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Abstract

The theory of strategic trade policy yields ambiguous recommendations for assistance to exporting firms in oligopolistic industries. However, some writers have suggested that investment subsidies are a more robust recommendation than export subsidies. We show that, though ambiguous in principle, the case for investment subsidies is reasonably robust in practice. Except when functional forms exhibit arbitrary non-linearities, it holds under both Cournot and Bertrand competition, with either cost-reducing or market-expanding investment, and with or without spillovers. Only if firms have strong asymmetries in their investment behaviour and engage in Bertrand competition is an investment tax clearly justified.

JEL: F12, L13.

Keywords: cost-reducing investment; export subsidies; market-expanding investment; R&D subsidies; strategic industrial policy; strategic trade policy.

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1. Introduction

Enormous sums are spent by governments throughout the world in direct assistance to private industry. Yet economic theory provides little guidance on how these funds should be disbursed. This is even true of the theory of strategic trade policy, the branch of international trade theory which comes closest to the concerns of governments seeking to foster "national champions". Even when the interests of consumers and foreign firms are ignored, the theory yields ambiguous recommendations for assistance to exporting firms in oligopolistic industries. Whereas Brander and Spencer (1985) demonstrated that an export subsidy is optimal when firms compete in quantities, Eaton and Grossman (1986) showed that if firms compete in prices then an export tax is optimal. Since there are no strong grounds for determining whether quantity or price competition is more plausible, the theory falls at the first hurdle in providing usable guidelines for policy making.

However, in recent years a different strand of the literature has reexamined this issue. In particular, it has considered the desirability of "industrial policy", in the sense of subsidies to spending (such as investment in marketing or R&D) which is incurred prior to direct market competition between firms. Brander (1995) conjectured that investment subsidies are a more robust recommendation than export subsidies. Evidence for this view has been provided by Spencer and Brander (1983), Bagwell and Staiger (1994), Maggi (1996) and Neary and Leahy (2000) in a number of special models.¹ All the studies which have been

¹ Spencer and Brander (1983) were the first to consider this issue, in a model where firms first invest in cost-reducing R&D and then engage in Cournot competition. Bagwell and Staiger (1994) considered the case of R&D which reduces costs stochastically, followed by Bertrand competition with linear demands. In the case which most resembles the deterministic one (where R&D reduces the mean but does not alter the variance of the cost distribution), they found that an R&D subsidy is optimal. Maggi (1996) considered a simplified version of the model of Kreps and Scheinkman (1983) in which second-period competition is Bertrand, but the outcome of the full game may resemble that of either a Cournot or a Bertrand one-period game, depending on the slope of the marginal cost function. Neary and Leahy (2000) provide a general framework for this literature, and stress the second-best nature of the case where an export subsidy is unavailable.
carried out to date find that, when exports cannot be subsidised directly, subsidies to investment are justified irrespective of the nature of product-market competition. Why one form of assistance should be apparently so robust while the other is not seems worthy of investigation.

A different reason for interest in industrial policy is practical. Explicit export subsidies are prohibited by the World Trade Organisation, but similar bans do not apply to investment subsidies. (Although they may be constrained in other ways: for example, the European Commission limits the extent of assistance which can be given to national firms.) Therefore from a policy perspective, it is more pertinent to consider the optimality of subsidies to investment than to exports. By contrast, most of the literature on policy towards oligopolistic firms in open economies has concentrated on trade policy, either in isolation or in conjunction with optimal industrial policy. (See Brander (1995) and Neary and Leahy (2000) for overviews.)²

In this paper we reexamine the robustness of industrial policy. In particular, we explore the robustness of the Brander conjecture, that positive investment subsidies are optimal irrespective of the nature of market competition between firms. We first present in Section 2 a relatively general model and derive the optimal investment subsidy. We then turn in Section 3 to a range of special cases and try to isolate the forces which work for and against subsidising investment. Section 4 concludes with a summary of results.

² Yet another consideration in favour of concentrating on industrial policy is that, before investment decisions have been made, governments are more likely to be able to commit to investment than to export subsidies. As Leahy and Neary (1996, 1999) and Grossman and Maggi (1998) have shown, intervention when governments cannot commit to subsidies may lead to lower welfare than free trade.
2. Optimal Industrial Policy in the General Model

2.1 The Firms' Decision Problem

We consider a model in which a home and a foreign firm compete on a single market. The full game is a three-stage one, with the home government setting the level of an investment subsidy \( s \) in the first stage. In the second stage, corresponding to the pre-market period, the home and foreign firms choose investment levels \( k \) and \( k^* \) respectively. Investment may reduce production costs, may shift the demand function facing the firm, or both. Finally, in the third stage, corresponding to the market period, the firms choose "actions" \( A \) and \( B \) respectively. These actions may be either outputs or prices. Each firm’s profits depend on its own and its rival's investment levels and market actions and on the home government's investment subsidy, \( s \). The home firm’s total profits \( \Pi \) are:

\[
\Pi(k,k^*,A,B,s) = \pi(k,k^*,A,B) + sk
\]  

