Title	Assessment of structural nonlinearities employing extremes of dynamic responses
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Publication date	2016-03-09
Publication information	Journal of Vibration and Control, 24 (1): 137-152
Publisher	Sage
Item record/more information	http://hdl.handle.net/10197/10446
Publisher's statement	Pakrashi, V., Fitzgerald, P, O'Leary, M., Assessment of structural nonlinearities employing extremes of dynamic responses, Journal of Vibration and Control, 24(1) pp. 137-152. Copyright © 2016 the Authors. Reprinted by permission of SAGE Publications.
Publisher's version (DOI)	10.1177/1077546316635935

ASSESSMENT OF STRUCTURAL NONLINEARITIES EMPLOYING EXTREMES OF DYNAMIC RESPONSES

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Abstract

A range of methodologies exist for estimating nonlinear responses of structural systems using numerical simulations. However, efforts in relation to experimental methods in this regard still warrant further investigation. This paper presents an approach for assessing structural nonlinearities using the extremes of dynamic responses of the structural system under consideration. The approach allows revisiting and parameter tuning of theoretical models of structures based on experimental studies. A single degree of freedom system was excited in this study using broadband input excitations and the output dynamic responses were measured using different devices. The type and extent of experimentation required for implementation of the presented technique was investigated along with the effects of the estimates of the measured variables and the effects related to different measurement devices.

Keywords: Extreme value, Gaussian white noise, structural dynamics, model testing, nonlinearity

1 Introduction

Assessment of extreme values of mechanical responses of structures is critical to their design and robustness against risk over their entire lifetime (Castillo, 2012). Environmental (Ditlevsen, 2002; O'Connor and O'Brien, 2005) or anthropogenic (Quilligan et al., 2012; Agarwal and Manuel, 2009) loadings are expressed probabilistically and extreme loadings agreed, which in combination with uncertainties in the structural system (Chen and Li, 2007) lead to estimates of extreme mechanical responses of structures. In this regard, assessment of extreme values of dynamic responses of structures has been a popular focus of study (Quilligan et al., 2012; Dueñas-Osorio and Basu, 2008; Radhika and Manohar, 2010) till date. Extreme values of input loads or output responses are modelled statistically (Ditlevsen, 2002; Sørensen and Toft, 2010; Andersen et al., 2012). The models may be formed by assuming a probabilistic distribution (Khan et al., 2006), through the use of some engineering judgement (O'Connor and O'Brien, 2005), or by fitting models to observed data (Naess and Gaidai, 2009; Dong et al., 2012). For a wide range of engineering applications, the assessment of extreme values relate to approximating observed or modelled responses of structures at their tails of distributions reasonably to asymptotic distributions like the Gumbel, Weibull or Frechet distributions which are essentially a special case of Generalised Extreme Value (GEV) distribution (Castillo, 2012). Consequently, a GEV fit can be appropriate for representing a wide range of situations where the tail of observed or simulated data significantly deviates from normality.

On the other hand, scaled models have been popular for a long time in structural engineering (Addis, 2013). Equivalence conditions of models with the actual system (Bhattacharya et al., 2011; Bilello et al., 2004), uncertainties in developed models and testing protocols for models (Adhikari et al., 2009; Michaelides and Fassois, 2013) have been investigated for various systems. It is observed that an appropriately chosen model testing can be extremely useful for a range of applications. In this regard, a structure exposed to broadband excitation is interesting, particularly when the spectrum of the excitation is approximately known or is agreed upon (Murtagh et al., 2008; Nichols et al., 2003; Wu and Law, 2011; Jaksic et al., 2014). The extreme responses of a structure may then be considered by assessing a long time history or by agglomerating a large number of shorter time histories of the dynamic responses under forced vibration. One of the important set ups for dynamical systems assessment in this regard would be excitation of the structure with a Gaussian white noise. Responses to excitations of this form have been previously used for structural health monitoring purposes (Cacciola et al., 2003).

While significant investigation has been carried out in the theoretical domain in terms of large scale simulations and efficient computational modelling, there is a gap in experimental approach in utilising extreme dynamic responses of structures through reasonably simple model testing in order to assess nonlinearities present in the structure. This paper explores the use of extreme dynamic responses to investigate and estimate nonlinearities in structural systems. A small scale physical model is employed in this study, which represents the global dynamics of the structural system under consideration (Moon, 2008). Utilising extreme dynamic responses does not necessarily require precise estimation or prediction of the extreme values of the dynamics of the system. The observed or simulated extremes fit well to the closest extreme value distribution, which can be helpful in assessing the nonlinearities in the system dynamics based on the deviation from linearity.

The results in this paper demonstrate how such estimated extremes of dynamic responses of a structure may be useful in assessing and modelling a nonlinear structural system, especially when it is excited by stochastic, broadband excitations. The type and the length of input excitation relating to a reduction of the number of tests, length of time series of the dynamic output of the system and the nature of the measurement of dynamic output were investigated in this connection. Tuning of theoretical models of dynamics of such structures using the proposed method and by taking experimental results into consideration is proposed in this paper too. Effects of variability in modelling the tails of dynamic responses are investigated as well.

2 Simulations on Linear and Weakly Nonlinear Systems with Cubic Stiffness

2.1 Probability Plots for Linear Dynamical System Responses

A Single Degree Of Freedom (SDOF) system is considered as

$$m\ddot{x} + c\dot{x} + kx = f(t) \tag{1}$$

or

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = a(t) \tag{2}$$

where *m* is the mass of the system, *c* is the linear viscous damping in the system, *k* is the stiffness of the system, *f(t)* is the time (*t*) dependent excitation force on the system, *x* is the displacement of the dynamic system with respect to its static deformed position, an overdot is a differentiation with respect to time, and $a(t)$ is the time dependent input acceleration. . The excitation $a(t)$ is considered as a Gaussian white noise, with mean absolute amplitude and absolute standard deviation both equal to unity. The force is kept without the choice of a unit so that the interpretation of the results is scalable.

Matlab computer programing software is used for numerical simulation (MathWorks, 2014). The force input is generated using 100000 data points and the system response is calculated using function ODE45 which is based on Runge Kutta 45. Matlab function 'linspace' is used to generate linearly spaced vector with 100000 data over the period of 3000sec. Hence, the displacement (x), velocity ($\dot{x}$) and acceleration ($\ddot{x}$) have been obtained with time series lengths of 100,000 for displacements and velocities. Both positive and negative tails have the same interpretation for vibration extremes since the magnitude is often more important than the sign of the output dynamic responses. The absolute values thus guide the distribution of response magnitudes at tails and the magnitudes of the responses have been considered in this paper for fitting various distributions, with a focus on the extreme responses. The fit of the distribution parameters is obtained following the method of Kotz and Nadarajah (2000) . The schematic of the simulated system is shown in Figure 1.

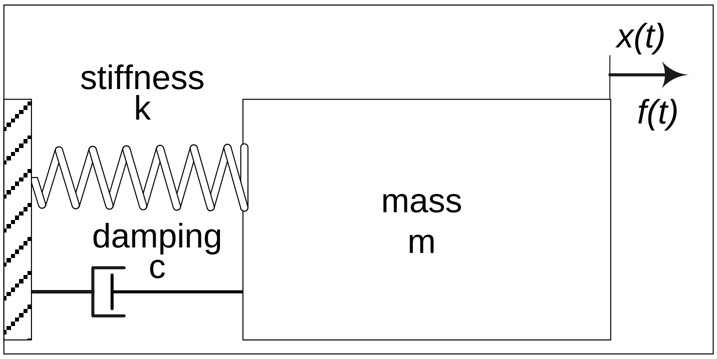

Figure 1. Schematic of SDOF symulated system subject to external force.

