DEMAND MANAGEMENT WITH RATIONING*

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ABSTRACT

This paper examines the impact of monetary and fiscal policies in both the Barro-Grossman model and a neo-Keynesian model which incorporates a bond market. It is shown that there is a unique Demand Management policy for every temporary equilibrium state which obviates the need to resort to 'supply-side' or wage-price policies. It emerges that these policies can contain counter intuitive elements.
1. Introduction.

The impact of fiscal policy in fixed-price models has been examined by numerous writers including Malinvaud (1977) and in a more general context, Muellbauer and Portes (1973). The latter modelled 'helicopter' money - i.e., lump-sum transfers and taxes - but did not explore the implications of the manner in which the use of this policy instrument differs from the application of fiscal policy. In a paper the main focus of which is the extension of fixed-price models to the theory of international monetary economics, Naylor (1983) studies the impact of a balanced increase in government purchases and lump-sum taxes which is an example of the range of demand-side fiscal policy combinations which are open to the political decision maker. Ehrenmann (1974) studies the production of public goods and a proportional income tax as policy options in the context of the three-market model of Barro and Grossman (1971). He concludes that both policies are, in general, required and identifies states of disequilibrium in which apparently perverse policies are applicable. Ehrenmann's model is ambiguous as to which policy combinations are required to deal with what Malinvaud subsequently styled classical unemployment, though Glaister (1981) suggests that this inconclusiveness arises from Ehrenmann's special assumption that the household's labour supply schedule is backward sloping. Hoel (1980) introduces a bond market with a flexible interest rate, thereby enabling the effect of
open-market operations and bond-financed fiscal policy to be studied. However, the labour supply is modelled as fixed and this precludes a sharp distinction between monetary and fiscal policy. Renassy (1932) also studies the effects of monetary and fiscal policy in a four-market model including a bond market with a flexible interest rate. However, he limits his examination to the traditional IS/LM case where the price of goods and the wage rate are inflexible in the downwards direction only. This excludes, for example, the regime of repressed inflation.

In this study, two models of temporary equilibrium with rationing are presented in which the complementary but distinct roles of monetary and fiscal policies are made theoretically and diagrammatically explicit. The first model is implicit in the work of Barro and Grossman (1971) as extended by Malinvaud (1977) and only considers monetary policy of the helicopter drop variety. A bond market with a flexible interest rate is incorporated in the second model in which open-market operations and pure bond-financed fiscal policy are examined as policy alternatives. In both models, it emerges that the choice between monetary and fiscal policy is not one of conflict but that in general, both policies are required to restore equilibrium. It is shown that there exists a combination of the two policies for every state of disequilibrium which will restore equilibrium without the need to have recourse to either 'supply-side' policies or prices and incomes policies. The
policy recommendations are not always obvious: by analogy with Swan (1955), there exist 'zones of perversity' where counter intuitive policy responses are required. For example, it is demonstrated that the correct response to a state of Keynesian unemployment could involve the use of contractionary fiscal policy in combination with expansionary monetary policy. In the second model, it is shown that in a state of repressed inflation, an expansionary fiscal policy can actually lead to a rise in output and employment contrary to the conclusion of Malinvaud (1977).

The plan of the paper is as follows. Section 2 outlines the first model which has no market for bonds. Section 3 generalises the first model by introducing a bond market with a flexible interest rate. Section 4 compares the models and examines their policy implications. Finally, Section 5 summarises the results and notes directions for future research.
2. The Model Without Bonds

2.1 The Agents

The economy comprises three agents: an aggregate household, an aggregate firm, and the government. A single good and homogeneous labour are traded against money. Expectations are exogenous and there is no foreign trade.

The household chooses its demand for the good (c), its supply of labour (l) and its end-period stock of money balances (m) subject to the budget constraint:

\[ p(c) = w l + I \]  \hspace{1cm} (2.1)

where \( p \) and \( w \) are the price of the good and the wage rate, respectively, and are exogenous. \( I \) is lump-sum income and is the sum of initial money balances, the amount of last period's profits which are paid to the household in the current period and transfer payments made by the government i.e.

\[ I = n_o + r_o + I \]  \hspace{1cm} (2.1a)

Though transfer payments can be negative (i.e. a lump-sum tax), it is assumed that taxes are never sufficiently high to make \( I \) non-positive. Leisure, current goods and future goods as reflected in savings are normal and are gross substitutes. The household's decision functions are as follows with signs of partial derivatives indicated beneath the exogenous variables:

\[ c = c(p, w, l) \quad l = l(p, w, I) \quad m = m(p, w, I) \]  \hspace{1cm} (2.2)

Lump-sum income is not a function of \( p \) but the explicit dependence of each of the three functions on \( p \) incorporates
a real balance effect. The household may face three different rationing regimes and the following additional functions are defined (*):

\[
\begin{align*}
\tilde{c} &= \hat{c}(p, w, I, \tilde{c}) \\
\tilde{m} &= \hat{m}(p, w, I, \tilde{c}) \\
\hat{I} &= \hat{I}(p, w, I, \tilde{c}) \\
\hat{m} &= \hat{m}(p, w, I, \tilde{c})
\end{align*}
\]  