(1)

where \( \pi \) represents operating profits (sales revenue less total costs) and \( sk \) is the firm's subsidy income. The foreign firm's profits are determined in the same way, except that we simplify by assuming that it does not receive any subsidies from its government. It therefore maximises the function:

\[
\pi^*(k,k^*,A,B)
\]  

(2)

The nature of interactions between firms is quite general in this specification. In particular, each firm’s profits may depend directly on its rival’s investment level as well as on its own. These cross-dependencies (of \( \pi \) on \( k^* \) and \( \pi^* \) on \( k \)) reflect direct spillovers between firms.

It is natural to assume that investment decisions are taken before decisions on prices or quantities. This in turn makes it natural to restrict attention to the case of a sub-game perfect Nash equilibrium. Facing a given home subsidy, firms choose their investment levels
in the pre-market period, taking into account the impact of these choices in the market period. Then, after investments are sunk, they choose their market actions given production costs and demands determined by past investment decisions. The market-period Nash equilibrium in actions is characterised by the first-order conditions:

$$\pi_A(k, k^*, A, B) = 0 \quad \text{and} \quad \pi_B^*(k, k^*, A, B) = 0$$

(3)

for the home and foreign firms respectively. The solutions, A and B, to (3) depend only on $k$ and $k^*$. (The subsidy only affects these actions indirectly through the investment decisions.) Hence operating profits can be written as functions of $k$ and $k^*$. The resulting "reduced-form" operating profit functions are denoted by circumflexes:

$$\hat{\pi}(k, k^*) = \pi[k, k^*, A(k, k^*), B(k, k^*)]$$

(4)

and

$$\hat{\pi}^*(k, k^*) = \pi^*[k, k^*, A(k, k^*), B(k, k^*)]$$

(5)

for the home and foreign firms respectively.

Consider next the investment decisions in the pre-market period. For the home firm, it maximises a reduced-form total profit function, equal to (4) plus subsidy revenue:

$$\hat{\Pi}(k, k^*, s) = \hat{\pi}(k, k^*) + sk$$

(6)

Its first-order condition for optimal choice of investment is therefore:

$$\hat{\Pi}_k(k, k^*, s) = \hat{\pi}_k(k, k^*) + s = 0$$

(7)

In words, the marginal return to investment, taking account of its strategic effect on the actions in the market period, plus the investment subsidy, must equal zero.

Similar derivations apply to the foreign firm, with the simplification that its profits do not depend directly on the investment subsidy. Maximising (5) gives the foreign firm’s first-
order condition for investment:

\[ \hat{\pi}^*(k, k^*) = 0 \]  \hspace{1cm} (8)

This is a single equation in the two investment levels only. Hence it implicitly defines the foreign firm’s investment reaction function, giving \( k^* \) as a function of \( k \). This function will play a crucial role below.

### 2.2 Welfare and the Optimal Investment Subsidy

Consider next the behaviour of the home government. In order to focus on strategic trade reasons for industrial policy we ignore domestic consumption and assume that all output is exported. Hence welfare equals the home firm's profits net of subsidy payments:

\[
W(k, k^*) = \hat{\Pi}(k, k^*, s) - sk = \hat{\pi}(k, k^*)
\]  \hspace{1cm} (9)

Because the welfare function is simply the home firm’s operating profits, it depends directly on home and foreign investment only and not on the subsidy. Totally differentiate (9) and make use of the first-order condition (7) to obtain:

\[
dW = -sk + \hat{\pi}_k dk^* \]  \hspace{1cm} (10)

This gives the change in welfare as a function of changes in investment levels. To solve this problem with two targets (\( k \) and \( k^* \)) and one instrument (\( s \)), it is helpful to think of the subsidy as giving the government direct control over home investment, while foreign investment responds according to the foreign firm’s investment reaction function. Setting the change in welfare \( dW \) equal to zero in (10) gives the solution for the optimal subsidy:
Following Brander (1995, Section 3.3.4), the sign of the optimal subsidy depends on two key terms. First is the effect of higher foreign investment on home profits, given by the derivative \( \hat{\pi}_k^* \). When this is negative, we say that foreign investment is "unfriendly" to the home firm. Second is the term \( dk^*/dk \), the slope of the foreign investment reaction function. When this is negative, so the reaction function is downward-sloping, we say, following Bulow et al. (1985), that foreign investment is a "strategic substitute" for home investment. Using the foreign firm’s first-order condition (8), the slope of the foreign reaction function can be written in turn as:

\[
\frac{dk^*}{dk} = - \frac{\hat{\pi}^*_{k,k}}{\hat{\pi}^*_{k,k} - \hat{\pi}^*_k} \tag{12}
\]

The denominator is negative from the foreign firm’s second-order condition for profit maximisation. Hence the crucial term is the numerator, which indicates whether foreign investment is a strategic substitute (numerator negative) or strategic complement (numerator positive) for home investment.

