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CROSS-BORDER MERGERS

AS INSTRUMENTS OF COMPARATIVE ADVANTAGE*

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Abstract

A two-country model of oligopoly in general equilibrium is used to show how changes in market structure accompany the process of trade and capital market liberalisation. The model predicts that bilateral mergers in which low-cost firms buy out higher-cost foreign rivals are profitable under Cournot competition. With symmetric countries, welfare may rise or fall, though the distribution of income always shifts towards profits. The model implies that trade liberalisation can trigger international merger waves, in the process encouraging countries to specialise and trade more in accordance with comparative advantage.

JEL: F10, F12, L13

Keywords: Comparative advantage; cross-border mergers; GOLE (General Oligopolistic Equilibrium); market integration; merger waves.

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CROSS-BORDER MERGERS
AS INSTRUMENTS OF COMPARATIVE ADVANTAGE

NON-TECHNICAL SUMMARY

Cross-border mergers are an increasingly important phenomenon in the world economy. They comprise well over half of all foreign direct investment (FDI), considerably more than greenfield FDI. They also constitute an increasing proportion of all mergers. For example, a recent study of 2,753 mergers worldwide from 1981 to 1998 finds an upward trend in the percentage of mergers which are cross-border, a trend which is particularly pronounced for EU countries in the 1990’s. Moreover, there is considerable anecdotal and other evidence suggesting that cross-border merger waves coincide with episodes of trade liberalisation and market integration. A study by the European Commission found that cross-border mergers were the dominant form of adjustment by European firms to the extension of the EU Single Market, and similar patterns have been found for the Mercosur economic union in Latin America.

Yet despite their real-world importance, the academic literature on cross-border mergers is tiny, both in absolute terms and relative to the enormous literature on greenfield FDI. Moreover, most authors who have tried to provide economic explanations for cross-border mergers have worked in a partial equilibrium framework, looking at the issues from the perspective of a single sector only. This has yielded important insights, but it fails to address the economy-wide issues raised by such mergers, such as their effects on income distribution or their implications for trade patterns.

The reasons for this neglect of the general-equilibrium aspects of cross-border mergers are not hard to find. The two dominant paradigms in international trade theory assume either perfect or monopolistic competition. While these differ in their assumptions about returns to scale and product differentiation, they model firms in the same way: as atomistic agents, which are in infinitely elastic supply, face no barriers to entry or exit, and do not engage in strategic interaction. This framework leaves very little scope for discussing mergers. A satisfactory theory linking trade liberalisation and mergers requires a theory of oligopoly, competition between small numbers of firms, in general equilibrium, but progress in this direction has been held back by the generally negative results in the literature.

In this paper, I use a model of oligopoly in general equilibrium I have recently developed to show how trade liberalisation can lead to cross-border merger waves. The model draws on the traditions of both industrial organisation and international trade theory. It allows for strategic interaction between firms, so permitting a game-theoretic approach to explaining merger activity. At the same time, it is a completely specified general equilibrium model, so making it possible to track the full effects of trade liberalisation on trade and production patterns. The problems of modelling oligopolistic interaction in general equilibrium which have plagued earlier work are avoided by assuming a continuum of oligopolistic sectors with economy-wide factor markets. Hence, while firms have market power in their own sector, they cannot influence factor prices or national income. Finally, since the model builds on the standard Ricardian framework, it permits an exploration of the process by which specialisation according to comparative advantage may or may not be helped by the rational decisions of individual oligopolistic firms.

To fix ideas, the model assumes that mergers do not lead to efficiency gains. Such
gains can arise from a variety of sources, such as cost savings via internal technology transfer, economies in the use of firm-specific assets, managerial synergies or the integration of pricing and marketing decisions on differentiated products. However, mergers may also raise costs, as different managerial and production structures and different corporate cultures have to be integrated. In any case, the empirical evidence on efficiency gains suggest that they are far from pervasive. In their absence, mergers take place for purely strategic motives. Firms seek to absorb their rivals in order to reduce competition in the market and to raise prices and profits. Such behaviour seems clearly harmful, but further analysis is needed to determine its economy-wide implications.

The key to the prediction of mergers in my framework is not general equilibrium *per se*, but rather cost differences between firms. However, the two go together, since in partial equilibrium there is usually no plausible basis for assuming firm heterogeneity. By contrast, in a Ricardian trade model, international differences in technology provide a natural reason for it.

The model predicts that international differences in technology generate incentives for bilateral mergers in which low-cost firms located in one country acquire high-cost firms located in the other. As a result, cross-border mergers serve as "instruments of comparative advantage". They facilitate more specialisation in the direction of comparative advantage, so moving production and trade patterns closer to what would prevail in a competitive Ricardian world. They also have implications for income distribution, putting downward pressure on wages, and so tilting the distribution of income towards profits at the expense of wages in both countries. As for aggregate welfare, the fall in wages puts downward pressure on prices in all sectors, which tends to increase the gains from trade in both countries. Potentially offsetting this, the sectors in which mergers occur become less competitive, so their prices tend to rise. Hence the full effect on welfare is ambiguous.

The results of this paper lend themselves to empirical testing. By construction, the model predicts that trade liberalisation encourages cross-border merger waves, consistent with the empirical evidence discussed above. In addition, the model makes a further empirical prediction: absent cost synergies, the pattern of cross-border mergers which results from market integration follows that of comparative advantage, in the sense that low-cost firms acquire high-cost foreign rivals. As a corollary, the model predicts that cross-border mergers and exports are complements rather than substitutes, in the sense that exporting sectors tend to be sources of rather than hosts for foreign direct investment. Finally, the model predicts that cross-border merger waves tend to reduce factor demands and so put downward pressure on the returns to productive factors. Most of there predictions are very different from, and more consistent with the available empirical evidence than, those of standard models of greenfield FDI.
1. Introduction

Cross-border mergers are an increasingly important phenomenon in the world economy. They comprise well over half of all foreign direct investment (FDI), considerably more than greenfield investment. They also constitute an increasing proportion of all mergers.\(^1\) Moreover, there is considerable anecdotal and other evidence suggesting that cross-border merger waves coincide with episodes of trade liberalisation and market integration.\(^2\) Yet the literature on cross-border mergers is tiny, both in absolute terms and relative to the enormous literature on greenfield FDI.\(^3\)

How to explain cross-border mergers? Both international trade theory and the theory of industrial organisation might be expected to throw some light on them, but neither provides a fully satisfactory explanation. Consider first the theory of industrial organisation. The enormous I.O. literature on mergers suggests two broad motives for mergers: an efficiency motive and a strategic motive. Efficiency gains can arise from a variety of sources, such as cost savings via internal technology transfer, economies in the use of firm-specific assets, managerial synergies, or the integration of pricing and marketing decisions on differentiated products. However, mergers may also raise costs, as different managerial and production structures and different corporate cultures have to be integrated. In any case, the empirical

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\(^1\) Gugler et al. (2003) study 2,753 mergers worldwide from 1981 to 1998, and find an upward trend in the percentage of mergers which are cross-border, a trend which is particularly pronounced for EU countries in the 1990’s. For example, the percentage of all mergers in Continental Europe which were cross-border rose from 24.2% in 1991-92 to 39.8% in 1997-98.

\(^2\) See European Commission (1996) and Chudnovsky (2000) for evidence that an increased importance of cross-border mergers coincided with greater market integration in the EU Single Market and Mercosur respectively.

\(^3\) For models of greenfield FDI, see Helpman (1984), Markusen (2002) and Helpman, Melitz and Yeaple (2004).
evidence on efficiency gains suggests that they are far from pervasive. As for the strategic motive, the benchmark case where symmetric firms engage in Cournot competition and there are no efficiency gains yields what is sometimes called the "Cournot merger paradox," due to Salant, Switzer and Reynolds (1983). They showed that mergers between identical firms are unprofitable unless the merged firms produce a very high proportion of pre-merger industry output: over 80% when demand is linear. Subsequent work has shown that mergers may be profitable if Cournot competition is extended in various ways, such as allowing cost synergies, convex demand or union-firm bargaining. It has also shown that the opposite problem, of too many mergers rather than too few, holds in Bertrand competition. However, it has not overturned the original Salant-Switzer-Reynolds result.

While the industrial organisation approach throws a lot of light on mergers, it provides an incomplete basis for understanding cross-border mergers. Especially if we want to relate cross-border mergers to an economy-wide shock such as trade liberalisation, partial equilibrium models which take demands and factor prices as given cannot tell the whole story. This suggests that a more promising route might start from the theory of international

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4 The study by Gugler et al. (2003) mentioned in footnote 1 finds that, relative to the median firm in the same 2-digit industry, acquiring firms had significantly higher profits but lower sales in each of the five years after the merger. They interpret these results as implying that the average profitable merger in their sample increased market power but did not yield significant cost savings.

5 From a large literature, see Perry and Porter (1985) and Farrell and Shapiro (1990) on cost synergies, Faúl-Oller (1997) and Leahy (2002) on demand convexities, Lommerud, Straume and Sørgard (2004) on union-firm bargaining, and Deneckere and Davidson (1985) on Bertrand competition. Recent papers by Faúl-Oller (2000), Macho-Stadler, Pérez-Castrillo and Porteiro (2002) and Toxvaerd (2002) model sequential mergers. Of these, the model of Faúl-Oller is closest in spirit to that of the present paper, but like all the papers mentioned it is cast in partial equilibrium.

