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1 Context and Problem Statement

The aim of this work is to model the flow of liquid as it spreads through a structured cavity (‘wicking’), as described schematically in Figure 1. The problem was posed by Analog Devices in the context of the 118th European Study Group with Industry, which was held in UCD in July 2018. The aim of the modelling exercise is to find the optimum structure morphology/size/porosity/materials for wicking/routing of liquid inside a cavity under various temperature and environment conditions?

Wicking of surfaces has many applications in the fields of biology, sensing and integrated chip cooling. As such, the existing literature on the subject is extensive. Therefore, the first objective of this report is to carry out a detailed literature review (Section 2), wherein we outline how the answers to many of the questions posed by Analog Devices can be answered by methods from the existing literature. Further refinements of this approach could be carried out, if any of the questions are not answered in this report. In this existing literature, the fluid that permeates the substrate is assumed to come from an infinite reservoir. Therefore, in section 3 we examine how including the finite volume of the liquid drop that permeates into the substrate affects the spreading dynamics. Additionally, we have also examined lubrication theory as a way of providing a very detailed and mathematically consistent description of wetting on rough surfaces, which we describe below in Section 4. Finally, our conclusions are presented in Section 5.

As the problem is fluid-dynamical in nature, we first of all identify flow parameters that determine which forces are relevant. We use

- The spreading velocity \( U \approx 1 \text{cm/10mins} = 10^{-5} \text{m/s} \)

![Figure 1: Schematic description of the model problem.](image)
The die size $R \approx 1 \text{ cm}$

- A range of drop sizes $r_0 \in 0.1 - 1 \text{ cm}$

We consider the problem in a water–air mixture, where the liquid (water) has density $\rho = 10^3 \text{ kg/m}^3$, viscosity $\mu = 10^{-3} \text{ Pa s}$, and liquid–air surface tension $\sigma \approx 7 \times 10^{-2} \text{ N/m}^2$. We obtain the following values for the Reynolds number $\text{Re}$, Bond number $\text{Bo}$, and Capillary number $\text{Ca}$:

$$\text{Re} = \frac{\rho UR}{\mu} \lesssim 10^{-1},$$

$$\text{Bo} = \frac{\rho g r_0^2}{\sigma} = 10^{-1} - 10,$$

$$\text{Ca} = \frac{\mu U}{\sigma} \lesssim 10^{-7}.$$

As such, in each of the considered models, gravity and surface tension are the key driving forces, and inertia is negligible.

## 2 Literature Review

*Chinna Deverapu was the main contributor to this section.*

### 2.1 Basics of liquid spreading on a microstructured surfaces

In order to understand the wicking on microstructured surfaces, it is helpful to start with the theory of wicking on the flat surfaces. Wetting on flat surfaces can be determined by an inverse relation between the contact angle and the surface tension of the flat surface. Young [1] formalized this relationship in 1805 as follows:

$$\gamma_{SG} = \gamma_{SL} + \gamma_{LG} \cos(\theta_c),$$  \hspace{1cm} (1)

where $\gamma_{SG}$, $\gamma_{SL}$, $\gamma_{LG}$ denote the surface tensions between the three phases: solid, liquid and gas, and $\theta_c$ denotes the contact angle as shown in Figure 2. However, most surfaces in practical applications are far from being flat and require more rigorous treatment similar to the problem-case posed by Analog Devices, *i.e.*, wicking of a
microstructures on a surface. For these rough surfaces, the contact angle of the liquid will be different than that of a flat surface case. For such rough surface cases, a modified contact angle, $\theta^*$, is given by the Cassie-Baxter equation [3]:

$$\cos (\theta^*) = r_f f \cos (\theta_c) + f - 1,$$

where $\theta_c$ is the contact angle of a liquid on a perfect flat surface and $f$ denotes the fraction of solid surface area wet by the liquid and $r_f$ denotes the roughness ratio of the wet surface area. Now, if $\theta^* < \theta_c < 90$, then a liquid drop on a microstructure surface starts to nucleate from the middle of the drop and reaches the edges of the drop. Therefore, in this limit, Cassie-Baxter model can be simplified as a Wenzel model. However, if the critical contact angle is above the contact angle of the flat surface, then the liquid on a microstructured surface spreads beyond the edges of the drop. Microstructured surfaces with such critical angle act as superhydrophilic surfaces as shown in Figure 3. Therefore, it can be inferred that surface roughness $r_f$ determines the critical contact angle $\theta^*$, which in turn determines if a rough surface is either a superhydrophobic surface or a superhydrophilic surface.

