The use of accelerometers in UAVs for bridge health monitoring

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**ABSTRACT:** Unmanned Aerial Vehicles (UAVs) technology has gained considerable popularity in bridge structural health monitoring for its strengths, such as low cost, safety and high energy efficiency. This paper envisions a scenario in which accelerometers are mounted onto UAVs, which then are able to gather acceleration signals by self-attaching to the bridge. However, battery life is an issue in UAVs with the subsequent limitation in the duration of the measurements. Therefore, this paper carries out a simulation on mode shape extraction from a short data burst by utilizing an output only technique, the so-called frequency domain decomposition (FDD). Modal assurance criterion (MAC) is used as a statistical indicator to check differences between the estimated mode shapes and the eigenvectors from finite element analysis. The short acceleration response is generated using a planar vehicle-bridge interaction system where the moving load is modelled as two quarter-cars and the bridge is modelled as a simply supported beam. The impact of signal noise, vehicle speed and signal duration on the accuracy of the estimated mode shapes is investigated. FDD is shown to achieve high values of MAC even for short data bursts. Damping ratio is identified as a significant source of MAC discrepancy in the extraction of mode shapes. The stiffness loss due to a crack is introduced in the beam to evaluate how damage affects the mode shape compared to operational effects. How the MAC values vary with crack location and damage severity is discussed for the first three mode shapes.

1. **INTRODUCTION**

Over the past few years, the application of Unmanned Aerial Vehicles (UAVs) to bridge Structural Health Monitoring (SHM) is becoming increasingly popular, due in part to its low cost and high energy efficiency, according to the survey by Chen et al. (2016). Furthermore, the application of UAVs could improve safety in the working environment as the need for individuals inspecting the condition of the bridge at a height is avoided. In addition, current UAV technology is able to get self-attached to the bridge soffit and to crawl along pre-established paths on the bridge upside down and/or underwater for taking acceleration measurements. Therefore, a fleet of such UAVs could be used to obtain the acceleration signal from the bridge while vehicles are passing. However, there are still some drawbacks, such as flight duration and limited sensor battery time, which hinder its further development. In some scenarios, only short amounts of data may be available due to these drawbacks. A question that remains to be answered is: “Can mode shapes be accurately obtained from short data bursts?” In order to address the latter, this paper analyses scenarios consisting of short-duration acceleration responses of a bridge caused by a moving load.

In the field of bridge SHM, it is usually difficult or costly to know the input force exciting the bridge. Therefore, output only techniques are easier to implement and often favoured over input-output techniques for extraction of the mode
shapes of the bridge. Operational Modal Analysis (OMA) aims to identify the structural properties based on vibration data measured when the structure is under its operating conditions, with no initial or known artificial excitation. For example, Weng et al. (2008) conduct an ambient vibration survey of a long-span cable-stayed bridge to obtain its dynamic characteristics using data collected through a novel wireless monitoring system. The first ten vertical mode shapes of a bridge deck are extracted successfully by using Stochastic Subspace Identification (SSI) and Frequency Domain Decomposition (FDD) from a 50 s long recorded ambient vibration velocity signal.

A review by Li and Hao (2016) presents other techniques that can be applied to obtain mode shapes from the bridge or vehicle response, such as the Hilbert Transform used by Yang et al. (2014) to extract the first three mode shapes from a 15 s long vehicle acceleration response. They check the influence of vehicle speed on mode shape extraction but limited to very low vehicle speeds (2, 4 and 8 m/s). The estimated mode shapes are compared to the theoretical ones through the Modal Assurance Criterion (MAC). In their case, the first three extracted mode shapes match very well with the theoretical ones for low speed (2 m/s), with MAC values over 0.998. However, for higher speeds (4 and 8 m/s), the MAC values tend to decrease, especially for higher modes. Even further, Malekjafarian and OBrien (2014) identify bridge dynamic properties using Short Time Frequency Domain Decomposition (STFDD) from responses measured in a vehicle crossing a bridge. In their paper, multi-stage measurements are used to obtain the bridge mode shapes from a 7.5 s long acceleration response due to two connected passing axles. The MAC values show almost fully consistent modes for the first two modes of the bridge (0.9998 and 0.9995, for the 1st and 2nd mode, respectively). In addition, the effect of noise, road profile and number of axles is also investigated, resulting in overall lower MAC values. It must be noted that the vehicle is assumed to be driven at an abnormally reduced speed (1 and 2 m/s).