6 There is also a small literature which examines international mergers in partial equilibrium, much of it concerned with issues of merger policy design. See Bertrand and Zitouna (2003), Falvey (1998), Francois and Horn (1998), Head and Ries (1997), Horn and Levinsohn (2001)
trade, with its almost two-centuries-old tradition of studying trade liberalisation in general
equilibrium. But here there is a different problem. The two dominant paradigms in trade
type assume either perfect or monopolistic competition. While these differ in their
assumptions about returns to scale and product differentiation, they model firms in the same
way: as atomistic agents, which are in infinitely elastic supply, face no barriers to entry or
exit, and do not engage in strategic interaction. This framework leaves little scope for
discussing mergers.7 A satisfactory theory linking trade liberalisation and mergers requires
a theory of oligopoly in general equilibrium, but progress in this direction has been held back
by the generally negative results of Gabszewicz and Vial (1972) and Roberts and

A third approach to mergers is to relate them to the state of the business cycle. For
example, Jovanovic and Rousseau (2003) construct a real-business-cycle model in which
merger activity intensifies in upturns. This effect may well be important in explaining many
real-world mergers. However, it does not explain why they should intensify in periods of
adjustment to trade liberalisation.

In this paper I use a model of oligopoly in general equilibrium introduced elsewhere
to show how trade liberalisation can lead to cross-border merger waves. (The model is
presented in Neary (2002); for a non-technical overview see Neary (2003).) The model draws

is to explore international mergers as the outcome of a cooperative game.

7 An exception is a recent paper by Marin and Verdier (2002) who use a principal-agent
framework to model the internal organisation of firms in general equilibrium with
monopolistic competition. They show that changes in the degree of competition, modelled
as an increase in the elasticity of substitution between goods, can induce a shift from an
eylrium with mostly agent-oriented organisations towards one with mostly principal-
oriented organisations, which they interpret as a merger wave. However, the mergers they
predict are solely vertical.
on the traditions of both industrial organisation and international trade theory. It allows for strategic interaction between firms, so permitting a game-theoretic approach to explaining merger activity. At the same time, it is a completely specified general equilibrium model, so making it possible to track the full effects of trade liberalisation on trade and production patterns. The problems of modelling oligopolistic interaction in general equilibrium which have plagued earlier work are avoided by assuming a continuum of oligopolistic sectors with economy-wide factor markets. Hence, while firms have market power in their own sector, they cannot influence factor prices or national income. In addition, since the model builds on the standard Ricardian framework, it permits an exploration of the process by which specialisation according to comparative advantage may or may not be helped by the rational decisions of individual oligopolistic firms. Finally, by concentrating on the case where mergers take place for strategic motives only, the model allows a clear focus on the possibility that mergers which do not yield efficiency gains may nonetheless lead to improved resource allocation.

The initial equilibrium in the paper is one where trade is free but cross-border mergers are not permitted. This can be thought of as a hypothetical step on the way from autarky to complete liberalisation of trade and foreign investment. Alternatively, it can be viewed as representing a situation such as prevailed in the European Union before the Single Market Act of 1992, in which trade barriers have been eliminated but restrictions on cross-border mergers and acquisitions remain in force. This free-trade equilibrium is then disturbed by an exogenous and unanticipated abolition of restrictions on cross-border mergers.

The first part of the paper considers mergers in a single industry, in which there are exogenous cost differences between home and foreign firms. Section 2 introduces the model, while Sections 3 and 4 show how trade liberalisation can induce cross-border mergers, both
when mergers take place in response to myopic incentives, and when firms contemplating a
merger anticipate the effects of other firms’ actions. The second part of the paper turns to
general equilibrium to show how costs are determined endogenously. Section 5 reviews the
model of general oligopolistic equilibrium introduced in Neary (2002), and Sections 6 and 7
explore its implications for the likelihood of cross-border mergers and for their effects on
resource allocation, income distribution and welfare.

2. Specialisation Patterns in the Absence of Mergers

Until Section 5, we consider a single sector in partial equilibrium. Hence, we can
suppress the sectoral index, and, like the participating firms themselves, we treat the cost and
demand parameters as exogenous.

Consider a sector with a small number of firms located in two countries, called
"home" and "foreign". The firms produce identical products and, following the liberalisation
of trade, engage in Cournot competition on an integrated world market. The world inverse
demand function they face is:

\[ p = a' - b'x \]  

This function is linear from the perspective of firms, though we will see later that it is highly
non-linear in general equilibrium. All firms in a given location have the same unit cost: \( c \)
for home firms and \( c^* \) for foreign. In autarky there are \( n \) firms at home and \( n^* \) abroad, with
\( \bar{n} \) equal to \( n \) and \( n^* \).\(^8\) Because of exogenous barriers to entry, trade liberalisation cannot
induce entry of new firms. However, it can induce exit, either because some firms become
unprofitable or (as we will see in the next section) because they are taken over.

\(^8\) Variables referring to the foreign country are denoted by an asterisk, while (where necessary
to avoid ambiguity) variables referring to the world as a whole are denoted by an over-bar.
Following Neary (2002), it is convenient to illustrate the possible regimes in \( \{c,c^*\} \) space. The condition for home firms to be profitable is easily stated. We abstract from fixed costs, since they would provide a trivial justification for mergers.\(^9\) Hence, profits are proportional to the square of output, \( \pi = b' y^2 \), and a positive level of profits is equivalent to a positive level of output. From the expression for home output, given by equation (28) in Appendix A, profits of a home firm will be positive if and only if its unit cost is less than or equal to a weighted average of the demand intercept and the unit cost of foreign firms, where the weight attached to the former is decreasing in the number of foreign firms:

\[
\begin{align*}
  c &\leq \xi_0 a' + (1-\xi_0)c^* \quad \text{where:} \quad \xi_0 = \frac{1}{n^*-1} < 1 \\
\end{align*}
\]

This condition holds both when the number of active foreign firms is strictly positive and when it is zero; in the latter case, the weight \( \xi_0 \) equals one, so home profitability requires that the unit cost of a home firm cannot exceed the demand intercept, the maximum price which consumers are willing to pay: \( c \leq a' \). These two conditions, along with corresponding profitability conditions for foreign firms (expressed in terms of \( \xi^*_0 \equiv 1/(n+1) \)), define four regions in \( \{c,c^*\} \) space, as illustrated in Fig. 1. (We defer discussion of the downward-sloping locus until Section 5.) If the costs of all firms exceed \( a' \), as in region \( O \), then the good is not produced. If firms in only one country have relatively low costs, then only they will produce it, as in regions \( H \) and \( F \) (where only home and foreign firms respectively can compete). Finally, region \( HF \) is a "cone of diversification" where both home and foreign firms can coexist: barriers to entry act as a surrogate for tariffs, allowing high-cost firms in one country to survive in the face of low-cost rivals in the other. In the competitive limit (as

\(^9\) By contrast, sunk costs of entry cannot be recouped and so have no effect on merger decisions.
\( \xi_0 \) and \( \xi_0^* \) approach zero), this region collapses to the 45° line, and the two countries specialise completely according to comparative advantage.

The discussion so far assumes that, in both countries, all firms which are profitable in free trade continue to produce. However, opening markets to international competition may generate incentives for mergers. The next sections show how the analysis must be amended when these incentives are taken into account.

3. Myopic Merger Incentives

We need to impose some structure on how firms decide to merge. I first make the plausible assumption that simultaneous negotiations between more than two firms face prohibitive transactions costs. Hence:

*Assumption 1: Only bilateral mergers can occur.*

This does not preclude sequential mergers, but it implies that they must consist of a sequence of bilateral mergers, each of which is desired by both parties.

My next assumption is that a merger must yield a surplus which is sufficient to compensate both participating firms. Since firms produce identical products and there are no tariffs or transport costs, there is no incentive for a firm to operate more than one plant, and so a merger implies that one of the participating firms is closed down. Hence the surplus following a merger equals the profits of the surviving firm less the initial profits of both firms. To fix ideas, consider the case where there are initially \( n \) home and \( n^* \) foreign firms, and a home firm is taken over. The surplus or net gain from such a merger is:
\[
G_{FH}(n,n^*) = \pi^*(n-1,n^*) - \pi^*(n,n^*) - \pi(n,n^*)
\]  

(3)

where \(\pi(n,n^*)\) and \(\pi^*(n,n^*)\) denote the profits of a home and foreign firm respectively, when there are \(n\) active home firms and \(n^*\) active foreign firms. Henceforward I assume that this must be strictly positive for a merger to occur.\(^{10}\)

**Assumption 2:** A merger will not take place if the gain defined in (3) is zero or negative.

Since the home firm ceases to produce, this merger can be thought of as a takeover of the home firm by the foreign firm, and I will use the terms "merger" and "takeover" interchangeably from now on. Assumption 2 should be interpreted as reflecting the inability of firms to borrow against future mergers. If such borrowing were allowed, then Assumption 1 could be circumvented. In particular, a merger to monopoly, since it ensures a sufficient surplus to compensate all firms, would always be possible.

