Title	Price-elastic demand in deregulated electricity markets
Authors(s)	Siddiqui, Afzal S.
Publication date	2004-04-30
Publication information	Siddiqui, Afzal S. “Price-Elastic Demand in Deregulated Electricity Markets” (April 30, 2004).
Series	Centre for Financial Markets working paper series, WP-04-03
Publisher	University College Dublin. School of Business. Centre for Financial Markets
Item record/more information	http://hdl.handle.net/10197/1155

Price-Elastic Demand in Deregulated Electricity Markets

Afzal S. Siddiqui

Department of Banking and Finance

Michael Smurfit Graduate School of Business

University College Dublin

Blackrock, Co. Dublin

Republic of Ireland

email: afzal.siddiqui@ucd.ie

30 April 2004

Abstract

The degree to which any deregulated market functions efficiently often depends on the ability of market agents to respond to fluctuating conditions. Many restructured electricity markets, however, have little demand response. We examine the implications for market operations when a risk-averse retailer's end-use con-

sumers are allowed to perceive real-time variations in the electricity price. Using a market-equilibrium model, we find that price elasticity both *increases* the retailer's revenue risk exposure and *decreases* the spot price. Consequently, the overall impact of price-responsive demand on the electricity forward price is ambiguous.

Keywords: Price-elastic demand, electricity deregulation, forward contracts.

JEL Classification Codes: L94, G13, D41.

1 Introduction

Due to their "natural monopoly" characteristics[1], infrastructure industries, e.g., those involving energy, telecommunications, and transportation, have traditionally been subject to government regulation. Within the set of infrastructure industries, electricity was especially suited to government regulation due to its lack of storability, the complex nature of its transmission, and to a lesser extent, economies of scale in its generation. In particular, electricity transmission, unlike other transportation networks, requires coördinated behavior to ensure that injections and withdrawals of electricity are continuously balanced.[2] As a result, electricity supply functions, such as generation and transmission, were kept *vertically integrated* under the auspices of a regulated entity that exclusively provided all

[1] These imply that costs decline with output and that a single extensive network is necessary to deliver the final product to consumers.

[2] This coördination is necessitated by Kirchhoff's laws, which state that alternating current (AC) follows the path of least impedance along a transmission system.

services within a given geographic region.

Although vertical integration allowed generation and transmission to be coördinated, it, nevertheless, turned the generation sector into a *de facto* monopoly. As a consequence, a potentially competitive generation sector[3] was encumbered by government regulation and its associated inefficiencies (see [9]). The desire to provide incentives for efficient operation of the electricity industry has, thus, meant unbundling of its various services. As identified in [11], the four main electricity supply functions provided by an investor-owned utility (IOU) were:

- **generation**: conversion of primary energy to electricity.
- **transmission**: transportation of electricity along meshed high-voltage wires to substations.
- **distribution**: transportation of electricity along low-voltage wires to customer meters.
- **retailing**: arrangements for billing, on-site support, and demand management.

Since the generation and retailing sectors are technologically amenable to competition, they have seen the promotion of competition. Indeed, economies of scale are either exhausted at current levels of production or are not applicable at all here (see [16]). These services are, thus, to be provided through the markets. For the IOUs, this has generally

[3]There exists little evidence that large companies are necessary to exploit economies of scale in generation (see [10]).

implied divestiture of their generation assets. The transmission and distribution sectors, however, continue to be regulated because of their "natural monopoly" characteristics. Outside of these very general guidelines, the actual measures implemented by electricity industry restructuring have varied considerably across states and countries (see [15] for more details).

In general, the introduction of competition into electricity generation sectors has led to some improvements in social welfare. For example, the England and Wales (E&W) electricity spot market, experienced average prices that were not as high as theoretically predicted, and the E&W competitive generation sector saw marked improvements in labor productivity within three years of deregulation (see [18]). Along with greater economic efficiency in the generation sector, however, deregulation has also introduced new problems, such as market power, high price volatility, and the competitive procurement of reserve generation, into an industry that was once insulated from competitive forces (see [5], [6], [7], [16], [14], and [13]).

The overarching problem that almost all electricity deregulation efforts have failed to address, however, is on the demand side. Indeed, the bill for electricity usage paid by end-use consumers is not related to the real-time (spot) price of electricity, even in most deregulated industries. In E&W, for example, about 5% of the total system load in 1996 was purchased by end-users who experienced periodic variations in the real-time price (see [16]). In California, retail rates were frozen in order to allow IOUs to recover sunk costs of investments in generating plants that had been made before the 1996

restructuring (so-called "stranded assets"), but would not be viable after deregulation. By not allowing demand to be price elastic, many electricity deregulation efforts stretch generation resources to the point where system stability is threatened. Indeed, due to an unresponsive demand side, electricity demand has to be met regardless of the cost. This, in turn, facilitates the exercise of market power, amplifies price volatility, and creates difficulties for the ISO in procuring ancillary services (AS), or reserves that can be dispatched quickly to meet real-time contingencies.

Unlike other competitive commodity markets, deregulated electricity markets had virtually no demand-side response because end-use consumers were exposed to a constant retail rate independent of market conditions. Under certain circumstances, the ISOs could act to reduce AS purchases and exercise interruptible load (IL) contracts, but in the extreme case of California, these mechanisms were only modestly successful at reducing load during 2000 because the high frequency of outages decreased customer response (see [12]). Consequently, when the supply side experienced shocks, the absence of price elasticity on the demand side resulted in wholesale prices that were substantially higher than both their historical levels and the frozen retail rate. Other factors, such as the lack of inventories, the long lead time required to add new generating capacity, and evidence of market power, exacerbated the problem and also contributed to the shutdown of the California Power Exchange (CalPX), which used to be the primary market for forward trading of electricity in California.

In theory, this problem could have been averted if end-use consumers were exposed

to real-time prices from the onset. A case for the adoption of real-time prices is made in [4], which postulates that their effect is to reduce the demand for electricity during peak hours. This then lowers the electricity spot price and reduces the need to build more power plants. Furthermore, the strategic role of generator hedging alone in reducing price volatility and mitigating market power is explicated in [2] and [17]. Thus, the combination of real-time pricing with long-term hedge contracts for electricity, which decrease the ability of generators to exercise market power, could have enabled the California markets to function cost-effectively.

Empirical work on price-elastic demand in electricity markets reveals the extent of the impact of a fully responsive demand-side. In [5], an empirical analysis of market power in California indicates that the elasticity of demand is a significant factor in mitigating the degree of market power. The extent of price-elastic demand in reducing prices and consumption is investigated in [8]. In the service territory of San Diego Gas & Electric (SDG&E), the retail rates to which end-users were exposed increased during the summer of 2000. It is estimated that a doubling of the retail rate results in a modest reduction in demand (approximately two percent). The fact that end-users were exposed to wholesale prices with a five-week lag and that a retroactive rate-freeze had been promised by politicians implied that the actual rate increase was not substantial. However, the inelastic nature of the electricity supply side for high levels of production implies that even modest shifts in demand will result in substantially lower prices.

Overall, while hedging instruments are commonplace in electricity markets and me-

tering technology for real-time pricing is becoming technologically feasible, in practice, there are restrictions on their usage. As a result, the impact of real-time pricing on forward markets is unclear. The objective of this paper is to assess the implications on electricity forward market operations, viz., the equilibrium price and optimal quantity traded, when a risk-averse retailer's end-use consumers are allowed to respond to real-time price signals. We model a perfectly competitive electricity industry, in which a spot market for electricity and forward markets for both electricity and AS exist. Using a spot price specification of end-use consumer response, we find that real-time pricing impacts the electricity forward price through the retailer's relative magnitudes of end-use consumer price elasticity and risk exposure.

The structure of this paper is as follows:

- in **section 2**, we describe the model of electricity production and markets.
- in **section 3**, we solve for the equilibrium prices and quantities in each market when end-use consumer demand depends on the spot price.
- in **section 4**, we summarize the main results and give direction for future research in this area.

2 Electricity Markets and Production

In this section, we model the markets for electricity and AS in order to assess the impact of price elastic end-user demand following the approach of [3] and [13] in which demand

is stochastic but completely price inelastic. We assume perfectly competitive[4] spot and forward markets for electricity and a forward market for one type of AS (as opposed to the four or more that actually exist in most markets). We analyze production decisions for only a single future time period because the non-storability of electricity creates markets that are effectively independent over time. For simplicity, we assume that all uncertainty is resolved before spot market decisions are made. Underlying this assumption is the fact that power companies are able to forecast demand in the immediate future, i.e., the next hour, with precision. Here, we also abstract from transmission constraints by supposing that electricity can be transmitted costlessly. Of course, in reality transmission bottlenecks play a significant role in determining the pattern of electricity generation and pricing. However, our focus is on the short-term strategies of market agents that will determine equilibrium prices rather than on congestion pricing. In addition, we ignore ramping constraints and unit commitment issues in order to focus solely on pricing decisions.