In this paper, a far shorter acceleration response than commonly found in the literature is used for extracting mode shapes. The response is obtained for a simply supported beam with a rectangular section that is traversed by two Quarter-Cars (QCs) at a constant speed. The differences between the estimated mode shapes by FDD and the eigenvectors from a Finite Element (FE) model are evaluated via the MAC. The effect of signal noise, vehicle speed, signal duration and damping ratio on MAC values is investigated and compared to the effect of damage.

Section 2 describes the Vehicle-Bridge Interaction (VBI) simulation model and the properties of the beam and the vehicle. Section 3 introduces the theory and background of FDD. The analyses of the healthy and damaged beams are conducted in Sections 4 and 5 respectively. Finally, Section 6 provides conclusions.

2. NUMERICAL SIMULATION

Figure 1(a) shows the two QCs crossing the beam model under investigation and Figure 1(b) shows the road profile used in the simulation. The bridge is assumed to have a very good road profile (Class “A”), as expected in a well-maintained highway, and the geometric spatial mean is $16 \times 10^6$ m$^3$ cycle$^{-1}$.
The QC consists of a sprung mass, \( m_s \), and an unsprung mass, \( m_u \). The unsprung mass connects to the road surface via a spring, \( K_t \), whereas the sprung mass is connected to the unsprung mass by another spring, \( K_s \), and a viscous damper, \( C_s \). The values of the parameters used in this VBI system are based on Cantero et al. (2008). Tables 1 and 2 give the properties of the bridge and vehicle respectively.

The sampling frequency is set at 1000 Hz for all simulations and it is assumed that vertical acceleration signals are acquired at every node of the model. Given the values in Tables 1 and 2, the time elapsed between the entry of the first QC and the exit of the second one, i.e. forced vibration, adds up to 1.08 s. The bridge acceleration signals in forced vibration constitute the short data bursts that will be analysed.

### Table 1: Properties of the bridge.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>( L )</td>
<td>20</td>
<td>m</td>
</tr>
<tr>
<td>Depth</td>
<td>( h )</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>Width</td>
<td>( b )</td>
<td>15</td>
<td>m</td>
</tr>
<tr>
<td>Mass per unit</td>
<td>( m )</td>
<td>37500</td>
<td>kg/m</td>
</tr>
<tr>
<td>2nd moment of area</td>
<td>( I )</td>
<td>1.25</td>
<td>m^4</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>( E )</td>
<td>( 3.5 \times 10^{10} )</td>
<td>MPa</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>( \zeta )</td>
<td>0.03</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Approach length at both ends</td>
<td>( L_{app} )</td>
<td>100</td>
<td>m</td>
</tr>
<tr>
<td>1st Frequency</td>
<td>( f_1 )</td>
<td>4.24</td>
<td>Hz</td>
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</tbody>
</table>

### Table 2: Properties of the vehicle.

<table>
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<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
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<tr>
<td>Sprung mass</td>
<td>( m_s )</td>
<td>13875</td>
<td>kg</td>
</tr>
<tr>
<td>Unsprung mass</td>
<td>( m_u )</td>
<td>1125</td>
<td>kg</td>
</tr>
<tr>
<td>Suspension stiffness</td>
<td>( K_s )</td>
<td>( 7.5 \times 10^5 )</td>
<td>N/m</td>
</tr>
<tr>
<td>Suspension damping</td>
<td>( C_s )</td>
<td>( 10 \times 10^3 )</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Tire stiffness</td>
<td>( K_t )</td>
<td>( 3.5 \times 10^6 )</td>
<td>N/m</td>
</tr>
<tr>
<td>Velocity</td>
<td>( v )</td>
<td>22.22</td>
<td>m/s</td>
</tr>
<tr>
<td>Vehicle Spacing</td>
<td>( d )</td>
<td>4</td>
<td>m</td>
</tr>
<tr>
<td>1st Frequency</td>
<td>( f_1 )</td>
<td>1.06</td>
<td>Hz</td>
</tr>
<tr>
<td>2nd Frequency</td>
<td>( f_2 )</td>
<td>9.79</td>
<td>Hz</td>
</tr>
</tbody>
</table>