Assumption 2 is only a necessary condition for a merger to take place, since it does not rule out forward-looking behaviour. However, I defer consideration of such behaviour until the next section. In the remainder of this section, I explore the implications of assuming that a positive value of \(G_{FH}(n,n^*)\) is sufficient as well as necessary for a merger to take place:

**Assumption 3:** A merger will take place if the gain defined in (3) is strictly positive.

\(^{10}\) The acquired firm must be paid at least its initial profits \(\pi(n,n^*)\) plus an infinitesimal amount if it is to agree to a merger, which is why \(G_{FH}(n,n^*)\) must be strictly positive for a merger to take place.
As we will see, Assumption 3 is consistent with cases where firms would earn higher profits by refraining from merging. Hence, I describe a positive value for $G_{FH}(n,n^*)$ as a myopic merger incentive.

Armed with Assumptions 1 to 3, I can now proceed to explore in more detail the circumstances when a merger will occur. Equation (3) provides some intuition for the Salant-Switzer-Reynolds result: if the two participating firms have the same costs, and hence the same initial outputs and profits, the profits of the acquiring firm (and of all firms like it) would have to double for a takeover to be profitable. This result continues to hold if other firms have costs which differ from those of the two firms involved in the merger. For completeness I state this formally (with proof in Appendix A), though Proposition 1 is closely related to the central result in Salant et al. (1983):\(^{11}\)

**Proposition 1:** A merger between two firms with the same unit cost (whether two home or two foreign firms), is never profitable, provided $n+n^* > 2$.

By contrast, the next result shows that, provided the cost differential between the two participating firms is sufficiently large, the gain to a takeover is strictly positive:

**Proposition 2:** A takeover of a home firm by a foreign firm meets the myopic merger criterion $G_{FH} > 0$ if and only if:

---

\(^{11}\) Relative to Salant et al., Proposition 1 is less general since it considers only bilateral mergers, but more general since it allows the unit costs of uninvolved firms to differ from those of the involved firms. Falvey (1998) shows that Proposition 1 holds for an arbitrary distribution of the marginal costs of non-participating firms.
Proposition 2 states that a takeover of a home firm will be profitable provided its unit cost exceeds a weighted average of the demand intercept and the unit cost of the acquiring firm. Key features of the result are that the condition for the myopic merger criterion to hold has the same form as the profitability condition, equation (2), and that the weight $\xi_1$ defined in (5) is positive but smaller than the weight $\xi_0$ in (2). Hence equation (4) defines a linear upward-sloping locus in Fig. 2 which lies strictly below that implied by (2). (This figure repeats the regions of specialisation from Fig. 1, with the boundaries of the HF region corresponding to zero profitability of home and foreign firms denoted by dotted lines). Introducing the possibility of takeovers has the effect of expanding the F region, adding to it a sub-region in Fig. 2 in which high-cost home firms can earn positive profits, but are vulnerable to a bilateral takeover by low-cost foreign rivals.

Intuitively, the reason for Proposition 2 is that outputs are strategic substitutes in Cournot competition, at least when demands are linear. (See Gaudet and Salant (1991) for further discussion.) Hence, a takeover which closes down one firm raises output and therefore operating profits for all surviving firms (including the acquiring firm). Provided the cost differential is sufficiently great, the profits of a low-cost foreign firm increase sufficiently to justify its taking over a high-cost home one.

\[ c > \xi_1 a' + (1 - \xi_1)c^* \tag{4} \]

where:

\[
\begin{align*}
\xi_1 &= \frac{\Phi - \Psi}{\phi}, \quad 0 < \xi_1 < \xi_0 \\
\Psi &= 2(n+1)\bar{n} + n^*(\bar{n}^2 - 1) \\
\phi &= 2n\bar{n} + (n^* + 1)(\bar{n}^2 - 1) = \psi + \bar{n}^2 - 2\bar{n} - 1
\end{align*} \tag{5} \]
An exactly similar argument applies to home takeovers. Similar derivations to those which yield Proposition 2, with the roles of the countries reversed, show that a takeover of a foreign firm by a domestic one is profitable if and only if:

\[ c^* > \xi_1^* \alpha' + (1 - \xi_1^*)c \]  \hspace{1cm} (6)

where \( \xi_1^* = (\phi^* - \psi^*)/\phi^* \), and \( \phi^* \) and \( \psi^* \) are defined in the same way as in (5), except that \( n \) replaces \( n^* \) and vice versa. Hence there is a similar region in Fig. 2 in which high-cost foreign firms can earn positive profits, but are vulnerable to a bilateral takeover by low-cost home rivals.

Next, we can ask what will be the effect of such a bilateral takeover on the incentives for further takeovers in the same industry. Because all outputs are strategic substitutes, the outputs, and hence the profits, of all surviving firms, both home and foreign, increase. Indeed, with linear demands, the outputs of all surviving firms rise by the same absolute amount. But the low-cost foreign firms have larger output to begin with. Hence, since profits are proportional to the square of output, a further takeover increases their profits by more. Formally, whenever \( G_{FH} \) is positive, it is decreasing in \( n \):

**Proposition 3:** A profitable takeover of a home firm increases the gain to a takeover of one of the remaining home firms by a foreign firm.

The proof is in Appendix A. It follows that a fall in \( n \) as a result of a takeover raises the incentive for a further takeover. This is the basis for the prediction of merger waves. A

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\(^{12}\) The increase equals the initial output of the takeover target divided by the initial number of firms in both countries: \( y(n-1,n^*) - y(n,n^*) = y'(n-1,n^*) - y'(n,n^*) = y(n,n^*)/\bar{n} \). See Lemma A.1 in Appendix A, with \( \bar{n} \) set equal to \( n - 1 \).
profitable takeover of one home firm by a foreign firm increases the gain to a takeover of one of the remaining home firms by a foreign firm in the same sector.

Because all home firms have identical costs, Proposition 3 implies that, if one of them is taken over, then all of them will be. Within sectors where a first takeover is profitable, further takeovers are even more profitable. Hence no high-cost firms survive. However, because of Assumption 2, the cone of diversification $HF$ does not vanish altogether. In that region, bilateral mergers would be profitable if there were fewer high-cost firms to begin with, i.e., if some bilateral mergers had already taken place. But, given the initial number of high-cost firms, the first bilateral takeover is not profitable, and so a merger wave is not initiated. A final implication of Proposition 3 is that, in the absence of cost synergies, encouraging "national champions" by promoting domestic mergers in high-cost sectors makes foreign takeovers more rather than less likely.

4. Forward-Looking Merger Incentives

So far, I have worked only with the myopic merger criterion embodied in Assumption 3. Putting this differently, the equilibria in the previous section are not sub-game perfect. This matters because it means that firms ignore the "after-you" or free-rider problem, which arises for both low- and high-cost firms. In the words of Stigler (1950), "Firms will do everything possible to encourage a merger, short of participating in one themselves." For low-cost foreign firms, all face the same takeover incentive $G_{FD}(n,n^*)$, but uninvolved firms reap the gains, $\pi'(n-1,n^*)-\pi'(n,n^*)$, without having to incur the cost of acquiring a home firm, $\pi(n,n^*)$. Hence no individual foreign firm may opt to engage in a takeover, if it believes that another foreign firm will do so first. High-cost home firms face a different free-rider problem. Since $\pi(n,n^*)$ is decreasing in $n$, each high-cost firm that is made a takeover offer
has a higher outside option than high-cost firms which have already been taken over. Hence, if payoffs are determined on the basis of the myopic merger criterion, (3), every high-cost firm would like to be taken over after rather than before other firms like it.

Two different responses can be given to this problem, and both have some validity. On the one hand, the model of the last section can be interpreted as making the empirical prediction that trade liberalisation increases the conditional probability of cross-border merger waves. A positive value of $G_{FH}(n,n^*)$ does not guarantee that a takeover will take place, partly because of free-riding but also because there are many other reasons why takeovers which promise net gains may not take place (especially if the gains are small), such as transactions costs, uncertainty, or managerial hubris. Nevertheless, it provides a clear-cut prediction which can be confronted with the data. On the other hand, it would be very desirable to construct a properly specified extensive-form game which is consistent with cross-border mergers taking place, even if this requires making some further assumptions. In the remainder of this section I outline one approach which meets this requirement.

I retain Assumptions 1 and 2 from the previous section, but drop Assumption 3. In its place, I put more structure on the way in which firms interact as follows:\footnote{For simplicity, I assume that no production takes place before all mergers are completed, and that the $n$ stages in the pre-production phase are sufficiently short so discounting can be ignored.}

**Assumption 4:** (a) The pre-production phase of the game lasts for $n$ stages;

(b) in each stage a randomly selected pair of firms, one high-cost and the other low-cost, meet; (c) at every such meeting, the low-cost firm makes a take-it-or-leave-it offer to the high-cost firm; and (d) all firms are risk-neutral.
Part (a) of Assumption 4 ensures that firms do not have an incentive to postpone indefinitely; part (b) borrows from the literature on search and matching the idea that friction is equivalent to the random arrival of trading partners; and parts (c) and (d) allow the pay-offs to both parties to be calculated explicitly.\(^\text{14}\)

With forward-looking behaviour, we need to solve for the full sequence of mergers at once. Consider the case where there are \(s\) stages and \(\tilde{n}\) high-cost home firms remaining in the pre-production phase of the game. (From Assumption 4, at most one high-cost firm is taken over in each stage. Hence \(s\) cannot exceed \(\tilde{n}\): \(s \leq \tilde{n}\).) Define the expected returns to the two types of firms in these circumstances as follows:

\[ R(s,\tilde{n},n) \] is the minimum reward a high-cost firm requires to agree to a takeover when \(s\) stages and \(\tilde{n}\) high-cost home firms remain.

\[ R'(s,\tilde{n},n) \] is the expected ex ante payoff to a low-cost firm entering the \(s\)'th last stage when \(\tilde{n}\) high-cost home firms remain.