Although market agents are assumed to face no uncertainty while making decisions in the real-time spot market, uncertainty exists at the forward market stage. In order to incorporate uncertainty and risk aversion into our model, we assume that the objective of each market agent i is to maximize its expected utility of profit function, which is $E_\omega[U(\pi_i(\omega))] \equiv E[\pi_i(\omega)] - \frac{A_i}{2}Var(\pi_i(\omega))$. Here, ω is a random variable that depicts the state of the world, which is unknown to the market agent when making forward market

[4]The degree to which the electricity markets are competitive is open to debate. Our concern, however, is more with pricing once market mechanisms are fully in place.

decisions but is realized before making spot market decisions. Naturally, agent i's profit $\pi_i(\omega)$ depends on the state of the world. $A_i > 0$ is a risk-aversion parameter that can differ across agent types.

Within this framework, we have three distinct types of agents who have various interests in the markets:

- $n \in \mathcal{Z}_+$ **generators**: generator p_i has $\alpha_{p_i} > 0$ megawatts (MW) of production capacity available for any given period.[5] It can use this capacity either to generate electricity and sell it into the electricity markets or to reserve the capacity and sell it into the AS forward market. For selling the output from $X^S_{p_i}$ MW of capacity into the electricity spot market, generator p_i receives the endogenously determined electricity spot price P^S_X. At the forward stage, if the generator sell the output from $X^F_{p_i}$ MW of capacity into the electricity forward market, it receives the endogenously determined electricity forward price P^F_X. If it sells $Y^F_{p_i}$ MW of capacity into the AS forward market, the generator receives the endogenously determined per MW AS forward price P^F_Y.

- $m \in \mathcal{Z}_+$ **retailers**: retailer r_j purchases electricity from the spot and forward markets and sells it to end-use consumers in its exclusive franchise area at a fixed unit price of $P_{r_j} \geq 0$. The total retail demand for electricity in its area, $X_{r_j}(P^S_X)$, is uncertain at the time of the decision to purchase forward and must be satisfied. However, the dependence of total retail demand on the electricity spot price formal-

[5] This is not really a maximal capacity, but is a parameter that indexes production costs.

izes the fact that end-users respond to real-time fluctuations in the spot price. This approximates how end-users can be induced to perceive spot prices as suggested in [4] even if in California, for example, most end-use consumers are guaranteed fixed per unit prices. The retailer takes the risk of purchasing from a volatile market, which would seem to imply that retailers would like to purchase forward contracts to lock in their purchase prices. Hence, retailer r_j's purchases in the spot and forward markets ($X^S_{r_j}$ and $X^F_{r_j}$, respectively) are used to meet its retail demand.

- **an ISO**: the ISO procures enough AS from the forward market to comply with the minimum levels required for reliability, Y_I. Usually, this implies that the amount of AS procured by the ISO is approximately a fixed percentage of overall electricity demand. The ISO, thus, acquires enough AS from the forward market (Y^F_I) to meet its requirements.

As we shall show in section 3, all agents act out of self-interest in order to maximize their respective expected utilities of wealth. Their interaction in the markets then determines equilibrium prices and positions for electricity and AS, which we analyze to determine how they are affected by allowing for price-elastic end-user demand.

3 Market Trading with Single-Stage Price Settlement

Here, we solve the optimization problems of the agents introduced in section 2. We use the market-equilibrium approach developed in [3] and extended in [13] to incorporate AS

trading. With two types of markets, i.e., forward and spot, we have two time stages in the model. At the forward market stage, we assume that agents maximize their respective expected utilities of wealth without knowledge of spot market conditions. Only at the spot market stage is ω revealed, and given the forward market transactions, the agents conduct spot market transactions. In order to solve this model, we proceed backwards by first evaluating the agents' spot market problems given that uncertainty has been resolved and that forward transactions are fixed. We then step back in time to determine the optimal forward quantities traded and the equilibrium prices.

Whereas in [3] and [13] the demand faced by any given retailer was inelastic, here we incorporate the approach of [1] and [2] by allowing demand to vary with price. In order to keep the analysis tractable, we specify demand to be a linear function of the spot price. Our model also addresses AS trading by requiring the ISO to purchase a certain amount of them in order to maintain system reliability during grid contingencies.

3.1 Spot Trading

At the spot market stage, since ω is known, agents approach their optimization problems without any uncertainty. Furthermore, because all forward positions ($X_{p_i}^{F*}$, $Y_{p_i}^{F*}$, $X_{r_j}^{F*}$, and Y_I^{F*}) and prices (P_X^{F*} and P_Y^{F*}) have been determined, we treat them as fixed. Hence, the only decision to be taken at this stage is how much to transact in the spot market.

Applying the notation and assumptions of section 2, the profit-maximization problem

of **generator** p_i as follows:

$$\pi^*_{p_i}(\omega, X^{F*}_{p_i}, Y^{F*}_{p_i}) = \max_{X^S_{p_i}} \{P^S_X X^S_{p_i} + P^{F*}_X X^{F*}_{p_i} + P^{F*}_Y Y^{F*}_{p_i} - \frac{\theta}{2\alpha_{p_i}}(X^S_{p_i} + X^{F*}_{p_i} + fY^{F*}_{p_i})^2\} \quad (1)$$

where $\pi^*_{p_i}(\cdot)$ is the maximized profit level, $\theta > 0$ is the per MW input (e.g., fuel) cost, and $0 \leq f \leq 1$ denotes the fraction of AS capacity sold that is called upon to generate.[6] Fuel cost is incurred only for actual electricity generation, i.e., to produce electricity sold as energy, and to operate any AS capacity that is specifically required by the ISO to generate in response to grid contingencies. Furthermore, the cost term exhibits the quadratic form, which implies increasing marginal costs of generation. Intuitively, this models the fact that as demand increases, less efficient sources of generation are brought on line. For the purposes of this model, we assume that continuous quadratic functions reasonably approximate generation costs, even though actual generation costs may be discontinuous.

The profit-maximization problem of **retailer** r_j is:

$$\begin{aligned} \pi^*_{r_j}(\omega, X^{F*}_{r_j}) &= \max_{X^S_{r_j}} \{P_{r_j} X_{r_j} - P^S_X X^S_{r_j} - P^{F*}_X X^{F*}_{r_j}\} \\ &\quad \text{subject to } X^S_{r_j} + X^{F*}_{r_j} \geq X_{r_j}(P^S_X) \end{aligned} \quad (2)$$

where $\pi^*_{r_j}(\cdot)$ is the maximized profit level, and $X_{r_j}(P^S_X) \equiv a_{r_j} - b_{r_j} P^S_X$ is the realized total electricity demand in the franchise area of retailer r_j. This demand is linear in

[6] Usually, per MW input costs vary with the level of production, but we abstract from that in order to maintain the tractability of the model. In addition, we assume that generators have sufficient capacity to meet system demand.

the spot price and deviates from its maximum possible value, a_{r_j}, in proportion to end-user responsiveness, $b_{r_j} > 0$.[7] While a_{r_j} is stochastic at the forward stage, b_{r_j} is always deterministic. As in most decentralized systems, the AS in our model are procured by the **ISO** to fulfill system requirements, and if they are called upon to generate, the output from the AS reserves are used by the ISO to satisfy grid contingencies. Since the ISO has no role in real time, we defer the presentation of its optimization problem to section 3.2.

Since generator p_i's problem is to decide how much electricity to sell into the spot market in order to maximize its profit, its **first-order necessary condition** is:

$$\frac{\partial \pi^*_{p_i}(\omega, X^{F*}_{p_i}, Y^{F*}_{p_i})}{\partial X^S_{p_i}} = 0$$

$$\Rightarrow P^S_X - \frac{\theta}{\alpha_{p_i}}(X^{S*}_{p_i} + X^{F*}_{p_i} + fY^{F*}_{p_i}) = 0$$

$$\Rightarrow X^{S*}_{p_i} = \frac{\alpha_{p_i}}{\theta} P^S_X - X^{F*}_{p_i} - fY^{F*}_{p_i} \tag{3}$$

Since the **second-order sufficiency condition** for this problem is also satisfied, $X^{S*}_{p_i}$ represents a global maximum:

$$\frac{\partial^2 \pi^*_{p_i}(\omega, X^{F*}_{p_i}, Y^{F*}_{p_i})}{\partial X^{S\,2}_{p_i}} = -\frac{\theta}{\alpha_{p_i}} < 0. \tag{4}$$

Retailer r_j, in contrast, *must* satisfy the retail demand in its area, $a_{r_j} - b_{r_j} P^S_X$. Its spot market purchases are, therefore, equal to the retail demand in its area less its forward purchases of electricity, i.e., $X^{S*}_{r_j} = a_{r_j} - b_{r_j} P^S_X - X^{F*}_{r_j}$.