For example, in Reference [4], the authors fabricated several micropillar devices on Si substrate by varying the dimensions of the pillars such as height ($h$), diameter ($d$), and pitch ($p$), as summarized in Figure 4. It has been found that the structure (pillar A) with critical contact angle, $\theta^* > \theta_c$, acted as hydrophobic, while the ones (Pillar B, C and D) with critical contact angle, $\theta^* < \theta_c < 90$, acted as superhydrophilic surfaces, confirming the above description relating the surface roughness and the wicking of microstructured surfaces.
2.2 Dynamics of the liquid spreading on microstructured surfaces

After determining the required surface roughness to make a superhydrophobic surface, it is important to study the dynamics of the wicking as it determines how fast a surface can be wicked. Many experimental findings in the literature [4, 5, 6, 3, 7, 8] found that liquid front on a micropillar surface spreads at a rate proportional to the square root of time. This observation is quite similar to liquid wicking inside a capillary tube. Single capillary tube wetting dynamics were first described by Washburn [9] as:

$$z = \sqrt{Dt}, \quad (3)$$

where $z$ is the penetration length of the liquid and $D$ is the coefficient of penetrance, i.e., the relative rate penetration of the liquid through a capillary tube or a lattice of micropillars.

Surprisingly, this simple relationship between the time and penetration length holds true for many experimental results of the wicking of microstructured surfaces. For example, Bico et al. [5] derived an empirical relation for the first time for liquid dynamics for the micropillar arrays by balancing the capillary pressure with the viscous force as:

$$z = \sqrt{\frac{2 \cos(\theta) - \cos(\theta_c)}{3 \beta} \frac{\gamma \delta}{\eta} t}. \quad (4)$$

It can be noticed that this relationship is quite similar to Washburn’s Law. These dynamics were experimentally verified with Si micropillar arrays as shown in Figure 5.

The more interesting observations came from the experimental study of the Ishino et al. [6]. They studied the wicking of micropillars in two regimes:

- Case 1 – the height of the pillars is less than the pitch;
- Case 2 – the height of the pillars is more than the pitch.

They found that in both the cases, liquid dynamics follow Washburn’s Law. In Case 1, liquid film progression on a micro-structured surface increases with the height of
Figure 6: (a) Micropillars on Si substrate from Ishino et al. [6]; (b) Plot to show that wicking on these Si pillars follow Washburn’s dynamics.

Figure 7: (a) Simplified model to understand the wicking on micropillars on Si substrate [3]. (b) Plot to show that this simplified model predicts that wicking on these Si pillars follows Washburn’s dynamics.

the pillars until the height of the pillar is less than the pitch. In Case 2, the pillar height does not influence the liquid film progression. The Washburn dynamics of these two cases are shown in Figure 6 and empirically expressed as:

$$z_1 = \sqrt{\frac{4\pi \gamma h^2 b}{3 \eta p^2 t}}, \quad z_2 = \sqrt{\frac{\gamma b}{\eta} (\ln(p/b) - 1.31)t}. \quad (5)$$

Furthermore, in Reference [4], the authors simplified the pillars geometrical parameters and developed a simple model to predict the liquid propagation on micropillar structures. In this model, they approximated the flow of fluid through the nanopillars as flow through open nanochannels that are of the same height and length (Figure 6(a)). Their experimental studies have also confirmed that wicking in micropillars follows Washburn’s dynamics (Figure 7) as expressed below:

$$z = \sqrt{\frac{2\gamma h \cos(\theta) - \cos(\theta_c)}{3\mu \beta \cos(\theta)}}t. \quad (6)$$

This simplified model predicts wicking on micropillars even more accurately than the previously described models of Bico et al. [5] and Ishino et al. [6].
2.3 Optimal parameters of the micropillars

Now that we understand the basics of wicking in micropillar arrays and understand that their dynamics follow Washburn’s Law, we aim to find if there exist any optimal parameters for the micropillar dimensions.