We can now state the criterion for a forward-looking merger wave:

\(^{14}\) Alternative sharing rules could be used instead of (c): for example, the Nash bargaining outcome is a natural candidate. This will alter the equilibrium but not significantly affect the conclusions reached below.
Proposition 4: Given Assumptions 1, 2 and 4, a merger wave will take place if the following expression is strictly positive:

\[ G_{FH}^{i}(n,n') = R'(n-1,n-1,n') - R'(n-1,n,n') - R(n,n,n') \]  (7)

Proof: The proof proceeds by calculating the payoffs to both types of firms when they expect the game to follow an equilibrium path along which a merger takes place at each stage, and then showing that such expectations are validated. Consider first the high-cost firms. We can calculate their minimum required return, which because of Assumption 4 (c) is also their supply price, as follows:

\[ R(s,\tilde{n},n') = \pi(\tilde{n},n'), \quad s = 1; \]  (a)

\[ \frac{s-1}{\tilde{n}} R(s-1,\tilde{n}-1,n') + \frac{1}{\tilde{n}} R(s-1,\tilde{n},n'), \quad 1 < s \leq \tilde{n} \]  (b)

Part (b) of (8) is a bivariate recursive equation with initial conditions given by (a). The proof is straightforward by backward induction. (Fig. 3 illustrates the reasoning for the last three stages.) In the final stage \( s=1 \), with \( \tilde{n} \) remaining high-cost firms, forward-looking behaviour is identical to myopic behaviour. If a high-cost firm refuses to sell it will receive \( \pi(\tilde{n},n') \).

Because of part (c) of Assumption 4, the low-cost firm has first-mover advantage in each meeting with a high-cost firm and so it appropriates all the surplus from a merger. Hence
the high-cost firm will actually sell for $\pi(\bar{n},n^*)$. This proves part (a) of Lemma 1. By contrast, in earlier stages (when $s$ exceeds 1), the high-cost firm will not be willing to sell for only its "current" profits $\pi(\bar{n},n^*)$. If it rejects a takeover offer in stage $s$ it will face in the subsequent stage $s-1$ a $1/\bar{n}$ probability of being made an offer and a $(\bar{n}-1)/\bar{n}$ probability of not being made an offer. Recalling from Assumption 4 (d) that all firms are risk-neutral, the expected value of this lottery must equal the price at which the high-cost firm is willing to sell in period $s$, which proves part (b) of the Lemma.

For later use, we note some useful properties of the function $R(s,\bar{n},n^*)$:  

**Corollary 1:** For all $s$, $R(s,\bar{n},n^*)$ is a weighted average of $\pi(m,n^*)$, $m = \bar{n}-s+1, \ldots, \bar{n}$.

**Corollary 2:** For all $\bar{n} > 2$ and $s > 1$, $\pi(\bar{n},n^*) < R(s,\bar{n},n^*) < \pi(\bar{n}-s+1,n^*)$.

**Corollary 3:** $R(s,\bar{n},n^*)$ is (a) increasing in $s$; (b) decreasing in $\bar{n}$; and (c) decreasing in $m$, where $s=\bar{n}=m$.

Consider next the low-cost firms. Their expected ex ante payoff is given by the following:

---

15 Strictly speaking, recalling footnote 10, it will only sell for $\pi(\bar{n},n^*)$ plus an infinitesimal amount. For convenience I ignore qualifications of this kind in the rest of this section.

16 The proofs are straightforward, and are given in a supplementary appendix to this paper, available at http://www.ucd.ie/~economic/staff/pneary/papers/crossbor.htm.
Lemma 2: Given Assumption 4, when a low-cost firm expects a takeover offer to be made and accepted in each future stage, its expected pay-off \( R'(s, \bar{n}, n^*) \) equals:

\[
\begin{align*}
(a) \quad & \pi'(\bar{n}, n^*), \quad s = 1; \\
(b) \quad & R'(s-1, \bar{n}-1, n^*) - \frac{1}{n^*} R(s, \bar{n}, n^*), \quad 1 < s \leq \bar{n}
\end{align*}
\]  

As with Lemma 1, this is proved by backward induction. With no remaining pre-production stages \( s=1 \) and \( \bar{n} \) remaining high-cost firms, a low-cost firm will earn profits of \( \pi'(\bar{n}, n^*) \) with certainty, which proves part (a) of Lemma 2. In earlier stages, a low-cost firm entering stage \( s \) faces a \( 1/n^* \) chance of being matched with a high-cost partner and an \( (n^*-1)/n^* \) chance of not being matched. In the former case, its expected future payoff assuming it makes an offer is \( R'(s-1, \bar{n}-1, n^*) \) less what it must pay to the high-cost firm, \( R(s, \bar{n}, n^*) \); in the latter case, assuming that a different low-cost firm makes a successful offer in this stage, it moves to the next stage with expected payoff \( R'(s-1, \bar{n}-1, n^*) \). Summing the expected payoffs weighted by the appropriate probabilities gives part (b) of the Lemma.

The final step in the proof is the forward-looking analogue of Proposition 3: 17

Lemma 3: If \( G'_{FH}(n, n^*) \) is positive, then \( G'_{FH}(\bar{n}, n^*) \) is also positive, for all \( \bar{n} < n \).

This ensures that, when \( G'_{FH}(n, n^*) \) is positive at the initial value of \( n \), each subsequent takeover (reducing \( n \) by one) leads in the next stage to a value for \( G'_{FH}(n, n^*) \) which is also positive, thus validating the expectations (which we assumed in proving Lemmas 1 and 2 are

\[\text{17 The analogy is only partial, since } G'_{FH}(n, n^*) \text{ does not always increase as } n \text{ falls.}\]
held by both types of firms) that all future merger offers will be made and accepted. I have failed to find an analytic proof of Lemma 3. However, an exhaustive search of parameter space confirms that it holds for all admissible cost levels and for all values of $n$ and $n^*$ up to fifty.\footnote{A Gauss program for the simulations is available at http://www.ucd.ie/~economic/staff/pneary/papers/crossbor.htm.} Since this result guarantees that a merger wave occurs provided the initial value of $G'_{FH}(n,n^*)$ is positive, it completes the proof of Proposition 4.

\[Q.E.D.\]

The expression for $G'_{FH}(n,n^*)$ in (7) emphasises the parallel with the myopic merger criterion in (3), and reduces to it when $s$ equals one. It is easier to explain intuitively when the terms in $R^*$ are expanded using (9), which gives:

\[G'_{FH}(n,n^*) = \pi^*(0,n^*) - \pi^*(1,n^*) - \frac{1}{n^*} \sum_{j=1}^{n-1} [R(n-j,n-j,n^*) - R(n-j,n-j+1,n^*)] - R(n,n,n^*)\] \hfill (10)

Consider a low-cost firm contemplating a takeover in the first stage ($s=n$), assuming that all future $n-1$ takeover offers are made and accepted. Equation (10) shows that making a takeover offer now affects its expected profits in two different ways. First, by reducing the final number of high-cost firms from one to zero, it tends to raise expected profits. This effect is given by the two terms on the first line of the right-hand side of (10). Second, it changes the payoffs to being matched in all future $n-1$ pre-production stages: with one less high-cost firm in each stage, the low-cost firm has a $1/n^*$ probability of having to pay an amount which, from Corollary 3, is greater. This effect is given by the sum of $n-1$ terms in
the second line. The first effect tends unambiguously to raise \( G_{FH}(n,n^*) \) relative to \( G_{FH}(n,n^*) \),\(^{19}\) but the second tends to reduce it. So it is not surprising that the myopic and forward-looking conditions cannot be uniquely ranked, though simulations show that \( G_{FH}(n,n^*) \) is larger in almost all cases.