(2.3)

The signs of \(\tilde{c}, \hat{I}, \tilde{c}, \) follow Neary and Roberts (1980) and the assumed signs of \(\tilde{m}, \) and \(\hat{m}, \) respectively imply that the marginal propensity to consume out of wage income, \(\tilde{c} \), and the marginal propensity to consume out of lump sum income, \(\hat{m} \), are both less than unity. \(\hat{I}, \) and \(\tilde{c}, \) are discussed in Hultenauer and Porter (1974); the sign of \(\hat{m} \) seems reasonable as it places an upper bound on \(\hat{I} \), and \(\tilde{c} \) follows by differentiating the household budget constraint. Neary and Roberts (1980) point out that because of competing income and substitution effects, \(\hat{c} \) cannot be signed unambiguously but following Neary (1980) it will be assumed that the substitution effect dominates. The signs of \(\hat{m}, \) and \(\tilde{c}, \) reflect the assumption that goods rationing causes 'forced

(*) We adopt the following convention: a bar over a variable indicates a constraint level; a circumflex indicates a goods constrained function; one tilde, a labour constrained function and double tilde, a function constrained in both markets. Finally, a subscript indicates a partial derivative unless otherwise indicated.
savings' by the household.

The firm produces the good \( y \) using labour as input \( e \) according to the technology:

\[
y = f(e) \quad \quad f' > 0
\]

Other inputs, including capital, are fixed in the short run. There is no investment or inventories so that the firm cannot be rationed in both the labour and goods market simultaneously. The current period's profit is distributed to the household at the beginning of the next period. The inclusion of current period dividends would introduce additional lump-sum income effects in the second model, blurring the distinction between the two different concepts of monetary policy. In order to facilitate the comparison of the two models, current period dividends are excluded from the first model also; it is easy to show that this does not substantially change the conclusions drawn from this model.

The government purchases goods \( q \) from the firm and makes transfer payments \( T \) (or levies lump-sum taxes) to the household; it is never rationed in the goods market; bonds are not issued and the government's net financial requirement is met through the issue of fiat money. Its budget constraint is:

\[
K - H = q + T
\]

where \( H \) is the money stock outstanding from the previous period. The restriction that lump-sum taxes are never so great as to make the household's lump-sum income non-positive
ensures that the money stock is always positive.

2.2 Notional equilibrium

The notional goods market (GMEL) and labour market (LMEL) equilibrium loci are defined by:

\[(\text{LMEL}) \quad e(p,w)=1(p,w,I) \quad (2.6)\]
\[(\text{GMEL}) \quad c(p,w,I)+g=y(p,w) \quad (2.7)\]

Our concern is to examine what values of \(g\) and \(T\) are consistent with notional equilibrium in each of the markets for given levels of \(p\) and \(w\). Neither the demand nor the supply of labour is affected by \(g\). There exists only one level of \(T\), which is the sole variable component of \(I\), that clears the labour market. (See figure 1) GMEL is downward-sloping with slope \(-1/c_w\) in \((T,g)\) space because both \(g\) and \(T\) increase goods demand without affecting the supply of goods. As figure 1 shows, there exist unique levels of \(g\) and \(T\) which define a Walrasian equilibrium \((E)\) for given \(p\) and \(w\). The notional loci partition the space into four regions of incipient market disequilibrium. In the diagram, the equilibrium level of \(T\) has been drawn as positive but without affecting the analysis, the equilibrium could involve lump-sum taxes.

2.3 Effective equilibria

The simplest way to consider how figure 1 is affected when agents take quantity constraints into account is to examine how each locus is changed.
Figure 1
LMEL(EDG)

That part of the LMEL which is to the right of the equilibrium in figure 1 coincides with conditions of excess demand for goods. Thus the household is rationed in the goods market and recalculates its supply of labour. Equation (2.6) can be rewritten as:

\[ e(p,w) = \hat{I}(p,w,I,\bar{c}) \]  \hspace{1cm} (2.8)

where \( \bar{c} = y(p,w) - g \)

The slope of the effective LMEL(EDG) in \((T,g)\) space is:

\[ \frac{\hat{I}}{\hat{c}} < 0 \]  \hspace{1cm} (2.9)

The negative slope of LMEL(EDG) indicates that when the quantity constraint is taken into account, the area of generalised excess demand expands.