Xiao et al. [7], proposed a universal scaling law to predict the optimal pillar dimensions by numerically simulating the meniscus formed by pillars (Figure 8(a)). Their model predicted the optimal dimensions of the pillar in terms of dimensionless parameters: $h/d$, $d/l$. The design guidelines for obtaining the optimum penetration length, D, have been summarised in Figure 8(b).

Finally, Shrivastava et al. [8] proposed more accurate universal scaling law for wicking in micropillar arrays. For this purpose, they employed COMSOL simulations to predict the pressure driven by the flow around an array of micropillars. Their model divides the optimal parameter space into four important regimes, where one of the regime is a lubrication regime (Figure 9(a)). According this study, it is possible to achieve the same rate of wicking by using many different combinations of the $h$, $d$ and $g$ as summarised in Figure 9(b). For example, for the same gap ($g$) between the pillars, the same value of $D$ is obtained for $h = 100 \mu m$ and $d = 5 \mu m$, for $h = 20 \mu m$ and $d = 20 \mu m$ as shown by the dotted line in Figure 9(b).

2.4 Discussion

Summarizing, it is possible to predict if a micropillar array with certain dimensions is hydrophilic or hydrophobic by evaluating the critical contact angle. Furthermore, it is found experimentally that the dynamics of the wicking in micropillar arrays follow Washburn’s Law. There are many studies that aimed to determine the coefficient of penetrance for the liquid flow on micropillar arrays. By varying the dimensions of the micropillars ($h$, $d$, and $g$) it is possible to obtain the same dynamics of liquid spreading on micropillar arrays. References [4, 6, 8] summarise the concept.
3 Finite-volume effects

Lennon Ó Náraigh was the main contributor to this section.

In the Washburn models presented in Section 2 it is assumed that the liquid feeding the porous substrate effectively comes from an infinite reservoir. In this section we present a simple model wherein this assumption is lifted, such that the liquid drop drains into the porous substrate. The physics is essentially the same as that in Section 2.

The approach is to customize and simplify the model of Starov [10]. The model setup is as follows:

- A drop of finite volume $V(t)$ spreads over the substrate, modelled here as a porous medium.
- The drop permeates into the medium and spreads through porous medium under capillary pressures.
- Two variables are required in the model, denoting the extent of the vanishing droplet and the extent of the fluid spread in the porous medium – respectively $L(t)$ and $\ell(t)$.

Hence, the volume loss by the droplet is expressed as

$$V = V_0 - \pi m \Delta \ell^2,$$

where $m$ is the porosity and $\Delta$ is the layer thickness – these are known a priori and depend explicitly on microstructure geometry. The model setup is shown in Figure 10. In contrast to the prior work by Starov, we simplify the mathematics considerably by assuming that the droplet behaves quasi-statically, such that at each instant in time the surface tension acts to give droplet its equilibrium shape. Hence, the droplet is described at each instance by its equilibrium contact angle $\theta_C$.