We can now investigate the implications of Proposition 4. A similar series of derivations to those which led to Proposition 2 in the last section now yields:

\*\*Proposition 5: A takeover of a home firm by a foreign firm meets the\*\*

forward-looking merger profitability criterion \( G_{FH}'(n,n^*) > 0 \) if and only if:

\[ c > \xi_2 a' + (1-\xi_2)c^*, \quad 0 < \xi_2 < \xi_0 \] \hfill (11)

Because of the complexity of the recursive equations defining \( R(n,n^*) \) and \( R^*(s,n^*) \), I have been unable to derive an explicit expression for \( \xi_2 \). However, Appendix A shows that the forward-looking merger profitability criterion exhibits the same form as the myopic criterion in the last section: highly non-linear in \( n \) and \( n^* \), but linear in \( a' - c \) and \( a' - c^* \). Once again, the requirement that a takeover be profitable implies that the costs of the home firm exceed a weighted average of the demand intercept and the cost of the foreign firm. It can therefore be illustrated in \( \{c,c^*\} \) space just as in the previous section. As for the likelihood of mergers under the two alternative assumptions about behaviour, I have already noted that simulations show that the forward-looking criterion \( G_{FH}' > 0 \) is usually though not always less stringent

\(^{19}\) From Corollary 2, \( R(n,n,n^*) \) is less than \( \pi(1,n^*) \). Hence, the sum of the first, second and final terms on the right-hand side of (10) is greater than \( G_{FH}(1,n^*) \), which from Proposition 3 is greater than \( G_{FH}(n,n^*) \).
than the myopic one $G_{FH} > 0$. If we retain Assumption 2, which implicitly means that firms face a myopic capital-market constraint that each merger must be profitable even if no further mergers take place, then the two criteria have identical implications. Alternatively, if we drop Assumption 2, then more mergers will usually take place when firms are forward-looking than when they are myopic. In both cases, the $H$ region expands at the expense of the $HF$ region when mergers are allowed.

A final issue which needs to be addressed is that, although firms are too small to influence economy-wide variables, if they are forward-looking they should be able to predict how such variables will be affected by takeovers occurring in many sectors. Hence it makes sense to assume that only ex post profitable takeovers will take place. This implies that firms anticipate the wage changes which the economy-wide takeovers will induce. So, the next step is to solve for the change in wages, which requires us to move from partial to general equilibrium.

5. Oligopoly in General Equilibrium

The model in the absence of mergers is similar to that presented in Neary (2002). Consider first the demand side. Utility is an additive function of a continuum of goods, with each sub-utility function quadratic:

$$U \{ x(z) \} = \int_0^1 [ax(z) - \frac{1}{2}bx(z)^2]dz$$ (12)

In each country there is a single representative consumer who maximises (12) subject to the budget constraint:
\[ \int_0^1 p(z)x(z)dz \leq I \]  

(13)

where \( I \) is aggregate income. This leads in each country to inverse demand functions for each good which are linear in own price, conditional on the marginal utility of income \( \lambda \), which is the Lagrange multiplier attached to the budget constraint:

\[
p(z) = \frac{1}{\lambda}(a - bx(z)), \quad \lambda[p(z), I] = \frac{a\mu_p - bl}{\sigma_p^2}
\]  

(14)

The effects of prices on \( \lambda \) are summarised by the first and second moments of the distribution of prices:

\[
\mu_p = \int_0^1 p(z)dz \quad \text{and} \quad \sigma_p^2 = \int_0^1 p(z)^2dz
\]  

(15)

Finally, in a free-trade world equilibrium, where the behaviour of each of two countries can be described by an aggregate utility function like (12), the world inverse demand curve for each good comes from equation (14) for the home country and the corresponding equation for the foreign country (assumed to have the same demand slope \( b \), but a possibly different intercept \( a' \)):

\[
p(z) = a' - b'x(z) \quad \text{where:} \quad a' = \frac{a}{\lambda}, \quad \tilde{a} = a - a^*, \quad \lambda = \lambda^* + \lambda, \quad b' = \frac{b}{\lambda}
\]  

(16)

Here \( \lambda \) is the world marginal utility of income, which provides the link between the actual demand functions in (16) and the perceived demand functions given by equation (1) in Section 2.

Next, to close the model, we need to explain how costs are determined in general equilibrium. Following Neary (2002), we adopt Ricardian assumptions about technology and factor markets: labour is the sole factor of production and is intersectorally but not...
internationally mobile; each country can potentially produce a continuum of goods indexed by \( z \in [0,1] \); while sectors differ in their unit labour requirements, denoted by \( \alpha(z) \) and \( \alpha^*(z) \) at home and abroad respectively. Hence the unit costs of firms in sector \( z \) are as follows:

\[
c(z) = w\alpha(z), \quad c^*(z) = w^*\alpha^*(z)
\]  

where \( w \) and \( w^* \) denote the home and foreign wages respectively. We assume that \( \alpha \) and \( \alpha^* \) are continuous in \( z \), and that sectors are ordered such that \( z \) is an index of foreign comparative advantage. To help the exposition it is often convenient to strengthen this last assumption and assume that \( \alpha(z) \) is increasing and \( \alpha'(z) \) is decreasing in \( z \), though this is not necessary for the results. 20

Inspecting equations (1), (2) and (17), as well as the expressions for sectoral outputs in Appendix A, equation (28), it can be seen that the values of real variables are homogeneous of degree zero in three nominal variables: the home and foreign wage rates \( w \) and \( w^* \), and the inverse of the world marginal utility of income, \( \bar{\lambda}^{-1} \). Putting this differently, the absolute values of these and all other nominal variables (e.g., prices and costs) are indeterminate. This is a standard property of real models, and it implies that we can choose an arbitrary numeraire or normalisation of nominal variables. It turns out to be most convenient to choose the world marginal utility of income as numeraire, so we work henceforward with wages normalised by it: \( W = \bar{\lambda}w \), \( W^* = \bar{\lambda}w^* \). These are not the same as real wages in the conventional sense (since tastes are not homothetic, so "real wages" cannot be

20 As discussed in more detail in Neary (2002), necessary and sufficient conditions for the model to be well-behaved are that \( y \) is decreasing in \( z \) at \( z=z^* \), which implies that \( w\alpha(z^*) > (1-\xi_0)w^*\alpha'(z^*)' \); and that \( y^* \) is increasing in \( z \) at \( z=z^* \), which implies that \( (1-\xi_0)w\alpha(z^*)' > w^*\alpha'(z^*)' \). In the competitive limit, these conditions collapse to an assumption made by Dornbusch, Fischer and Samuelson (1977) that \( \alpha(z)/\alpha'(z) \) is increasing in \( z \). Geometrically, they imply that the cost locus intersects each of the boundaries of the HF region at most once.
defined independent of the level of utility), though they come close. A better interpretation is "marginal real wages", since they equal nominal wages deflated by the marginal cost of utility.

We can now state the equations which define the free-trade equilibrium in the absence of mergers. First, the threshold sectors at home and abroad, denoted \( \tilde{z} \) and \( \tilde{z}^{*} \) respectively, are determined implicitly by setting (2) and the corresponding equation for the foreign country to equalities.\(^{21}\) This is illustrated in Fig. 1, where the downward-sloping line denotes an arbitrary distribution of home and foreign costs, conditional on wages in the two countries. As shown, the home country produces positive outputs in all sectors for which \( z \) is below \( \tilde{z} \), while the foreign country specialises in sectors for which \( z \) is above \( \tilde{z}^{*} \). Next, wages are determined by the conditions for full employment at home and abroad. That for the home country is as follows:

\[
L = \int_{0}^{\tilde{z}} \alpha(z) n y(W; z; n, 0) dz + \int_{\tilde{z}}^{\tilde{z}^{*}} \alpha(z) n y(W^{*}, z; n, n^{*}) dz \tag{18}
\]

where, from equation (28) in the Appendix, the levels of output in sectors which do not and which do face foreign competition equal respectively:

\[
y(W; z; n, 0) = \frac{\tilde{\alpha} - W\alpha(z)}{b(n+1)} \quad \text{and} \quad y(W^{*}, z; n, n^{*}) = \frac{\tilde{\alpha} - (n^{*}+1)W\alpha(z) + n^{*}W\alpha^{*}(z)}{b(n+n^{*}+1)} \tag{19}
\]

The full-employment condition for the foreign country, analogous to (18), completes the specification of the model.

\(^{21}\) Equation (2) and the corresponding equation for the foreign country could hold as strict inequalities for all \( z \). This would happen for example if all home and foreign sectors were identical. However, since the focus of the paper is on how cross-border mergers shift the threshold of specialisation, we will assume throughout that both \( \tilde{z} \) and \( \tilde{z}^{*} \) lie strictly within the \([0, 1]\) interval.
6. Effects of Mergers on Specialisation Patterns and Income Distribution

To solve the model explicitly, I concentrate on the case of countries which are the same size but exhibit symmetric inter-sectoral differences. This is analytically convenient, and also gives a stylised representation of an empirical paradigm sketched by Davis and Weinstein (2002): significant differences in sectoral relative efficiencies prevail even between high-income countries with similar factor endowments. Equal size and symmetry imply that countries have the same endowments: \( L=L^* \); the same tastes: \( a=a^*=\frac{1}{2}a \); the same industrial structure: \( n=n^* \); and technology distributions which are "mirror images" of each other: \( \alpha(z) = \alpha^*(1-z) \), for all \( z \). In equilibrium they therefore have the same marginal utility of income: \( \lambda = \lambda^* = \frac{1}{2} \lambda^* \); the same wage: \( W = W^* \); and symmetric threshold sectors: \( \tilde{z} = 1 - \tilde{z}^* \).