### 2.3 Inside the Black Box

So far, this is just as in Brander (1995): the sign of the optimal subsidy depends solely on "friendliness" and "strategic substitutability", expressed in terms of the first and second derivatives of the reduced-form profit functions. Brander’s conjecture that investment subsidies are a robust policy rule relies on the presumption that investments (especially in cost-reducing R&D) are likely to be unfriendly and strategic substitutes, the two negative signs cancelling to give a positive sign for the optimal subsidy in (11).
To investigate whether these presumptions are general, we need to go further by restating these derivatives in terms of the derivatives of the original profit functions, (1) and (2). Going inside the "black box" of the reduced-form profit functions leads to more rather than less complicated expressions for the key terms in the optimal subsidy formula (11). However, it allows us to focus on the underlying determinants of optimal industrial policy and also paves the way for considering a range of special cases in the next section.

Consider first the friendliness term $\pi^{*}_{k*}$. Using the definition of the reduced-form profit function $\hat{\pi}$ in (4), this can be expressed as follows:

$$\hat{\pi}_{k*} = \pi_{k*} + \pi_{B}B_{k*} \quad (13)$$

(where $\pi_{A}$ has been set to zero since $A$ is chosen optimally in the third stage.) Friendliness therefore depends on two effects. First is a pure spillover effect, represented by the term $\pi_{k*}$. Second is a strategic effect, which depends on how increased foreign investment affects home profits, through its effect on the foreign firm’s own action in the market period. From Fudenberg and Tirole (1984), we know that many kinds of investment have a negative strategic effect, tending (in their terminology) to make the investing firm "tough" by lowering its rival’s profits. But other kinds of investment make the investing firm "soft". (We will see examples of both types in Section 3.) So there is no presumption that investment levels are unfriendly, even in the absence of spillovers.3

Consider next the strategic substitutability term. Return to the foreign firm’s marginal profitability of investment, equation (8), but now use (5) to express it in terms of the

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3 Fudenberg and Tirole’s categorisation of investment differs from Brander’s in two respects. First, they do not consider pure spillover effects, as given by the first term on the right-hand side of (13). Second, they assume a qualitative symmetry of the effects of market-period actions on rival’s profits, so that $\pi_{A}^{*}$ has the same sign as $\pi_{B}$. As we shall see in Section 3, this is a crucial assumption.
derivatives of the original profit function:

$$\pi_{k^*} = \pi_{k^*} + \pi_A^* A_{k^*}$$  \hspace{1cm} (14)

(where once again the direct effect $\pi_B^*$ has been set to zero since $B$ is chosen optimally in the third stage.) Differentiating this with respect to $k$ gives:

$$\hat{\pi}_{k^*} = \pi_{k^*} + \pi_{A^*} A_k + \pi_{k^*B} B_k$$

$$+ \left( \pi_{A_A^*} A_{k^*} + \pi_{AB} B_k \right) A_k + \pi_{A^*} A_{k^*k}$$  \hspace{1cm} (15)

In a sense, equation (15) is the main result of the paper. However, for the present, its implications are largely negative. Without further restrictions there is little basis for determining the sign of this expression.\(^4\) For example, the first two terms in each line, $\pi_{k^*k}$ and $\pi_{A^*}$, reflect the role of spillovers whereby changes in home investment $k$ directly affect the responsiveness of the foreign firm’s profits to changes in $k^*$ and $A$. As we will see, spillovers have very different effects depending on whether they arise on the production side (such as R&D spillovers) or the demand side (such as marketing or consumer switching cost spillovers). Even if we rule out direct spillovers, so that $\pi^*$ does not depend directly on $k$ and thus $\pi_{k^*k} = \pi_{A^*k} = 0$, there is still a key ambiguity in the final term in (15), $A_{k^*k}$. This term can never be signed in general since it depends on the third derivatives of the home and foreign profit functions.

