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Monopoly and Competition in Addictive Markets

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MONOPOLY AND COMPETITION IN ADDICTIVE MARKETS

Abstract

This paper models monopoly markets on which demand is addictive as defined by Stigler and Becker; and hence dynamic. It is shown that the monopolists' equilibrium is unique and stable and the time path taken to reach it is also unique. The comparative statics and comparative dynamics of these markets are analysed. It is proved formally that the harmfully (beneficially) addictive monopolist will produce to the left (right) of the equality between marginal revenue and marginal cost, and that the latter may produce where marginal revenue is negative. The model is applied to policy problems in drug-related crime. It is shown that supply-directed policies result in more crime in both long and short runs under monopoly than under competition. The reverse holds true under demand-directed policies. Demand-directed policies cause less crime than supply-directed policies, whether competition or monopoly obtains.

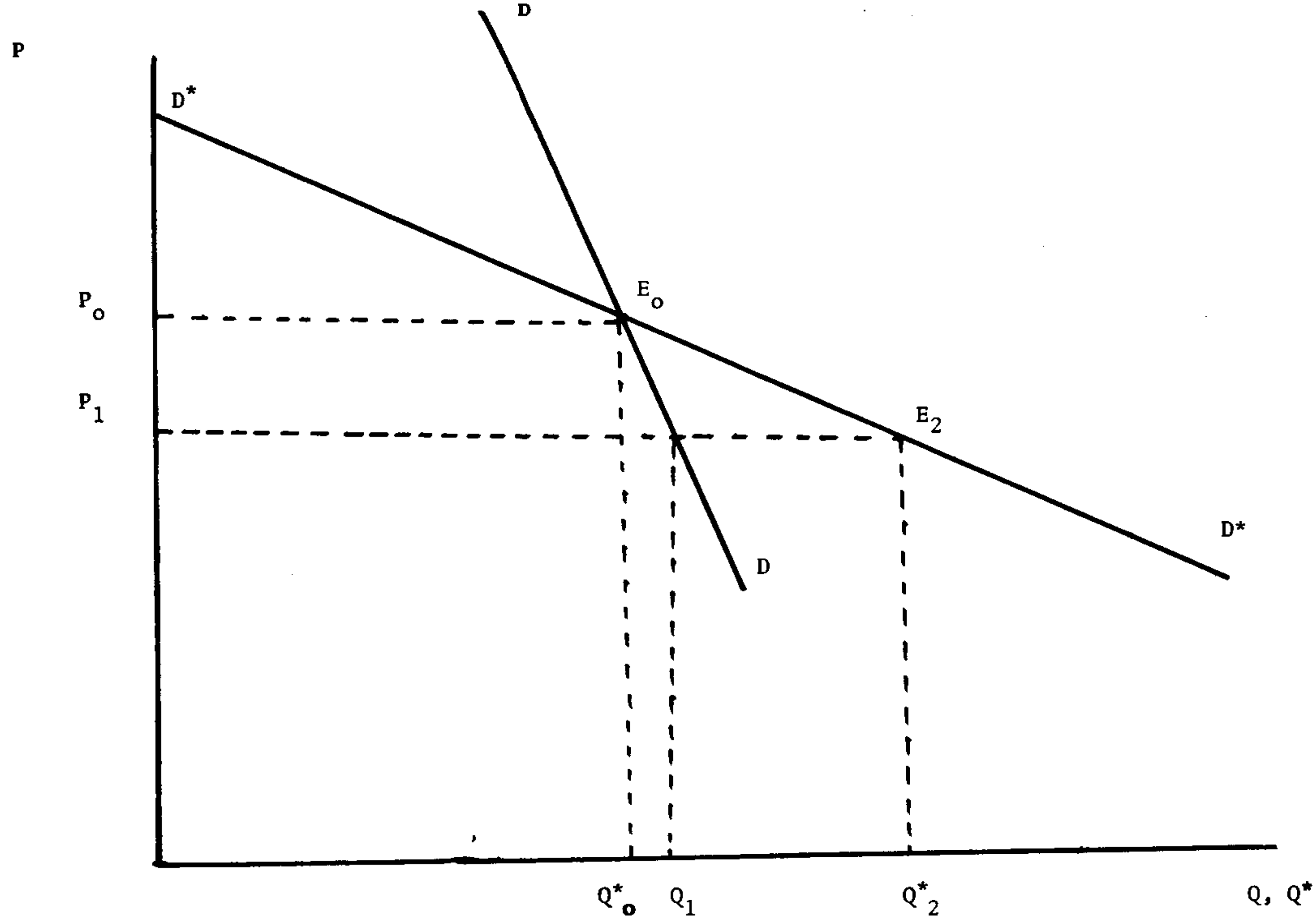

FIG. I

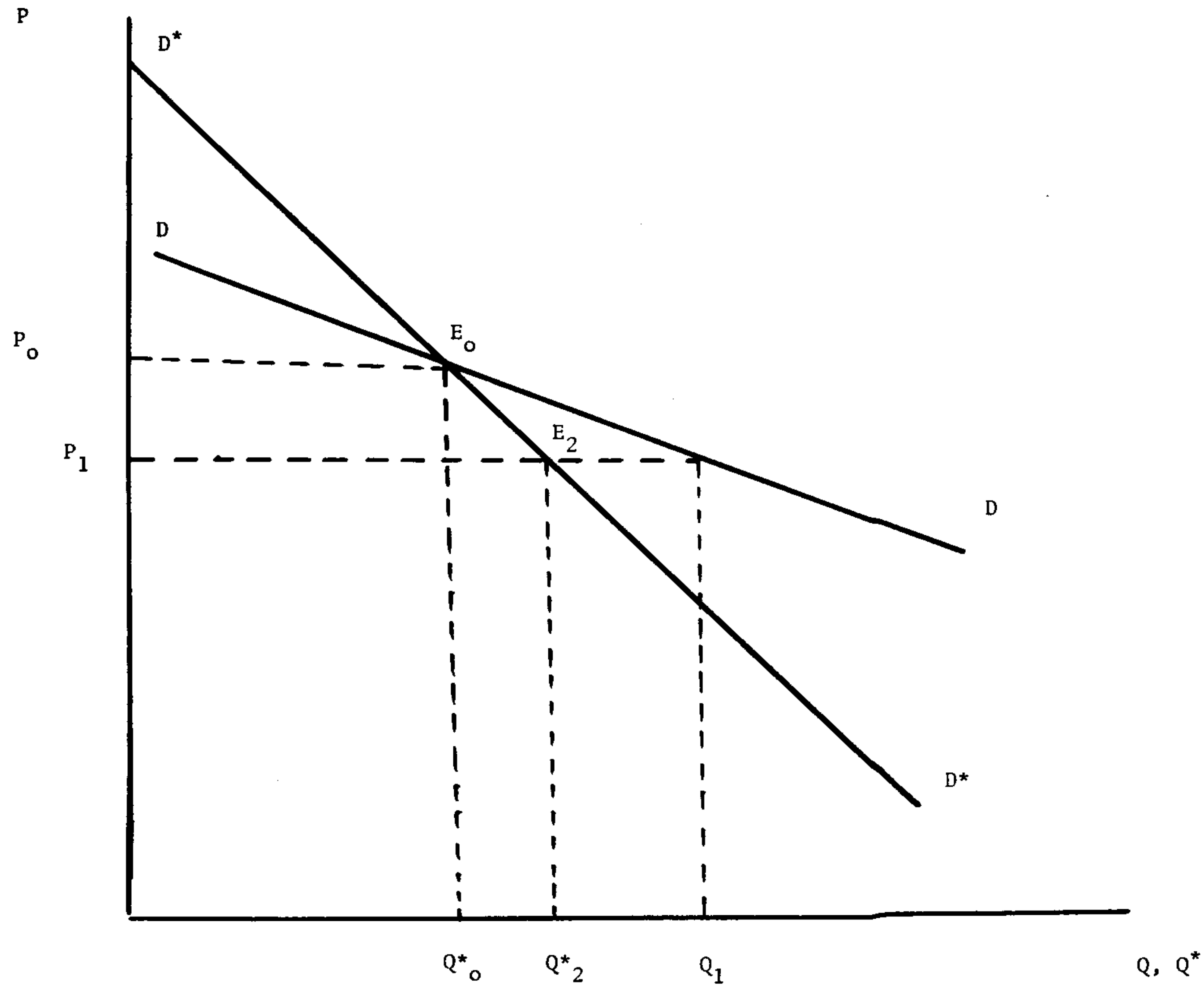

FIG. II.

where Q^* is long-run or target consumption. Long-run equilibrium at E_o implies that consumption is just sufficient to offset depreciation and to maintain consumption capital intact. Through E_o there runs a short-run demand curve DD, steeper than D*D*. Let price fall to P_1. Short-run consumption rises to Q_1. Consumption is more than sufficient merely to maintain capital, which is now being accumulated. The increase in consumption capital raises demand, and the short-run demand curve will shift rightwards, continuing to do so until long-run equilibrium is reached at a new target level of consumption capital, and at a new target level of consumption, Q^*_2.