The effects of mergers on equilibrium wages can be explained intuitively by considering how they affect the demand for labour at initial wages. In some sectors, high-cost firms in the home country are bought out by low-cost foreign rivals, while in other sectors the converse happens. In both countries there are expanding and contracting sectors. However, at the initial wages, expanding firms increase their output by only a fraction of the output of the firms which are taken over. (Recall footnote 12.) In addition, the expanding firms have lower labour requirements per unit output than the contracting ones. Aggregating over all sectors, the total demand for labour therefore contracts in both countries at the initial wages. Hence, wages must fall to restore labour-market equilibrium.

To see this formally, consider the two equations which define the equilibrium. (To help the exposition we refer to these as equations for the home country, though nothing hinges on this. Similar equations apply to the foreign country but, with symmetry, they are identical to (20) and (21).) First is the equation for the home threshold sector, or the "extensive margin". Whether this is the threshold for zero profits, or for profitable takeovers
under either myopic or forward-looking assumptions, we have seen in equations (2), (4) and (11) that it takes the same form, highly non-linear in \( n \) and \( n^* \), but linear in \( c \) and \( c^* \) and hence in \( \alpha \) and \( \alpha^* \). Hence, with or without mergers, it can be written as follows:

\[
G(W, \tilde{z}; \xi) = W\alpha(\tilde{z}) - \xi\hat{a} - (1-\xi)W\alpha^*(\tilde{z}) = 0
\]

where \( \xi \) takes different values depending on the context. The signs under the arguments of \( G(.) \) indicate the signs of the corresponding partial derivatives.\(^\text{22}\) Higher values of both \( W \) and \( \tilde{z} \) make specialisation or profitable takeovers more likely, while higher values of \( \xi \), the weight attached to the demand intercept, make them less likely. Hence the loci corresponding to equation (20) for different values of \( \xi \) are downward-sloping in \( \{W, \tilde{z}\} \) space, as illustrated in Fig. 4, and shift downwards as \( \xi \) falls.

The second equation is the labour-market equilibrium condition. With symmetric countries, this is simply (18) with \( n=n^* \) and with the indexes of the home and foreign threshold sectors summing to unity: \( \tilde{z}+\tilde{z}^*=1 \). This can be written as follows:

\[
L(W, \tilde{z}) = L
\]

where once again the signs under the arguments indicate the signs of the corresponding partial derivatives. To confirm these, consider first the derivative of the labour demand function (18) with respect to \( W \):

\(\text{22}\) With symmetry, \( \tilde{z} \) is always greater than \( \frac{1}{2} \), and so \( \alpha(\tilde{z}) \) is always greater than \( \alpha'(\tilde{z}) \). Hence \( G_w \), which equals \( \alpha(\tilde{z})-(1-\xi)\alpha'(\tilde{z}) \), is positive. As for \( G_z \), which equals \( W\alpha(\tilde{z})'(1-\xi)W\alpha^*(\tilde{z})' \), this must be positive at \( \xi = \xi_0 \), from the assumption in footnote 20, and from analogous assumptions at other values of \( \xi \). Finally, \( G_{\xi} \), which equals \( -(\tilde{z}-W\alpha'(\tilde{z})^* \), must be negative from the requirement that foreign firms are profitable in the post-merger equilibrium.
where $\sigma_1^2$, $\sigma_2^2$ and $\gamma^2$ denote appropriately truncated moments of the home and foreign technology distributions and are defined in Appendix B. Inspecting equation (22) shows that a higher wage both at home and abroad encourages many but not necessarily all sectors to shed labour at the intensive margin: some low-cost home sectors may gain more from the higher wages which their foreign rivals have to pay than they lose from higher home wages. However, summing across all sectors, the aggregate demand for labour is unambiguously decreasing in $W$. (See Appendix B for details.)

It is intuitively obvious that labour demand also falls when the extensive margin contracts. To see this, differentiate (18) with respect to $\tilde{z}$:

$$L_{\tilde{z}} = n \left[ \alpha(1-\tilde{z}) \left( y(W,1-\tilde{z};n,n^*) - y(W,1-\tilde{z};n,0) \right) + \alpha(\tilde{z}) y(W,\tilde{z};n,n^*) \right]$$  \hspace{1cm} (23)

This simplifies to give:  \hspace{1cm} (24)

$$L_{\tilde{z}} = n \left[ \alpha(\tilde{z}) - \frac{n}{n^*+1} \alpha^*(\tilde{z}) \right] y(W,\tilde{z};n,n^*)$$

With symmetry, the expression in square brackets is positive; and $y(W,\tilde{z};n,n^*)$ is positive at

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23 The relevant expression in (22) is $(n+1)\alpha(z)-n\alpha^*(z)$. Recalling footnote 22, this is positive for all $z$ greater than $\frac{1}{2}$, but may be negative (implying that an equal increase in home and foreign wages raises output and labour demand in the relevant sector) for low values of $z$.

24 To derive (24), use symmetry to replace $\alpha(1-\tilde{z})$ by $\alpha^*(\tilde{z})$, and $y(W,1-\tilde{z};n,n^*)$ by $y^*(W,\tilde{z};n^*,n)$. The expression in curly brackets in (23) can then be written in terms of home output $y(W,\tilde{z};n,n^*)$ using Lemma A.1, with $\tilde{n}$ set equal to $n$. 

26
points to the left of the \( G(.;\xi_0) \) locus in Fig. 4 and zero where it meets the locus.\(^{25}\) Hence the labour-market equilibrium locus is upward-sloping to the left of the \( G(.;\xi_0) \) locus and horizontal where it cuts it, as shown in Fig. 4.

The effects of cross-border mergers on wages and on the extensive margin can now be deduced from Fig. 4. Free-trade equilibrium in the absence of mergers is at point \( A_0 \), the intersection of the \( L \) and \( G(.;\xi_0) \) loci. When mergers take place, the \( G(.;\xi_0) \) locus shifts to the left, but the \( L \) locus is unaffected. At initial wages (the partial equilibrium case considered in Sections 3 and 4), this reduces the demand for labour in both countries. In general equilibrium, assuming labour markets are perfectly flexible, wages are bid down, which raises the profitability of marginal high-cost firms, putting them out of reach of takeovers. Hence the general-equilibrium repercussions working through labour markets dampen, though they cannot reverse, the tendency towards merger waves. The same outcome is illustrated from a different perspective in Fig. 5. The dashed line indicates the cost distribution corresponding to the equilibrium in the absence of mergers. As wages fall, this cost locus shifts inwards as shown, and so the range of sectors which remain in the \( HF \) region is greater than an analysis based on fixed wages would predict, though still less than if no mergers had occurred. For example, in the sector represented by point \( B \) in Fig. 5, home firms are just on the margin of being taken over at initial wages. The fall in wages shifts the point corresponding to this

\(^{25}\) Extending the argument, the labour-market equilibrium locus is downward-sloping at points to the right of the \( G(.;\xi_0) \) locus. Of course, such points correspond to negative values of home output and so are irrelevant to equilibrium analysis. They are of interest, however, from the perspective of establishing the stability of equilibrium. Out of equilibrium, such points generate incentives for a reduction in \( \tilde{z} \); and (in the no-mergers case where \( \tilde{\xi} = \xi_0 \)) the converse holds for points to the left of the \( G(.;\xi_0) \) locus. Similarly, points above the \( L \) locus correspond to unemployment and generate incentives for reductions in \( W \), and conversely for points below the \( L \) locus. Combining these dynamics shows that the intersection point of the \( L \) locus and the \( G(.;\xi_0) \) locus is a stable equilibrium, and a similar argument applies to the intersection point of the \( L \) locus and each of the other \( G(.;\xi) \) loci.
sector inwards along a ray from $B$ to $B'$, where home firms are now too profitable to be taken over.

Wages fall not only relative to the numeraire but also relative to profits. Profits rise even if wages do not change, since this is a necessary condition for mergers to take place. The fall in wages raises profits further. (See Appendix C for a formal demonstration.) Hence:

**Proposition 6:** When the two countries are symmetric, both forward-looking and myopic mergers lower the share of wages in national income.

So mergers have an unambiguous effect on the functional distribution of income.

7. Mergers and Welfare

Finally, we wish to establish the effect of mergers on aggregate welfare. It is instructive to begin with a partial-equilibrium perspective. In this view, only changes in sectors where mergers occur need be taken into account; the elimination of some firms raise prices in those sectors, which in turn lowers consumer surplus; but since aggregate profits rise, the net effect on welfare is ambiguous. In fact, it can be shown that this ambiguity can be resolved in the present case. Since only high-cost firms are eliminated, the increase in production efficiency ensures that the rise in profits dominates the fall in consumer surplus. Hence, if aggregate welfare is measured by the sum of profits and consumer surplus, it unambiguously rises.26

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26 This effect was first noted by Lahiri and Ono (1988). Adapting their terminology to the present context, closing down minor firms raises welfare. A full derivation is given in the supplementary appendix.
However, this partial-equilibrium approach is misleading when we consider an economy-wide shock such as the elimination of restrictions on cross-border mergers. In general equilibrium only consumers matter, and if there are benefits from more efficient resource allocation they show up not in higher profits but in lower prices throughout the economy. We can see this by calculating explicitly the indirect utility function corresponding to the direct utility function (12). Substituting from the direct demand functions (and ignoring a constant) gives:

\[ \bar{U} = a^2 - (\lambda \sigma_p)^2 \]  

(25)

Thus welfare depends inversely on the second moment of the marginal-utility-of-income-weighted price distribution, and we need to determine the effects of cross-border mergers on this expression.