These considerations cast doubt on the suggestion of Brander (1995) that investment levels are always likely to be strategic substitutes. However, it would be going too far to rest a case for non-robustness of the positive-investment-subsidy policy rule on such

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\(^4\) The terms giving the effects of investment on market-period actions ($A_k$, $B_k$, etc.) can be calculated explicitly, as in equation (27) in the Appendix. However, substituting these solutions into (15) does not help in either interpreting or signing the expression.
considerations. The theory of oligopoly has many examples of results which are not robust to changes in functional form but which are nevertheless accepted as convenient rules of thumb.\textsuperscript{5} To throw more light on the issue we need to turn instead to consider the sign of the optimal subsidy under plausible special assumptions, while keeping equations (13) and (15) in mind.

3. Some Special Cases

We begin with a benchmark case in which competition is Cournot, investment serves to reduce costs, there are no spillovers and firms are symmetric. We then consider the effects of relaxing these assumptions in turn. Throughout, we concentrate on simple functional forms: linear demands and investment functions and quadratic investment cost functions. Rather than solving each game in full, we need only calculate the derivatives of the profit functions (using the results in the Appendix) and substitute them into the general expressions (13) and (15). The results are summarised in Table 1.

3.1 Cost-Reducing Investment followed by Cournot Competition

R&D incurs quadratic costs in the pre-market period, equal to $\gamma k^2/2$ and $\gamma k^*^2/2$ for the home and foreign firms respectively. The benefits accrue in the form of reductions in the marginal cost parameters $c$ and $c^*$ (assumed to be independent of output):

$$
\begin{align*}
    c &= c_0 - \theta k \\
    c^* &= c_0 - \theta k^*
\end{align*}
$$

(16)

As for demands, we allow for differentiated products, with the prices of home and foreign

\textsuperscript{5} Examples include the assumption that outputs are strategic substitutes in Cournot competition and strategic complements in Bertrand competition, both of which can fail to hold for particular configurations of the second derivatives of the demand functions.
goods denoted by $p$ and $q$ respectively:

$$p = a - b(x + ey)$$
$$q = a^* - b(y + ex)$$

(17)

(where $e \leq 1$ is an inverse measure of the degree of product differentiation). Under these assumptions it is immediate that investment levels are unfriendly: an increase in foreign investment raises the equilibrium level of foreign output in the market period, and this directly reduces home sales and profits. Less immediate, though just as intuitive, is that investment levels are strategic substitutes. A rise in home investment lowers foreign sales, which, because the benefits of lower costs are spread more thinly, directly reduces the marginal profitability of foreign investment (the first, non-strategic, term on the right-hand side of (14)). A rise in home investment also reduces the foreign firm’s return from pushing the home firm down its output reaction function, so reducing the second strategic term on the right-hand side of (14)). For both reasons, higher home investment lowers the marginal profitability of foreign investment, and so investment levels are strategic substitutes. It follows immediately that the optimal policy is an investment subsidy. By encouraging more home investment, this reduces equilibrium foreign investment, which in turn shifts profits towards the home firm.

It is instructive to look at the explicit expressions for the two key terms. (They can

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6 Allowing for differentiated products makes little difference in the Cournot case. The extra notation is justified since it facilitates comparison with the Bertrand case, where we need to assume that products are differentiated in order to guarantee the existence of an equilibrium in pure strategies.

7 Under the linear-quadratic assumptions made here, many of the terms in (15) vanish, and the expression reduces to: $\pi^{*}_{1+k} = (\pi^{*}_{1+k} + \pi^{*}_{1+k}A_{1+k})B_k$. These two remaining terms give respectively the changes in the non-strategic and strategic terms in (14) induced by an increase in $k$. Their values can be read from the first row of Table 1.

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be read from the first row of Table 1, setting the spillover parameter \( \phi \) equal to zero.) The friendliness term in equation (13) can be written as:

\[
\hat{\pi}_k = -\frac{e}{4-e^2} b x
\]

This confirms that investment levels are unfriendly. Similarly, the strategic substitutability term from equation (15) is:

\[
\hat{\pi}^*_k = -\frac{e}{4-e^2} \mu \eta \gamma
\]

So investment levels are strategic substitutes. Moreover, it is clear that the determinants of friendliness and strategic substitutability are very similar. We will see this pattern recurring below.

The linear-quadratic model of this sub-section is a special case of that of Spencer and Brander (1983), who allowed for a non-linear demand function, and showed that, with appropriate additional restrictions, an investment subsidy remains optimal. Unfortunately, this robustness does not extend to the other models we consider.