GMEL(ESL)

That part of GMEL which is below the equilibrium in figure 1 coincides with conditions of excess supply of labour. Thus, the household is rationed in the labour market and will recalculate its demand for goods. Equation (2.7) can be rewritten as:

\[ \tilde{c}(p,w,I,\bar{I}) + g = y(p,w) \]  \hspace{1cm} (2.10)

where \( \bar{I} = e(p,w) \)

The slope of the effective GMEL(ESL) in \((T,g)\) space is:

\[ -1/\tilde{c}_I < 0 \]  \hspace{1cm} (2.11)

From Neary and Roberts (1980), we know that \( \tilde{c}_I > c_I \) and therefore \( -1/\tilde{c}_I > -1/c_I \). Thus, the GMEL pivots to the right in figure 1 when the quantity constraint is taken into account, expanding the area of generalised excess supply.
LMEL(ESG) and GMEL(EDL) - 'Joint Locus'

That part of the LMEL which is to the left of the equilibrium in figure 1 coincides with conditions of excess supply of goods. The firm's demand for labour is determined through the production function by the aggregate demand for goods. Equation (2.6) can be rewritten as:

\[ f^{-1}(c(p,w,I)+g)=l(p,w,I) \]  \hspace{1cm} (2.12)

Similarly, that part of the GMEL which is above the equilibrium in figure 1 coincides with conditions of excess demand for labour. The firm's supply of goods is determined through the production function by the supply of labour. Equation (2.7) can be rewritten as:

\[ c(p,w,I)+g=f(l(p,w,I)) \]  \hspace{1cm} (2.13)

Since there are no inventories in the model, it is obvious that LMEL(ESG) and GMEL(EDL) are identical. To see this, note that equation (2.13) is obtained by applying the function \( f(.) \) to both sides of equation (2.12). The slope of the common locus in \((T,g)\) space is:

\[ -1/(c_1-f') < 0 \]  \hspace{1cm} (2.14)

It follows immediately that this slope is less than the slope of the notional LMEL and greater than the slope of the notional GMEL. Consequently, both the areas of generalised excess demand and excess supply expand.

2.4 The complete model

If one collects the results of Section 2.3 and reconstructs figure 1, the result is shown by the solid
lines in figure 2. As in figure 1, the loci are drawn as straight lines: there is no reason to believe that the relationships are linear but it obviously simplifies the diagrams. The regions of generalised excess supply and excess demand have expanded and are labelled 'K' and 'R' respectively. It is shown in the Appendix that the slope of LMEL(EDG) is greater than the slope of GMEL(ESL) if leisure and money are weakly separable from goods in the household utility function, giving rise to a region of effective excess supply in the labour market and excess demand in the goods market, labelled 'C'. It is useful to construct isoemployment or equivalently iso-output contours throughout the space.

C region

Under the classical regime, employment is constant because it is determined by the notional demand for labour which is invariant with respect to the policy instruments:

\[ \dot{I} = e(p,w) \]  \hspace{1cm} (2.15)

K region

Under Keynesian unemployment, the level of employment is determined by the firm's effective demand for labour which is itself constrained by the level of employment:

\[ \dot{I} = f^{-1}(\tilde{c}(p,w,I,\dot{I}) + g) \]  \hspace{1cm} (2.16)

Recalling that \( f' > w/p \), when firms are rationed, it is easy to show that employment increases through the demand multiplier in response to an increase in \( g \) or \( T \):

\[ \frac{\delta \dot{I}}{\delta g} = 1/(f' - \tilde{c} \dot{I}) > 0; \quad \frac{\delta \dot{I}}{\delta T} = \frac{\tilde{c} \dot{I}}{(f' - \tilde{c} \dot{I})} > 0 \]  \hspace{1cm} (2.17)
The necessary and sufficient condition for the derivations in (2.17) to have positive signs is \( f' > \frac{-c}{\hat{L}} \), which is assured by the assumption that the marginal propensity to consume out of wage income is less than unity. Thus the slope of isoemployment lines in the K region is the same as the slope of \( \text{LMEL(BSL)} \):
\[
-\frac{1}{\tilde{c}} < 0 \quad (2.18)
\]

R region

Under repressed inflation, the level of employment is determined by the household's effective supply of labour which is itself constrained by a shortage of goods:
\[
\tilde{I} = \tilde{l}(p, z, I, \tilde{c}) \quad (2.19)
\]

where \( \tilde{c} = f(I) - \alpha \)

Employment falls through the supply multiplier in response to an increase in \( \alpha \) or \( \tilde{z} \):
\[
\frac{\delta I}{\delta \alpha} = \frac{-c}{\hat{L}} \quad (1 - \frac{c}{\hat{L}} f') < 0; \quad \frac{\delta I}{\delta \tilde{z}} = \frac{-c}{\hat{L}} \quad (1 - \frac{c}{\hat{L}} f') < 0 \quad (2.20)
\]

The necessary and sufficient condition for the derivatives in (2.20) to have negative signs is \( \frac{\hat{L}}{\hat{c}} < 1 / f' \). A necessary condition is that the marginal propensity to work following a relaxation of the goods market constraint (\( \hat{L} / \hat{c} \)) is less than unity. The latter condition is assured by our assumption of forced savings, \( \hat{m} < 0 \). Thus the slope of the isoemployment lines in the R region is the same as the slope of \( \text{LMEL(EDG)} \):
\[
\frac{\hat{L}}{\hat{c}} < 0 \quad \hat{L}/\hat{c} \quad (2.21)
\]