Because of symmetry, it is sufficient to consider the second moment of prices evaluated over half of the technology distribution:

\[ \frac{1}{2} \sigma_p^2 = \int_{\gamma_0}^{\tilde{z}} p(W, z; n, n')^2 dz + \int_{\tilde{z}}^{1} p(W, z; 0, n')^2 dz \]  

(26)

Since prices are increasing in wages, it follows that \( \sigma_p^2 \) is increasing in \( W \). To establish how it varies with the extensive margin, partial equilibrium intuition is perfectly adequate: at given wages, mergers must raise prices in all sectors where they occur, so \( \sigma_p^2 \) is decreasing in \( \tilde{z} \). Iso-welfare loci are therefore upward-sloping in the relevant region of Fig. 6 (i.e., to the left of the \( G(\cdot; \tilde{z}_0) \) locus). Appendix D shows further that they are horizontal where they cross the \( G(\cdot; \tilde{z}_0) \) locus, just like the full-employment locus. However, it has not proved possible to determine which of these loci is steeper, so the full effect of mergers on welfare is unambiguous. Fig. 6 illustrates the case where the wage responsiveness of labour demand
is greater than that of the second moment of prices, so welfare is increased by the merger wave. But the possibility of the opposite occurring cannot be ruled out.

8. Conclusion

In this paper I have used a model of oligopoly in general equilibrium presented elsewhere to throw light on the phenomenon of cross-border mergers. The key to the prediction of mergers in my framework is not general equilibrium per se, but rather cost differences between firms. However, the two go together, since in partial equilibrium there is usually no plausible basis for assuming firm heterogeneity.\(^{27}\) By contrast, in a Ricardian trade model, international differences in technology provide a natural reason for it.

The model predicts that international differences in technology generate incentives for bilateral mergers in which low-cost firms located in one country acquire high-cost firms located in the other. As a result, cross-border mergers serve as "instruments of comparative advantage". They facilitate more specialisation in the direction of comparative advantage, so moving production and trade patterns closer to what would prevail in a competitive Ricardian world. They also have implications for income distribution, putting downward pressure on wages, and so tilting the distribution of income towards profits at the expense of wages in both countries. As for aggregate welfare, the fall in wages puts downward pressure on prices in all sectors, which tends to increase the gains from trade in both countries. Potentially offsetting this, the sectors in which mergers occur become less competitive, so their prices tend to rise. Hence the full effect on welfare is ambiguous.

The results of this paper lend themselves to empirical testing. By construction, the

\(^{27}\) For an alternative approach to firm heterogeneity, see Melitz (2003) and (in the context of greenfield FDI) Helpman, Melitz and Yeaple (2004).
model predicts that trade liberalisation encourages cross-border merger waves, consistent with the evidence cited in the Introduction. In addition, the model makes a further empirical prediction: absent cost synergies, the pattern of cross-border mergers which results from market integration follows that of comparative advantage, in the sense that low-cost firms acquire high-cost foreign rivals. As a corollary, the model predicts that cross-border mergers and exports are complements rather than substitutes, in the sense that exporting sectors tend to be sources of rather than hosts for foreign direct investment. Finally, the model predicts that cross-border merger waves tend to reduce factor demands and so put downward pressure on the returns to productive factors. Most of there predictions are very different from, and more consistent with the available empirical evidence than, those of standard models of greenfield foreign direct investment. (Baldwin and Ottaviano (2001) is a notable exception.)

Of course the model presented here is subject to many qualifications. The assumption of no efficiency gains may seem especially unrealistic at first blush. Rather than defending it by referring to the empirical evidence (such as that summarised in footnote 4), it can be seen as a methodologically convenient simplification, which highlights the finding that mergers motivated by purely strategic (and so anti-competitive) motives lead to a rationalisation of production in the socially desirable direction, and may even increase welfare. Further work is needed to explore the robustness of the conclusions presented here to relaxing this and other assumptions. The present paper is unlikely to be the last word on the theory of cross-border mergers in general equilibrium, but at least it appears to be the first.
Appendix A

Preliminaries: The setting is a homogeneous-good Cournot oligopoly where all firms have constant marginal costs, and face the (perceived) linear inverse demand curve (1). Throughout Appendix A, we consider a single industry in partial equilibrium. Hence the arguments $W, W^*$ and $z$ can be suppressed. In the pre-takeover equilibrium, there are $n=n+n^*$ firms, of which $n$ home firms have marginal cost $c$ and $n^*$ foreign firms have marginal cost $c^*$. The first-order conditions are $b'y(n,n^*)=p(n,n^*)-c$ and $b'y^*(n,n^*)=p(n,n^*)-c^*$. Solving these for industry output and price:

$$\bar{y}(n,n^*) = \frac{n(a'-c)+n^*(a'-c^*)}{b'(n+n^*+1)}, \quad p(n,n^*) = \frac{a'+nc+n^*c^*}{n+n^*+1} \quad (27)$$

These in turn can be used to solve for the outputs of each firm:

$$y(n,n^*) = \frac{(n^*+1)(a'-c)-n^*(a'-c^*)}{b'(n+n^*+1)}, \quad y^*(n,n^*) = \frac{(n+1)(a'-c^*)-n(a'-c)}{b'(n+n^*+1)} \quad (28)$$

We can now state a key lemma, which gives the effects on outputs of surviving firms of takeovers which eliminate a subset of home firms. Let $\tilde{n}$ be the number of surviving home firms. Then:

**Lemma A.1:** Closing down $n-\tilde{n}$ home firms increases the output of all remaining firms (both home and foreign) by the same amount, equal to a constant times the initial output of each home firm:

$$y(\tilde{n},n^*) - y(n,n^*) = y^*(\tilde{n},n^*) - y^*(n,n^*) = \frac{n-\tilde{n}}{n+n^*+1} y(n,n^*) \quad (29)$$

The proof is immediate from (28).
Proof of Proposition 1: Consider first a takeover of a home firm by another home firm. From Lemma A.1, with \( \tilde{n} \) equal to \( n-1 \), the post-takeover output of each remaining home firm, \( y(n-1,n^*) \), is proportional to their pre-takeover output \( y(n,n^*) \). Hence, recalling that profits are proportional to the square of output, the gain to such a takeover can be written as follows:

\[
\frac{1}{b'} G_{HH}(n,n^*) = y(n-1,n^*)^2 - 2y(n,n^*)^2 = \left( \frac{\tilde{n}+1}{\tilde{n}} \right)^2 - 2 \frac{\tilde{n}+1}{\tilde{n}} y(n,n^*)^2 = -\tilde{n}^2 + 2\tilde{n} + 1 \quad (30)
\]

This is negative for \( \tilde{n} > 2 \), which proves that a takeover of a home firm by another home firm is never profitable, irrespective of the level of foreign firms’ costs. Similar reasoning shows that a takeover of a foreign firm by another foreign firm is never profitable. This proves Proposition 1.

Proof of Proposition 2: Next, consider a takeover of a home firm by a foreign firm. The myopic merger criterion (3) can be expressed in terms of outputs as follows:

\[
\frac{1}{b'} G_{FH}(n,n^*) = y^*(n-1,n^*)^2 - y^*(n,n^*)^2 - y(n,n^*)^2 \quad (31)
\]

The key step is to factorise the difference between the pre- and post-takeover squared output levels of the foreign firm so that (31) can be rewritten as follows:

\[
\frac{1}{b'} G_{FH}(n,n^*) = [y^*(n-1,n^*) - y^*(n,n^*)][y^*(n-1,n^*) - y^*(n,n^*)] - y(n,n^*)^2 \quad (32)
\]

Using Lemma A.1, with \( \tilde{n} \) equal to \( n-1 \), to express \( y^*(n-1,n^*) \) in terms of \( y^*(n,n^*) \) gives an explicit expression for \( G_{FH} \) in terms of pre-takeover outputs only:

\[
\frac{1}{b'} G_{FH}(n,n^*) = \frac{1}{\tilde{n}^2} y(n,n^*)[2\tilde{n}y^*(n,n^*) - (\tilde{n}^2 - 1)y(n,n^*)] \quad (33)
\]
The sign of $G_{FH}$ depends only on the expression in square brackets, which is linear in outputs. Using (28) to eliminate $y(n, n^*)$ and $y^*(n, n^*)$ from this expression yields:

$$G_{FH}(n, n^*) = \frac{\phi y(n, n^*)}{n^2(n+1)} \left[ c - \xi_1 a' - (1 - \xi_1) c^* \right]$$

from which equation (4) in Proposition 2 follows immediately. (The parameters $\phi$ and $\xi_1$ are defined in equation (5).)

It is immediately clear that $\xi_1$, equal to $1 - \psi/\phi$, is positive. The final step in the proof

is to show that $\xi_1$ is less than $\xi_0$. Substituting for $\xi_1$ from (5), the inequality to be proven is equivalent to $(n^*+1)(\phi - \psi) < \phi$. Substituting for $\phi - \psi$ from (5), this in turn is equivalent to

$(n^*+1)(\bar{n}^2 - 2\bar{n} - 1) < \phi$, which, from the definitions of $\psi$ and $\phi$ in (5), is always true.