3.2 Bertrand Competition

How are these results affected when firms engage in price or Bertrand competition

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8 Here and later, as in our earlier work (Leahy and Neary (1996) and Neary and Leahy (2000), we use two composite parameters to express the results in a more compact way. The first, \( \eta \), measures the non-strategic relative return to investment: \( \eta = \theta / b \gamma \). In the absence of strategic behaviour, the first-order condition for investment is: \( \pi_k = \theta x - \gamma k = 0 \). Hence the non-strategic return to investment is \( \theta \); while its cost can be measured by the induced fall in price: \( dp/dk = (\partial p/\partial x)(\partial x/\partial k) = b \gamma \theta \). The ratio of these gives \( \eta \). The second parameter, \( \mu \), measures the strategic component in the marginal return to investment per unit output: \( \mu = 4/(4-e^2) \). The first-order condition including the strategic effect is \( \pi_k = \mu \theta x - \gamma k = 0 \), so when (as here) \( \mu \) exceeds one, there is strategic over-investment. Both of the parameters \( \eta \) and \( \mu \) must be positive.
rather than quantity or Cournot competition? To examine this case it is convenient to switch from inverse to direct demand functions: 9

\[ x = \alpha - \beta (p-eq) \]
\[ y = \alpha^* - \beta (q-ep) \]  

(20)

It is well-known that actions (prices in this case) are now strategic complements rather than strategic substitutes: \( \pi_{AB}^* \) is positive rather than negative. Less well-known, though clear from a careful reading of Bagwell and Staiger (1994), is that investment subsidies remain optimal, if exports cannot be directly taxed or subsidised. The detailed chain of causation is different. For example, higher home investment now lowers rather than raises the foreign firm’s action (its price). But the effect on foreign profits is the same as in the Cournot case: they fall, so investment levels are unfriendly. Similarly, the different terms in the key second cross-derivative (15) have little in common (compare the first and second rows in Table 1) yet it remains true that investment levels are strategic substitutes. Explicit calculation (using the expressions in the second row of Table 1, with the spillover parameter \( \phi \) set equal to zero) shows that the expressions for the friendliness and strategic substitutability terms are identical to those in the Cournot case, equations (18) and (19) (except that \( \varepsilon \) replaces \( e \)). Hence an investment subsidy is still optimal.

3.3 Market-Expanding Investment

The next application we consider is to the case of market-expanding investment. The

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9 If \( e<1 \) and all demand parameters are independent of investment levels, the direct demand functions in (20) can be derived from the inverse demand functions in (17), and \( \varepsilon = e \). However, with market-expanding investment as in Sections 3.3 and 3.4 below, the two sets of demand functions are not directly related. Hence we do not require that \( \varepsilon \) equal \( e \) in all cases.
simplest way to model this is to assume that investment raises the intercepts of the demand functions. For the inverse demand functions this implies:

\[ a = a_0 + \theta k \quad \text{and} \quad a^* = a_0 + \theta k^* \]  

(21)

Similarly for the direct demand functions: \(^{10}\)

\[ a = a_0 + \beta \theta k \quad \text{and} \quad a^* = a_0 + \beta \theta k^* \]  

(22)

As in the case of cost-reducing investment we assume quadratic investment costs: \(\gamma k^2/2\) and \(\gamma k^*^2/2\) for the home and foreign firms respectively. However, we assume that marginal production cost \(c\) is constant for both firms.

It is easy to show that, when firms choose quantities, the game with market-expanding investment is effectively identical to that with cost-reducing investment. In both cases the only direct effect of investment is to raise the price-marginal-cost margin; for example, the home firm’s is: \(p - c = a_0 - c_0 - b(x + ey) + \theta k\), irrespective of whether \(\theta\) reflects cost-reducing or market-expanding investment. Hence, there is no need for a separate row in Table 1 corresponding to this case: all the terms in the expressions are unchanged. In particular, the friendliness and strategic substitutability terms continue to be given by equations (18) and (19) respectively, and so the optimal investment subsidy is still positive.

Matters are more interesting when firms compete in prices. A rise in foreign investment now raises the price at which the foreign firm can sell its product. This price increase translates directly into a rise in home profits. So, for the first time, investment levels are friendly. Formally, the relevant expression is almost identical to that in the case of cost-reducing investment with Bertrand competition, except that its sign is reversed:

\(^{10}\) Though recall from an earlier footnote that these specifications of the inverse and direct demand functions are not mutually compatible. The absence of investment spillovers in one implies that they must be present in the other.
This might suggest that the optimal investment subsidy is now negative. But this is not so, because investment levels now turn out to be strategic *complements*. The reason is that all the arguments of Section 3.1 are reversed. For example, a rise in home investment raises foreign sales, so tending to *increase* the marginal profitability of foreign investment. Once again, the relevant expression is identical to that in the case of cost-reducing investment with Bertrand competition, except that its sign is reversed:

\[
\hat{\pi}_{k^*} = \frac{e}{4-e^2}2\theta x
\]

Hence, when investment is market-expanding and firms compete on price, the relationships between home and foreign investment are diametrically opposite to those in the previous cases we have considered. Since the changes are mutually offsetting, a positive investment subsidy is once again optimal.