Fig. II. illustrates the case of beneficial addiction. The short-run demand curve DD is flatter than the long-run demand curve D*D*. A fall in price from P_o to P_1 raises short-run consumption to Q_1. Capital accumulates and reduces demand, the short-run demand curve shifts leftwards and a new long-run equilibrium is in time reached at Q^*_2.

Interpret the demand curves of Figs. I and II as market demand curves, and P^o and P^1 as horizontal marginal cost curves, and these diagrams can do duty to analyse competitive addictive markets. (Their stability is proved in the next section). The dynamics and comparative statics of these markets are straightforward.

The analysis is more difficult under monopoly. Since the slopes of the long and short-run demand curves differ, then the monopolist can never equate marginal cost simultaneously with both long and short-run marginal revenues. Furthermore, the monopolist will never be in equilibrium if long-run marginal cost (LRMC) equals long-run marginal revenue (LRMR). Were the two equal, then the monopolist, in a maleficently addictive market could, at a trivial loss of long-run profit, reduce quantity, increase short-run price, and by this stratagem increase the present value of profit. Therefore the monopolist, in long-run equilibrium (if such exists) will produce where LRMR exceeds LRMC. A formal proof of this proposition is provided in Section III.

But precisely where will that equilibrium occur ? The value of short-run profit depends (given the discount rate and the other parameters) on its duration, and hence on the speed of adjustment of the short-run demand curve. Therefore, only a well-specified dynamic intertemporal model can answer the question. This paper attempts such a model. [1)]

The plan of the paper is now set out. The succeeding section develops the formal model and from it fashions the analytical tools employed in the subsequent phase-diagrammatic analyses. For the case of maleficent addiction, Sections II and III between them provide an analytical solution for the monopolist's equilibrium, which is shown to be unique and to lie to the left of the equality of LRMR and LRMC. Equilibrium is stable and the time-path followed by price and quantity to attain it is unique if the monopolist maximizes profit. It is shown that the value of the monopoly right is greater, the higher is the arbitrary initial price. The comparative-static and comparative-dynamic effects of a rise in marginal cost are analysed. Section IV uses the analytical armoury of the earlier sections to evaluate, to the extent that theory can, some policies directed at drug-related crime. Under plausible assumptions, anti-narcotics policies which are directed towards reducing *supply* will result in more crime under competition than under monopoly. The reverse, again under plausible assumptions, is true of policies directed at reducing *demand*. Whether competition or monopoly obtains, demand-directed policies cause less crime than supply-directed policies, both in the short and long runs. Section V establishes, for the case of beneficent addiction, propositions equivalent (mutatis where appropriate mutandis) to those of Sections II and III for maleficent addiction. Inter alia, it shows that a monopolist in a beneficially addictive market may produce where marginal revenue is negative, even though marginal cost is positive, another conclusion at variance with that of the textbooks.

II. THE MODEL

In this section we formalize the model described in the introduction. We show that competitive equilibrium is unique and stable. Then we set up and solve the monopolist's intertemporal maximization problem. We show that the time paths followed by the monopolists, beneficial and maleficent, in Q and Q^* space are unique and derive the equations of the respective stable paths, and of the singularities along which the rates of change of Q and Q^* are zero. These are the tools which will be used for the phase-diagrammatic analyses of the rest of the paper.

The model is:

$$(1) \quad Q = \gamma + \alpha Q^* - \beta P \qquad ; \quad \gamma, \beta > 0.$$

For maleficent addiction : $0 < \alpha < 1$.