Figure 2a presents the probability plot of the velocity response of the system while Figure 2b presents the probability plot of the acceleration response of the system.

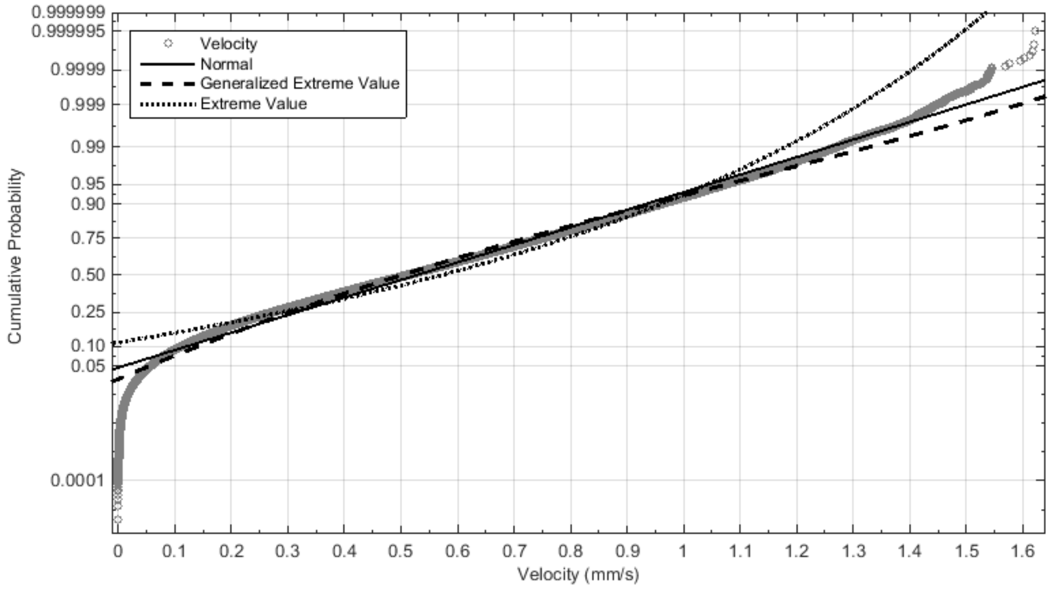

Figure 2a. Probability plot for velocity responses of a SDOF simulated system excitated by Gaussian white noise.

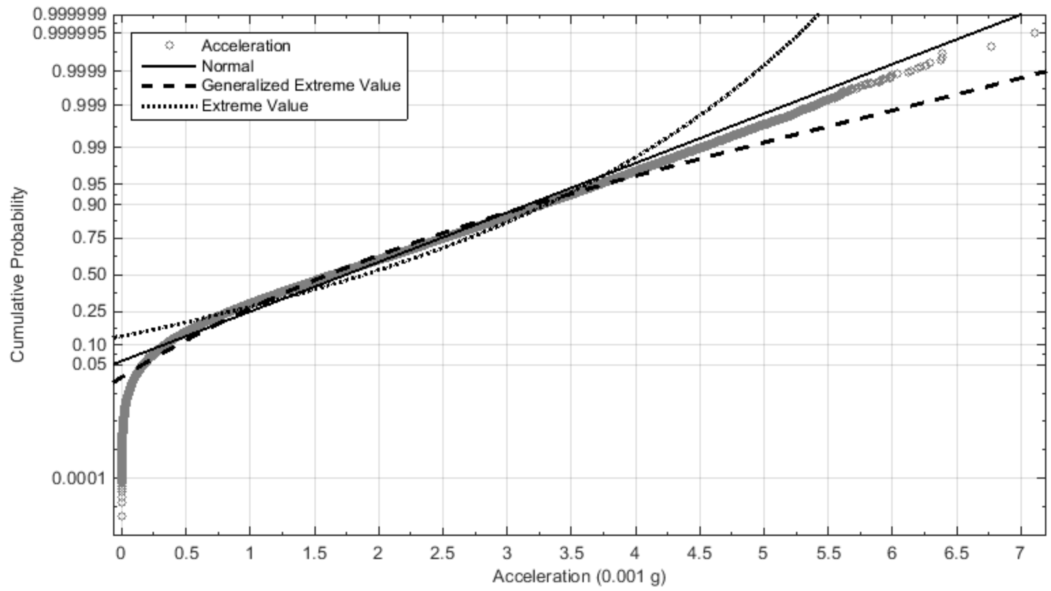

Figure 2b. Probability plot for acceleration responses of a SDOF symulated system excitated by Gaussian white noise.

The Normal distribution fits the extreme values and the rest of the data very well. This is due to the fact that the excitation is Gaussian and the structure acts as a linear filter, thereby preserving the output Gaussianity for a linear system. The deviation from exact matching near zero value is due to conversion of negative response amplitude to positive numbers. The Normal fit is compared with an Extreme Value (Weibull) and a Generalised Extreme Value (GEV) distribution fit and it is trivially found that these fits are not appropriate for the linear model.

The simulations consider acceleration and velocity since these are often obtained as primary measured quantities for dynamical systems using accelerometers or Laser Doppler Vibrometers respectively. Derived quantities, on the other hand tend to contain errors related to numerical procedures (Stiros, 2008), which would be related to numerical integration in the current discussion when deriving displacements.

2.2 Probability Plots for Nonlinear Dynamical System Responses

A wide number of structural systems are nonlinear. Cubic nonlinearities present in the stiffness term are quite common (Jaksic and Pakrashi, 2013) in this regard. A SDOF system with cubic nonlinearity in the stiffness term may be represented as

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x + \alpha x^3 = a(t) \qquad (3)$$

where the degree of nonlinearity is dictated by the value of the α parameter. For weak cubic nonlinearities (α=0.01), the probability plot for acceleration response is given in Figure 3.

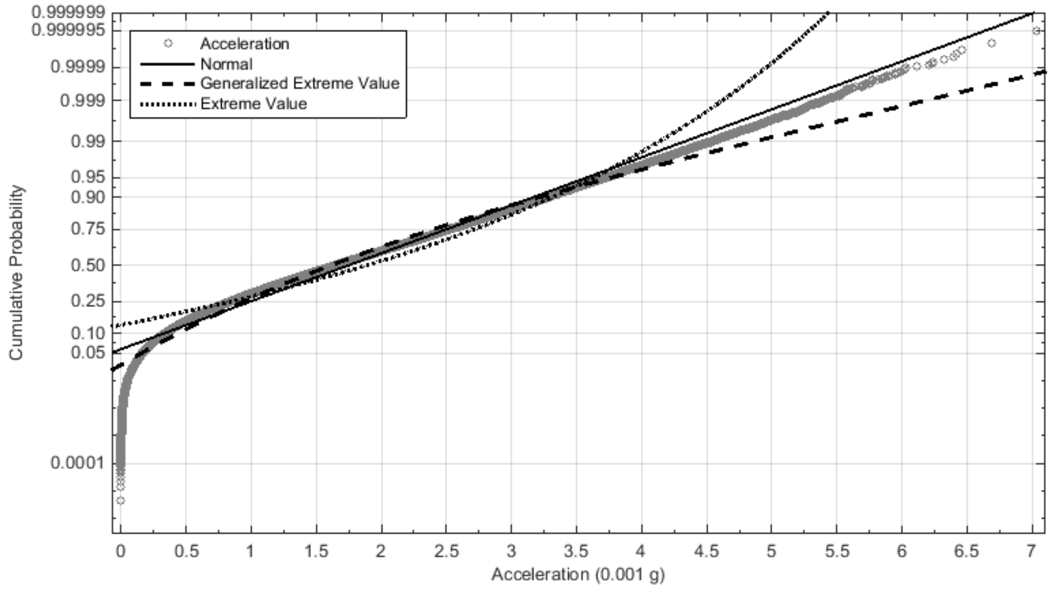

Figure 3. Probability plot for acceleration responses of a weakly nonlinear symulated system with cubic stiffness (α=0.01) excitated by Gaussian white noise.