Using equation 3 together with the retailer's purchase requirement, we now solve for the equilibrium electricity spot price. In our model, the **market-clearing conditions**

[7] This indexes the price elasticity of retail demand.

are:

$$\sum_{i=1}^{n} X_{p_i}^{S*} + \sum_{i=1}^{n} f Y_{p_i}^{F*} = \sum_{j=1}^{m} X_{r_j}^{S*} \tag{5}$$

$$\sum_{i=1}^{n} X_{p_i}^{F*} = \sum_{j=1}^{m} X_{r_j}^{F*} \tag{6}$$

$$\sum_{i=1}^{n} Y_{p_i}^{F*} = Y_I^{F*} \tag{7}$$

Equation 5 states that in order for an equilibrium to occur in the electricity spot market, the total sales by the generators *plus* the total AS calls equal the total purchases by the retailers. Similarly, equations 6 and 7 ensure that total supply equals total demand in the forward markets for electricity and AS, respectively.

Solving for the equilibrium spot price, we obtain:

$$P_X^{S*} = \frac{\theta a}{\alpha + \theta b} \tag{8}$$

where $\alpha \equiv \sum_{i=1}^{n} \alpha_{p_i}$, $a \equiv \sum_{i=1}^{n} a$, $b \equiv \sum_{i=1}^{n} b$, and total system retail demand is $X_R \equiv \sum_{j=1}^{m} X_{r_j} = \frac{\alpha}{\alpha+\theta b} a$. The details of this derivation are left for appendix A. Intuitively, the electricity spot price is simply the pro-rated cost of meeting the overall electricity retail demand. Since this is a perfectly competitive market with no uncertainty at this stage, all generators are compensated at the marginal cost of production. The implication of a price-elastic demand at this stage is that the equilibrium spot price is lower here than in [13] as end-users respond to it by reducing consumption. By letting $b_{r_j} = 0$, for $j = 1, \ldots, m$, we recover the spot price from [13].

By substituting equation 8 into equation 3 and the retailer's purchase requirement, we obtain the optimal quantities sold and purchased in the spot market by generator p_i and retailer r_j, respectively:

$$X_{p_i}^{S*} = \frac{\alpha_{p_i} a}{\alpha + \theta b} - X_{p_i}^{F*} - f Y_{p_i}^{F*} \tag{9}$$

and

$$X_{r_j}^{S*} = a_{r_j} - \frac{b_{r_j} \theta a}{\alpha + \theta b} - X_{r_j}^{F*} \tag{10}$$

Compared to the case with no price response, here both quantities are reduced. In particular, generator p_i's equilibrium output is its pro-rated share of the total system retail demand (which is now reduced due to price elasticity) less its forward commitments. Similarly, retailer r_j's equilibrium purchase is the retail demand in its area (again, this is lower than in the case without price elasticity) less the quantity purchased forward.

3.2 Forward Trading

After having analyzed the spot market transactions, we now evaluate the agents' forward transactions. By maximizing their respective expected utilities of profit, the agents reveal the quantities of electricity and AS that they transact through the forward market. Applying the market-equilibrium conditions, we then assess equilibrium forward prices for both electricity and AS to examine the impact of price-elastic demand. Unlike spot market trading, at the forward stage, the random variable ω is not known.

Accounting for this uncertainty, we express generator p_i's profit as:

$$\begin{aligned}\pi_{p_i}(\omega) = \quad & P_X^{S*}(\omega)X_{p_i}^{S*}(\omega) + P_X^F X_{p_i}^F + P_Y^F Y_{p_i}^F \\ & -\frac{\theta}{2\alpha_{p_i}}(X_{p_i}^{S*}(\omega) + X_{p_i}^F + f(\omega)Y_{p_i}^F)^2 \end{aligned} \tag{11}$$

Setting $X_{p_i}^F = 0$ and $Y_{p_i}^F = 0$, we define the *unhedged* profit level:

$$\rho_{p_i}^*(\omega) = P_X^{S*}(\omega)X_{p_i}^{S*}(\omega) - \frac{\theta}{2\alpha_{p_i}}X_{p_i}^{S*^2}(\omega) \tag{12}$$

Substituting in equation 9 with $X_{p_i}^{F*} = 0$ and $Y_{p_i}^{F*} = 0$, we obtain:

$$\begin{aligned} \rho_{p_i}^*(\omega) &= \frac{\theta}{\alpha}X_R(\omega)\frac{\alpha_{p_i}}{\alpha}X_R(\omega) - \frac{\theta}{2\alpha_{p_i}}(\frac{\alpha_{p_i}}{\alpha}X_R(\omega))^2 \\ \Rightarrow \quad \rho_{p_i}^*(\omega) &= \frac{\alpha_{p_i}\theta}{2\alpha^2}X_R^2(\omega) \end{aligned} \tag{13}$$

By using equations 8 and 13 together with equation 9 as usual, we obtain:

$$\pi_{p_i}(\omega) = \quad \rho_{p_i}^*(\omega) + X_{p_i}^F(P_X^F - P_X^{S*}(\omega)) + Y_{p_i}^F(P_Y^F - f(\omega)P_X^{S*}(\omega)) \tag{14}$$

Employing the expected utility of profit function with the same absolute risk-aversion parameter $A_P > 0$ for all generators, we express generator p_i's forward stage optimization problem:

$$\begin{aligned} &\max_{X_{p_i}^F, Y_{p_i}^F}\{E[\rho_{p_i}^*(\omega)] + X_{p_i}^F(P_X^F - E[P_X^{S*}(\omega)]) + Y_{p_i}^F(P_Y^F - E[f(\omega)P_X^{S*}(\omega)]) \\ &-\frac{A_P}{2}Var(\rho_{p_i}^*(\omega) + X_{p_i}^F(P_X^F - P_X^{S*}(\omega)) + Y_{p_i}^F(P_Y^F - f(\omega)P_X^{S*}(\omega)))\} \end{aligned} \tag{15}$$

The first-order necessary conditions imply:

$$X_{p_i}^{F*} = \frac{P_X^F - E[P_X^{S*}(\omega)]}{A_P Var(P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} - \frac{Y_{p_i}^{F*}Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \tag{16}$$

and

$$Y_{p_i}^{F*} = \frac{P_Y^F - E[f(\omega)P_X^{S*}(\omega)]}{A_P Var(f(\omega)P_X^{S*}(\omega))} + \frac{Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))} - \frac{X_{p_i}^{F*} Cov(P_X^{S*}(\omega), f(\omega)P_X^{S*}(\omega))}{Var(f(\omega)P_X^{S*}(\omega))} \tag{17}$$

Intuitively, equation 16 indicates that generator p_i increases its forward sales of electricity either:

- in response to an increase in the forward price relative to the expected spot price, or
- to reduce the covariation of its unhedged profit with the spot price

Moreover, its electricity forward sales decrease if it increases its AS commitments. Analogously, its forward sales of AS as described by equation 17 are motivated by the desire for higher mean profit and lower variance of profit.[8]

Solving equations 16 and 17 simultaneously, we isolate expressions for the amount of electricity and AS sold forward by generator p_i:

$$X_{p_i}^{F*} = \frac{1}{Z(f(\omega),a(\omega),\theta,\alpha,b)}[\frac{(P_X^F - E[P_X^{S*}(\omega)])Var(f(\omega)a(\omega))}{A_P}$$
$$+Cov(\rho_{p_i}^*(\omega), P_X^{S*}(\omega))Var(f(\omega)a(\omega)) - \frac{(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Cov(a(\omega), f(\omega)a(\omega))}{A_P}$$
$$-Cov(\rho_{p_i}^*(\omega), f(\omega)P_X^{S*}(\omega))Cov(a(\omega), f(\omega)a(\omega))] \tag{18}$$

and

$$Y_{p_i}^{F*} = \frac{1}{Z(f(\omega),a(\omega),\theta,\alpha,b)}[\frac{(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Var(a(\omega))}{A_P}$$

[8] We assume that the fraction of AS required to generate, $f(\omega)$, is independent of the total system retail demand, $X_R(\omega)$.

$$+Cov(\rho^*_{p_i}(\omega), f(\omega)P^{S*}_X(\omega))Var(a(\omega)) - \frac{(P^F_X - E[P^{S*}_X(\omega)])Cov(a(\omega), f(\omega)a(\omega))}{A_P}$$

$$-Cov(\rho^*_{p_i}(\omega), P^{S*}_X(\omega))Cov(a(\omega), f(\omega)a(\omega))] \tag{19}$$

where

$$Z(f(\omega), a(\omega), \theta, \alpha, b) \equiv \frac{\theta^2}{(\alpha+\theta b)^2}\left[Var(a(\omega))Var(f(\omega)a(\omega)) - Cov^2(a(\omega), f(\omega)a(\omega))\right]$$

(which also equals $\frac{\theta^2}{(\alpha+\theta b)^2}Var(f(\omega))Var(a(\omega))E[a^2(\omega)]$). Together, these expressions reveal that generator p_i increases forward sales of one product if either its forward price increases relative to its expected spot price or the covariance between its spot price and unhedged profits increases. Conversely, it reduces forward sales of one product if the other product either becomes relatively more profitable or offers greater relative risk hedging opportunities.