The zeroed lines in figure 2 illustrate the isoemployment contours. A non-differentiability occurs at
the boundary between the $K$ and $R$ regions. The continuity of the contours at this boundary is guaranteed by the assumption that the constrained goods demand and labour supply functions are continuous in the constraint levels. Within the closure of the $C$ region, the maximum voluntarily attainable level of employment, conditional on the given values of $u$ and $w$, obtains. As one moves outside of this region, in any direction, the level of employment falls monotonically.
1. The Model with Bonds and a Flexible Interest Rate

3.1 The agents

The main difference between this model and the previous discussion is that a second asset—bonds(b)—can be chosen by consumers. Throughout the discussion, even when effective equilibria are considered, the price of bonds(q, the inverse of the interest rate) is flexible. The model differs from Hool(1980) in that the household's labour supply is variable. In addition, Hool considered a multiplicity of households as distinct from the aggregate consumer which is studied here.

The household's budget constraint differs from the specification in section 2.1 in that lump-sum income is replaced by a wealth variable which is in three parts: initial money holdings, corresponding to I in the first model; the value of initial holdings of bonds (ah0), and the value of interest paid (b0) on the latter.

\[ nc+q0+t+m+w+q(q+1)b0+I \]  \hspace{1cm} (3.1)

To facilitate diagrammatic exposition and to sharpen the distinction between the different concepts of monetary policy in the two models, there are no lump-sum taxes so I is constant in money terms throughout the discussion. Following Hool, the explicit dependence of household decision functions on initial assets is suppressed though this relationship is reflected in functional dependence on p and q. Bonds are normal and are gross substitutes for the other decision variables of the household. This is summarised in
the following notional and quantity-constrained bond demand functions:

\[ b = b(p, w, q); \quad \tilde{b} = \tilde{b}(p, w, q, \tilde{q}) \]  
\[ \hat{b} = \hat{b}(p, w, q, \hat{q}); \quad \tilde{\hat{b}} = \tilde{\hat{b}}(p, w, q, \tilde{\hat{q}}) \]  

(3.2)

In addition, gross substitutability implies \( q > 0, q < 0, n_q > 0 \) and these signs are assumed the same in the case of quantity-constrained functions. Flood (1980) points out the likelihood of \( \hat{b_p} < 0 \), which is followed here and \( \tilde{b_p} < 0 \) also appears reasonable.

The firm neither issues nor holds bonds and consequently the interest rate is not an argument in the profit function. A government bond is a promise to pay by way of interest one unit of money in perpetuity. There are two independent policy instruments: open-market operations and bond-financed fiscal policy. The government's budget constraint is:

\[ M = M_0 q (B_0 - B_1) + B_1 q \]  

(3.3)

where \( M_0 \) and \( B_0 \) are the initial money stock and the initial number of bonds outstanding. The constraint states that the change in the money stock is the amount required to finance net purchases of bonds by the government, interest payments on the initial stock of bonds and current government purchases of goods.

3.2 Notional equilibrium

The notional goods market and labour market equilibrium loci are defined by:

\[ (LMEL) \quad e(p, w) = I(p, w, q) \]  

(3.4)
(GMEL) \[ c(p, w, q) + g = y(p, w) \] \[ (3.5) \]

To identify which combinations of M and g will clear each of these markets in the notional sense, for given values of p and w, it is necessary to examine how q is affected by changes in the policy instruments. Once the government selects M and g as the two independent policy instruments, the value of net purchases or sales of bonds is determined as a residual from the Government budget constraint. The interest rate adjusts to ensure that the stock of bonds is held voluntarily by the household. Differentiating the government budget constraint, setting \( dp = dw = 0 \) and rearranging:

\[ dq = \frac{dM}{(b_0 - b_1 - qb)} - \frac{pdg}{(b_0 - b_1 - qb)} \] \[ (3.6) \]

From the household budget constraint, \( b_0 - b_1 - qb \) is positive:

\[ b_0 - b_1 - qb = m - w l > 0 \] \[ (3.6a) \]

This reflects the fact that a rise in interest rates (i.e., a fall in q) reduces the flow demand for additional bond holdings i.e.

\[ \delta q (b_0 - b_1) / \delta q = b_0 - b_1 - qb \] \[ (3.7) \]

Thus an increase in M alone leads to a fall in interest rates whereas an increase in g alone tends to raise interest rates which is the 'crowding out' effect of expansionary fiscal policy.