It is observed that for a weakly nonlinear system with cubic nonlinearity still preserves the near linear filtering characteristics of the structure and the response is still close to Gaussian. Output dynamic responses due to an input would differ significantly from Gaussianity if the filter is nonlinear, which relates to the nonlinearity present in the system in this case. Figure 4 presents the probability plot for acceleration where the nonlinearity in the system is significantly higher for both softening and hardening spring conditions (α=1.0 and -1.0 respectively).

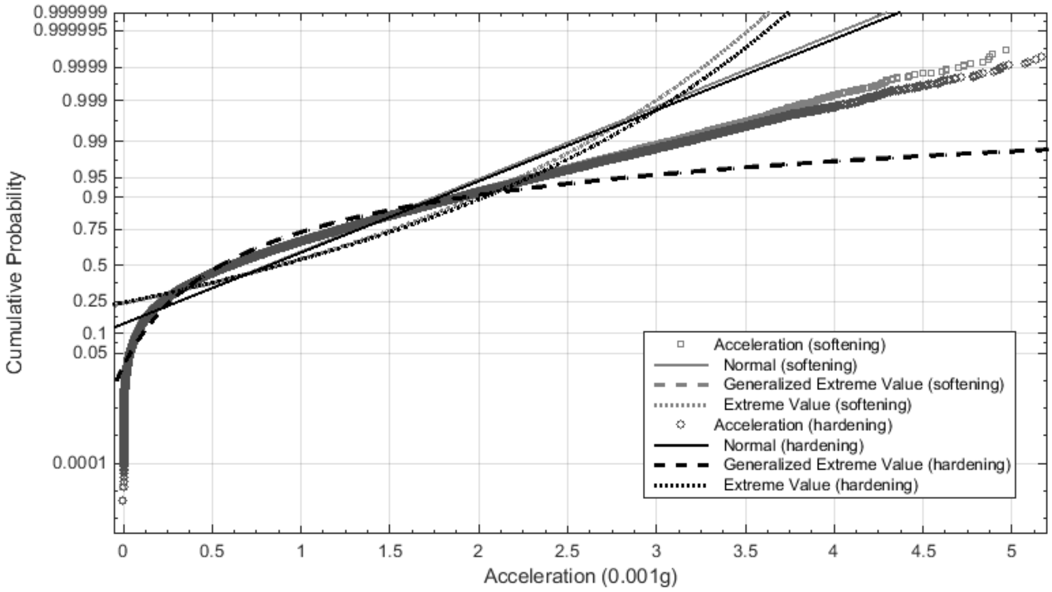

Figure 4. Probability plot for acceleration responses of a strongly nonlinear symulated system with cubic stiffness, softening (α=1.0) and hardening (α=-1.0) spring conditions excitated by Gaussian white noise.

The extreme value responses deviate significantly from a Normal distribution and it is important to assess how the extreme tails represent a closer fit in terms of extreme value distributions. The softening and hardening extremes match in terms of their magnitudes indicating the lack of skewness in the distribution of the response magnitudes. A hardening spring situation is thus sufficient to present the concept proposed and the subsequent plots only consider a hardening spring condition. Additionally, a hardening spring model is often more commonplace in structural engineering than a softening spring model.

A comparison with other distributions presented in Figure 4 indicates that a Generalised Extreme Value (GEV) distribution can be more appropriate than a standard Extreme Value (EV) distribution for a wide range of cases. The choice of fit can be carried out even with a preliminary idea regarding the type of dynamics the structure might exhibit. Where information is not available from the structure, an extreme value fit would fundamentally depend on the fitting of the tail of the histogram of dynamic responses.

The frequency contents of time domain responses of nonlinear systems, as opposed to linear systems (Jaksic and Pakrashi, 2013) can provide a visual basis for the current approach considering the contribution of extreme responses for assessing nonlinearity. However, such a frequency domain response will not be illustrative of situations where a sudden stiffness change takes place (Pakrashi and Ghosh, 2009; Jaksic et al., 2012).

2.3 Stability of Histogram of Dynamic Responses and the Length of Time Series

The histogram of the output dynamic response time series measured has to be stable to be used for estimation of extreme values. Independent of the linearity or nonlinearity of the system, the input excitation is considered as Gaussian in this case and consequently the individual tests or pooled test results can be considered under such circumstances. A long, individual test will then be equivalent to a number of short individual tests and vice-versa since the input excitation characteristics remain unchanged. Consequently, an idea regarding the number of points required in the time series of the measured dynamic response to obtain a stable histogram fit is required. An investigation was carried out where the simulate time series of dynamic response of the structure was decimated to the point where the histogram fit significantly deviated from a reasonable value of convergence. The threshold of deviation can be set based on what accuracy is required by the original objective of the application on the actual dynamical system. For a linear system, the distribution fits, in a least square sense, were stable up to time series lengths of around 1000 data points, but not for lengths of the order of 1000 data points and over. Table 1 provides the equations of the fits to illustrate the dependence of the length of the time series on the stability of the output histograms of dynamic responses. The cumulative distribution function fitted to simulations is represented as *F(x)*. It is to be noted here that the results also reflect the effects of the discretized time steps considered for Gaussian white noise. Independent of the interaction between these time-step effects of inputs and outputs, such analyses provide guidelines for the minimum number of points required within a time series to fit the extremes of dynamic response histograms reasonably.

Time Series Length	**Distribution**	**Equation**
1000	GEV	$F(x)=\frac{1}{0.958364}\left(1+\frac{x-1.27039}{0.958364}\right)^{-0.0936559+1} e^{-\left(1+\frac{x-1.27039}{0.958364}\right)}$
1000	Normal	$F(x)=\frac{1}{\sqrt{2\pi(1.10796)^2}} e^{\frac{-(x-1.75064)^2}{2(1.10796)^2}}$
100	GEV	$F(x)=\frac{1}{1.03088}\left(1+\frac{x-1.37942}{1.03088}\right)^{-0.150774+1} e^{-\left(1+\frac{x-1.37942}{1.03088}\right)}$
100	Normal	$F(x)=\frac{1}{\sqrt{2\pi(1.84657)^2}} e^{\frac{-(x-1.84657)^2}{2(1.15142)^2}}$

Table 1. Histogram fits of output acceleration response of a linear system excited by Gaussian white noise using different distributions for varying time series lengths.

Nonlinear simulations (α=1.0) were also carried out for varying time series lengths to assess the length of data required to obtain a relatively stable probabilistic fit of the extreme values. Table 2 presents the summary of the fits. As observed in Table 1, the Normal distribution is characterised by a mean and a standard deviation that are estimated from the time series data, while the GEV is characterised by an additional shape parameter (K) which decides on the type of the extreme value distribution to follow based on its sign. The standardised cumulative distribution function of the GEV may be expressed as

$$F(x) = e^{-\left(1+K\frac{(x-\mu)}{\sigma}\right)^{-1/K}}, \quad 1+K\frac{(x-\mu)}{\sigma} > 0, \; K \neq 0$$
$$F(x) = e^{-e^{-\frac{(x-\mu)}{\sigma}}}, \; K = 0 \qquad (4)$$

where μ is the mean and σ is the standard deviation of the dynamic response. Distributions associated with *K>0* is known as the Fréchet class and includes typical fat tailed distributions like the Pareto, Cauchy, Student-t, and mixture distributions. For *K*=0, the distribution falls under the Gumbel class and includes the normal, exponential, gamma, and lognormal distributions. On the other hand, for *K*<0, the distribution class is Weibull.