A simple geometric argument (slender droplets, Figure 11) then gives The above
The equation is then combined with Equation (7) to produce
\[
\frac{dL}{dt} = \frac{1}{3} \left( \frac{4}{\pi V^2 \theta_C} \right) \frac{1}{3} \frac{dV}{dt} \left( \frac{4}{\pi (V_0 - \pi m \Delta l^2)^2 \theta_C} \right)^{1/3} \frac{d\ell}{dt}. 
\]
\[ (8) \]

We further model the substrate morphology as a porous medium of thickness $\Delta$ and porosity $m$, these values are known \textit{a priori} and are set by microstructure geometry. We apply Darcy’s law for the pore pressure $p$ and the lateral creeping velocity $u$ to give
\[
\frac{1}{r} \partial_r \left( r \partial_r p \right) = 0, \quad u = -\frac{K_p}{\mu} \frac{\partial p}{\partial r}, 
\]
where $K_p$ is the permeability. The partial differential equation (9) is solved in an annular region $L < r < \ell$. The boundary conditions at the annulus boundaries are prescribed by physical intuition:

- \textbf{At} $r = \ell$ the capillary pressure in the microstructure drives creeping flow:
  \[ p(r = \ell) = -p_c = -2\gamma/a_s, \]

- \textbf{At} $r = L$ the droplet pressure drives liquid from droplet into the microstructure:
  \[ p(r = L) = p_d. \]

To simplify the model equation, an assumption is made that $p_d \ll p_c$. This can be justified \textit{a posteriori}. The annular domain of Equation (9) and the boundary conditions are shown schematically in Figure 12.
The model is now assembled in final form as follows. Darcy’s law is applied at the front \( r = \ell \)

\[
  u(r = \ell) = -\frac{K_p \partial p}{\mu \partial r}
  \bigg|_{r=\ell} = \frac{d\ell}{dt}.
\]

Hence,

\[
  \frac{d\ell}{dt} = \frac{K_p p_c/\mu}{\ell \ln(l/L)}.
\]

We remark here that the permeability \( K_p \) can be prescribed using the Kozeny–Karman model [11]

\[
  K_p \propto \frac{m^2 a_*^2}{(1 - m)^2}.
\]

As such, the model now reduces down to two coupled ODEs:

\[
  \begin{align}
  \frac{dL}{dt} &= -\frac{2\pi^{2/3}}{3} m \Delta \ell \left[ \frac{4}{\pi (V_0 - \pi m \Delta \ell_0^2) \theta C} \right]^{1/3} \frac{d\ell}{dt}, \\
  \frac{d\ell}{dt} &= \frac{K_p p_c/\mu}{\ell \ln(l/L)}.
  \end{align}
\]

A sample numerical solution of these equations is shown in Figure 13.

Even without such quantitative knowledge we can obtain explicit information regarding optimizing the spreading, as the ODEs are governed by the single timescale

\[
  \tau = \frac{\ell_*^2 \ln(\ell_*/L_*)}{K_p p_c/\mu},
\]

where \( \ell_* \) is the maximum extent of the creeping flow when \( V(t) = 0 \), i.e., \( V_0 = \pi m \Delta \ell_*^2 \). Also, \( L_* \) is the scale of the base of the initial drop. With the Kozeny–Karman model for \( K_p \), we see that the timescale for the liquid seepage process is a function of key variables:

\[
  \tau = \tau(m, \Delta, \mu, V_0, L_*, a_*, \sigma).
\]

The new knowledge of these dependencies can be used to make the liquid seepage process go as fast as desired.
Figure 13: Sample numerical solution of the ODEs (10), with parameters given in Table 1. The profile of the droplet/front is shown at times \( t = 1, 2, 3, 4, 5, 6 \) s.

\[
L^* = \frac{L(t=0)}{10^{-2} m} \\
V_0 = \frac{(2/3)\pi (L^*/2)^3}{10^{-1} N/m} \\
\theta_C = \frac{\pi}{4} \\
\gamma = \frac{\Delta}{10} \\
a_\ast = \frac{2\gamma}{a_\ast} \\
p_c = \frac{m^2 a^2 / (1 - m)^2}{10^{-3} Pa \cdot s} \\
\mu = \frac{m a^2 / (1 - m)^2}{10^{-3} Pa \cdot s}
\]

Table 1: Parameters for the numerical solution of the ODEs in Figure 13.
Figure 14: Sketch of droplet spreading over a topological substrate. The substrate profile \( z = \eta(x) \) is smooth. The droplet has contact points at \( x = a(t) \) and \( x = b(t) \) with free-surface profile \( H(x,t) \). The fluid has density \( \rho \), surface tension \( \sigma \), and viscosity \( \mu \).