For beneficent addiction : $\alpha < 0$.

$$(2) \quad \dot{Q}^* = \phi(Q - Q^*) \qquad ; \quad \phi > 0.$$

$$(3) \quad C = \theta Q \qquad ; \quad \theta > 0.$$

Furthermore, to ensure a positive Q^* and P in long-run equilibrium:

$$\frac{\gamma}{\beta} > \theta.$$

Q is quantity, Q^* is long-run or target demand, C is total cost and the dot indicates a time-derivative. Equation (1) is the short-run demand curve, the position of which is determined by Q^*, which changes only over time, as specified by (2) according to the difference between actual consumption Q, and Q^*. If one sets

$$\dot{Q}^* = 0, \quad \text{then}$$

$$(4) \qquad Q = Q^*.$$

Upon substitution for Q from (4) into (1) then one obtains the equation of the long-run demand curve D*D* :

$$(5) \qquad P = \frac{\gamma}{\beta} - \frac{(1-\alpha)}{\beta} Q^*$$

and D D is steeper (flatter) then D*D* if addiction is harmful (beneficial). To keep matters simple (3) supposes that marginal cost is constant.

We now establish stability of equilibrium under competition. Set marginal cost θ equal to P and substitute in (1), then substitute for Q from the resulting equation into (2) :

$$(6) \qquad \dot{Q}^* = \phi\,\beta[\frac{\alpha}{\beta} - \theta] \quad - \phi\,(1-\alpha)\,Q^*$$

For both harmful and beneficial addiction, long-run equilibrium is globally stable (and ∴ unique). Long-run equilibrium output is :

$$(7) \qquad Q^* = \frac{\beta(\frac{\gamma}{\beta} - \theta)}{(1-\alpha)}$$

which as one might expect, is lower under beneficial than under maleficent addiction.

We now extend the model to the case of a monopolistic market. The monopolist's problem is to choose Q and Q* so as to maximize the present value of profit,

$$(8) \qquad (P - \theta)\,Q$$

subject to the constraint in (2). Hence the maximand is :

$$(9)\quad H = \int_0^\infty e^{-rt}\left[\frac{\gamma}{\beta} + \frac{\alpha}{\beta} Q^* - \frac{1}{\beta} Q - \theta\right] Q + \lambda\phi(Q-Q^*) + \dot{\lambda}\, Q^*\, dt$$

where r is the exogenously given discount rate.[2)]

Maximizing with respect to Q and Q^* yields :

$$(10)\quad \frac{\delta H}{\delta Q} = e^{-rt}\left[\left(\frac{\gamma}{\beta} + \frac{\alpha}{\beta} Q^* - \frac{1}{\beta} Q\right) - \theta - \frac{1}{\beta} Q\right] + \lambda\phi = 0$$

$$(11)\quad \frac{\delta H}{\delta Q^*} = e^{-rt}\,\frac{\alpha}{\beta} Q - \lambda\phi + \dot{\lambda} = 0$$

Differentiating (10) with respect to time we obtain :

$$(12)\quad \dot{\lambda} = \frac{r}{\phi} e^{-rt}\left[\frac{\gamma}{\beta} + \frac{\alpha}{\beta} Q^* - \frac{1}{\beta} Q - \theta - \frac{1}{\beta} Q\right] - \frac{e^{-rt}}{\phi}\,\frac{\alpha}{\beta}\,\dot{Q}^* + \frac{e^{-rt}}{\phi}\,\frac{2}{\beta}\,\dot{Q}$$

Substituting for $\dot{\lambda}$ from (12) into (11), having substituted for $\dot{Q}^*$ from (2) into (12) yields:

$$(13)\quad \lambda\phi = e^{-rt}\left\{-\frac{r}{\phi}\frac{2}{\beta} Q + \frac{\alpha}{\beta}\left(1 + \frac{r}{\phi}\right) Q^* + \frac{1}{\phi}\frac{2}{\beta}\dot{Q}\right\}$$

Now eliminate $\lambda\phi$ from (13) and (10) to obtain the differential equation (14), which, together with (2), describes a system of two first-order linear differential equations in Q and Q^* :

$$\begin{array}{l}(14)\\ (2)\end{array}\quad \begin{bmatrix}\dot{Q}\\ \dot{Q}^*\end{bmatrix} = \begin{bmatrix}-\left(\frac{\gamma}{\beta} - \theta\right)\left(1 + \frac{r}{\phi}\right)\frac{\phi\beta}{2}\\ 0\end{bmatrix} + \begin{bmatrix}(\phi + r) & -\alpha\left(\phi + \frac{1}{2} r\right)\\ \phi & -\phi\end{bmatrix}\begin{bmatrix}Q\\ Q^*\end{bmatrix}$$

Call the above matrix A. The trace of A is:

$$\text{(15)}\quad \text{tr. (A)} = r > 0$$

The determinant is

$$\text{(16)}\quad \det(A) = \phi[\phi(\alpha - 1) + r(\tfrac{\alpha}{2} - 1)] < 0$$