	100,000 Data points						
	μ	σ			*K*	μ	σ
Normal	0.85393	0.711704		GEV	0.285906	0.433389	0.46811
	10,000 Data points						
Normal	0.842987	0.697123		GEV	0.269936	0.429274	0.46865
	1,000 Data points						
	μ	σ			*K*	μ	σ
Normal	0.848127	0.709112		GEV	0.280215	0.437889	0.462465
	100 Data points						
	μ	σ			*K*	μ	σ
Normal	0.850529	0.733462		GEV	0.38519	0.414652	0.431999

Table 2. Histogram fits of output acceleration response of a nonlinear system (α=1.0) excited by Gaussian white noise using different distributions for varying time series lengths.

It is observed that reasonably sized time series outputs can provide an adequate description of a histogram fit and time series lengths as small as 1000 data points may be useful.

2.4 Importance of Distribution Tails and Benchmarked Responses

White noise is not necessarily the only input excitation that may be used to assess the response extremes in relation to the type of system considered. Broadband, benchmarked excitations may also be used in this regard. In fact, when a dynamical system is calibrated through differential equations or a physical model, the deviation of distribution tails can be investigated by considering some benchmark responses. As an example, an accelerogram of the El Centro earthquake in California (Rana and Soong, 1998; Ghosh and Basu, 2004) is considered in this paper (see Figure 5a) as an excitation on the system with a cubic nonlinearity (α=1.0) and the probability plot for acceleration response is presented in Figure 5b. The deviation of the system from linearity at the extremes can be demonstrated. The distribution of the input earthquake may not be Gaussian and consequently, the distribution of dynamic responses after being filtered by a linear system will not be Gaussian but would assume a distribution similar to the distribution of the input acceleration. As long as the input acceleration is significantly broadbanded, it can still be used for assessing nonlinearities once a benchmark is set for the linear case using simulations or a well-controlled experiment.

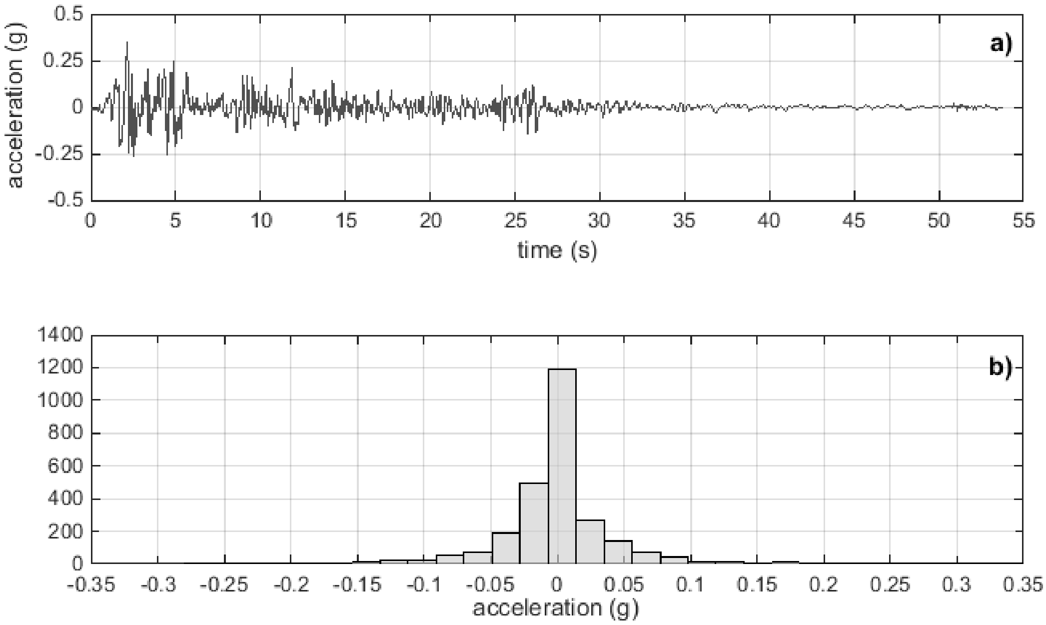

Figure 5a. a) Ground acceleration over the time and b) histogram distribution of ground acceleration caused by EL Centro earthquake.

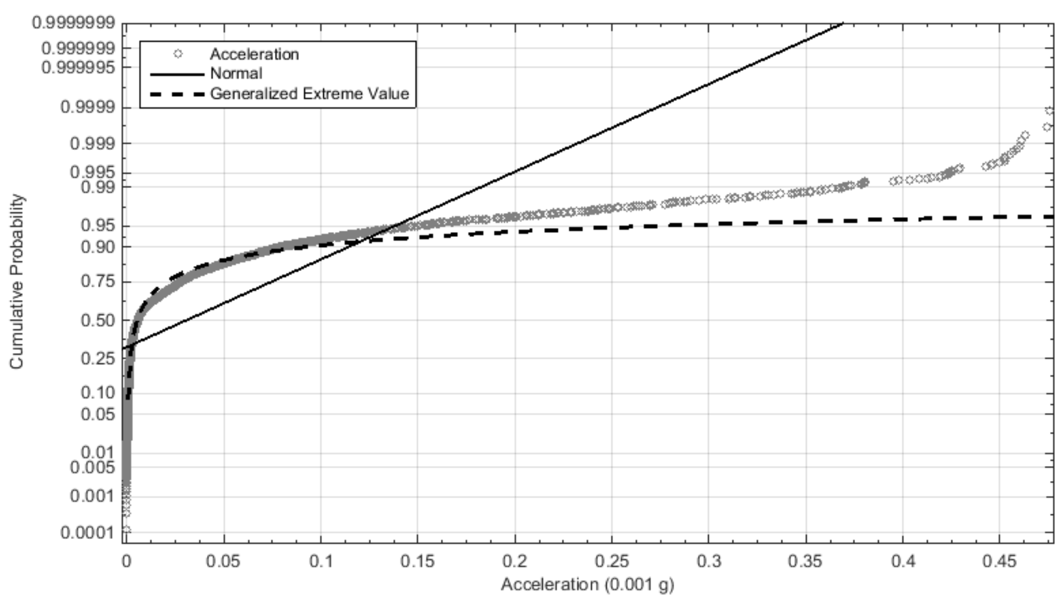

Figure 5b. Probability plot for acceleration responses of a strongly nonlinear system with cubic stiffness (α=1.0) excitated by a benchmark El Centro earthquake acceleration input.