### 3.4 Investment Spillovers

So far, we have considered the case where each firm’s investment affects only its own cost or demand function. But inter-firm spillovers are plausible for both kinds of investment, and they might be expected to alter the relationship between profitability and rival investment levels.\(^{11}\) Consider for example the case in which investment is cost-reducing. The model is as before except that each firm’s marginal cost parameter is reduced not only by its own investment but also by its rival’s:

\[
\hat{\pi}_{k^*} = \frac{e}{4-e^2}\mu \eta \gamma
\]

---

\(^{11}\) For the case of cost-reducing investment and Cournot competition, this was noted by Henriques (1990) in a comment on d’Aspremont and Jacquemin (1988).
where $\phi$ is the spillover parameter. (Demand spillovers are modelled similarly: see Table 1 for details.\textsuperscript{12}) It is intuitively obvious that, for sufficiently high $\phi$, investment levels may become friendly: in equation (13), the spillover effect (whereby foreign investment directly lowers home costs and so raises home profits) may be sufficient to offset the strategic effect.

Does this mean that the earlier conclusions about optimal investment policy are reversed? Surprisingly, the answer is no. While it is true that investment levels may be friendly rather than unfriendly, exactly the same parameter values which give this outcome also imply that they become strategic complements rather than strategic substitutes. Clearly this conclusion is sensitive to the special linear-quadratic assumptions we have made. But it is remarkable that it holds for all the specifications we have considered so far: Cournot or Bertrand competition, with either cost-reducing or market-expanding investment. To avoid unnecessary taxonomy, we simply refer to Table 1, where the detailed expressions are given. The conclusion is thus that, though firm conduct and market behaviour are greatly affected by investment or demand spillovers, an investment subsidy continues to be the optimal policy in all cases.

3.5 Asymmetric Firms

So far we have assumed that investment \textit{either} reduces marginal costs or increases market demand. Of course, some forms of investment may do both. Even in this case, our

\begin{align}
c &= c_0 - \theta(k + \phi k^*) \\
c^* &= c_0 - \theta(k^* + \phi k)
\end{align} \tag{25}

\textsuperscript{12} One difference with demand spillovers is that $\phi$ may be negative rather than positive. The only restriction which must hold is that its value cannot be such that the marginal return to investment per unit output, $\mu$ (given in the final column of Table 1), is negative.
analysis extends straightforwardly to show that the optimal industrial policy is still a subsidy, provided the investments affect both firms symmetrically. However, consider instead an asymmetric case. Assume for example that the home firm's investment is cost-reducing but the foreign firm’s is market-expanding. (Ignore spillovers for the moment.) Assume also that firms compete on price. Then from Section 3.3 we know that foreign investment is friendly to the home firm. The foreign firm's investment raises its price which then raises home profit. But we also know from Section 3.3 that home investment is a strategic substitute for foreign investment, tending to reduce the rival’s marginal profitability.\footnote{Note that investments are strategic substitutes for the foreign firm, so its investment reaction function slopes downwards. But they are strategic \textit{complements} for the home firm, so its investment reaction function slopes upwards. It can be checked that the equilibrium is nevertheless stable.} Hence for the first time we have a simple and not unrealistic example where the home government should tax investment. The resulting decrease in home investment leads to an increase in the rival firm's market-expanding investment thus increasing both the foreign and home prices. Home profits may rise or fall (helped by higher foreign investment, harmed by the tax) but the gain in tax revenue ensures that home welfare rises.

A different example where an investment tax is optimal is where spillovers are asymmetric. Now, irrespective of the nature of competition or the type of investment, there exist parameter values which make investment levels friendly for one firm but strategic substitutes for the other. In the case of Cournot competition, for example, this is true when the parameter measuring the extent of spillovers to the foreign firm, $\phi^*$, lies on the opposite side of the term $e/2$ from the parameter measuring the extent of spillovers to the home firm, $\phi$. Such a configuration would arise in the plausible case where the foreign firm has a technological advantage and so has less to gain from spillovers than the home firm, so
ϕ<e/2<ϕ. Now a tax on the home firm leads it to reduce its investment which encourages the foreign firm to invest more (because investments are strategic substitutes for the foreign firm); this in turn tends to raise home profits (because investments are friendly for the home firm). Once again, therefore, we see that it is asymmetries which are crucial in overturning the presumption that an investment subsidy is optimal.