In the labour market, only labour supply depends on q. An increase in M cuts interest rates and thereby cuts the household's labour supply. An increase in q raises interest rates which has the reverse effect. Thus both policy
Figure 3
instruments must move in the same direction to maintain labour market equilibrium and so the notional LMEL is upward-sloping in \((M,g)\) space with slope \(P\). Clearly there is only one value for \(q\) (given \(p\) and \(w\)) which clears the labour market and the notional LMEL is, therefore, an iso-interest rate line. From equation (3.6) it is clear that all iso-interest rate lines are parallel to the notional LMEL. In the goods market, only the demand side is affected by the policy instruments. An increase in \(M\) cuts interest rates and raises private consumption. An increase in \(g\) has two effects: it directly raises aggregate demand but, by increasing interest rates, private consumption is reduced. By differentiating the household budget constraint, it is clear that the first effect dominates (i.e. private spending is not fully crowded out). Thus the policy instruments must move in opposite directions to maintain goods market equilibrium and the notional GMEL is downward-sloping in \((M,g)\) space with slope:

\[ p - \left( b - \frac{qb}{c} \right) < 0 \]  

(3.8)

In figure 3, the policy space is divided into four regions of incipient market disequilibrium. A unique combination of \(M\) and \(g\) leads to the Walrasian equilibrium (E) given \(p\) and \(w\).

3.3 Effective equilibria

As in model 1, we now examine how the loci in figure 3 are changed when quantity constraints are taken into account. The new feature of this model is that we have to solve for \(q\) from the government budget constraint and indeed
the determination of \( q \) has to be reexamined for each set of constraint conditions.

LMEL(EDG)

That part of the LMEL which is above and to the right of the GMEL in figure 5 lies in a region of excess demand for goods. Consequently, the household is goods constrained. Equation (3.4) can be rewritten as:

\[
\hat{e}(p,w) = \hat{l}(p,w,q,\hat{c})
\]

where \( \hat{c} = y(p,w) - q \)  

(3.9)

and \( dq = dM/(b_0 - b_1 - q\hat{b}_q) - (p+q\hat{b}_c)dg/(b_0 - b_1 - q\hat{b}_q) \)

The slope of the effective LMEL(EDG) in \((M,q)\) space is:

\[
p + q\hat{b}_c + ((b_0 - b_1 - q\hat{b}_q)\hat{l}/\hat{i}) \geq 0
\]

(3.10)

The positivity of \( b_0 - b_1 - q\hat{b}_q \) can be derived by differentiating the goods constrained household budget constraint and it ensures that the slope of the LMEL(EDG) is less than \( p \). Thus, the LMEL rotates clockwise in figure 3, expanding the area of generalised excess demand. The necessary and sufficient condition for the slope to be positive is:

\[
\frac{\text{MPW}}{||\text{MPS}||} < \left(\frac{e^k}{\hat{e}}\right)^r(R^m)(\hat{\text{APS}})\]

(3.11)

where MPW, the marginal propensity to work following the relaxation of the goods market constraint equals \( \hat{w}_c / \hat{p} \).

MPS, the marginal propensity to save following the relaxation of the goods market constraint equals \( \hat{m}_c / \hat{p} \).

\( e^g \), the goods-constrained interest elasticity of the

*\(|x|\) indicates the absolute value of \( x \).*
supply of labour equals \( r_{r}/\dot{\bar{l}} \).

\[ e_{r}^{m} \text{the goods-constrained interest elasticity of the demand for savings in the form of money equals } \frac{r_{r}}{(\dot{m} - m_{q})}. \]

\[ \text{APS, the average propensity to save out of wage income equals } \frac{(\dot{m} - m_{q})}{w \dot{\bar{l}}}. \]

**GMEL(ESL)**

That part of the GMEL which is below and to the right of the LME in figure 3 coincides with conditions of excess supply of labour. Households are labour constrained and equation (3.5) can be rewritten as:

\[ \dot{c}(p, w, q, \bar{l}) + g = y(p, w) \]

where \( \bar{l} = e(p, w) \) \hspace{1cm} (3.12)

and \( dq = dM/(b_{0} - b_{1} - q\tilde{b}_{q}) - (p)dg/(b_{0} - b_{1} - q\tilde{b}_{q}) \)

The slope of effective GMEL(ESL) in \((M, g)\) space is:

\[ p - ((b_{0} - b_{1} - q\tilde{b}_{q})/\dot{c}_{q}) < 0 \] \hspace{1cm} (3.13)

The sign of the slope and the positivity of \( b_{0} - b_{1} - q\tilde{b}_{q} \) can be derived by differentiating the labour constrained household budget constraint. A necessary condition for the slope of effective GMEL(ESL) to be greater than the slope of notional GMEL(ESL) in (3.8) is:

\[ c_{q}/\dot{c}_{q} < m_{q}/\dot{m}_{q} \] \hspace{1cm} (3.14)

The condition requires that the interest elasticity of the demand for goods falls proportionally less under the impact of the constraint in the labour market than does the interest elasticity of the demand for money. Thus GMEL rotates anti-clockwise in figure 3, expanding the area of generalised excess supply.
LMEL(ESG) and GMEL(EDL) - 'Joint Locus'

That part of the LMEL which is below and to the left of GMEL in figure 3 co-exists with excess supply of goods and the part of GMEL which is above and to the right of LMEL lies in conditions of excess demand for labour. As in the first model, these two loci are identical in the absence of inventories and equations (3.4) and (3.5) can be rewritten as:

\[ c(p,w,q) + g = f(l(p,w,q)) \]  \hspace{1cm} (3.15)

where dq is determined from equation (3.6)

The slope of this locus in \((M,g)\) space is:

\[ p - \frac{(b_0 - b_1 - qb_q)}{(c_q - f^1_q)} < 0 \]  \hspace{1cm} (3.16)

Comparison with (3.8) shows that this slope is greater than the slope of the notional GMEL. The sign of the slope follows from differentiating the household budget constraint and recalling that firms are rationed. Consequently both the areas of generalised excess demand and excess supply have expanded.