Using the description from section 3.1 and substituting in the binding constraint from equation 2, we express retailer r_j's profit as:

$$\begin{aligned}
\pi_{r_j}(\omega) = \quad & P_{r_j}X_{r_j}(P^{S*}_X(\omega)) - P^F_X X^F_{r_j} - P^{S*}_X(\omega)X^{S*}_{r_j}(\omega) \\
\Rightarrow \pi_{r_j}(\omega) = \quad & P_{r_j}X_{r_j}(P^{S*}_X(\omega)) - P^F_X X^F_{r_j} - P^{S*}_X(\omega)(a_{r_j}(\omega) \\
& -b_{r_j}P^{S*}_X(\omega) - X^F_{r_j}) \\
\Rightarrow \pi_{r_j}(\omega) = \quad & (P_{r_j} - P^{S*}_X(\omega))(a_{r_j}(\omega) - b_{r_j}P^{S*}_X(\omega)) + (P^{S*}_X(\omega) - P^F_X)X^F_{r_j}
\end{aligned} \tag{20}$$

Letting $\rho^*_{r_j}(\omega) \equiv (P_{r_j} - P^{S*}_X(\omega))[a_{r_j}(\omega) - b_{r_j}P^{S*}_X(\omega)]$ be the unhedged profit level for retailer r_j, we rewrite equation 20 as:

$$\pi_{r_j}(\omega) = \quad \rho^*_{r_j}(\omega) + (P^{S*}_X(\omega) - P^F_X)X^F_{r_j} \tag{21}$$

The optimization problem of retailer r_j is to select the amount of electricity to purchase (or sell) forward in order to maximize its expected utility of profit, where $E_\omega[U(\pi_{r_j}(\omega))] \equiv E[\pi_{r_j}(\omega)] - \frac{A_R}{2}Var(\pi_{r_j}(\omega))$ and $A_R > 0$, the retail analog of A_P, is common to all retailers. Mathematically, this becomes:

$$\max_{X_{r_j}^F}\{E[\rho_{r_j}^*(\omega)] + X_{r_j}^F(E[P_X^{S*}(\omega)] - P_X^F) - \frac{A_R}{2}[Var(\rho_{r_j}^*(\omega)) \\ +X_{r_j}^{F^2}Var(P_X^{S*}(\omega)) + 2X_{r_j}^F Cov(\rho_{r_j}^*(\omega), P_X^{S*}(\omega))]\} \tag{22}$$

The resulting first-order necessary condition implies:

$$X_{r_j}^{F*} = \frac{E[P_X^{S*}(\omega)]-P_X^F}{A_R Var(P_X^{S*}(\omega))} - \frac{Cov(\rho_{r_j}^*(\omega),P_X^{S*}(\omega))}{Var(P_X^{S*}(\omega))} \tag{23}$$

Similar to equation 16, equation 23 indicates that retailer r_j's forward purchases increase in response to the bias in the spot price over the forward price. Furthermore, its forward purchases are reduced (increased) if there exists positive (negative) covariance between its unhedged profits and the electricity spot price.

The ISO's optimization problem is different from that of generator p_i or retailer r_j. Unlike other agents, the ISO has no active role in the spot market. Therefore, since the ISO faces no tradeoff between spot and forward trading, its optimization problem is not affected by risk aversion and is simply:

$$\begin{aligned} \pi_I^*(Y_I^F) &= \max_{Y_I^F} \{-P_Y^F Y_I^F\} \\ &\quad \text{subject to } \; Y_I^F \geq Y_I \equiv \gamma E[X_R(\omega)] \end{aligned} \tag{24}$$

where $\pi_I^*(\cdot)$ is the maximized profit level, Y_I^F is the amount of AS purchased by the ISO from the forward market, $0 \leq \gamma \leq 1$ is the AS requirement as a fraction of expected

total retail demand, and Y_I is its total purchase requirement.[9] Equation 24 indicates that the ISO only has to purchase enough AS forward to satisfy the forecasted reserve requirements. Hence, the ISO's transaction is:

$$\begin{aligned} Y_I^{F*} &= \gamma E[X_R(\omega)] \\ \Rightarrow Y_I^{F*} &= \frac{\gamma\alpha}{\alpha+\theta b} E[a(\omega)] \end{aligned} \tag{25}$$

Since the use of electricity and AS forwards by market agents is influenced by the covariance between unhedged profits and the spot price, We now evaluate these terms explicitly:

Lemma 3.1

$$Cov(\rho^*_{p_i}(\omega), P_X^{S*}(\omega)) = \frac{\alpha_{p_i}\theta^2}{2(\alpha+\theta b)^3} Cov(a^2(\omega), a(\omega))$$

Lemma 3.2

$$Cov(\rho^*_{p_i}(\omega), f(\omega)P_X^{S*}(\omega)) = \frac{\alpha_{p_i}\theta^2}{2(\alpha+\theta b)^3} Cov(a^2(\omega), f(\omega)a(\omega))$$

Lemma 3.3

$$Cov(\rho^*_{r_j}(\omega), P_X^{S*}(\omega)) = \frac{\theta P_{r_j}}{\alpha+\theta b} Cov(a_{r_j}(\omega), a(\omega))$$

[9] We assume that generation from AS reserves is equal in proportion across all generators, i.e., if generator p_i sold $Y_{p_i}^{F*}$ MWs of AS, then the ISO orders it to generate $f(\omega)Y_{p_i}^{F*}$ MWs of electricity in the spot market (in addition to its electricity sales through the spot and forward energy markets, $X_{p_i}^{S*}$ and $X_{p_i}^{F*}$, respectively).

$$
\begin{aligned}
&-\frac{\theta^2}{(\alpha+\theta b)^2}Cov(a_{r_j}(\omega)a(\omega),a(\omega)) \\
&+\frac{\theta^3 b_{r_j}}{(\alpha+\theta b)^3}Cov(a^2(\omega),a(\omega)) \\
&-\frac{\theta^2 b_{r_j}P_{r_j}}{(\alpha+\theta b)^2}Var(a(\omega))
\end{aligned}
$$

The proofs follow from [13]. By substituting lemmas 3.1, 3.2, and 3.3 into equations 18, 19, and 23, we obtain optimal reaction functions for generator p_i and retailer r_j:

$$
\begin{aligned}
X_{p_i}^{F*} &= \frac{1}{Z(f(\omega),a(\omega),\theta,\alpha,b)}[\frac{(P_X^F - E[P_X^{S*}(\omega)])Var(f(\omega)a(\omega))}{A_P} \\
&+\frac{\alpha_{p_i}\theta^2 Cov(a^2(\omega),a(\omega))Var(f(\omega)a(\omega))}{2(\alpha+\theta b)^3} \\
&-\frac{(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Cov(a(\omega),f(\omega)a(\omega))}{A_P} \\
&-\frac{\alpha_{p_i}\theta^2 Cov(a^2(\omega),f(\omega)a(\omega))Cov(a(\omega),f(\omega)a(\omega))}{2(\alpha+\theta b)^3}]
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
Y_{p_i}^{F*} &= \frac{1}{Z(f(\omega),a(\omega),\theta,\alpha,b)}[\frac{(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Var(a(\omega))}{A_P} \\
&+\frac{\alpha_{p_i}\theta^2 Cov(a^2(\omega),f(\omega)a(\omega))Var(a(\omega))}{2(\alpha+\theta b)^3} \\
&-\frac{(P_X^F - E[P_X^{S*}(\omega)])Cov(a(\omega),f(\omega)a(\omega))}{A_P} \\
&-\frac{\alpha_{p_i}\theta^2 Cov(a^2(\omega),a(\omega))Cov(a(\omega),f(\omega)a(\omega))}{2(\alpha+\theta b)^3}]
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
X_{r_j}^{F*} &= \frac{E[P_X^{S*}(\omega)] - P_X^F}{A_R Var(P_X^{S*}(\omega))} + \frac{Cov(a(\omega)a_{r_j}(\omega),a(\omega))}{Var(a(\omega))} \\
&-\frac{(\alpha\theta b)P_{r_j}Cov(a_{r_j}(\omega),a(\omega))}{\theta Var(a(\omega))} + b_{r_j} - \frac{\theta b_{r_j}Cov(a^2(\omega),a(\omega))}{(\alpha+\theta b)Var(a(\omega))}P_{r_j}
\end{aligned} \tag{28}
$$

According to equation 26, generator p_i increases its forward electricity sales if either the forward price is higher than the expected spot price or spot market profit covaries positively with the spot market price. Since the latter implies extreme positive retail demand realizations, i.e., intervals during which retailers avoid purchases at high spot prices, generator p_i is induced to increase its forward sales. Similarly, it reduces forward electricity sales if AS are relatively more lucrative. Its AS forward sales are analogously affected by the relative values of the AS forward price and expected spot price, the desire for removing covariation in spot market profit, and the relative attraction of electricity forward sales.