4. Summary and Conclusion

In this paper we have reexamined the rationale for direct investment subsidies to firms competing against foreign rivals in oligopolistic markets. We first considered a model with general functional forms, and derived the standard result that the sign of the optimal subsidy depends on two key questions: first, whether investments are "friendly", in the sense that higher foreign investment raises home profits; and second, whether investments are "strategic substitutes", in the sense that higher home investment raises the marginal profitability of foreign investment. We then related these two considerations to the properties of the underlying profit functions and showed that neither can be signed unambiguously. This suggests that the theoretical case for a positive investment subsidy is not robust.

The ambiguity of the general model is a puzzle in itself, since all previous studies have found that, when direct assistance to exports is ruled out, positive investment subsidies are justified. We therefore turned to some special models, to try and isolate the features which work in favour of industrial policy. We began with a benchmark case in which competition is Cournot, investment serves to reduce costs, there are no spillovers and firms are symmetric. Following Spencer and Brander (1983), this yielded clear-cut results: investment levels were both unfriendly and strategic substitutes, or, in plainer language, an increase in investment by one firm reduced its rival’s profits both in total and at the margin.
As a consequence, a domestic subsidy to investment was justified: by increasing home investment, this reduced foreign investment (because of strategic substitutability) which in turn raised home profits (because of unfriendliness).

We then considered the consequences of relaxing each of the benchmark assumptions in turn. Switching from Cournot or quantity competition to Bertrand or price competition affects the detailed workings of the model, but leaves the central policy conclusion unaffected. More surprisingly, with market-expanding rather than cost-reducing investment, and with inter-firm spillovers, an investment subsidy remains optimal. This despite the fact that the mechanisms operating in the various cases were very different. For example, with either market-expanding investment or strong spillovers, investments may be friendly and strategic complements. However, these two reversals of the benchmark case always occur for the same parameter values and so they offset each other.

The only cases we found where an investment tax was warranted was where the two firms were asymmetric, either in the sign of the spillovers between them or in the effects of their investments. Thus, for example, if home investment tends to expand the size of the market (and competition is Bertrand) both firms benefit, so investments are friendly; if at the same time foreign investment tends to reduce costs, then investments are strategic substitutes. In such cases a tax on home investment would be welfare-increasing.

Why is it that investment subsidies are optimal in so many special cases (including all those examined so far in the literature), whereas the general formulae are so ambiguous? One heuristic explanation is that the two key concepts tend to be closely associated in simple cases: "friendliness" refers to the effect of one firm’s investment on its rival’s total profits, whereas "strategic substitutability" refers to the effect of one firm’s investment on its rival’s marginal profits. With simple functional forms these two concepts tend to have the same
sign. Arbitrary non-linearities in investment or demand functions can always be found which will lead to their having different signs. But as we have noted, this would be a weak basis for a case against investment subsidies.

It need hardly be stressed that the issues considered in this paper concern only part of the objections which have been raised to the interventionist thrust of strategic trade policy. Even in the benchmark case of Cournot competition, subsidisation of either exports or investment may not be optimal if there are many home firms, if foreign governments also subsidise, if some of the additional profits are captured by domestic factors, or if the opportunity cost of public funds exceeds unity. (See Brander (1995) for a review of these arguments and further references.) Moreover, it is surely true that much assistance to private industry is driven in practice by special-interest politics.

A different objection to investment subsidies is made in our earlier work (Neary and Leahy (2000)), where we considered the jointly optimal choice of investment and export subsidies. We stressed there that in these models there are two separate grounds for intervention: first, to restore efficiency in investment; and, second, to optimally manipulate the foreign firm by subsidising or taxing home exports. Attaining the "first-best" outcome therefore requires two instruments, both an investment and an export subsidy or tax.\(^\text{14}\) Hence, if exports cannot be subsidised (or taxed), the case for subsidising investment alone is only a second-best one. We also presented some simulation results which suggest that, even when an investment subsidy is optimal, it is unlikely to bridge much of the gap between the free-trade and first-best welfare levels; and the optimal subsidy rate is unlikely to be large.

For all these reasons, the practical case for strategic industrial policy is questionable.

\(^{14}\) We write "first-best" in inverted commas, since the oligopolistic market structure is taken as given and only domestic welfare is considered.
Nevertheless, it is striking that the qualitative policy recommendation is relatively unaffected
by changes in assumptions about demand, technology and firm behaviour. Although positive
subsidies to investment are not a very robust recommendation, they are considerably more
robust than positive subsidies to exports.