3.4 The complete model

The solid lines in figure 4 summarise the results of section 3.3. The regions of generalised excess supply (K) and excess demand (R) have expanded and a positive slope for the effective LMEL(EDG) guarantees the existence of a classical regime (C). Since the RHS of equation (3.11) is positive by our assumptions, and since MPW is negative for sufficiently large displacements from equilibrium by Neary and Roberts (1980), then LMEL(EDG) will have a positive
slope for at least part of its range. In figure 4 the locus is drawn as positive to emphasise this point. Isoemployment lines are again constructed throughout the space and are illustrated by the scored lines in figure 4. As in the first model, employment is constant throughout $C$.

K region

In the region of Keynesian unemployment, employment is:

$$\bar{i} = f^{-1}(\tilde{c}(p, w, q, \bar{l}) + g)$$

where

$$dq = dM/B^K - (p - q\tilde{b}_q - \tilde{b}_q \tilde{l})dg/B^K$$

$$B^K = b_c - \tilde{b}_l - q\tilde{b}_q - q\tilde{b}_q \tilde{l}$$

$$\tilde{l}_q = \delta \bar{i}/\delta q | q \text{ constant}$$

$$\tilde{l}_g = \delta \bar{i}/\delta g | q \text{ constant}$$

(3.17)

An increase in $M$ cuts interest rates, thereby raising private consumption which works through the demand multiplier to increase employment:

$$\delta \bar{l}/\delta M = \bar{c}_q / (B^K (f' - \bar{c}_q)) > 0$$

(3.18)

The positive sign of $B^K$ can be derived by differentiating the household budget constraint and manipulating. An increase in $g$ has two opposing effects: it directly increases aggregate demand and through the demand multiplier raises employment but it also increases interest rates which leads initially to a contraction of private consumption:

$$\delta \bar{l}/\delta g = \text{slope of GMEL(ESG)}(-\delta \bar{l}/\delta M) > 0$$

(3.19)

where slope of GMEL is given by (3.13)

and $-\delta \bar{l}/\delta M$ is given by (3.18)

As in the first model, the slope of the isoemployment lines in the Keynesian region is the same as the slope of the
GMEL(ESG).

R region

Under repressed inflation, employment is:

\[ \hat{I} = l(p, w, q, \tilde{c}) \]

where \( \tilde{c} = f(\hat{I}) - g \)

and \( dq = dM/B^R - (p - \hat{q}_c \cdot \tilde{c}_q) dq/B^R \) \hspace{1cm} (3.20)

\[ B^R = b_0 - b_1 - \hat{q}_q \cdot q - \hat{q}_c \cdot \tilde{c}_q \]

\[ \tilde{c}_q = \delta \tilde{c}/\delta q | q \text{ constant} \]

\[ \tilde{c}_g = \delta \tilde{c}/\delta g | q \text{ constant} \]

When \( M \) is increased, the interest rate falls, thereby lowering the household's effective labour supply.

\[ \delta \hat{I} / \delta M = \lambda_q / (B^R (1 - \hat{c}^R f')) < 0 \] \hspace{1cm} (3.21)

By the continuity of the iso-employment contours \( \delta \hat{I} / \delta M \) must be negative and consequently \( B^R \) must be positive given \( \hat{c}^R < 1/f' \).

An increase in \( g \) has two opposing effects. On the one hand, interest rates rise tending to increase the household's effective labour supply. On the other hand, the excess demand for goods increases tending to contract the household's labour supply.

\[ \delta \hat{I} / \delta g = (\text{slope of LMEL(EDG)}) (-\delta \hat{I} / \delta M) \] \hspace{1cm} (3.22)

where slope of LMEL(EDG) is given by (3.10)

and \( \delta \hat{I} / \delta M \) is given by (3.21)

In the first model and in for example, Malinvaud (1977), only the goods constraint effect is present and increased government spending cuts employment under repressed inflation (see equation 2.20). In this model, the effect of increasing government spending is ambiguous. The stronger the
interest rate effect on labour supply the more likely is the result of the first model to be reversed. The sign of (3.22) depends on the sign of the slope of the LMEL(EDG) which is governed by (3.11). It is clear from (3.21) and (3.22) that the slope of the iso-employment lines in the R region is the same as the slope of the LMEL(EDG).