4 Lubrication Theory

James Herterich was the main contributor to this section.

We analyse droplet spreading over a rigid surface. In particular, we apply the 2-dimensional model of Savva & Kalliadasis [12] for droplet spreading over a smooth surface. This can be thought of as giving a more theoretical understanding of ‘effective’ contact angles for rough surfaces than the one given in Section 2 (e.g., Equation (2)). The theory is based on prior work by Savva & Kalliadasis [12].

The model setup is shown in Figure 14. The substrate profile \( z = \eta(x) \) about zero has contact points at \( x = a(t) \) and \( x = b(t) \), and the droplet has shape \( H(x,t) \). The theoretical approach is to analyse the problem on different scales to determine the overall shape and contact-line dynamics. The leading-order observer free-surface profile develops quasi-statically as \( H_0(x,a(t),b(t)) \). We do not consider higher-order terms as the leading-order profile and contact points are sufficient to analyse droplet spreading. We input different substrate profiles to determine the extent of spreading.

4.1 Model

A small reduced Reynolds number simplifies the governing flow equations. We assume a small contact angle and a long-wave approximation (the slope of the droplet is small everywhere). A slip length \( \lambda \), defined as the distance from the contact point over which fluid–structure interaction is important, is used to remove the contact line singularity. This allows the droplet to spread on the substrate. Combining these assumptions, the resulting Stokes equations can be reduced to a single film-thickness equation for the droplet profile \( H(x,t) \),

\[
\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} \left( H^2 (H + \lambda) \frac{\partial^3 (H + \eta)}{\partial x^3} \right) = 0. \tag{11}
\]
The droplet makes an angle with the substrate at the contact points, the microscopic static contact angle \( \alpha_s \) (Figure 15). The value of \( \alpha_s \) is determined by the chemistry of the liquid–air–substrate interaction. However, an observer sees an effective equilibrium contact angle \( \theta \), combining the microscopic contact angle and the gradient of the substrate (defined by an angle \( \tan \gamma = \partial_x \eta \)), such that

\[
\partial_x H = -\tan \theta_a, \quad \partial_x H = \tan \theta_b,
\]

at contact points \( a \) and \( b \), respectively. However, since

\[
\partial_x H = \tan(\alpha_s + \gamma),
\]

we can determine \( \theta_a \) and \( \theta_b \) using the microscopic contact angle and substrate gradient

\[
\tan \theta_a = \frac{1 + \tan^2 \alpha_s \eta'_a^2}{1 + \tan^2 \alpha_s \eta'_a^2}, \quad (15)
\]
\[
\tan \theta_b = \frac{1 + \tan^2 \alpha_s \eta'_b^2}{1 - \tan^2 \alpha_s \eta'_b^2}, \quad (16)
\]

where \( \eta'_a = \partial_x \eta(x = a) \) and \( \eta'_b = \partial_x \eta(x = b) \).

Finally, the amount of fluid is conserved, taken (for simplicity) to be 2,

\[
\int_b^a H \, dx = 2. \quad (17)
\]

An asymptotic expansion of the thin-film equation (11) determines \( H_0(y, a(t), b(t)) \), using contact points \( a \) and \( b \) and mass conservation (17),

\[
H_0 = \frac{3}{2} \left( \frac{2}{a - b} + \bar{\eta} - \frac{1}{2}(\eta_a + \eta_b) \right) (1 - y^2) \\
+ \frac{1}{2} (\eta_a(1 + y) + \eta_b(1 - y)) - \eta \left( \frac{1}{2}(a - b)y + \frac{1}{2}(a + b) \right), \quad (18)
\]
where \( y \) is a mapping of \( b < x < a \) such that \(-1 < y < 1\),

\[
x = \frac{1}{2}(a - b)y + \frac{1}{2}(a + b),
\]

and

\[
\bar{\eta} = \frac{1}{a - b} \int_b^a \eta(\xi) \, d\xi,
\]

is the average substrate height under the droplet.