2.5 Summary of Numerical Studies

The numerical studies carried out indicate that the presence of nonlinearity in a structural system is reflected in the extreme value fits of the histograms of dynamic responses of the structure. The direction and amount of deviation of such fit from a linear structure are indicative of the type and extent of nonlinearity present in the system. Additionally, it is also observed that such extreme value fits can be achieved with reasonably manageable numbers of points in the dynamic response time series data when the input excitation to the system is relatively broadband. Under such circumstances, extreme value fit is observed to have a potential to track the presence and extent of nonlinearity in a dynamic system where only the output response is known and where the inputs to the system or the system characteristics are either vague or unknown. This approach lends to the development of equivalent physical models of a more complex dynamic systems under consideration where the equivalence is established from the matching of extreme value fits, without the requirement of details related to the input excitation of the model itself. The physically developed model can then be analysed relatively easily to develop calibrated equations, which would govern the global responses of the original system. When such calibrated equations are related to the physics of the system, the technique can be useful in obtaining simplified equations of more complex dynamics of a system when a specific output is being considered. Additionally, when a linear or a nonlinear benchmark is available, a relatively long time series response of the structure during an event (e.g. earthquake) can be used to assess and quantify the nonlinearities present in the post-event dynamic system. Nonlinearities after an event are often related to damage and thus, for continuous monitoring, the approach can be related to a structural health monitoring implementation. The numerical observations presented in this section lead to the requirement of carrying out an illustrative experiment and also investigating experimentally the effects of sudden incidents occurring within the system along with effects related to measurement devices for providing implementation guidelines for practising engineers.

3 Experimental Setup

To assess how experimental responses may be used to utilise the extreme value responses of a system through model testing, a Single Degree Of Freedom system (SDOF) is physically modelled as 0.1kg mass cart attached to six springs, three on either side connected to fixed supports. Calculated equivalent stiffness of the combined springs at the beginning of the experiment was k = 0.378 N/mm. The input force is represented by the movement of a vibration bench and the response is represented by the movement of the cart. Figure 6 represents the schematic of the experimental setup.

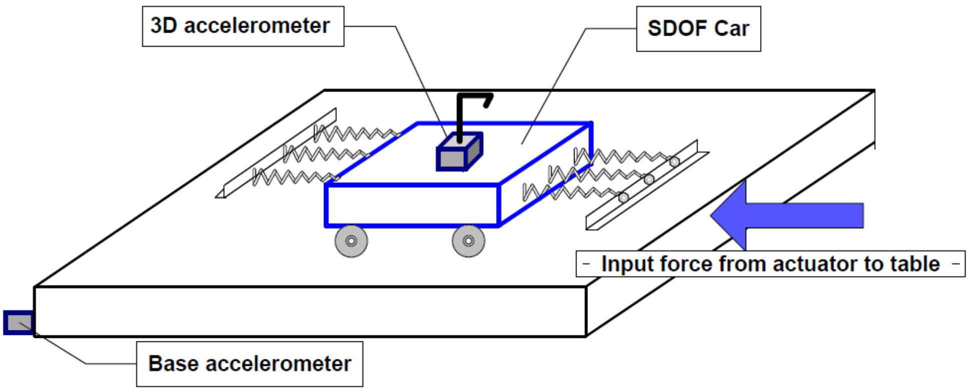

Figure 6. Schematic of SDOF car experimental setup.

Parameters of the physically modelled system for the experiment can be related to parameters in the differential equation used to model an SDOF. For example, the equivalent spring stiffness can be related to the stiffness term in the SDOF differential equation. The structural damping ratio may have to be assumed but for such damping only the transient vibration is significantly affected. A reasonable choice of the damping ratio may be carried out by observing the transients (Lamarque et al., 2000; Prandina et al., 2009). The structural damping is usually considerably below critical damping and the system undergoes forced vibration. Consequently, small disparities between assumed damping for simulation and the actual damping in the system will not affect their forced vibration responses significantly. When assuming the damping of the simulated model, free vibration decay of the physically built model was considered and an estimated value of 2% equivalent viscous damping ratio was used. For system identification related to accurate estimates of damping ratios in from small amplitude broadband excitation, especially for small nonlinearities, the approach of (Rüdinger and Krenk, 2004) can also be useful when long time series are available from experiments.

The input acceleration was measured by an accelerometer placed on the table. An accelerometer placed directly on the cart measured the acceleration output while a Laser Doppler Vibrometer (LDV) measured the velocity of the vibrating SDOF system. Velocity Data from the LDV can be differentiated to get acceleration data which in turn could be compared with the acceleration data from the accelerometer on the cart itself. However, such differentiation can also lead to an amplification of noise within the measurement. Figure 7 presents a photograph of the experimental setup.

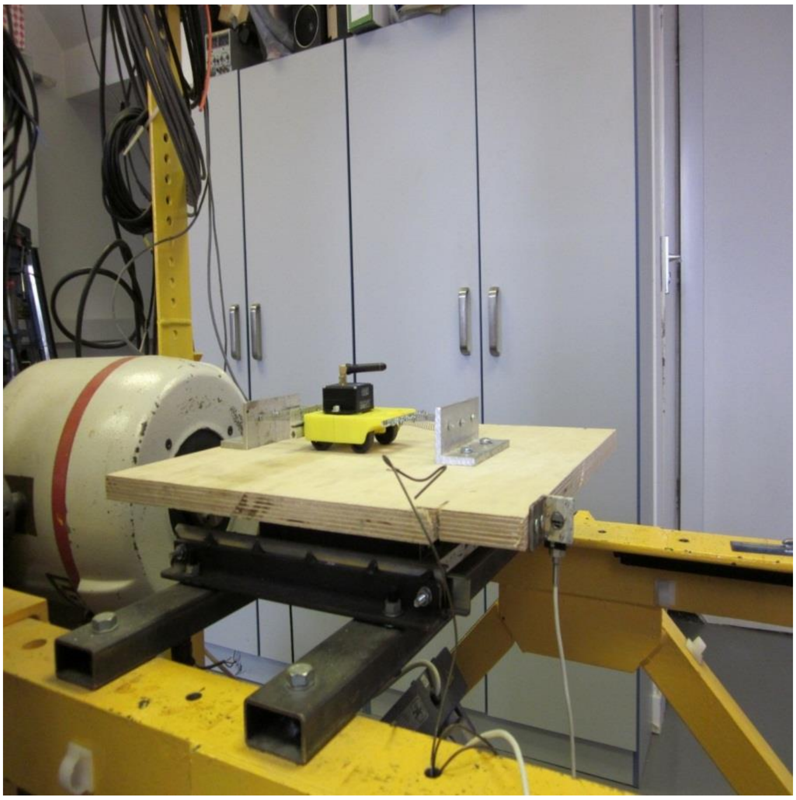

Figure 7. Photograph of SDOF car experimental setup.

The LDV recorded data at a rate of 830Hz and the accelerometers recorded data at a rate of 128Hz. Gaussian white noise excitation was chosen for these experiments to correspond to the simulations. Previous tests on structural monitoring employing extreme value fits of dynamic responses (Worden et al., 2002) have observed the existence of non-Gaussian tails in those experimental responses as an indicator of limitation in damage detection algorithms developed using assumptions of Gaussianity, but have not investigated utilising such non-Gaussian tails. The example of input acceleration and accelerometer measurements with coherence function estimates as the first indication of the nonlinearities present is given in Figure 8.