**Proof of Proposition 3:** To prove the proposition, we treat the number of home firms as a continuous variable. Differentiate (28) with respect to $n$:

$$\frac{dy(n, n^*)}{dn} = \frac{dy^*(n, n^*)}{dn} = -\frac{1}{n+1} y(n, n^*), \quad \frac{dy^*(n-1, n^*)}{dn} = -\frac{\bar{n}+1}{\bar{n}^2} y(n, n^*) \quad (35)$$

The first equation in (35) restates Lemma A.1 for $\bar{n}=n-1$ in continuous form: to a first-order approximation, a takeover of a home firm (a fall in $n$) leads to identical increases in the pretakeover output of home and foreign firms. The second equation in (35) shows that it leads to a larger increase in the post-takeover output of a foreign firm. To see the implications of this for the profits of all three types of firms and hence for the gains from a takeover, differentiate $G_{FH}$ from (31) with respect to $n$: 

34
\[
\frac{1}{2b'} \frac{dG_{FH}(n,n^*)}{dn} = \frac{y'(n-1,n^*) dy'(n-1,n^*)}{dn} - \frac{y'(n,n^*) dy'(n,n^*)}{dn} - y(n,n^*) \frac{dy(n,n^*)}{dn}
\]

\[
= - \frac{1}{n^2(n+1)} y(n,n^*) \left[ (\tilde{n}+1)^2 y'(n-1,n^*) - \tilde{n}^2 y'(n,n^*) - \tilde{n}^2 y(n,n^*) \right]
\]

(36)

As in (33), the sign of this depends only on the expression in square brackets, which is linear in outputs. Using Lemma A.1, with \( \tilde{n} \) equal to \( n-1 \), to eliminate \( y'(n-1,n^*) \) yields:

\[
\frac{dG_{FH}(n,n^*)}{dn} \propto -\tilde{n}(2\tilde{n}+1) y(n,n^*) + [\tilde{n}^3-(\tilde{n}+1)^2] y(n,n^*)
\]

(37)

Finally, we wish to evaluate this at points where the gain to a takeover is strictly positive. From (33), this is equivalent to:

\[
G_{FH}(n,n^*) > 0 \iff 2\tilde{n} y'(n,n^*) = (\tilde{n}^2-1) y(n,n^*) + Z
\]

(38)

where \( Z > 0 \). Substituting into (37) gives:

\[
\frac{dG_{FH}(n,n^*)}{dn} \propto -(3\tilde{n}^2+2\tilde{n}+1) y(n,n^*) - (2\tilde{n}+1)Z
\]

(39)

This is negative, which proves the result. Note also that, from (5), \( \frac{d\xi_1}{dn} \) is proportional to the right-hand side of (39) with \( Z=0 \). An increase in the gain to a takeover shifts downwards the boundary between the \( F \) and \( HF \) regions, as asserted in the text.

**Proof of Proposition 5:** The proof is similar to that of Proposition 2. Consider the expression for \( G'_{FH}(n,n^*) \) in (10). By expressing the difference in profits as the difference in squared outputs, the first two terms in this expression can be written as follows:
Using Lemma A.1, \( y(1,n^*) \) can be shown to be proportional to \( y(n,n^*) \). As for the remaining terms in (10), from Corollary 1 to Lemma 1, they are linear in \( y(\tilde{n},n^*) \), for \( \tilde{n}=1, \ldots, n \). Repeated use of Lemma A.1 allows all of these to expressed as multiples of \( y(n,n^*) \). Hence this common factor can be extracted from the expression, leaving (after substituting for outputs from (28)) a term which is affine in \( a' - c \) and \( a' - c' \). Hence the requirement that \( G'_{FH}(n,n^*) \) be positive can be expressed in the linear form given by (11).

### Appendix B

With symmetry, the truncated second moments of the home and foreign technology distributions, and their truncated uncentred covariance in equation (22) are defined as follows:

\[
\begin{align*}
\sigma_1^2 &= \int_0^{1-\xi} \alpha(z)^2 \, dz - \int_1^{1-\xi} \alpha'(z)^2 \, dz \\
\sigma_2^2 &= \int_{1-\xi}^{\xi} \alpha(z)^2 \, dz = \int_{1-\xi}^{\xi} \alpha'(z)^2 \, dz = 2 \int_{\gamma_1}^{\xi} \alpha(z)^2 \, dz = 2 \int_{1-\xi}^{\gamma_1} \alpha'(z)^2 \, dz \\
\gamma^2 &= \int_{1-\xi}^{\xi} \alpha(z) \alpha'(z) \, dz = 2 \int_{1-\xi}^{\gamma_1} \alpha(z) \alpha'(z) \, dz = 2 \int_{\gamma_1}^{\xi} \alpha(z) \alpha'(z) \, dz
\end{align*}
\]

(41)

Note that:

\[
\begin{align*}
\sigma_2^2 - \gamma^2 &= \gamma_2 \int_{1-\xi}^{\xi} \left[ \alpha(z)^2 + \alpha'(z)^2 - 2\alpha(z)\alpha'(z) \right] \, dz \\
&= \gamma_2 \int_{1-\xi}^{\xi} \left[ \alpha(z) - \alpha'(z) \right]^2 \, dz > 0
\end{align*}
\]

(42)

from which it follows that \( L_w \) in (22) must be negative as claimed.

### Appendix C

To prove Proposition 6, consider the effects of mergers on total profits in general equilibrium. From symmetry, total profits in the home country equal half of total profits in
the world, which in turn equals total profits for all firms in sectors where $z$ exceeds $\frac{1}{2}$:

$$\Pi(W, \tilde{z}) = n \int_{\tilde{z}}^{\infty} \pi(W, z; n, n) \, dz + n \int_{\tilde{z}}^{\infty} \pi^*(W, z; n, n) \, dz + n \int_{\tilde{z}}^{1} \pi^*(W, z; 0, n) \, dz$$

(43)

This is clearly decreasing in $W$. Differentiating with respect to $\tilde{z}$:

$$\Pi_{\tilde{z}} = -n \left[ \pi^*(W, \tilde{z}; 0, n) - \pi^*(W, \tilde{z}; n, n) - \pi(W, \tilde{z}; n, n) \right]$$

(44)

From Proposition 3, the expression in brackets is greater than the myopic gain from a merger, $G_{FH}(n, n^*)$, in (3). Hence we can conclude that $\Pi$ is decreasing in $\tilde{z}$ for all $\tilde{z}$ in the range $\tilde{z}_1 < \tilde{z} < \tilde{z}_0$. Proposition 6 follows immediately.

Appendix D

To see how welfare varies with the threshold sector, $\tilde{z}$, differentiate (26):

$$\frac{d\sigma_p^2}{d\tilde{z}} = \frac{d}{d\tilde{z}} \left[ \frac{V_2}{1} \right] = \frac{p(W, \tilde{z}; n, n^*)^2}{n + n^*} - \frac{p(W, \tilde{z}; 0, n^*)^2}{n^* + 1}$$

(45)

This difference in squares can be simplified by factorising it and by using Cournot equilibrium prices from (27), which in general equilibrium can be written as:

$$p(W, z; n, n^*) = \frac{\tilde{a} + nW_{\alpha}(z) + n^*W_{\alpha^*}(z)}{n + n^* + 1}, \quad p(W, z; 0, n^*) = \frac{\tilde{a} - n^*W_{\alpha^*}(z)}{n^* + 1}$$

(46)

Substituting this into (45) gives:

$$\frac{d\sigma_p^2}{d\tilde{z}} = -\frac{2b'n}{n^* + 1} \left[ p(W, \tilde{z}; n, n^*) + p(W, \tilde{z}; 0, n^*) \right] y(W, z; n, n^*)$$

(47)

This is zero along the $G(\cdot; \tilde{z}_0) = 0$ locus in Fig. 6 and negative to the left of it, as stated in the text.
References


Fig. 1: Equilibrium Production Patterns for a Given Cost Distribution

Incentives for foreign firms to take over home

Incentives for home firms to take over foreign

Fig. 2: Takeover Incentives
### Fig. 3: Proof of Lemma 1 by Backward Induction

(Fractions in boxes give the probability in stage $s$ that a high-cost firm will face the takeover offer indicated by the corresponding arrow in stage $s-1$)

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<th>1</th>
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<td>$R(3, 3, n^+)$</td>
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</table>

### Fig. 4: Simultaneous Determination of Wages and the Extensive Margin

\[ G(W, \tilde{z}; \xi) = 0 \]
\[ L = L(W, \tilde{z}) \]
\[ G(.; \tilde{z}_0) = 0 \]
\[ G(.; \tilde{z}_1) = 0 \]
Fig. 5: Wage Adjustments Dampen Cross-Border Merger Waves

Fig. 6: Mergers May Increase Welfare

\[ G(W, \tilde{z}; \xi) = 0 \]
\[ L = L(W, \tilde{z}) \]
\[ \bar{U} = U (W, \tilde{z}) \]
\[ G(\cdot; \xi_0) = 0 \]
\[ L = L(\cdot) \]