Appendix

Totally differentiating the first-order conditions for actions given in (3):

\[
\begin{bmatrix}
\pi_{AA} & \pi_{AB} \\
\pi_{BA}^* & \pi_{BB}^*
\end{bmatrix}
\begin{bmatrix}
dA \\
\vdB
\end{bmatrix}
= -
\begin{bmatrix}
\pi_{Ak}^* dk + \pi_{Ak}^* \cdot dk^* \\
\pi_{Bk}^* dk + \pi_{Bk}^* \cdot dk^*
\end{bmatrix}
\tag{26}
\]

The diagonal elements in the left-hand-side coefficient matrix are negative from the home and
foreign second-order conditions for actions; the off-diagonal elements are negative if and only
if the two actions are strategic substitutes; and the determinant (denoted by $\Delta$) must be
positive for stability. Solving (26) gives the derivatives of the solutions for $A(k,k^*)$ and
$B(k,k^*)$:

\[
\begin{bmatrix}
A_k & A_{k^*} \\
B_k & B_{k^*}
\end{bmatrix}
= \frac{1}{\Delta}
\begin{bmatrix}
-\pi_{BB}^* \pi_{Ak}^* + \pi_{AB}^* \pi_{Bk}^* & \pi_{AB}^* \pi_{Bk}^* - \pi_{BB}^* \pi_{Ak}^* \\
\pi_{BA}^* \pi_{Ak}^* - \pi_{AA}^* \pi_{Bk}^* & -\pi_{AA}^* \pi_{Bk}^* + \pi_{BA}^* \pi_{Ak}^*
\end{bmatrix}
\tag{27}
\]

If there are no spillovers, the second term in each expression in the right-hand-side matrix
vanishes, since the cross-derivatives $\pi_{Bk}^*$ and $\pi_{Ak}^*$ are zero.
References


### Table 1: Determinants of the Optimal Investment Subsidy in Some Special Cases

\[ \pi = (p - c)x - \gamma k^2/2 \quad \pi^* = (q - c^*)y - \gamma k^*^2/2 \]

<table>
<thead>
<tr>
<th>Friendliness: ( \hat{\pi}_{k^*} &gt; 0? )</th>
<th>Strategic Substitutability: ( \hat{\pi}_{k^* k} &lt; 0? )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi_k^* )</td>
<td>( \pi_B )</td>
</tr>
</tbody>
</table>
| CR+C and ME+C: | \( \phi \theta x \) | \(- bex \) | \( \frac{2-\phi}{b(4-e^2)} \theta \) | \( -\frac{e-2\phi}{b(4-e^2)} \theta 0 \) | \( 0 \) | \( \theta \) | \(-\frac{e-2\phi}{b(4-e^2)} \theta \) | \( 0 \) | \(-bex \) | \(-\frac{e-2\phi}{4-e^2} \mu \gamma \) | \( \frac{2(2-e^2)}{4-e^2} \)
| CR+B: | \( \phi \theta x \) | \( ex \) | \(-\frac{2+\phi}{4-e^2} \theta \) | \(-\frac{e-2\phi}{4-e^2} \theta 2\theta x \) | \( \beta \epsilon \theta \) | \(-\beta \theta \) | \(-\frac{e-2\phi}{4-e^2} \theta \) | \( \beta \epsilon \theta \) | \( \beta \epsilon \) | \(-\frac{e-2\phi}{4-e^2} \mu \gamma \) | \( \frac{2(2-e^2)}{4-e^2} \)
| ME+B: | \( \phi \theta x \) | \( ex \) | \( \frac{2+\phi}{4-e^2} \theta \) | \( \frac{e+2\phi}{4-e^2} \theta 2\theta x \) | \( 0 \) | \( \beta \theta \) | \( \frac{e+2\phi}{4-e^2} \theta \) | \( 0 \) | \( \beta \epsilon \) | \( \frac{e+2\phi}{4-e^2} \mu \gamma \) | \( \frac{2(2-e^2)}{4-e^2} \)

Notes: The table shows the values of the individual terms in the general expressions for "friendliness" and "strategic substitutability" of investment ((13) and (15) respectively) in the special cases indicated in the first column.

C: Cournot competition: \( p = a - b(x + ey) \), \( q = a^* - b(y + ex) \)
B: Bertrand competition: \( x = \alpha - \beta(p - \epsilon q) \), \( y = \alpha^* - \beta(q - \epsilon p) \)
CR: Cost-reducing investment: \( c = c_0 - \theta(k + \phi k') \), \( c^* = c_0 - \theta(k^* + \phi k) \)
ME: Market-expanding investment:
- ME+C: \( a = a_0 + \theta(k + \phi k') \), \( a^* = a_0 + \theta(k^* + \phi k) \)
- ME+B: \( \alpha = \alpha_0 + \beta\theta(k + \phi k') \), \( \alpha^* = \alpha_0 + \beta\theta(k^* + \phi k) \)
\( \eta \): Relative return to investment: \( \eta = \theta^2/b \) in C; \( = \beta\theta^2/\gamma \) in B
\( \mu \): Strategic component in the marginal return to investment per unit output (with no strategic investment, \( \mu \) equals one)