Meanwhile, retailer r_j increases electricity forward purchases if either the electricity forward price is less than the expected spot price or extreme positive demand realizations in its area covary positively with those for the entire industry. Alternatively, it reduces its forward purchases if retail revenues increase with the spot price (see the second term of equation 28. The effect of price elasticity is to induce both an increase (because its retail revenues now covary more with the spot price) and a decrease (because end-users reduce consumption) in the demand for forwards. As in [3] and [13], retailers have some degree of differentiation because both their levels of end-user price responsiveness and correlation of local demand with industry-wide demand vary. This heterogeneity is what drives retailers' desire for risk reduction and impacts forward prices accordingly.

3.3 Equilibrium Forward Prices

By using the market-clearing conditions (equations 5, 6, and 7) together with the agents' optimal forward reaction functions (equations 26, 27, 28, and 25), we assess the equilibrium forward prices for electricity and AS:

$$\begin{aligned} P_X^{F*} &= E[P_X^{S*}(\omega)] + \frac{\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta} \\ &\quad -\frac{\theta}{\alpha\eta}\left[\sum_{j=1}^{m} P_{r_j}\beta'_{r_j} - E[P_X^{S*}(\omega)](1+\gamma E[f(\omega)])\right] Var(X_R(\omega)) \end{aligned} \tag{29}$$

and

$$\begin{aligned} P_Y^{F*} &= E[f(\omega)P_X^{S*}(\omega)] + \frac{E[f(\omega)]\theta^2 Skew(X_R(\omega))}{2\alpha^2\eta} \\ &\quad -\frac{\theta}{\alpha\eta}\left[\sum_{j=1}^{m} P_{r_j}\beta'_{r_j} - E[P_X^{S*}(\omega)](1+\gamma E[f(\omega)])\right] E[f(\omega)]Var(X_R(\omega)) \\ &\quad +\frac{\theta\gamma E[P_X^{S*}(\omega)]Var(f(\omega))E[X_R^2(\omega)]}{\eta'\alpha} \end{aligned} \tag{30}$$

As in [13], $\eta \equiv \frac{n}{A_P} + \frac{m}{A_R}$ reflects the number of firms trading in the electricity markets and their degree of risk aversion; $\eta' \equiv \frac{n}{A_P}$ reflects the number of generators and their degree of risk aversion; $\beta_{r_j} \equiv \frac{Cov(X_{r_j}(\omega), X_R(\omega))}{Var(X_R(\omega))}$ is the extent to which demand in retailer r_j's franchise area is correlated with total retail demand; and

$$\beta'_{r_j} \equiv \beta_{r_j}\left(\frac{\alpha+\theta b}{\alpha}\right) - \frac{\theta}{\alpha} b_{r_j} \tag{31}$$

The latter term accounts for the change in the covariation of retailer r_j's demand with total retail demand due to price elasticity. We leave derivation of the prices for appendix B and now discuss their intuitive properties.

Equation 29 is similar in structure to the electricity forward price in [3] and [13]. Specifically, the forward price differs from the expected spot price by two terms related to statistical aspects of the total retail demand. The skewness of total retail demand increases the forward price from the expected spot price because of the retailers' desire to avoid spot market purchases during times of extreme positive demand realizations. Since retailers would like to avoid making purchases during such periods of high prices, they shift their electricity purchases from the spot to the forward market. This then induces an increase in the quantity of electricity supplied forward by generators and results in the forward price's being increased from the expected spot price.

In contrast, more profitable spot market retailing decreases the forward price from the expected spot price. As retailers decrease forward purchases, the forward price decreases in proportion to the variability of industry-wide demand. Intuitively, the greater the volatility in total retail demand (and by extension, in the spot price), the less likely are generators to decrease their quantity of electricity supplied forward when its demand decreases. Hence, without an offsetting decrease in the quantity of electricity supplied in the forward market, the forward price of electricity plummets in comparison to a case in which generator are more willing to reduce forward output. Figure 1 illustrates this effect, with supply curve S' representing the state of the world with a more volatile spot market. If demand shifts from D to D', then the resulting equilibrium price is lower with supply curve S' (the one in a more volatile spot market) than with S.

The impact of end-user price elasticity on the electricity forward price is twofold:

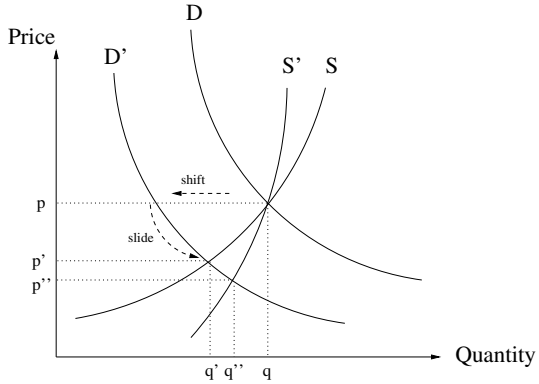

Figure 1: Impact of Increased Retailer Spot Market Profit on Forward Price

- a *direct* effect which increases the forward price
- an *indirect* effect which decreases the forward price

From equation 31, we note that the former effect arises because end-users now respond to fluctuations in the spot price, corresponding to the $\frac{\theta}{\alpha} b_{r_j}$ term. This in turn makes retail revenues more dependent on the spot price, which induces retailers to increase forward purchases of electricity to offset this increased spot market risk exposure. The resulting increase in demand for forward electricity then drives up its equilibrium price. The consequence of price responsiveness, however, is that the electricity spot price is now lower than it would be with inelastic demand. Therefore, this decreases electricity demanded forward, which then decreases the equilibrium forward price in proportion to the correlation of local demand with industry-wide demand. Indeed, the more its local demand varies with industry-wide demand, the more retailer r_j is affected by the decrease in the spot price. This industry-wide phenomenon can be traced to the $\frac{\alpha+\theta b}{\alpha}$ term in equation 31, which reflects the decreased spot price. Hence, the effect of price

elasticity on the electricity forward price is ambiguous since it depends entirely upon whether the direct or indirect effect is stronger.

Intuitively, if β_{r_j} is large relative to $\frac{b_{r_j}}{b}$, then the price responsiveness of end-users in *other* retailers' areas is chiefly responsible for the decrease in spot price. In effect, retailer r_j is put in a position of simply reacting to the price responsiveness of others by reducing its own forward purchases. The high correlation of its local demand with industry-wide demand forces it to do so. By contrast, if β_{r_j} is small relative to $\frac{b_{r_j}}{b}$, then retailer r_j has the luxury of not being forced to react to the decreased spot price. In this case, it is more motivated by the direct effect of price elasticity, which increases the risk exposure of its retail revenues to the spot price. Indeed, its own end-users reduce consumption and decrease the spot price, therefore, inducing it to increase its forward purchases. In addition, end-user price responsiveness impacts the forward price by decreasing the total retail electricity demand compared to its level in [13]. This then reduces the impact of both the skewness and variance terms in equation 29. By letting $b_{r_j} \to 0$ for $j = 1, \ldots, m$ in equation 30, we recover the result of [13] in which there is no end-user price response.

Equation 29 can be expressed as:

$$P_Y^{F*} = \frac{\theta\gamma}{\eta'\alpha} E[P_X^{S*}(\omega)] Var(f(\omega)) E[X_R^2(\omega)] + E[f(\omega)] P_X^{F*} \tag{32}$$

This represents the per MW AS forward price and has two terms: the first is a *capacity payment* that compensates the generator (at the electricity spot price) for its opportunity costs and the second is an *energy payment* for electricity generated. The up-front payment is made because by reserving capacity instead of offering electricity in the spot market,

the generator loses revenue due to foregone electricity sales from the reserves that are not called. For the energy component, since the ISO is effectively contracting forward, the generator is compensated at the forward price. Moreover, because on average only a fraction $E[f(\omega)]$ of the reserves sold into the AS forward market will be called upon to generate, this payment compensates the generator for an equivalent amount of energy sold into the forward electricity market. Hence, by thinking of AS as call options, we interpret P_Y^{F*} in terms of the classic two-part call option payoff, which includes a guaranteed up-front payment and a contingent payment if the option is exercised.

