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Modal analysis of a bridge using short-duration accelerations

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Abstract

The application of unmanned aerial vehicle technology to bridge structural health monitoring has become a hot research topic due to its low cost, safety and high energy efficiency. However, flight duration and battery life are substantial technical limitations. Is a short data burst sufficient for damage detection? This paper intends to answer this question by developing a novel approach based on frequency domain decomposition to obtain the mode shapes from a short data burst. Then, the modal assurance criterion is used as an indicator of the differences between the estimated mode shapes from the short data burst and the exact eigenvectors from finite element analysis. Here, the short data burst is obtained from the simulated acceleration response of a bridge beam model due to the crossing of two quarter-cars. A new damage indicator based on the modal assurance criterion profile along the beam is proposed to locate and quantify damage.

1. Introduction

The use of unmanned aerial vehicle technology for damage detection has been attracting more and more attention over the past few years for its outstanding advantages (Chen et al. 2016). However, there are still some disadvantages, such as flight duration and battery life, which hinder its further development. In some scenarios, only short amounts of data may be available due to these drawbacks, and there is need to clarify the potential of a short data burst for damage detection.

A Structural Health Monitoring (SHM) system may be able to reach one of the following four levels: Level 1) identification of damage; Level 2) Level 1 and localization of damage; 3) Level 2 and quantification of damage and; Level 4) Level 3 and prediction of remaining life. So far, many attempts have been made for achieving levels 1 and 2 successfully, but achieving levels 3 and 4 is still challenging, even more for a short data burst. Dynamic properties are often used for SHM purposes, i.e., natural frequencies, mode shapes and damping ratios. For example, Dahak et al. (2019) locate damage using only the changes in measured natural frequencies and the curvature mode shapes vectors. OBrien and Malekjafarian (2016) develop a damage index based on mode shape squares extracted by applying Short Time FDD on a 4.0 s long acceleration signal, which proves to work efficiently in a theoretical simulation. Cao et al. (2017) review the use of damping for structural damage detection.
This paper proposes an innovative damage indicator based on the Modal Assurance Criterion (MAC) profile, to detect damage location and quantify damage severity from a short data burst. For a given Vehicle-Bridge Interaction (VBI) system and damage model, there is a unique MAC profile in terms of damage location and severity for each mode. By combining the first three modes, it is expected that only one combination of values for damage location and severity, will satisfy a given set of MAC values. The principle behind this approach is investigated using the theoretical acceleration response of a bridge beam model to two Quarter-Cars (QCs).

2. Numerical simulation

Figure 1 shows the two QCs crossing the simply supported bridge beam model under investigation. The finite element bridge model is discretized into 20 beam elements. The acceleration response of the bridge is obtained with a scanning frequency of 1000 Hz at 17 nodes, spaced every 1 m, for the purpose of extracting the mode shapes. The road surface of the bridge is assumed to be very good (class ‘A’ according to ISO standards). Each QC consists of two masses, including the axle, \( m_u \), and body of a vehicle, \( m_s \). Vehicle suspensions are represented by a spring, \( K_s \), and damper, \( C_s \), system, whereas tires are modelled by means of a spring, \( K_t \). Tables 1 and 2 give the properties of the bridge and vehicle, respectively, based on Cantero et al. (2008). In a practical situation, it may not be possible to choose the more ideal free vibration, and only forced vibration is considered here with a total signal duration of 1.08 s.

![Figure 1: VBI system.](image-url)

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<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Length</td>
<td>( L )</td>
<td>20</td>
<td>m</td>
</tr>
<tr>
<td>Depth</td>
<td>( h )</td>
<td>1</td>
<td>m</td>
</tr>
<tr>
<td>Width</td>
<td>( b )</td>
<td>15</td>
<td>m</td>
</tr>
<tr>
<td>Mass per unit</td>
<td>( m )</td>
<td>37500</td>
<td>kg/m</td>
</tr>
<tr>
<td>2(^{nd}) moment of area</td>
<td>( I )</td>
<td>1.25</td>
<td>m(^4)</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>( E )</td>
<td>( 3.5 \times 10^{10} )</td>
<td>MPa</td>
</tr>
<tr>
<td>Damping ratio</td>
<td>( \zeta )</td>
<td>0.03</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Approach length at both ends</td>
<td>( L_{\text{app}} )</td>
<td>100</td>
<td>m</td>
</tr>
<tr>
<td>1(^{st}) Frequency</td>
<td>( f_1 )</td>
<td>4.24</td>
<td>Hz</td>
</tr>
</tbody>
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<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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<tr>
<td>Sprung mass</td>
<td>( m_s )</td>
<td>13875</td>
<td>kg</td>
</tr>
<tr>
<td>Unsprung mass</td>
<td>( m_u )</td>
<td>1125</td>
<td>kg</td>
</tr>
<tr>
<td>Suspension stiffness</td>
<td>( K_s )</td>
<td>( 7.5 \times 10^{5} )</td>
<td>N/m</td>
</tr>
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</table>
### 3. Modal technique for analysis of short data burst