<u>both</u> for maleficent addiction $(0 < \alpha < 1)$ and beneficent addiction $(\alpha < 0)$. I.e, for each case we have a saddlepoint. There is therefore only one stable path for Q and Q^* which it were well that the monopolist, in the interest of maximizing profit, should follow. The equation of this stable path, which is derived in the appendix, is

$$\text{(17)}\quad Q = (1 - \frac{m_2}{n_2})\bar{Q}^* + \frac{m_2}{n_2} Q^*$$

where the bar indicates a steady state value, and

$$\text{(18)}\quad \frac{m_2}{n_2} = \frac{\alpha(\phi + \frac{1}{2} r)}{(\phi + r) - \lambda_2}$$

Setting $\dot{Q} = 0$ in (14) one gets :

$$\text{(19)}\quad Q = \frac{(\frac{\gamma}{\beta} - \theta)(1 + \frac{r}{\phi})\frac{\phi\beta}{2}}{(\phi + r)} + \frac{\alpha(\phi + \frac{1}{2}r)}{(\phi + r)} Q^*$$

Equations (17), (18), (19) and (4) (the forty-five degree line in Q, Q^* space which specifies a zero rate of change of Q^*) constitute the tool-kit with which we shall attempt to analyse addictive markets, to which task we now bend ourselves.

III. HARMFUL ADDICTION

The dynamics of the harmfully addictive monopolistic market are illustrated in Fig. III. The curve labelled Q^* Q^* is the curve of equation (4) and shows combinations of Q and Q^* which yield a zero rate of change of Q^*. Its slope is unity, since a one-unit increase in Q will raise $\dot{Q}^*$ by ϕ, as will a one-unit fall in Q^*, and in long-run equilibrium the two are equal. The curve labelled $Q\ Q$ is the curve of (19), along which $\dot{Q}$ is zero. Its slope is :

$$(20) \qquad \frac{\delta Q}{\delta Q^*}\bigg|\,\dot{Q} = 0 \quad = \quad \frac{\frac{\alpha}{2}\phi(2 + \frac{r}{\phi})}{\phi + r}$$

which is less than unity, since we have shown that :

$$\det(A) \quad = \quad \phi[\ -\ (\phi + r) + \frac{\alpha}{2}\,\phi(2 + \frac{r}{\phi})\ ] < 0$$

which implies that the denominator of (20) exceeds the numerator. Hence QQ is flatter than $Q^*\ Q^*$. Given the direction of the arrows in the picture then it is clear that the slope of the stable arm S^oS^o in Fig. III. must be flatter than that of QQ, as indeed it is, since its slope, given in (18) differs from (20) only in the addition of the positive number $-\ \lambda_2$ (where λ_2 is the negative root) to the denominator.

We have established that the monopolist will settle down in the long run to a unique quantity and therefore to a unique price. [3] The time-path followed to attain that long-run equilibrium is also unique. Stability is assured by the assumption that profit is maximized.

