y = 0$ which falls under the action of gravity against a stationary steel sheet on which the fluid velocity is zero.
4.1 Governing equations

We adopt Cartesian coordinates $x = (x, y, z)$ with the $x$-axis spanning the width of the sheet directly below the air knife, the $y$-axis aligned with the direction of motion of the steel sheet, and $z$ in the transverse direction as shown in Figure 6(b). We consider a layer of molten zinc of width $x = b(y)$ in the $x$-direction and thickness $h(x, y)$ in the $z$-direction, in contact with the steel sheet at $z = 0$. The flow in the region below the air knife is governed by the steady-state Navier-Stokes equations

$$\nabla \cdot \mathbf{u} = 0,$$  \hspace{1cm} (10a)

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot (\mu [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]) + \rho g,$$  \hspace{1cm} (10b)

These equations should be supplemented by boundary conditions. At the top of the backflow region, we impose the thickness and flux of zinc directly below the air knife, i.e.

$$u = 0, \quad v = V_0, \quad h = H, \quad \text{on} \quad y = 0.$$  \hspace{1cm} (11)

On the free surfaces $z = h(x, y)$ and $x = b(y)$ we impose kinematic and dynamic boundary conditions (which are conditions on the motion and stress at the free surface respectively). In principle we should take into account the effect of surface tension at these free edges. On the free surface $z = h(x, y)$ the curvature is small and the effect of surface tension relative to inertial effects is of size $\epsilon \Sigma^{-1} \ll 1$, where

$$\Sigma = \frac{\rho V_0^2 L}{\gamma},$$  \hspace{1cm} (12)

so we neglect surface tension effects on $z = h(x, y)$ and impose no-flux and no-stress conditions

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad [\mathbf{\sigma}]^+ \cdot \mathbf{n} = 0,$$  \hspace{1cm} (13)

where $\mathbf{n}$ is the normal to the zinc surface and $\mathbf{\sigma} = -\mathbf{p} + \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ is the stress. In the neighbourhood of the edge of the zinc layer, $x = b(y)$, the sheet surface adjusts rapidly to meet the steel at a prescribed contact angle. The curvature here is much larger than in the bulk of the sheet, so that surface tension may play a role at leading order. We therefore impose

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad [\mathbf{\sigma}]^+ \cdot \mathbf{n} = \gamma \kappa \mathbf{n},$$  \hspace{1cm} (14)

on $x = b(y)$, where $\kappa$ is the curvature of the zinc surface, $\gamma$ is surface tension at the zinc-air interface, and $[\mathbf{\sigma}]^+$ denotes the jump in stress across the surface.
4.2 Non-dimensionalisation

In this problem there are two characteristic length scales, the width of the sheet and the thickness of the back-flowing zinc, which is of size $O(\text{mm})$. We introduce an aspect ratio $\epsilon = H/L \ll 1$, where $H$ is the thickness of zinc directly below the air knife and $L$ the width of the steel sheet. We then non-dimensionalise via

\begin{align}
(x, y) &= L(x', y'), \quad z = \epsilon L z', \quad t = \frac{L}{V_0} t', \\
u &= V_0 (u', v'), \quad w = \epsilon V_0 w', \quad p = p_{\text{atm}} + P p',
\end{align}

where $V_0$ is the speed of zinc at the top of the backflow region and $\rho$ is its density, and the scaling pressure $P$ has still to be chosen. Substituting these dimensionless variables into the governing equations (10) and dropping the primes from the notation we arrive at

\begin{align}
\nabla \cdot \mathbf{u} &= 0, \\
\hat{\text{Re}} \left( \frac{\epsilon^2 L P}{\mu V_0} \mathbf{u} + \epsilon^2 \mathbf{v} \right) &= \left( -\frac{\epsilon^2 L P}{\mu V_0} p + \epsilon^2 u_x \right)_x + \epsilon^2 (u_y + v_x)_y + (u_z + \epsilon^2 w_x)_z, \\
\hat{\text{Re}} \left( \frac{\epsilon^2 L P}{\mu V_0} \mathbf{v} + \epsilon^2 \mathbf{w} \right) &= \epsilon^2 (u_y + v_x)_x + \left( -\frac{\epsilon^2 L P}{\mu V_0} p + \epsilon^2 v_y \right)_y + (v_z + \epsilon^2 w_y)_z, \\
\hat{\text{Re}} \left( \frac{\epsilon^2 L P}{\mu V_0} \mathbf{w} + \epsilon^2 \mathbf{u} \right) &= (u_z + \epsilon^2 w_x)_x + (v_z + \epsilon^2 w_y)_y + \left( -\frac{P L}{\mu V_0} p + 2w_z \right)_z,
\end{align}