3.5 Iso interest rate contours

In section 3.2, it was pointed out that the slope of iso interest rate lines in notional (M, g) space is p. From (3.17), it is clear that the slope of the iso-interest rate lines in the K region is:

$$p - \frac{(q^b_k)}{(f' - \tilde{r}_k)}$$  \hspace{1cm} (3.23)

Similarly, (3.20) yields the slope of the iso-interest rate lines in the R region:

$$p + \frac{(q^b_C)}{(1 - f' \tilde{l}_C)}$$  \hspace{1cm} (3.24)

In the classical region:

$$dq = dM/(b_0 - b_1 - q^b_0) - (p + q^b_C)dg/(b_0 - b_1 - q^b_q)$$  \hspace{1cm} (3.25)

and the iso-interest rate lines have slope:

$$p + \frac{q^b_C}{q^b_C}$$  \hspace{1cm} (3.25)

(3.23), (3.24) and (3.25) are all less than p, the notional slope but it can easily be shown from the household budget constraint that the iso-interest lines in K and C still have positive slopes. In R, it can be demonstrated that the slope of iso-interest lines is greater than the slope of LMEL(EDG) so that the former is positive if the latter is positive. Thus the relative impact of the fiscal over the monetary instrument on interest rates is consistently diminished when
rationing is taken into account. In figure 5, the scored lines are iso interest rate contours: lines to the right and below the equilibrium correspond to higher interest rates and vice versa. Though it is not intended to compare the slopes in the different regions, they are, in general, different.
4. Policy Considerations and Comparison of Models

4.1

Following the approach of Swan (1955), Figures 6 and 7 are constructed by imposing a cross with centre $E$ on figures 2 and 4 respectively with the isoemployment contours omitted for clarity. In each case, seven zones emerge naturally from the boundaries of the crosses and disequilibrium regimes. There is a remarkable similarity between the two diagrams despite the fundamental differences in the interpretation of the monetary policy instruments.

Each zone is characterised by the policy prescription which is required for the restoration of the Walrasian equilibrium. The important feature is that the policy remedy is in general not uniform within a given disequilibrium state. For example, there are three distinct policy zones within the Keynesian region in both figures 6 and 7. One of these zones in each diagram indeed requires the traditional Keynesian approach: use both instruments in an expansionary direction. However there exist non-trivial zones of policy perversity. The shaded areas in both figures 6 and 7 constitute such zones in the Keynesian region: despite the existence of Keynesian unemployment, it is inappropriate to expand both policy instruments. There is no policy combination involving the expansion of government purchases which can avoid causing a classical depression. The very un-Keynesian perverse policy remedy is to cut government purchases combined with a relaxation of
monetary policy.

The obvious distinction between figures 6 and 7 is the difference in the signs of the slopes of the R/C boundaries (LMEL(EDG)). Of course if (3.11) does not hold the slope of this locus in figure 7 would be negative and the comparability of the two models would be complete. However as was pointed out in section 3.4, the boundary has a positive slope for at least part of its range thereby eliminating the symmetry between the regions of repressed inflation and Keynesian unemployment which is implied in Malinvaud (1977) and is a feature of the first model.'

The problems of perverse policy zones and the ambiguity of the boundary between the regions of repressed inflation and classical unemployment can be managed in the context of the assignment problem which was first raised by Mundell (1962). The target of maintaining equilibrium in the labour market can be assigned to the monetary authority as follows: Increase M if there is unemployment;
Decrease M if there is a shortage of labour;
Similarly the goods market can be assigned to the fiscal authority:
Increase g if there is a glut of goods;
Decrease g if there is a shortage of goods;
This assignment is illustrated by the arrows in figures 6 and 7: it is stable in both models.*

4.2

It is interesting to examine the meaning of a balanced
Figure 8
budget expansion in the context of the first model.

\[ pg = -T \]  \hspace{1cm} (4.1)

In figure 8, the effective loci of figure 1 have been positioned in the fourth quadrant. It has already been stated that there is no presumption that the equilibrium level of T is either negative (lump-sum taxes) or positive (transfer payments). A positive equilibrium level of T merely reinforces the points which follow. The dotted 'balanced budget locus' in figure 8 has slope \(-p\) and it is the locus of combinations of government purchases and lump-sum taxes which lead to a balanced budget. Policy combinations below the locus correspond to a budget surplus and those above the locus correspond to a budget deficit. In general, the equilibrium will not lie on this line and changes in the fixed levels of p and w would be required to ensure that it did. This immediately suggests the policy implication that if the government is financially constrained, in the sense that both policy instruments cannot be manipulated independently, some form of prices and incomes or 'supply-side' policy would be required in order to eliminate macroeconomic disequilibrium. In figure 8, the balanced budget locus has been drawn so that it intersects the K and C regions.

* For the assignment to be strictly stable, both models should be developed to include inventories; this would generate regions of underconsumption.
only. There is, however, no reason to assume that a balanced budget is not consistent with repressed inflation and figure 8 illustrates the primary point which is that a balanced budget is consistent with a multiplicity of disequilibrium regimes, depending on the nominal level at which the budget is set. Though a budget surplus may give rise to repressed inflation, at least part of the Keynesian region must lie below the balanced budget locus. This is because sufficiently high taxes and low levels of government purchases will always give rise to Keynesian disequilibrium. This is the sense in which a budget surplus is more likely to give rise to Keynesian unemployment. Conversely a budget deficit is more likely to give rise to repressed inflation.