3. FREQUENCY DOMAIN

DECOMPOSITION

FDD, first presented by Brincker et al. (2001), is one of the most popular OMA techniques. The relationship between the unknown input \( m \) and the measured response \( n \) can be defined as:

\[
[G_{nn}(j\omega)] = [H(j\omega)]^H [G_{mm}(j\omega)] [H(j\omega)]^T
\]

(1)

where \( [G_{nn}(j\omega)] \) is the power spectral density (PSD) matrix of the input \( m \), \( [G_{nn}(j\omega)] \) is the PSD matrix of the response \( n \) and \( [H(j\omega)] \) is the Frequency Response Function (FRF) matrix. The superscript “\( T \)” indicates the transpose of the matrix, whereas the superscript “\( H \)” indicates the complex conjugate of the matrix. It should be noted that the mode shapes obtained from FDD are unscaled. More details about FDD can be found in Brincker et al. (2001).

The Modal Assurance Criterion (MAC) is a statistical indicator that is very sensitive to differences in the mode shapes (Pastor et al. (2012)). The MAC values range between 0 and 1, where 1 represents fully consistent mode shapes and 0 indicates that the modes are not consistent. The MAC is calculated as the normalised scalar product of two vectors \( \{\phi_{est}\} \) and \( \{\phi_{FE}\} \) as per Eq. (2).

\[
MAC = \frac{|\{\phi_{est}\}^T \{\phi_{FE}\}|^2}{\{\phi_{est}\}^T \{\phi_{est}\} \{\phi_{FE}\}^T \{\phi_{FE}\}}
\]

(2)
where \( \{ \varphi_{est} \} \) and \( \{ \varphi_{FE} \} \) represent the mode shapes extracted by applying FDD to the signals and obtained from eigenvalue analysis, respectively. The latter is adopted as the true mode shape so that the MAC serves as an indicator for the accuracy of the former. The analysis will focus on the first three mode shapes of the structure since higher modes are difficult to obtain in a real case. Figure 2 shows the eigenvectors \( \{ \varphi_{FE} \} \), which are normalised to 1.

**Figure 2: First three eigenvectors of the bridge.**

### 4. ANALYSIS OF HEALTHY BEAM

#### 4.1. Influence of noise on mode shapes

The short data burst is contaminated with random noise to simulate real measurements. The noise is obtained from a normal distribution with \( \mu = 0 \) and a standard deviation related to the level of noise, which is characterised by the signal-to-noise ratio (SNR). Here, SNR is defined by González et al. (2008):

\[
SNR = \frac{\sigma_Y}{\sigma_n} \tag{3}
\]

where \( \sigma_Y \) and \( \sigma_n \) are the standard deviation of the noisy and real accelerations, respectively. The SNR and the amount of noise in the signal are inversely proportional, i.e. high SNRs imply low levels of noise and vice-versa. In this paper, 10%, 20%, 30% and 40% noise levels are simulated, representing SNR values of 10, 5, 3.3 and 2.5, respectively.

Figure 3 shows the first three normalised mode shapes obtained from applying FDD to the short data bursts. For 10% noise, the difference between the estimated and true mode shapes is not significant, with the MAC values being 0.9999 (1st mode), 0.9997 (2nd mode) and 0.9995 (3rd mode). The MAC value is close to 1, but not exactly 1, due to noise, operational effects and damping. When increasing the noise to 40%, the MAC value decreases from 0.9995 to 0.9915 for the 3rd mode. While the MAC value will decrease with an increase in noise, these changes appear to be more noticeable in higher modes. Nonetheless, it must be noted that higher values of MAC can also be found for higher modes due to the random nature of noise.