where subscripts denote differentiation, $\hat{\text{Re}} = \epsilon^2 \rho LU/\mu$ is a rescaled Reynolds number and $\tilde{g} = \epsilon \rho g L^2/\mu V_0$ is a rescaled Stokes number. Unlike the upper region considered in §2, we cannot make the usual lubrication approximation $\text{Re} \ll 1$ here. Since the sheet is thicker in this lower region ($H = O(10^{-3} \text{m})$), we must consider the case $\text{Re} = O(1)$ and retain inertial effects at leading order in our governing equations. We also assume $\tilde{g} = O(1)$ so that the zinc undergoes acceleration due to gravity. We choose $P = \rho U^2$ to balance with inertia in the in-plane momentum equations (17b)–(17c). The boundary conditions on $y = 0$ become

\begin{align}
u &= 0, \quad v = h = 1,
\end{align}

and the boundary conditions at the free surface $z = h(x, y)$ become

\begin{align}
u h_x + v h_y = w,\quad (19a)
\end{align}
\( h_x (-\hat{Re}p + 2\epsilon^2 u_x) + \epsilon^2 h_y (u_y + v_x) = u_z + \epsilon^2 w_x, \)  
(19b)

\( \epsilon^2 h_x (u_y + v_x) + h_y (-\hat{Re}p + 2\epsilon^2 v_y) = v_z + \epsilon^2 w_y, \)  
(19c)

\( h_x (u_z + \epsilon^2 w_x) + h_y (v_z + \epsilon^2 w_y) = -\epsilon^{-2}\hat{Re}p + 2w_z. \)  
(19d)

At \( x = b(y) \) we take the radius of curvature of the sheet to be \( H = \epsilon L \) and impose

\[
\begin{align*}
  u &= vb_y, & \text{(20a)} \\
  \begin{pmatrix}
    -\epsilon^{-2}\hat{Re}p + 2u_x - b_y (u_y + v_x) \\
    u_y + v_x - b_y (-\epsilon^{-2}\hat{Re}p + 2v_y) \\
    \epsilon^{-1}u_x + \epsilon w_x - b_y (\epsilon^{-1}v_x + \epsilon w_y)
  \end{pmatrix} &= \epsilon^{-1} Ca^{-1} \begin{pmatrix} 1 \\ -b_y \\ 0 \end{pmatrix}. & \text{(20b)}
\end{align*}
\]

### 4.3 Thin-sheet limit

We exploit the fact that the zinc layer is very thin compared to the size of the sheet by considering the asymptotic limit \( \epsilon \to 0 \). We perform an asymptotic expansion in powers of \( \epsilon \). Evaluating the transverse momentum equation (17d) at leading order and imposing the no-stress condition (19d) we find that the leading-order pressure is simply

\( p = 0. \)  
(21)

Evaluating the mass balance (17a) and in-plane momentum equations (17b)–(17c) at leading order and integrating across the sheet thickness we determine a reduced system of equations for the in-plane velocities \( u \) and \( v \) and the sheet thickness \( h \) (note that we must go to next order in the transverse momentum equation (17d) to provide closure)

\[
\begin{align*}
  \left( \int_0^h u \, dz \right)_x + \left( \int_0^h v \, dz \right)_y &= 0, & \text{(22a)} \\
  \Re \left[ \left( \int_0^h u^2 \, dz \right)_x + \left( \int_0^h u v \, dz \right)_y \right] &= -\Re \int_0^1 p_x \, dz + u_z(h) - u_z(0), & \text{(22b)} \\
  \Re \left[ \left( \int_0^h u v \, dz \right)_x + \left( \int_0^h v^2 \, dz \right)_y \right] &= -\Re \int_0^1 p_y \, dz + v_z(h) - v_z(0) - \hat{g}h, & \text{(22c)}
\end{align*}
\]

These equations do not yield much insight in their current form, but we make progress by following the example of Shkadov [12] and introducing an
ad-hoc parabolic profile for the in-plane velocity. We impose no-slip and no-stress conditions at the wall and free surface respectively to arrive at

\[ u = q_1 z(z - 2h), \quad v = q_2 z(z - 2h), \]

where \( q_1 \) and \( q_2 \) are flux components to be determined. For a discussion of this approach see [3]. We note that these are the exact profiles that would be obtained in the case \( \hat{R}e \ll 1 \). Substituting these profiles into the integrated equations (22) yields

\begin{align}
\left(q_1 h^3\right)_x + (q_2 h^3)_y &= 0, \\
\frac{8}{15} \hat{R}e \left[q_1^2 h^5\right]_x + (q_1 q_2 h^5)_y &= 2q_1 h, \\
\frac{8}{15} \hat{R}e \left[q_1 q_2 h^5\right]_x + (q_2^2 h^5)_y &= 2q_2 h - \tilde{g} h
\end{align}