If figures 2 and 8 are considered together it follows immediately that a balanced budget expansion will increase employment in the K region if and only if the slope of the balanced budget locus is greater than the slope of the isoemployment contour:

\[-p > -1/\theta_1\]  \hspace{1cm} (4.2)

This requires that the labour constrained marginal propensity to consume out of lump-sum income is less than unity, a condition which is assured by our assumptions. Similarly, the effect of a balanced increase in the budget in the R region depends on a comparison of the slopes of the balanced budget locus and the isoemployment contours. It is shown in the Appendix that if leisure and money are weakly separable from goods in the utility function, the marginal
propensities to work following changes in the goods market constraint and lump-sum income are the same in absolute value. This implies:

\[-p = \frac{\hat{i}_c}{\hat{i}_I}\] (4.3)

i.e. the slopes of the balanced budget locus and isoemployment loci in region R are the same. Thus a balanced budget expansion in conditions of repressed inflation has no effect on employment and output.

The equivalent of a balanced budget in the second model is the financial constraint that no bonds may be issued. This implies that the government must finance its current expenditures including the payment of interest on outstanding debt by money creation alone:

\[\Delta M = M_1 - M_0 = \pi g + b_0\] (4.4)

In the context of figures 4 and 5, this implies that policy options are restricted to a straight line with slope p through the initial temporary equilibrium position. As in the first model, there is no reason to believe that the Walrasian equilibrium will be attainable and 'supply-side' or prices and incomes policy will be required. However, expansionary policy in the Keynesian region increases employment because the isoemployment lines are negatively sloped and expansionary policy under repressed inflation unambiguously decreases employment because the slope of the isoemployment lines are less than p from equation (3.10). From equations (3.23), (3.24) and (3.25), the iso-interest rate lines in all regions have slopes less than
Thus expansionary policy always leads to a decline in interest rates.
Conclusion

In this paper, a number of simple diagrams have been constructed to illustrate the effects of monetary and fiscal policies in macroeconomic disequilibrium models. Though the models enable us to consider the impact of the two policies together in the spirit of traditional IS/LM analysis, a more faithful Keynesian approach would incorporate rationing in the bond market - the liquidity trap. In common with the existing literature on static fix-price models, distributional effects are ignored. Though there is no foreign trade, it is likely that Neary (1980) could be readily cast into the framework of this study. Both models involve a single period only and this prevents the consideration of expectations and investment. Nevertheless, the paper has gone some distance towards clarifying the roles of monetary and fiscal policy in disequilibrium models. It emerges that both demand management instruments are, in general, required: that 'supply-side' or prices and incomes policies are only necessary if governments are constrained in their freedom to adjust the monetary and fiscal instruments independently; and that knowledge of the disequilibrium regime alone is an inadequate guide to policy formulation unless the assignment discussed in section 4.1 is applied. A particularly interesting result is that the explicit introduction of a bond market can reverse the effect of fiscal policy on employment in conditions of repressed inflation.
Appendix

A1. Comparison of Marginal Propensities to Work

The marginal propensity to work following a relaxation of the goods market constraint = \( \hat{w}_c/p \).

The marginal propensity to work following an increase in lump-sum income = \( \hat{w}_l \).

From Neary and Roberts (1980):

\[
\hat{I}_c = \hat{I}_c^c + \hat{I}_I (\hat{p} - \tilde{p}) \tag{A.1}
\]

\[
\hat{I}_I = l_I - \hat{c}_I \c I
\]

where \( \hat{I}_c^c = I_c^c/c_c^c \) and \( \hat{p} = \text{"virtual" price of goods}.^*

After some manipulations, it is easy to show:

\[
\hat{w}_c/p + \hat{w}_l = (w/p)(c_c^c)^{-1}(c_m^m - c_m^c) \tag{A.2}
\]

From Geary and Morishima (1973), a sufficient condition for the final bracketed term to be zero is that leisure and money are weakly seperable from goods in the utility function.

A2. Comparison of the Slopes of LMEL(EDG) and GMEL(ESL) in the Model Without Bonds.

From (2.9) the slope of LMEL(EDG) is \( \hat{I}_c/\hat{I}_I \); from (A2), \( \hat{I}_c/\hat{I}_I = -p \); from (2.11), the slope of GMEL(ESL) is \( -1/c_I \).

*The superscript 'c' indicates a Slutsky compensated price slope.
Assume the slope of $GMEL(ESL)$ is greater than the slope of $LMEL(EDG)$:

Thus $p_{c1} > 1$.

This contradicts our assumption that the labour constrained marginal propensity to consume out of lump-sum income is less than unity. Thus the slope of $GMEL(ESL)$ is less than the slope of $LMEL(EDG)$, guaranteeing the existence of a classical region.
References