3.4 Optimal Forward Positions

By inserting equations 29 and 30 into equations 26, 27, and 28, we obtain the agents' optimal forward positions:

$$\begin{aligned} X_{p_i}^{F*} &= \frac{\alpha_{p_i}}{\alpha} E[X_R(\omega)] + \left[\frac{1}{2\eta A_P} + \frac{\alpha_{p_i}}{2\alpha}\right] \frac{Skew(X_R(\omega))}{Var(X_R(\omega))} \\ &\quad - \frac{\alpha}{\eta\theta A_P}\left[\sum_{j=1}^{m} P_{r_j}\beta'_{r_j} - E[P_X^{S*}(\omega)](1+\gamma E[f(\omega)])\right] \\ &\quad - \frac{\gamma E[f(\omega)]E[X_R(\omega)]}{n} \end{aligned} \tag{33}$$

$$Y_{p_i}^{F*} = \frac{\gamma E[X_R(\omega)]}{n} \tag{34}$$

$$\begin{aligned} X_{r_j}^{F*} &= E[X_{r_j}(\omega)] + \frac{Coskew(X_{r_j}(\omega), X_R(\omega))}{Var(X_R(\omega))} \\ &\quad - \frac{Skew(X_R(\omega))}{2\eta A_R Var(X_R(\omega))} - \frac{\alpha}{\theta}\beta'_{r_j}\left[P_{r_j} - E[P_X^{S*}(\omega)]\right] \\ &\quad + \frac{\alpha}{\theta\eta A_R}\left[\sum_{j=1}^{m} P_{r_j}\beta'_{r_j} - E[P_X^{S*}(\omega)](1+\gamma E[f(\omega)])\right] \end{aligned} \tag{35}$$

$$Y_I^{F*} = \gamma E[X_R(\omega)] \tag{36}$$

where β'_{r_j} is as defined in equation 31.

The decisions of all risk-averse agents are affected in part by motives for hedging, which then have primary and feedback implications for the forward positions. Consider generator p_i's forward sales of electricity in equation 33: they differ from its pro-rated share of expected total retail demand by three risk-related terms. The first, proportional to skewness of total retail demand, increases its forward sales because positively skewed demand spurs retailers to shift purchases into the forward market, thereby putting upward pressure on the forward price. This is relieved when generators increase the quantity of electricity sold forward. In contrast, the second term arises out of retailers' desire to avoid spot market risk by selling forward (or, reducing forward demand). This, however, decreases the forward price in proportion to retailers' spot market profitability, and in equilibrium, reduces the quantity of electricity sold forward by generators. Finally, the third term represents the AS reserves called upon to generate, and thus, decreases the electricity available to sell forward. The equilibrium quantity of AS reserves sold by generator p_i simply equals its pro-rated share of AS requirements, as indicated in equation 34. In sum, it equals the total AS purchased by the ISO in equation 36 (see [13] for further discussion of AS).

Similarly, retailer r_j's forward purchases of electricity are motivated by risk hedging. Equation 35 indicates that retailer r_j's forward purchases deviate from the expected local demand in its area by four terms. First, its forward purchases are increased

by the coskewness of local demand with industry-wide demand because higher coskewness implies greater spot market purchase costs. It is, therefore, beneficial for it to increase forward purchases to offset the risk from such events. The cumulative effect of such a response, however, is to bid up the electricity forward price. Consequently, retailer r_j reduces its quantity of electricity purchased forward. To see this, note that $Skew(X_R(\omega)) \equiv \sum_{j=1}^{m} Coskew(X_{r_j}(\omega), X_R(\omega))$. Thus, the skewness term captures the industry-wide effect of behavior motivated by the coskewness term. As discussed in section 3.3, retailer r_j offsets the risks due to increased retail profitability by selling more electricity forward, which is equivalent to decreasing its forward purchases. The fourth term of equation 35 captures this effect. However, in decreasing its forward purchases, retailer r_j puts downward pressure on the forward price which results in an increase in the quantity of electricity demanded forward. This feedback effect accounted for by the fifth term of equation 35 and illustrated in figure 1, where (p', q') represents the new equilibrium.

The overall effect of price elasticity not clear. Its direct effect is to decrease the impact of both the fourth and fifth terms in equation 35. The former results in increased forward purchases of electricity because price elasticity induces a dependency in retailer r_j's revenues with its costs (the spot price) which it offsets by diversifying its costs. Meanwhile, the latter results in reduced forward purchases of electricity because price elasticity reduces downward pressure on the forward price. Indirectly, price elasticity increases the effect of both the fourth and fifth terms in equation 35. The former follows from the fact

that price elasticity reduces the spot price, thereby causing an even greater decrease in forward purchases which hedges the risk due to increased spot market profitability. As a feedback effect, this increases the downward pressure on the electricity forward price, hence leading to the latter effect, i.e., an increase in the quantity of electricity purchased forward. Thus, the ultimate consequence of price elasticity on the forward price and trading is ambiguous. While the effect of price elasticity *may* be to increase forward purchases, recall from equation 10 that any such increases are offset by decreased spot purchases. Hence, the overall effect of price elasticity is to decrease total consumption of electricity.

4 Conclusions

One of the problems with deregulated electricity markets is hypothesized to be the absence of price responsiveness on the demand side. Unlike other competitive entities, many deregulated electricity industries are incomplete since only suppliers are able to receive and respond to fluctuations in market prices. The resulting failure to allocate electricity without often resorting to random rationing, i.e., rolling blackouts in California, was thought to be a natural consequence of this deficiency. Allowing end-users to respond to price signals, however, is a means to alleviate this inefficiency. Specifically, end-user price elasticity should decrease electricity consumption during peak hours and reduce the electricity forward price.

In order to determine the effect of price elasticity on forward prices, we use a market-

equilibrium model based loosely on a decentralized paradigm, incorporating AS requirements and an ISO. Under the assumption of perfect competition, we introduce price elasticity into the demand side by allowing end-users of electricity retailers to perceive and respond to the real-time (spot) price via a linear relationship. The electricity forward price is affected by this change through two related channels. First, increased covariation between retailers' revenues and costs in the spot market induces them to increase forward purchases of electricity in order to remove this additional risk exposure. Second, price elasticity at the real-time stage reduces the spot price relative to its level with no price elasticity, which engenders retailers to decrease forward purchases since spot purchases become more attractive. Hence, the overall effect is ambiguous because the latter response decreases the electricity forward price, while the former increases it. Nevertheless, total consumption of electricity decreased as a result of price elasticity.

For future work, we would like to extend the model to allow end-user demand to be price responsive at the forward stage. This feature will enable a retailer to "lock in" part of the demand in its area, thereby reducing the need to use electricity forwards for hedging. Another line of research is to use experimental economics to determine under which circumstances each effect of price elasticity dominates. From a theoretical point of view, our model could benefit from the introduction of an oligopolistic supply-side with a competitive fringe. Indeed, the results in this paper are driven primarily by differences among retailers, hence variation among generators is likely to add to the explanatory power of the model.

References

[1] Allaz, B. (1992), "Oligopoly, Uncertainty and Strategic Forward Transactions," *International Journal of Industrial Organization*, 10: 297-308.

[2] Allaz, B. and J.-L. Vila (1993), "Cournot Competition, Forward Markets and Efficiency," *Journal of Economic Theory*, 59: 1-16.

[3] Bessembinder, H. and M.L. Lemmon (2002), "Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets," *Journal of Finance*, 57(3): 1347-1382.

[4] Borenstein, S. (2002), "The Trouble With Electricity Markets and California's Electricity Restructuring Disaster," *Journal of Economic Perspectives*, 16(1): 191-212.

[5] Borenstein, S. and J. Bushnell (1999), "An Empirical Analysis of Market Power in California's Electricity Industry," *Journal of Industrial Economics*, 47(3): 285-323.

[6] Borenstein, S. , J. Bushnell, and C.R. Knittel (1999), "Market Power in Electricity Markets: Beyond Concentration Measures," *The Energy Journal*, 20(4): 65-83.

[7] Borenstein, S. , J. Bushnell, and F. Wolak (2002), "Measuring Market Inefficiencies in California's Deregulated Wholesale Electricity Market," *American Economic Review*, 92(5): 1376-1405.

[8] Bushnell, J. and E. Mansur (2001), "The Impact of Retail Rate Deregulation on Electricity Consumption in San Diego," POWER Working Paper PWP-082, University of California Energy Institute, Berkeley, CA, USA.

[9] Deng, S. (1999), "Financial Methods in Competitive Electricity Markets," Ph. D. thesis, IEOR Department. University of California, Berkeley, CA.

[10] Joskow, P. L. (1987), "Productivity Growth and Technical Change in the Generation of Electricity," *The Energy Journal*, 8(1): 17-38.

[11] Joskow, P. L. (1997), "Restructuring, Competition, and Regulatory Reform in the U. S. Electricity Sector," *Journal of Economic Perspectives*, 11(3): 119-138.