(24a, 24b, 24c)

At the boundary of the sheet \( x = b(y) \) we integrate the free edge conditions (20) across the sheet thickness. The no-flux condition (20a) becomes

\[ a_1 = a_2 b_y \text{ on } x = b(y). \]

The appropriate dynamic condition is most easily obtained by changing to a coordinate system embedded in the sheet edge, with \( s \) being arclength along the sheet edge, \( \hat{t} \) the unit tangent vector to the sheet edge in the \( xy \)-plane, \( n \) the perpendicular distance from the sheet edge and \( \hat{n} \) the unit normal vector to the sheet edge. Our condition of no tangential stress becomes at leading order

\[ \frac{\partial u_s}{\partial n} = 0, \]

where \( u_s \) is the velocity along the sheet edge. Imposing the normal stress condition is somewhat trickier, since pressure dominates and a correction to pressure enters at the same order as the velocity gradient. It was not possible to derive the correct condition in the limited time frame of the study group, so we instead bypassed this issue by imposing a velocity gradient

\[ \frac{\partial u_n}{\partial n} = \Gamma, \]

where \( u_n \) is the velocity normal to the free surface, and \( \Gamma \) is representative of the force due to surface tension (and should be some multiple of \( \text{Ca}^{-1} \)).

We stress that this is not a carefully derived condition and is not the correct way of imposing the dynamic conditions, it is an ad-hoc assumption designed simply to give an idea of the necking-in profile of the cascading sheet.
4.4 Results

We solved the governing equations (24)–(27) using the method of lines for different values of $\Gamma$ and $\tilde{g}$. Representative profiles of the sheet edge are shown in Figure 7(a). As expected, the sheet necks in further as the liquid sheet falls further, and the magnitude of the necking-in phenomenon increases with increasing $\Gamma$ (see Figure 7(b)). These results demonstrate that the cascading fluid will indeed neck in in the presence of surface tension. This is in contrast to the $\epsilon^2 \text{Re} \ll 1$ case, where the cascading fluid retains uniform width at all orders. However, calculating solutions for larger values of the surface tension is challenging, and this is most likely a result of our incorrect implementation of the surface tension effect at the free edge.

Evidence provided by Wuppermann Staal suggests that the edge temperature is higher in wider sheets (where bananas occur). Since viscosity decreases with increasing temperature, one might expect the necking-in phenomenon to be more prevalent in wide sheets, so it is a possible candidate for the origin of the defect. However, it is not obvious how exactly necking in would give rise to defects, and further investigation is required.

![Figure 7](image)

Figure 7: (a) Edge profile $b(x)$ of the a liquid sheet falling down a vertical wall, for different values of $\Gamma$, (b) total distance necked in for varying $\Gamma$.

5 Conclusions

This proved to be a very rich problem. The equations derived for the development of the molten coating approximate very closely the steady-state
behaviour experienced in practice and this was extended to consider the two dimensional edge effects of differential pressure. The resulting calculations suggest that it is not this that causes the bananas to grow, and the marginal stability of the flow also suggests that it is also not a mechanism for a dip to grow into a dry spot.

The removal of this as a possible cause leaves surface tension, dewetting and the Marangoni effect or the instability on the edge of the sheet in the back flow as possible causes of the initiation and growth of the bananas. A quantitative analysis of the surface tension parameters does show some promise as a mechanism for the expansion of the banana once it has formed. It is more difficult to determine whether this or dewetting are the cause of the bare patches.

A simple model for the backflow of molten zinc beneath the air knife suggests that the combined effects of inertia and surface tension are enough to induce necking of the falling film. Further investigation into this problem should seek to resolve the ad-hoc imposition of surface tension at the edge of the falling film, as it was not clear to the study group (in the time available) what the correct boundary condition in the thin sheet limit should be.

It remains an open question to determine the coupling, if any, between this flow and the flow of molten zinc being drawn up out of the bath by the metal sheet. We note that it may be the case that the dynamics of the cascading fluid are a result of these competing flows, and the effects due to inertia and surface tension are less important than assumed here. Steps to resolve this problem might include a full 3-dimensional simulation of the flow beneath the air knife, or a simple 2-dimensional model of the competing flows which encompasses the correct physics. An analysis of the flow very near to the edge of the sheet might also be revealing. In any event, it is a strong possibility that it is in this region that the bananas originate and a more detailed analysis is worthwhile.