**Figure 3: Impact of noise on normalised mode shapes by FDD: (a) 1st, (b) 2nd and (c) 3rd mode.**

#### 4.2. Influence of vehicle speed on mode shapes

This section investigates the impact of vehicle speed on mode shape extraction. Speed values of 2, 4, 8, 16 and 32 m/s are considered here. The first three normalised mode shapes are shown in
Figure 4 for each value of speed. With low speeds such as 2 and 4 m/s, the MAC value lies between 0.9997 and 1.000, indicating fully consistent mode shapes. For 32 m/s, those values decrease to 0.9992, 0.9937 and 0.9905 for the 1st, 2nd and 3rd mode, respectively. The results support that a low vehicle speed represents a better scenario for extracting mode shapes, more noticeably the higher the mode.

and true mode shapes. The MAC values for the first three mode shapes are in the range of 0.9993 to 1.000, considering all durations whether they include free vibration or not. In particular, it is worth noting that the estimation of the 1st mode is insensitive to the signal duration as shown in Figure 5(a).

![Figure 4: Impact of speed on normalised mode shapes by FDD: (a) 1st, (b) 2nd and (c) 3rd mode.](image)

4.3. Influence of signal duration on mode shapes

Figure 5 compares the mode shapes estimated by FDD and those resulting from eigenvalue analysis for a duration of the response varying from 1.08 s to 5.08 s in increments of 1 s. Therefore, the forced vibration short data burst is extended by also including free vibration. However, there are no significant differences between the estimated and true mode shapes. The MAC values for the first three mode shapes are in the range of 0.9993 to 1.000, considering all durations whether they include free vibration or not. In particular, it is worth noting that the estimation of the 1st mode is insensitive to the signal duration as shown in Figure 5(a).

![Figure 5: Impact of signal duration on normalised mode shapes by FDD: (a) 1st, (b) 2nd and (c) 3rd mode.](image)

5. ANALYSIS OF DAMAGED BEAM

5.1. Modelling of damage

Various approaches for crack modelling are available in the literature. This paper uses the stiffness reduction for a single crack proposed by Sinha et al. (2002). In this case, the crack is located at $x = 8$ m of the VBI model, i.e., between elements 8 and 9, as illustrated by Figure 6.
For a given beam depth, \( h \), the damaged depth, \( h_d \), takes a value of \( h_d = \lambda h \), where \( \lambda \) is the damage level. Following Sinha’s approach, the effective length, \( l_d \), is equal to \( 1.5h \), over which beam stiffness varies linearly until reaching a maximum loss at the crack location. Figure 7 shows the stiffness distribution throughout the beam for crack depths that are between 10% and 50% of the beam depth.

**Figure 7: Stiffness profile for five levels of damage.**

5.2. Impact of damage on mode shapes

Simulations in this section are conducted based on the parameters shown in Tables 1 and 2, using noise-free signals and the damage model introduced in section 5.1. Figure 8 shows the first three normalised mode shapes for each damage scenario. In the case of 10% damage, the MAC values associated with the 1st, 2nd and 3rd modes are 0.9999, 0.9993 and 0.9990, respectively. When the damage increases to 50%, the MAC values further decrease to 0.9988 (1st mode), 0.9976 (2nd mode) and 0.9895 (3rd mode). The largest change is felt in the 3rd mode, i.e., a MAC decrease of 0.0095 from 10% to 50%. Overall, these results indicate that MAC is not too sensitive to localised stiffness losses. Nonetheless, the MAC values for higher modes (3rd mode) are more sensitive than those for lower modes (1st mode).