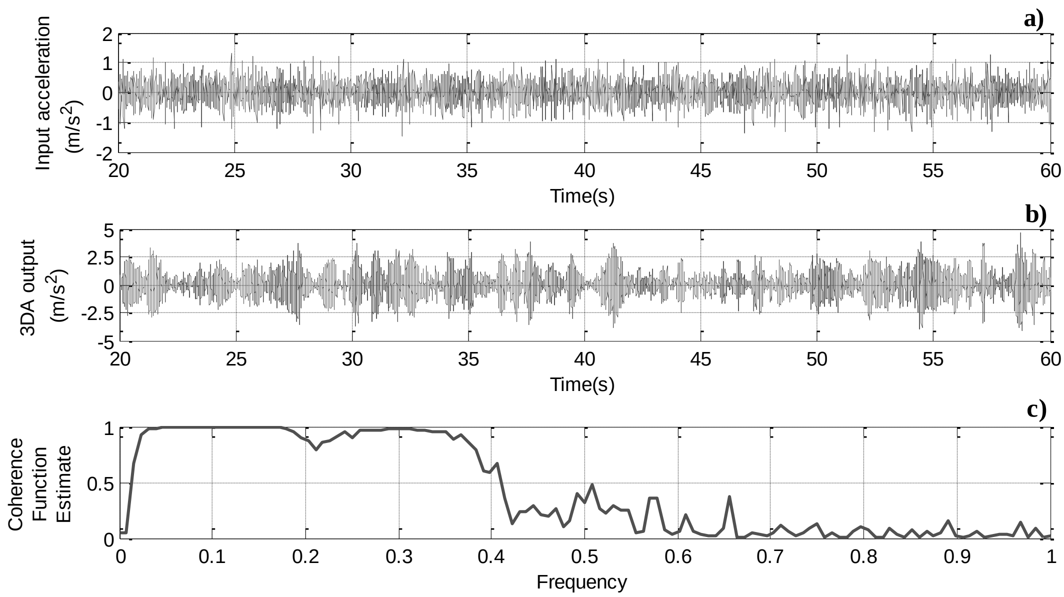

Figure 8. SDOF cart experiment: a) shaker table input acceleration, b) accelerometer output in direction of movement, and c) coherence function estimate.

4 Experimental Results

4.1 System Calibration by Integrating Physical Experimentation and Numerical Simulation

Although the springs are linearly calibrated in tension under static loading and to a certain displacement, the combined system is not guaranteed to exhibit linear model behaviour as obtained from linear or weakly nonlinear simulations. Figure 9a presents the acceleration response in the form of a probability plot for the physically modelled system excited by Gaussian white noise. The deviation from linearity at extreme values is apparent.

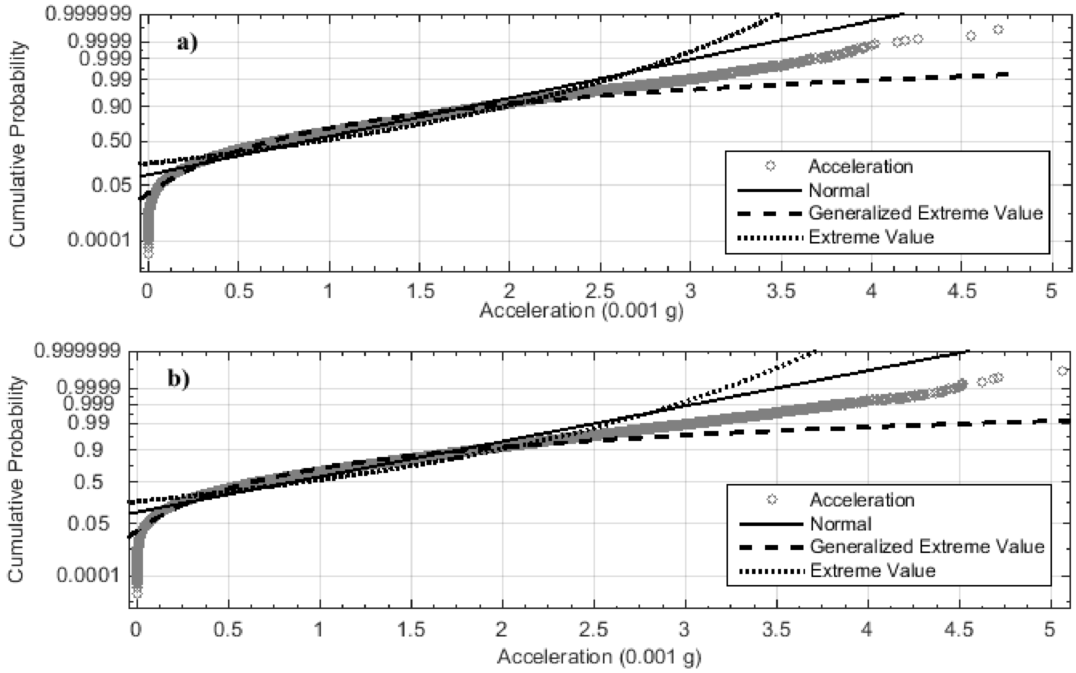

Figure 9. Probability plot for acceleration responses of a) a physically modelled SDOF system excitated by a Gaussian white noise input and b) a symulated system mached with experimentation.

These experiments allow matching of extreme dynamic responses between model simulation and model testing. The initial estimated behaviour of the original system may be linear and then based on model testing, the differential equation of the system may be altered so that the simulated extremes match the experimentation as much as possible under the assumption that a GEV fit reasonably represents the extreme values of the observed and simulated dynamic responses. Once the differential equation of the global dynamics is matched with experimentation, simulations may be performed to investigate a wide range of applications of the system, including the effects of uncertainty based on assumed distributions associated with parameters that model the differential equation. Qualitative understanding of the dynamical system under consideration can be helpful in this regard. For example, the qualitative information regarding the possibility of originally assumed linear system actually being represented through a soft spring is not difficult to assess after initial tests for a wide range of structures. In this paper, the model simulation that was observed to fit well with this experimental system through trial and error can be expressed as equation 3 (α=1.0) in this paper. The simulated probability plot matching the experimentation is presented in Figure9 8, which is a very close match of Figure 9b.

It is observed that a Normal distribution is not suitable for these cases to represent the responses at extremes. GEV underestimates the responses and the system should be designed to be below that underestimated response. Consequently, GEV is conservative in terms of attaining safety if exceedance of levels of responses is associated to failure characteristics of the system. These deviations act as a guideline of the extent of nonlinearity present in the system. The probability of exceedance can be determined form the probability plot. By selecting an arbitrary exceedance of 4.1 (in units of 0.001g) the exceedance probability is 99.99% in this case. Such levels of exceedance and the probabilities connected to the levels can be related to the fragility curves for the performances of structures (Quilligan et al. 2012). The tail of Figure 9b is zoomed in Figure 10 for a clearer understanding. The GEV is not necessarily the best possible fit, but is adopted since it has a technical basis for assessing extremes and because it is conservative. Non-parametric fits may provide higher conformity with tails of data, but are too sensitive to the particular time series that is measured. Consequently, such fits can be too specific to the particular simulated or experimental dataset under consideration and the variability of estimated parameters for such non-parametric fits for different sets of dynamic responses would be larger than the effects of nonlinearity in the global dynamics of the system and would tend to mask it. Additionally, when non-parametric fits are represented through higher order polynomials, the probability plot tends to include a number of changes in slope that are dependent on the specific dataset, thus creating a confusion even at the level of visually interpreting the probability plot.