It is clear that the shear and pressure dominate the coating thickness and so it would seem that maintaining the shape of these as close as possible to the edge of the sheet (maintaining two dimensionality of the flow) would be the most desirable strategy. To this end, a perpendicular baffle (rather than one angled over or away from the sheet) would seem most appropriate. This should at least reduce the “fatter” back flow near the edges and reduce the edge instability in the back flow.

Future work will be used to confirm and refine some of the preliminary results. It would be desirable to find surface trajectories for more realistic pressure/shear terms. Using the full partial differential equation could show how an initial distortion will evolve: it could be used to consider whether
small ripples could cause the line/wave defects that have also been observed.

In summary, the work at MISG revealed that the evolution of the bananas after their formation is most likely due to the Marangoni effect (due to variation in surface tension), while the initial formation may be either due to dewetting of the surface or some edge instability in the flow as it enters the air knife after passing the separation point beneath which there is a backflow. Further calculations will be necessary to fully understand how this defect may occur and how it evolves, but a sound start and clear future directions have been identified in this study group.

A Typical values used;

The list below gives typical values used in this work, e.g. [1, 9].

- \( C_p \) = specific heat of zinc alloy coating \( \approx 388 \text{ J kg}^{-1}\text{K}^{-1} \)
- \( C_{ps} \) = specific heat of steel \( \approx 700 \text{ J kg}^{-1}\text{K}^{-1} \)
- \( d \) = half-width of the steel strip \( \approx 10^{-3} \text{m} \)
- \( g \) = gravitational acceleration \( \approx 9.8 \text{ m s}^{-2} \)
- \( \gamma_0 \) = surface tension of coating layer \( = 0.7821 - 0.17 \times 10^{-3}(T - 462^\circ\text{C}) \text{ N/m}; (0.7566 \text{ N/m for } T = 477^\circ\text{C}) \)
- \( h_0 \) = length scale of the thickness of the coating layer \( \approx 10^{-5} \text{ m} \)
- \( k_a \) = thermal conductivity of air \( \approx 0.026 \text{ Wm}^{-1}\text{K}^{-1} \)
- \( k_s \) = thermal conductivity of steel \( \approx 30 \text{ Wm}^{-1}\text{K}^{-1} \)
- \( k \) = thermal conductivity of zinc alloy coating \( \approx 100 \text{ Wm}^{-1}\text{K}^{-1} \)
- \( L \) = length scale in upwards direction - half-width of air jet \( \approx 5 \times 10^{-3} \text{m} \)
- \( \mu \) = dynamic viscosity of the coating \( \approx 3.254 \times 10^{-3} \text{ kg m}^{-1}\text{s}^{-1} \text{ for } T = 477^\circ\text{C}. \)
- \( \rho \) = density of the coating \( \approx 3 \times 10^3 \text{ kg m}^{-3} \)
- \( \rho_s \) = density of steel \( \approx 6508 \text{ kg m}^{-3} \text{ for } T = 477^\circ\text{C}. \)
- \( U \) = upward speed of the metal sheet \( \approx 2.5 \text{ m s}^{-1} \)
- \( U_a \) = maximum centerline speed of the air jet \( \approx 28 \text{ m s}^{-1} \)
• $T_M$ = melting point of zinc alloy coating $\approx$ 420 °C
• $T_B$ = typical bath temperatures $\approx$ 460 °C

Using these values, the scalings for the pressure and shear stress are

• Pressure scaling $\frac{\mu U}{\epsilon^2 L} \approx 10^5$ kg m$^{-1}$s$^{-2}$
• Shear scaling $\frac{\mu U}{\epsilon L} \approx 600$ kg m$^{-1}$s$^{-2}$

and the non-dimensional quantities that appear are

• Capillary number (Surface tension) $Ca = \frac{\mu U}{\gamma_0} \approx 2 \times 10^{-2}$
• Reynolds number $Re = \frac{UL}{\mu} \approx 12$
• Stokes number $S = \frac{\rho g b^2}{\mu U} \approx 0.001$
• Length ratio $\epsilon \approx 2 \times 10^{-3}$

For the Marangoni number:

• $d\gamma /dT \approx -0.17 N/m/^\circ C$
• thermal conductivity $k = 164$ W m$^{-1}$K$^{-1}$
• density of the zinc at 477°C: $\rho = 6553$ kg m$^{-3}$
• $C_p$ = specific heat of zinc alloy coating $\approx$ 479.9 J kg$^{-1}$K$^{-1}$
• thermal diffusivity $alpha = \frac{k}{\rho c_p} (m^2 s^{-1})$

Acknowledgements

We are grateful to the industry representatives for their support and enthusiasm. We also thank the other people who participated in this project who included

References