The leading-order profile (18) is determined by the contact-line dynamics. The fluid–air–structure interaction is important here, dominated by slip. Hence, this is approached by the microscopic dynamics. Using matched-asymptotic expansions, the contact-point dynamics are determined by a set of ODEs

\[
\dot{a} = \frac{\delta_a I_b + \delta_b \phi_a I_0}{I_a I_b - \phi_a \phi_b I_0^2},
\]

\[
\dot{b} = -\frac{\delta_b I_a + \delta_a \phi_b I_0}{I_a I_b - \phi_a \phi_b I_0^2},
\]

where \( \phi_a \) and \( \phi_b \) are the apparent contact angles set by \( \partial_y H_0 \) at \( y = \pm 1 \),

\[
\phi_a = \frac{2}{a - b} \left( \frac{6}{a - b} + 3\bar{\eta} - (2\eta_a + \eta_b) + \frac{1}{2} \eta_a'(a - b) \right),
\]

\[
\phi_b = \frac{2}{a - b} \left( \frac{6}{a - b} + 3\bar{\eta} - (\eta_a + 2\eta_b) - \frac{1}{2} \eta_b'(a - b) \right),
\]

and interpreted as mesoscopic contact angles. The other expressions are given by

\[
\delta_a = \frac{1}{3} (\phi_a^3 - \theta_a^3),
\]

\[
\delta_b = \frac{1}{3} (\phi_b^3 - \theta_b^3),
\]

\[
I_a = \log \left( \frac{\theta_a a - b}{\lambda} \right) + \int_{-1}^{1} \frac{1}{1 - y} \left( \frac{\phi_a^3 (a - b)^3 (1 - y^2)^4}{128 H_0^3 (1 - y)} - 1 \right) \, dy,
\]

\[
I_b = \log \left( \frac{\theta_b a - b}{\lambda} \right) + \int_{-1}^{1} \frac{1}{1 + y} \left( \frac{\phi_b^3 (a - b)^3 (1 - y^2)^4}{128 H_0^3 (1 + y)} - 1 \right) \, dy,
\]

\[
I_0 = \frac{\phi_a \phi_b (a - b)^3}{128} \int_{-1}^{1} \frac{(1 - y^2)^3}{H_0^3} \, dy.
\]

Here, \( \delta_a \) and \( \delta_b \) give the contact-line dynamics \( \dot{a} \) (21) and \( \dot{b} \) (22) a form that resembles a generalization of Tanner’s law for spreading on a non-flat substrate. We refer the reader to Saxton et al. [13] for another generalization when an evaporating droplet spreads on a flat surface. Although there are clear differences, this is not dissimilar as evaporation is non-uniform across a droplet, much like the gradient of our substrate.

Now we need only specify the substrate profile \( \eta(x) \) to determine the contact-line dynamics (21, 22), and ultimately the droplet shape \( H_0 \) (18).
Figure 16: Spreading of a droplet from \( t = 0 \) to \( t \to \infty \) over a flat substrate \( \eta(x) \equiv 0 \) (dashed). An equilibrium shape is reached with \( \theta_{a,b} = \pi/4 \).

### 4.2 Examples

We briefly consider three examples of substrate topography \( \eta(x) \) from flat and undulating profiles, and discuss the resulting droplet spreading. In all cases, the droplet is initially in contact at \( a = 1, b = -1 \).

The flat topography takes \( \eta(x) \equiv 0 \). Hence all gradients are \( \partial_x \eta = 0 \) and the equilibrium angles (15,16) are \( \theta_{a,b} = \pi/4 \). This means that spreading eventually stops, in this case approximately doubling in extent (Figure 16). The droplet does not completely spread across the substrate. The outer solution \( H_0(x,a(t),b(t)) \) spreads symmetrically, as expected.