[12] Marnay, C. , K. Hamachi, M. Khavkin, and A.S. Siddiqui (2001), "Direct Participation of Electrical Loads in the California Independent System Operator Markets During the Summer of 2000," *Proceedings of the European Council for an Energy Efficient Economy Summer Study 2001*, Mandelieu, France.

[13] Siddiqui, A.S. (2003), "Managing Electricity Reliability Risk Through the Forward Markets," *Networks and Spatial Economics*, 3:225-263.

[14] Siddiqui, A.S. , C. Marnay, and M. Khavkin (2000), "Excessive Price Volatility in the California Ancillary Services Markets: Causes, Effects, and Solutions," *The Electricity Journal*, 13(6): 58-68.

[15] Wilson, R. (2002), "Architecture of Power Markets," *Econometrica*, 70(4): 1299-1340.

[16] Wolak, F.W. (2000), "Market Design and Price Behavior in Restructured Electricity Markets: An International Comparison," in *Pricing in Competitive Electricity Markets*, edited by A. Faruqui and K. Eakin, Kluwer Academic Publishers, Boston, MA.

[17] Wolak, F.W. (2000), "An Empirical Analysis of the Impact of Hedge Contracts on Bidding Behavior in a Competitive Electricity Market," *International Economic Journal*, 14(2): 1-39.

[18] Wolfram, C.D. (1999), "Measuring Duopoly Power in the British Electricity Spot Market," *American Economic Review*, 89: 805-826.

Appendix A: Solving for the Equilibrium Spot Market Price

Substituting equation 3 and the retailers' purchase requirements into equation 5, we obtain:

$$\sum_{i=1}^{n} \frac{\alpha_{p_i}}{\theta} P_X^S - \sum_{i=1}^{n} X_{p_i}^{F*} - \sum_{i=1}^{n} f Y_{p_i}^{F*} + \sum_{i=1}^{n} f Y_{p_i}^{F*}$$
$$= \sum_{j=1}^{m} (a_{r_j} - b_{r_j} P_X^S) - \sum_{j=1}^{m} X_{r_j}^{F*}$$

Making use of equation 6, and letting $\alpha \equiv \sum_{i=1}^{n} \alpha_{p_i}$, we arrive at the following:

$$(\frac{\alpha}{\theta} + b) P_X^S = a \tag{37}$$

This is equivalent to equation 8.

Appendix B: Solving for Equilibrium Forward Prices

We now solve for P_X^{F*} by inserting equations 26 and 28 into equation 6:

$$\frac{n(P_X^F - E[P_X^{S*}(\omega)])Var(f(\omega)a(\omega))}{A_P Z(f(\omega), a(\omega), \theta, \alpha, b)}$$
$$+ \frac{\alpha\theta^2 Cov(a^2(\omega), a(\omega)) Var(f(\omega)a(\omega))}{2(\alpha + \theta b)^3 Z(f(\omega), a(\omega), \theta, \alpha, b)}$$
$$- \frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Cov(a(\omega), f(\omega)a(\omega))}{A_P Z(f(\omega), a(\omega), \theta, \alpha, b)}$$
$$- \frac{\alpha\theta^2 Cov(a^2(\omega), f(\omega)a(\omega)) Cov(a(\omega), f(\omega)a(\omega))}{2(\alpha + \theta b)^3 Z(f(\omega), a(\omega), \theta, \alpha, b)}$$

$$
\begin{aligned}
&= \frac{m(E[P_X^{S*}(\omega)] - P_X^F)}{A_R Var(P_X^{S*}(\omega))} - \frac{\alpha \sum_{j=1}^m P_{r_j} Cov(a_{r_j}(\omega), a(\omega))}{\theta Var(a(\omega))} \\
&+ \frac{\sum_{j=1}^m Cov(a_{r_j}(\omega)a(\omega), a(\omega))}{Var(a(\omega))} - \frac{\theta b Cov(a^2(\omega), a(\omega))}{(\alpha+\theta b) Var(a(\omega))} \\
&+ \sum_{j=1}^m b_{r_j} P_{r_j}
\end{aligned}
$$

By letting $\eta \equiv n/A_P + m/A_R$, $\eta' \equiv n/A_P$, and $\beta_{r_j} \equiv \frac{Cov(a_{r_j}(\omega), a(\omega))}{Var(a(\omega))}$, and using the fact that $Cov(a^2(\omega), a(\omega)) \equiv Skew(a(\omega)) + 2E[a(\omega)]Var(a(\omega))$ and $P_X^{S*}(\omega) = \frac{\theta}{\alpha+\theta b} a(\omega)$, we obtain:

$$
\begin{aligned}
&(P_X^F - E[P_X^{S*}(\omega)]) \left[\frac{n Var(f(\omega)a(\omega))}{A_P Z(f(\omega), a(\omega), \theta, \alpha, b)} + \frac{m\alpha^2}{A_R \theta^2 Var(a(\omega))} \right] \\
&= \frac{Cov(a(\omega), f(\omega)a(\omega))}{Z(f(\omega), a(\omega), \theta, \alpha, b)} [\frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])}{A_P} \\
&+ \frac{\alpha\theta^2 Cov(a^2(\omega), f(\omega)a(\omega))}{2(\alpha+\theta b)^3}] + Cov(a^2(\omega), a(\omega))[\frac{1}{Var(a(\omega))} \\
&- \frac{\alpha\theta^2 Var(f(\omega)a(\omega))}{2(\alpha+\theta b)^3 Z(f(\omega), a(\omega), \theta, \alpha, b)} - \frac{\theta b}{(\alpha+\theta b) Var(a(\omega))}] \\
&- \frac{(\alpha+\theta b)\sum_{j=1}^m P_{r_j}\beta_{r_j}}{\theta} + \sum_{j=1}^m b_{r_j} P_{r_j} \\
\Rightarrow\ & \frac{(P_X^F - E[P_X^{S*}(\omega)])}{Z(f(\omega), a(\omega), \theta, \alpha, b)} \left[\frac{n Var(f(\omega)a(\omega))}{A_P} + Var(f(\omega))E[a^2(\omega)]\frac{m}{A_R} \right] \\
&= \frac{Cov(a(\omega), f(\omega)a(\omega))}{Z(f(\omega), a(\omega), \theta, \alpha, b)} [\frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])}{A_P} \\
&+ \frac{\alpha\theta^2 Cov(a^2(\omega), f(\omega)a(\omega))}{2(\alpha+\theta b)^3}] \\
&+ \frac{Cov(a^2(\omega), a(\omega))}{Z(f(\omega), a(\omega), \theta, \alpha, b)} [\frac{\theta^2 Var(f(\omega))E[a^2(\omega)]}{(\alpha+\theta b)^2} - \frac{\alpha\theta^2 Var(f(\omega)a(\omega))}{2(\alpha+\theta b)^3} \\
&- \frac{\theta^3 b Var(f(\omega))E[a^2(\omega)]}{(\alpha+\theta b)^3}] - \frac{(\alpha+\theta b)}{\theta} \sum_{j=1}^m P_{r_j}\beta_{r_j} + \sum_{j=1}^m b_{r_j} P_{r_j} \\
\Rightarrow\ & (P_X^F - E[P_X^{S*}(\omega)])[\frac{n}{A_P}(Var(f(\omega))Var(a(\omega)) + (E[f(\omega)])^2 Var(a(\omega)) \\
&+ (E[a(\omega)])^2 Var(f(\omega))) + \frac{m}{A_R} Var(f(\omega))E[a^2(\omega)]] \\
&= Cov(a(\omega), f(\omega)a(\omega))[\frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])}{A_P}
\end{aligned}
$$

$$+\frac{\alpha\theta^2 Cov(a^2(\omega), f(\omega)a(\omega))}{2(\alpha+\theta b)^3}] + Cov(a^2(\omega), a(\omega))[\frac{\theta^2 Var(f(\omega))E[a^2(\omega)]}{(\alpha+\theta b)^2}$$

$$-\frac{\theta^3 b Var(f(\omega))E[a^2(\omega)]}{(\alpha+\theta b)^3} - \frac{\theta^2\alpha}{2(\alpha+\theta b)^3}(Var(f(\omega))Var(a(\omega))$$

$$+(E[f(\omega)])^2 Var(a(\omega)) + (E[a(\omega)])^2 Var(f(\omega)))]$$

$$-\frac{(\alpha+\theta b)Z(f(\omega), a(\omega), \theta, \alpha, b)}{\theta}\sum_{j=1}^{m} P_{r_j}\beta_{r_j} + Z(f(\omega), a(\omega), \theta, \alpha, b)\sum_{j=1}^{m} b_{r_j}P_{r_j}$$

$$\Rightarrow \quad (P_X^F - E[P_X^{S*}(\omega)])[\frac{n}{A_P}Var(f(\omega))E[a^2(\omega)] + \frac{n}{A_P}(E[f(\omega)])^2 Var(a(\omega))$$