Frequency Domain Decomposition (FDD), introduced by Brincker et al. (2001), is one of the most popular operational modal analysis techniques. The relationship between the unknown input \( m \) and the measured response \( n \) can be defined as:

\[
\begin{align*}
[\mathbf{G}_n(j\omega)] &= [H(j\omega)]^H [\mathbf{G}_m(j\omega)] [H(j\omega)]^T
\end{align*}
\]

where \([\mathbf{G}_m(j\omega)]\) is the Power Spectral Density (PSD) matrix of the input \( m \), \([\mathbf{G}_n(j\omega)]\) is the PSD matrix of the response \( n \) and \([H(j\omega)]\) is the frequency response function matrix. The superscript “\(^T\)” indicates the transpose of the matrix, whereas the superscript “\(^H\)” indicates the complex conjugate of the matrix. The MAC value is calculated as the normalized scalar product of two vectors \( \{\varphi_{est}\} \) and \( \{\varphi_{eig}\} \) as shown by Equation (2), where \( \{\varphi_{est}\} \) and \( \{\varphi_{eig}\} \) represent the mode shape extracted by FDD from the signal and the mode shape obtained from eigenvalue analysis of the healthy structure, respectively. The MAC values lie between 0 and 1, with 0 representing no consistent and 1 indicating fully consistent mode shapes. The MAC value is very sensitive to differences in the mode shapes (Pastor et al. 2012).

\[
\text{MAC} = \frac{|(\{\varphi_{est}\})^T \{\varphi_{eig}\}|^2}{(\{\varphi_{est}\})^T (\{\varphi_{est}\}) (\{\varphi_{eig}\})^T (\{\varphi_{eig}\})}
\]

### 4. Damage model and damage detection method

#### 4.1 Damage model

Various approaches for crack modelling are available in the literature. This paper uses the stiffness reduction for a single crack proposed by Sinha et al. (2002). For a given beam depth, \( h \), the damaged depth, \( h_d \), takes a value of \( h_d = \lambda h \), where \( \lambda \) is the damage level. Following Sinha’s approach, the effective length, \( l_d \), is equal to \( 1.5h \), over which stiffness varies linearly until reaching a maximum loss at the crack location. Figure 2 illustrates an example, where the crack is located between elements 4 and 5 of the bridge model.

![Figure 2: Damage details.](image-url)
4.2 Method to locate and quantify damage

Figure 3 shows the 3D MAC profile versus damage location and severity for the first three modes. The x- and y-axes represent the damage location and damage severity, respectively. Damage severity is varied from 0% to 50% with a 5% increment. The z-axis represents the MAC values. As severity increases, changes in MAC values are more significant the higher the mode order.

Figure 3: MAC profile vs crack location and severity: (a) 1st, (b) 2nd and (c) 3rd mode.

A function is assumed to represent the relationship between the MAC value and another two independent real variables, damage location and damage severity, as shown in Equation (3). Then, for a given damage location, \(u\), and damage severity, \(v\), there is a unique value of the \(MAC\) for each mode.

\[
MAC = f(u, v) \tag{3}
\]

However, it is clear from Figure 3 that there can be several solutions \((u, v)\) in each mode for a given \(MAC\) value, i.e., the inverse problem does not have a unique solution. Therefore, \(MAC\) values from more than one mode are needed to find the combination of values \((u, v)\) that solve all equations simultaneously (Equation (4)).

\[
\begin{align*}
MAC_1 &= f_1(u, v) \\
MAC_2 &= f_2(u, v) \\
MAC_3 &= f_3(u, v)
\end{align*} \tag{4}
\]

where, \(f_1\), \(f_2\) and \(f_3\) represent the MAC profile for the first three modes. There will be one combination of values \((u_0, v_0)\) for given values of \(MAC_1\), \(MAC_2\) and \(MAC_3\), that will satisfy the system of Equations (4).

5. Results and discussion

A simple case study is used to demonstrate how the method works. In this case, the crack is located at element number 4 with 30% damage, i.e., \(u = 4\ m\), \(v = 30\%\). The first three MAC values are 0.9992 \((MAC_1)\), 0.9928 \((MAC_2)\), and 0.9757 \((MAC_3)\). Figures 4(a), (b) and (c) show the horizontal planes corresponding to these MAC values plotted over the isometric view of Figure 3. All possible solutions \((u, v)\) will be contained in the intersection of the horizontal planes with the 3D MAC profile.
profiles. The views in Figures 4(d), (e) and (f) clearly show the combinations of location and severity where the horizontal plane (i.e., a horizontal line in this view) will intersect the MAC profile. If there is not intersection for a given location, this means damage is not possible there. If there was intersection for a given location, this location must be intersected in the MAC profiles for the other two modes with the same value of damage severity to truly represent damage.

Figure 4: Damage localization and severity quantification for a given $\text{MAC}_1$, $\text{MAC}_2$ and $\text{MAC}_3$: (a, d) 1st, (b, e) 2nd and (c, f) 3rd mode.

The results from Figure 4 support that there can be several solutions $(u, v)$ for a given MAC value when considering only a single mode. Figure 5 presents all potential solutions for the given MAC values. By cross-checking the potential solutions obtained for each of the three modes, it is possible to find the only pair of values $u_0 = 4 \, \text{m}$, $v_0 = 30\%$, that solves the system of Equations (4).

Figure 5: Solutions for given MAC values.
6. Conclusions

In this paper, the FDD has been applied to extract mode shapes from a short data burst (i.e., 1.08 s) in forced vibration. The short data burst has been obtained from the bridge acceleration response of a VBI system consisting of two QC's and a damped beam finite element model. MAC has been used to assess the differences between the mode shapes of the damaged beam estimated by FDD, and the healthy beam obtained from eigenvalue analysis. Based on the MAC profile, a new damage indicator that locates and quantifies damage has been proposed. For a given VBI system and damage model, there is a unique MAC profile for each mode. The MAC profiles for the first three modes have been used to obtain the only values of damage location and severity that satisfy a given set of MAC values. The new method has been demonstrated on the theoretical response of a finite element beam with a single-crack damage successfully. Multiple-crack damages, various damage models, changes in excitation forces or the impact of noise need to be assessed next.

7. Acknowledgements

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References