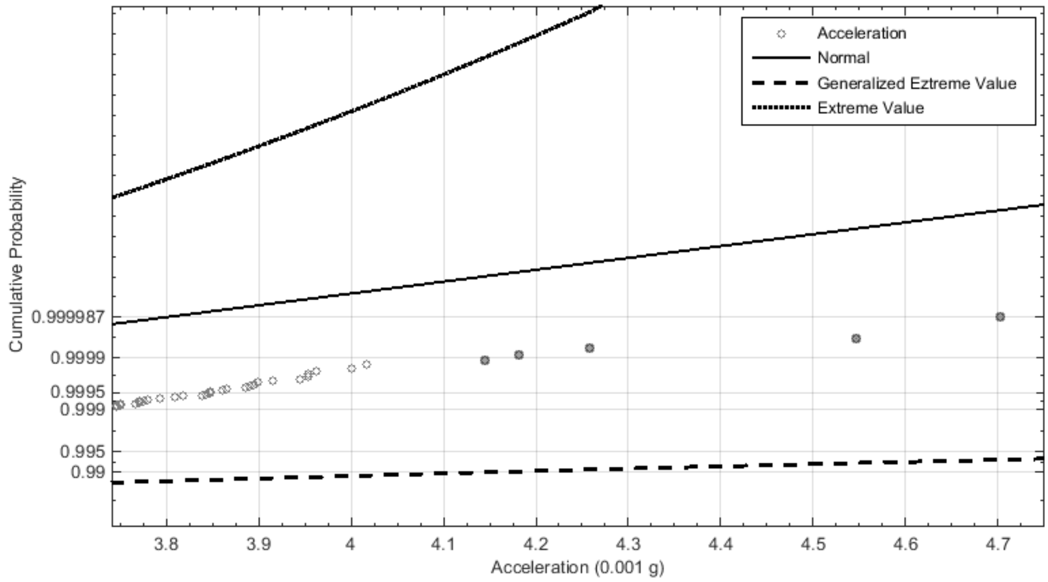

Figure 10. Probability plot for acceleration responses of a matched simulated system with experimentation (zoomed figure 9b tail) with highlighted extreme values for a preselected exceedance threshold.

4.2 Experiments on Sudden Changes in System Stiffness

Springs were carefully sheared instantaneously for experimentally creating sudden change in system stiffness while the physically modelled SDOF system was being excited by external white noise. Changes in system stiffness are related to indicators of structural failures and extreme responses were investigated for the SDOF system with altered equivalent stiffness. The equivalent spring stiffness before the break was 0.378 N/mm. After the first break, the equivalent spring stiffness was 0.303 N/mm and after the second break the equivalent spring stiffness was 0.249 N/mm. The probability plots were separately analysed for data after the first and second break as follows. The first break occurred 13s into the experiment. The extreme values for such situation are highlighted in Figure 11. The distribution of the responses in and around the sudden occurrence of shearing of springs is a non-trivial problem due to the introduction of high frequency components around the instant of this occurrence and is outside the scope of consideration when practically attempting to work on the entire dataset. However, as long as a reasonable representation of extremes using GEV is adopted, such events are still reflected in the probability plot.

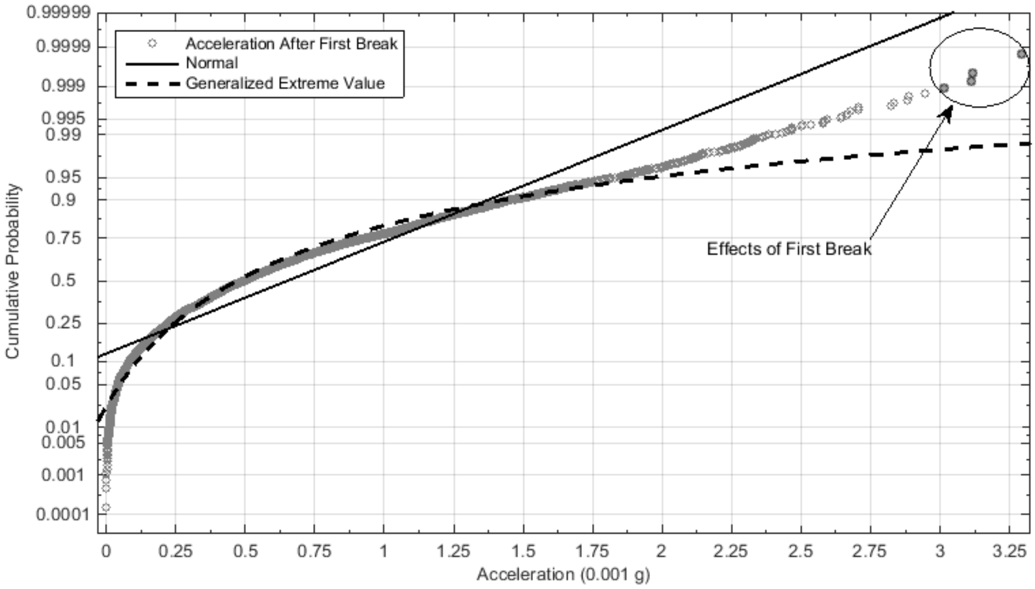

Figure 11. Probability plot for acceleration responses of the experimental SDOF system excited by Gaussian white noise with extremes highlighed for a sudden loss of stiffness due to the sudden shearing of one spring (13sec into the experiment).

Selecting an arbitrary exceedance value of 3 (units of 0.001g) the probability of failure may be interpreted as 0.1%.

A second spring was sheared 38 s into the experiment, thus reducing the equivalent spring stiffness of the system to 0.249 N/mm. The probability plot is displayed in Figure 12. An exceedance of 3 is selected again to compare with Figure 11 and the probability of failure was interpreted as 0.5%, which is greater than 0.1%. To distinguish between the effects of the first and the second break, the data fits must be monitored with respect to a previous state or an agreed benchmark, which can be conveniently taken as an undamaged state. Consequently, for sudden change in a system, the detection horizon in time will be related to how frequently the distribution fit is being carried out. On the other hand, a fit carried out after an event has taken place can be useful to assess the effect of such an event in terms of estimated nonlinearity.

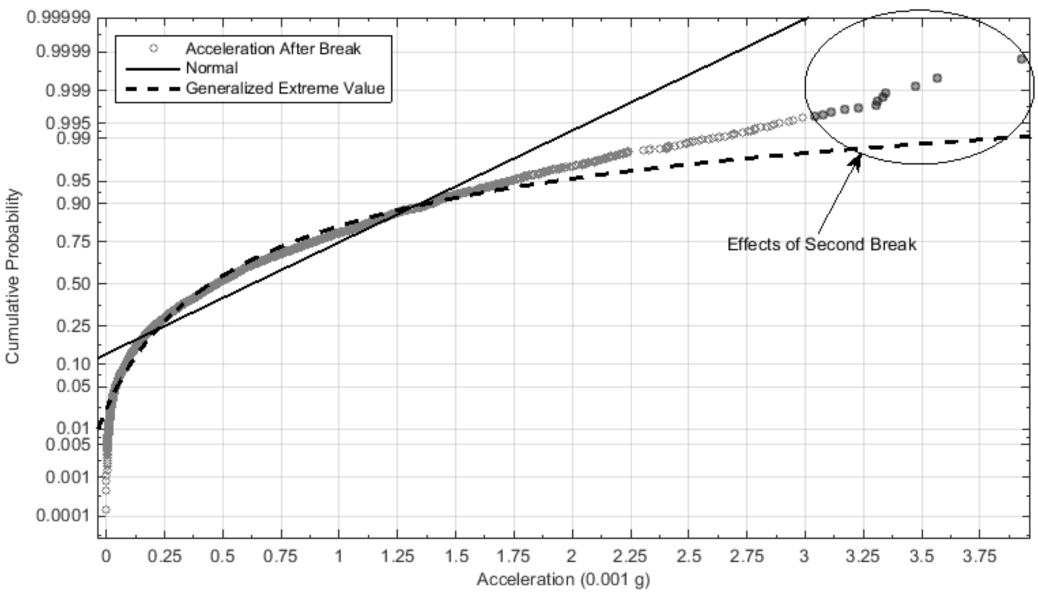

Figure 12. Probability plot for acceleration responses of the experimental SDOF system excited by Gaussian white noise for a sudden loss of stiffness due to the sudden shearing of two springs (38sec into experiment).