$$+\frac{m}{A_R}Var(f(\omega))E[a^2(\omega)]]$$

$$= \frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(a(\omega))}{A_P}$$

$$+\frac{\alpha\theta^2 Cov(a^2(\omega), a(\omega))(E[f(\omega)])^2 Var(a(\omega))}{2(\alpha+\theta b)^3}$$

$$+Cov(a^2(\omega), a(\omega))[(\frac{\theta^2}{(\alpha+\theta b)^2} - \frac{\theta^3 b}{(\alpha+\theta b)^3} - \frac{\alpha\theta^2}{2(\alpha+\theta b)^3})Var(f(\omega))E[a^2(\omega)]$$

$$-\frac{\alpha\theta^2(E[f(\omega)])^2 Var(a(\omega))}{2(\alpha+\theta b)^3}]$$

$$-\frac{(\alpha+\theta b)Z(f(\omega), a(\omega), \theta, \alpha, b)}{\theta}\sum_{j=1}^{m} P_{r_j}\beta_{r_j} + Z(f(\omega), a(\omega), \theta, \alpha, b)\sum_{j=1}^{m} b_{r_j}P_{r_j}$$

$$\Rightarrow \quad (P_X^F - E[P_X^{S*}(\omega)])[\eta Var(f(\omega))E[a^2(\omega)] + \eta'(E[f(\omega)])^2 Var(a(\omega))]$$

$$= \frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(a(\omega))}{A_P}$$

$$+\frac{\alpha\theta^2 Cov(a^2(\omega), a(\omega))(E[f(\omega)])^2 Var(a(\omega))}{2(\alpha+\theta b)^3}$$

$$-\frac{\alpha\theta^2 Cov(a^2(\omega), a(\omega))(E[f(\omega)])^2 Var(a(\omega))}{2(\alpha+\theta b)^3}$$

$$+\frac{\theta^2 Cov(a^2(\omega), a(\omega))Var(f(\omega))E[a^2(\omega)]}{(\alpha+\theta b)^2}[1 - \frac{\theta b}{\alpha+\theta b} - \frac{\alpha}{2(\alpha+\theta b)}]$$

$$-\frac{(\alpha+\theta b)Z(f(\omega), a(\omega), \theta, \alpha, b)}{\theta}\sum_{j=1}^{m} P_{r_j}\beta_{r_j} + Z(f(\omega), a(\omega), \theta, \alpha, b)\sum_{j=1}^{m} b_{r_j}P_{r_j}$$

$$\Rightarrow \quad (P_X^F - E[P_X^{S*}(\omega)])[\eta Var(f(\omega))E[a^2(\omega)] + \eta'(E[f(\omega)])^2 Var(a(\omega))]$$

$$= \frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(a(\omega))}{A_P}$$

$$
\begin{aligned}
&+\frac{\alpha\theta^2 Var(f(\omega))E[a^2(\omega)]Skew(a(\omega))}{2(\alpha+\theta b)^3} \\
&+\frac{\alpha\theta^2 Var(f(\omega))E[a^2(\omega)]E[a(\omega)]Var(a(\omega))}{(\alpha+\theta b)^3} \\
&+Z(f(\omega),a(\omega),\theta,\alpha,b)\sum_{j=1}^{m} P_{r_j}\left[b_{r_j}-\beta_{r_j}\frac{(\alpha+\theta b)}{\theta}\right] \\
\Rightarrow\quad & (P_X^F - E[P_X^{S*}(\omega)])[\eta Var(f(\omega))E[a^2(\omega)]+\eta'(E[f(\omega)])^2 Var(a(\omega))] \\
&=\frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(a(\omega))}{A_P} \\
&+\frac{\alpha\theta^2 Var(f(\omega))E[a^2(\omega)]Skew(X_R(\omega))}{2(\alpha+\theta b)^3} \\
&-\frac{(\alpha+\theta b)Z(f(\omega),a(\omega),\theta,\alpha,b)}{\theta}\left[\sum_{j=1}^{m} P_{r_j}\beta_{r_j}-E[P_X^{S*}(\omega)]\right] \\
&-bZ(f(\omega),a(\omega),\theta,\alpha,b)E[P_X^{S*}(\omega)]+Z(f(\omega),a(\omega),\theta,\alpha,b)\sum_{j=1}^{m} b_{r_j}P_{r_j} \\
\Rightarrow\quad & P_X^{F*}=E[P_X^{S*}(\omega)]+\frac{n(P_Y^{F*}-E[f(\omega)P_X^{S*}(\omega)])E[f(\omega)]Var(a(\omega))}{A_P(\eta Var(f(\omega))E[a^2(\omega)]+\eta'(E[f(\omega)])^2 Var(a(\omega)))} \\
&+\frac{\alpha\theta^2 Var(f(\omega))E[a^2(\omega)]Skew(a(\omega))}{2(\alpha+\theta b)^3(\eta Var(f(\omega))E[a^2(\omega)]+\eta'(E[f(\omega)])^2 Var(a(\omega)))} \\
&-\frac{(\alpha+\theta b)Z(f(\omega),a(\omega),\theta,\alpha,b)}{\theta(\eta Var(f(\omega))E[a^2(\omega)]+\eta'(E[f(\omega)])^2 Var(a(\omega)))}\left[\sum_{j=1}^{m} P_{r_j}\beta_{r_j}-E[P_X^{S*}(\omega)]\right] \\
&-\frac{Z(f(\omega),a(\omega),\theta,\alpha,b)}{\eta Var(f(\omega))E[a^2(\omega)]+\eta'(E[f(\omega)])^2 Var(a(\omega))}\left[bE[P_X^{S*}(\omega)]-\sum_{j=1}^{m} b_{r_j}P_{r_j}\right] \quad (38)
\end{aligned}
$$

We now arrive at a similar expression for P_Y^{F*} by inserting equations 27 and 25 into equation 7:

$$
\begin{aligned}
&\frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Var(a(\omega))}{A_P} \\
&+\frac{\alpha\theta^2 Cov(a^2(\omega),f(\omega)a(\omega))Var(a(\omega))}{2(\alpha+\theta b)^3} \\
&-\frac{n(P_X^F - E[P_X^{S*}(\omega)])Cov(a(\omega),f(\omega)a(\omega))}{A_P} \\
&-\frac{\alpha\theta^2 Cov(a^2(\omega),a(\omega))Cov(a(\omega),f(\omega)a(\omega))}{2(\alpha+\theta b)^3}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\alpha\gamma}{\alpha+\theta b} E[a(\omega)] Z(f(\omega), a(\omega), \theta, \alpha, b) \\
\Rightarrow \quad & \frac{n(P_Y^F - E[f(\omega)P_X^{S*}(\omega)])Var(a(\omega))}{A_P} = \frac{\alpha\gamma E[a(\omega)] Z(f(\omega), a(\omega), \theta, \alpha, b)}{\alpha+\theta b} \\
& - \frac{\alpha\theta^2 Cov(a^2(\omega), f(\omega)a(\omega)) Var(a(\omega))}{2(\alpha+\theta b)^3} \\
& + \frac{n(P_X^F - E[P_X^{S*}(\omega)])Cov(a(\omega), f(\omega)a(\omega))}{A_P} \\
& + \frac{\alpha\theta^2 Cov(a^2(\omega), a(\omega)) Cov(a(\omega), f(\omega)a(\omega))}{2(\alpha+\theta b)^3} \\
\Rightarrow \quad & P_Y^F - E[f(\omega)P_X^{S*}(\omega)] = \frac{\alpha\gamma E[a(\omega)] Z(f(\omega), a(\omega), \theta, \alpha, b)}{(\alpha+\theta b)\eta' Var(a(\omega))} \\
& + \frac{(P_X^F - E[P_X^{S*}(\omega)])Var(a(\omega))E[f(\omega)]}{Var(a(\omega))} \\
& + \frac{\alpha\theta^2 Cov(a^2(\omega), a(\omega)) Var(a(\omega)) E[f(\omega)]}{2(\alpha+\theta b)^3 \eta' Var(a(\omega))} \\
& - \frac{\alpha\theta^2 Cov(a^2(\omega), f(\omega)a(\omega)) Var(a(\omega))}{2(\alpha+\theta b)^3 \eta' Var(a(\omega))} \\
\Rightarrow \quad & P_Y^{F*} = E[f(\omega)P_X^{S*}(\omega)] + \frac{\alpha\gamma\theta^2 E[a(\omega)] Var(f(\omega)) E[a^2(\omega)]}{\eta'(\alpha+\theta b)^3} \\
& + E[f(\omega)](P_X^{F*} - E[P_X^{S*}(\omega)]) \qquad (39)
\end{aligned}
$$

By solving equations 38 and 39 simultaneously, we arrive at equations 29 and 30.