We consider an undulating topography \( \eta(x) = 0.1 \cos 2x \), with the centre of the droplet placed above a peak of the topography. The droplet spreads symmetrically, as expected (Figure 17). However, equilibrium is again reached before spreading can cover the substrate completely. The extent of spreading has increased to approximately 2.25 times the initial radius. The droplet shape \( H_0 \) has two peaks as equilibrium is reached above this topography.

Finally, we consider an undulating topography \( \eta(x) = 0.1 \sin 2x \), so that the centre of the droplet is out of phase with the topography (compared to the previous example). The spreading is non-symmetric (Figure 18). The droplet translates in each direction, and does not reach equilibrium. Multiple equilibria are explored in a hysteresis-like effect. The greatest extent of the droplet occurs at \( t \approx 28.25 \). Again, it does not spread completely across the substrate.

### 4.3 Discussion

A droplet spreads across an impermeable surface. We investigate how changes in substrate topography affect the extent of spreading. The basic equation structure
Figure 17: Spreading of a droplet centred at 0 from $t = 0$ to $t \to \infty$ over an undulating substrate $\eta(x) = 0.1 \cos 2x$ (dashed). An equilibrium shape is reached with two peaks.

Figure 18: Spreading of a droplet centred at 0 over an undulating substrate $\eta(x) = 0.1 \sin 2x$ (dashed). A non-symmetric droplet profile does not settle down to an equilibrium, with the greatest extent occurring at $t \approx 28.25$. 
for the observer droplet shape (18),
\[ H_0 = f_H(a, b, \eta_a, \eta_b, \bar{\eta}), \tag{30} \]
takes macroscopic quantities, while the contact-line dynamics (21,22)
\[ \{\dot{a}, \dot{b}\} = f_{\{a,b\}}(a, b, \eta_a, \eta_b, \eta'_a, \eta'_b, \bar{\eta}, \theta_a, \theta_b, \alpha_s, \lambda), \tag{31} \]
requires also the microscopic quantities that contain the chemistry of the fluid–air–substrate interaction. Both scales are important in this problem.

We analyse droplet spreading over three different topographical situations: flat (symmetric spreading), in-phase undulating (symmetric spreading), and out-of-phase undulating (asymmetric spreading). The extent of droplet spreading changes in each situation (figures 16, 17, and 18), but not significantly. A varying impermeable topography does not result in the droplet covering the whole substrate. Eventually, the microscopic chemistry of the fluid–air–structure interaction dominates to reach an equilibrium (or reverse spreading on one end in order to reach equilibrium).

Sharper gradients in topography would not increase spreading significantly. While travelling down a sharp gradient will speed up the dynamics, there is a balance with the subsequent travelling up a sharp gradient. Global changes in gradation (sharp in some areas, transitioning to smoother in others) may work, but it loses the generality of the initial placement of the droplet.

The model, however, can be used in conjunction with spreading on top of a permeable topography that wicks the fluid through a sub-domain.

5 Conclusions

Based on the literature review in Section 2, we are able to answer some of the questions posed by Analog Devices in the Study Group:

- **Is there an optimum distribution for a large scale wicking structure?**

- **What should be the pillars height, diameter and structure (cylindrical or hexagonal etc.)?**

  Yes, there exists an optimal parameter design space. The practical implementation of realising such optimized microstructures is restricted only by fabrication methods.

- **Combine the pillars with a connecting microstructure to improve wicking?**

  We do not think that is necessary.

- **What form is most efficient for the microstructure for the movement of liquid?**

- **Is there an optimum porosity and pore size/shape? Circular, square etc?**

  Literature suggests that circular pillar shape should be the best shape, due to its smooth surface profile. But shape of the pillars does not seem to be significant and can be accounted through roughness value.
• How much effect does the thickness of the microstructure affect the wicking?

If what is meant is periodicity, then this is just one of the parameters in determining the wicking dynamics and has a more or less same influence such as the height and diameter of pillars.

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References