4.3 Effects of Measurement Devices

When dynamical systems are instrumented, a number of different devices are often incorporated. These different devices may measure the same quantity or the measurement of one may be employed to derive another quantity that is measured directly by a different device. A comparison of the applicability of the proposed method for different measurement devices is investigated next. For this purpose, the acceleration responses from the accelerometer and the derived acceleration responses from the LDV by differentiating velocity responses are considered. This is obtained using FFT analyser that can easily calculate acceleration by using calculation function provided by manufacturer (Polytec, 2011). Figure 13a presents the probability plot of the acceleration response from experimental data for Gaussian white noise excitation, while Figure 13b presents the probability plot of the derived acceleration responses from LDV for the same experiment. The two devices are observed to match well even for extreme value fits. This is also indicative of the low amount of noise in the LDV. None of the springs is damaged in this experiment.

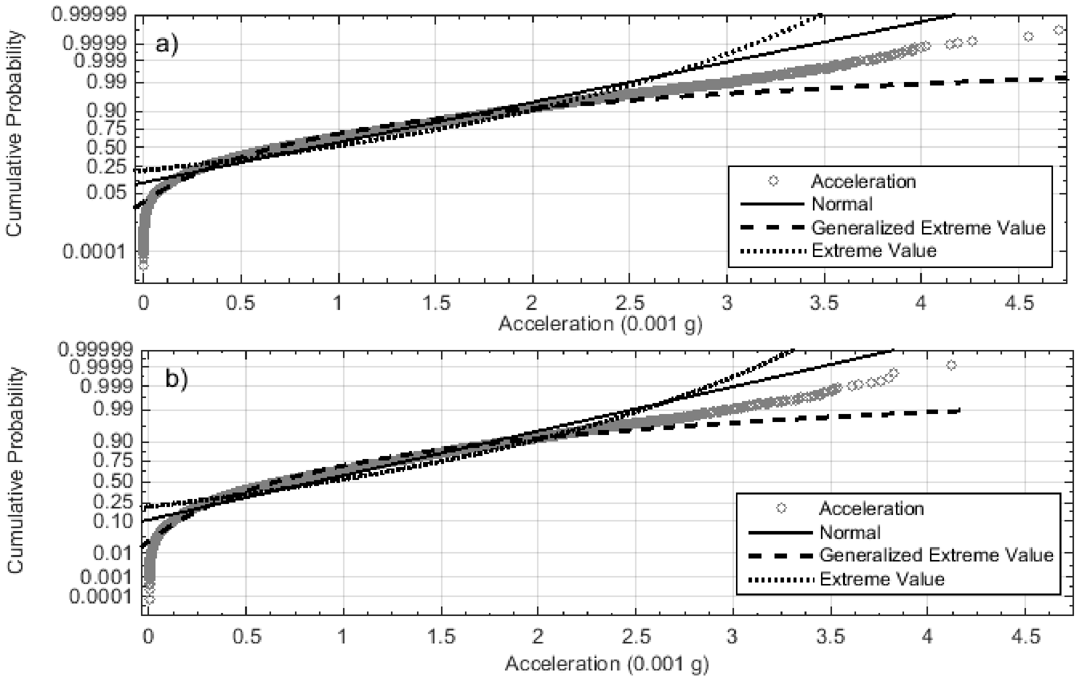

Figure 13. Probability plot for acceleration responses of the experimental SDOF system excited by Gaussian white noise measured using: a) an accelerometer and b) LDV.

4.4 Discussions Related to Implementation

For the practising engineer this approach has benefits for implementation in a wide range of situations. The expectation is that the governing dynamic system is known at least at a vague level so that a situation where different sources of nonlinearity are all strongly affecting the dynamic response can be avoided. For a wide range of cases in structural engineering, the dynamic responses can be related mainly to second order linear or cubic stiffness systems with low levels of equivalent viscous damping ratio. Tests carried out on full-scale structures can utilise the histogram of responses for model fitting or updating over time when short-term, periodical monitoring or long-term, continuous monitoring are carried out. Extreme response fits of structures can be compared against a known benchmark or against its own evolution over time in the case of long-term monitoring following an event of damage or rehabilitation and the effects of such changes can then be quantified through change of such fits. For structures monitored during an event (Pakrashi et al., 2013), this approach can consequently capture the cumulative change in the structure during such response, while accounting for errors related to modelling of the structure, as long as the measurement noise in such signal remains reasonably low with respect to the effects of the event.

Sometimes only the design of a structure is available in terms of a scaled physical model (Jaksic et al., 2015a). Here, the proposed approach can be used to fit the governing equation of such a design from laboratory based tests. On the other hand, when a numerical model of a design concept is available (Staino et al., 2012; Arrigan et al., 2011), the method can help create a small scale physical model that captures the global dynamics of the system and subsequent up-scaled models then have a benchmark physical model to match against.

Finally, this approach is also appropriate for monitoring a structure where noise may be an issue (Pakrashi et al., 2009) due to the fact that the deviation of extreme value fit will be expected from actual phenomenon affecting the structure as opposed to broadband noise. Exceptions to this comment relate to flicker noise, instantaneous or sharp changes in structures and the situation where the level or nature of measurement noise changes rapidly between the fitted responses under comparison.

For reasonable extreme value fits from numerical simulations or from experiments, it is advised to obtain time series with significant number of data points and for an input which is broadband and preferably one where the spectral content of the input is well defined. The length of the time series can be related to a longer time interval of simulation or testing as well as to a higher sampling rate. However, a higher sampling rate may attract issues related to measurement noise and the sampling rate and time interval under consideration should be selected based on the dynamics of the system observed and the level of noise present.

Ambient vibration based techniques (Ivanovic et al., 2000) or other tests utilising white noise or broadband input excitation is often standard for testing structures and this method is expected to be particularly helpful when considering exposure of structures to natural hazards including effects of earthquake (Takewaki et al., 2011), wind (Quilligan et al., 2012) and waves (Jaksic et al., 2015b) for system identification or monitoring purposes. With the importance of offshore renewable energy recognised worldwide, a wide range of structural designs for wind and wave energy are extremely flexible and behave nonlinearly while in operation in the presence of high wind and wave. The proposed method can be particularly suitable for such structures. For multi degree of freedom (MDOF) systems (Quek et al., 1999), this approach is feasible as well, but the exact implementation guidelines cannot be trivially extended from SDOF system as presented in the paper since the MDOF system responses may well deviate from GEV and a number of aspects, including coupling of modal responses will come into play.

5 Conclusions

The paper presents a method of using extreme value dynamic responses of structures dynamics to assess, quantify and model nonlinearity present in it. A hybrid method comprising of numerical simulations and experimental testing has been adopted for this purpose. Broadband input excitation to the structure is recommended for successful implementation of the proposed method. For a white noise excitation or excitations with well-defined frequency domain representation, a longer run time of experiments can replace the need of carrying out a large number of experiments to obtain a stable extreme value distribution fit. Approximately guessed theoretical model of structural dynamics can be retuned and corrected employing the proposed method in conjunction with experimentation. Sudden changes in structural stiffness are observed to have an effect in the extreme value distributions of dynamic responses. The proposed method can also be used for dynamical systems where the detailed designs are still at conceptual level, but where the governing differential equations related to the dynamics are available. Examples presented in this paper act as an initial guideline for such purposes.

Acknowledgements:

The Irish Research Council for Science, Engineering and Technology (IRCSET)

Marine Research Energy Ireland (MaREI), grant number 12/RC/2302, Science Foundation Ireland (SFI)

SFI Advance award 14/ADV/RC3022 Science Foundation Ireland (SFI)

Polytec Ltd. for providing the RSV -150 Remote Sensing Vibrometer

Prof. Sayan Gupta, Indian Institute of Technology – Madras, for providing suggestions on the initial draft.

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