**Figure 8: Impact of damage on normalised mode shapes by FDD: (a) 1st, (b) 2nd and (c) 3rd mode.**

5.3. Impact of crack location on MAC

In Section 5.2, the position of the crack was fixed at 8 m. Here, the MAC values are calculated varying the longitudinal position of the crack on the beam. As in the previous analysis, damage severity is varied as well. Figure 9 shows the MAC value versus damaged location for different damage levels and the first three mode shapes. For a given crack location, MAC clearly decreases as damage severity increases regardless of the mode shape. In the case of the 1st mode, two troughs, at \( \frac{1}{4} \) and \( \frac{3}{4} \) span, are found for the MAC value. On the other hand, the values of MAC tend to increase for damage locations at mid-span and near the supports. Troughs also exist in representations of
MAC for 2\textsuperscript{nd} and 3\textsuperscript{rd} modes when the damage is located close to the points with the highest amplitude of the mode shape. However, the MAC values are almost equal to 1 for damage locations near modal nodes (i.e., supports for the 1\textsuperscript{st} mode, supports and mid-span for the 2\textsuperscript{nd} mode, etc.).

![Figure 9: MAC vs crack location and severity in damped beam: (a) 1\textsuperscript{st}, (b) 2\textsuperscript{nd} and (c) 3\textsuperscript{rd} mode.](image)

5.4. Impact of damping on MAC

The pattern of MAC value versus damaged location shown in Figure 9 is not symmetrical with respect to mid-span. The latter can be attributed, to some extent, to damping. This can be seen in Figure 10, which shows MAC versus damaged location for the same scenarios as Figure 9, but in the case of an undamped beam. Troughs are identified at the same locations as in Figure 9, but this time, MAC values are roughly symmetrical with respect to mid-span. Slight deviations can still be noticed but they should disappear if longer free vibration signals or a finer discretisation of the model were used.

![Figure 10: MAC vs crack location and severity in undamped beam: (a) 1\textsuperscript{st}, (b) 2\textsuperscript{nd} and (c) 3\textsuperscript{rd} mode.](image)

6. CONCLUSIONS

This paper has analysed the impact of signal noise, vehicle speed and signal duration on the estimation of mode shapes by applying FDD on a short data burst (i.e., 1.08 s) in forced vibration. The short data burst has been obtained from the acceleration response of a vehicle-bridge interaction system consisting of two QCs and a damped beam model. MAC has been used to assess the differences between the mode shapes estimated by FDD and the eigenvectors from the FE model for scenarios distinguished by noise, signal duration and vehicle speed. Finally, a
damaged beam has been considered to assess how damage severity and location affect MAC.

The analysis of MAC values has shown that lower vehicle speeds (i.e., less than 16 m/s) lead to a significantly more accurate extraction of mode shapes in forced vibration. If the duration of the signal consisting of forced vibration was extended by a ratio of 4 allowing for free vibration, there is not a relevant improvement in MAC values compared to low vehicle speeds and a signal consisting solely of forced vibration. In addition, Gaussian noise does not have a noticeable impact on the results unless considering levels of noise greater than 30% and the 3rd mode. Regarding the analyses of the damaged bridge, it has been found that the detection of cracks depends mainly on its location. For instance, given a vehicle speed of 22.22 m/s and a noise-free signal, cracks of 10% of the beam depth could only be clearly identified at locations between 2-18 m for the 1st mode, between 2-9 m and 11-18 for the 2nd mode, and between 2-6 m, 7-13 m and 14-18 m for the 3rd mode. Outside of these ranges, MAC values have not exhibited a significant decrement regardless of damage severity. The latter is due to the sensitivity of the mode shape with respect to the relative position of the damage and, to some extent, also to noise and vehicle speed concealing the effect of damage. By comparing the MAC values in Section 5 obtained for a damaged beam, with those in Section 4 for a healthy beam with varying noise, vehicle speed and signal duration, the reliability of the method in identifying a specific damage severity on a particular location could be established for given values of velocity, signal duration and noise.

7. ACKNOWLEDGEMENTS
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8. REFERENCES