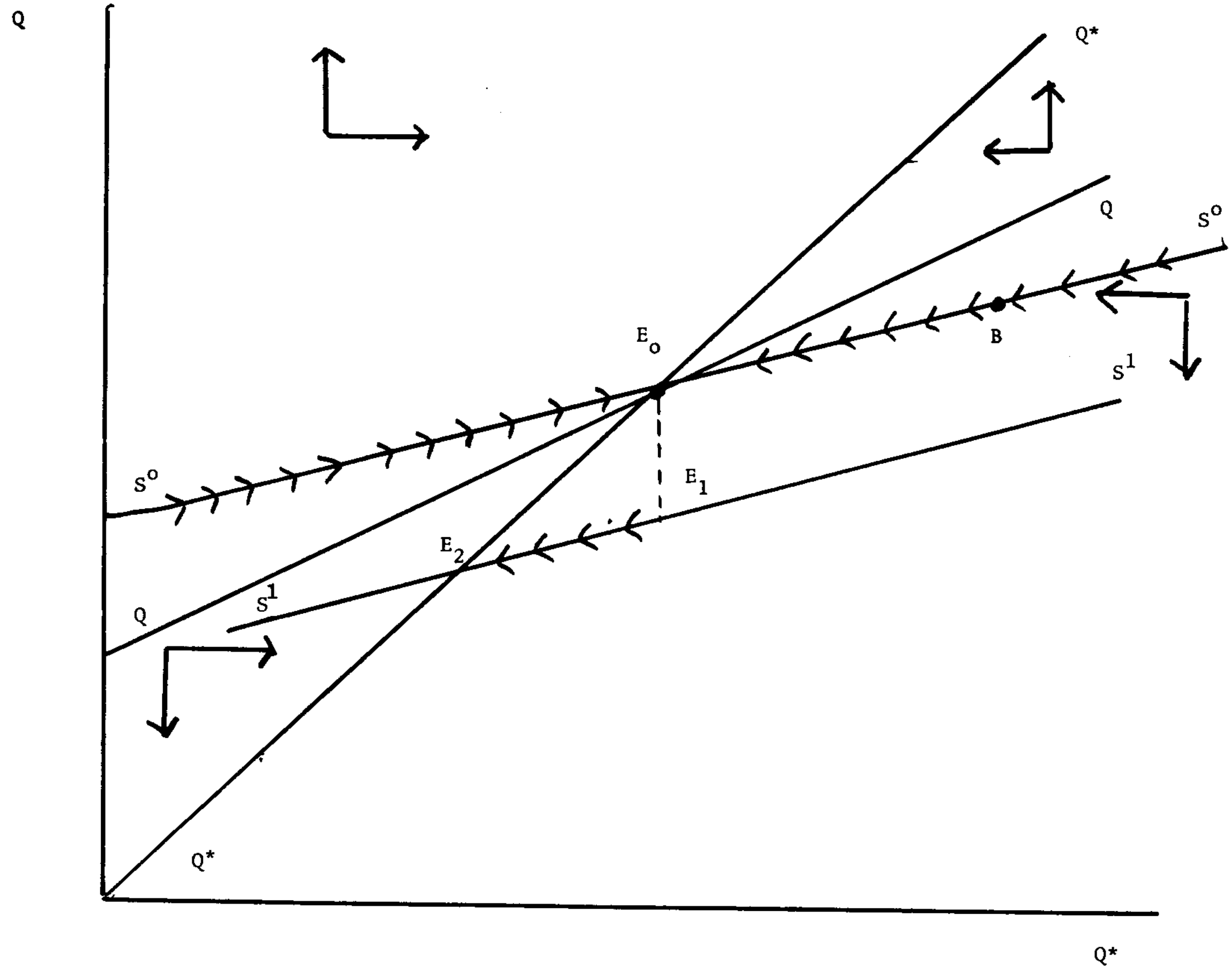

FIG. III.

Consider first the monopolist's long-run equilibrium. Solving equations (18) and (19) for $\bar{Q}^*$ we have

$$(21) \qquad \gamma - \frac{2}{\beta}(1-\alpha)\,\bar{Q}^* = \theta + \frac{\frac{\alpha}{\beta}\frac{r}{\phi}}{(1+\frac{r}{\phi})}\,\bar{Q}^*$$

But the sum of the two terms on the left hand side is LRMR, and θ is LRMC. Thus our equilibrium occurs where :

$$(22) \qquad \text{LRMR} = \text{LRMC} + \frac{\frac{\alpha}{\beta}\frac{r}{\phi}}{(1+\frac{r}{\phi})}\,\bar{Q}^*$$

In general, in a malificently addictive market, equilibrium will occur to the left of the intersection of LRMC and LRMR, i.e. at a lower output than that of the static monopolist. The smaller is the interest rate r, or the larger is the adjustment speed ϕ, then the smaller will be the difference between the equilibrium of our static and dynamic monopolists. In the limit, when either r is negligible or ϕ infinite, the dynamic monopolist will, in the long-run, produce where LRMC = LRMR.

The reason for this general result is to be found in the dynamics of the addictive market. Suppose the monopolist were operating to the left of the steady-state equilibrium described by (22). Since LRMR there exceeds LRMC the monopolist can increase long-run profit by expanding his long-run output Q^*, that is by moving down the long-run demand curve D^* D^*. But according to equation (2) he can only do so by raising Q above Q^*, that is by cutting price and moving down the short-run demand curve at his current Q^* and foregoing current profit. His decision to expand is thus an investment decision. He will continue to expand until the investment cost is just counterbalanced by the increment to the present value of future profit. If the adjustment speed ϕ is infinite the monopolist need not drop his price along the short-run demand curve but may move along $D^* D^*$, incurring no investment costs in adding to future profit. If r is zero, then the present value of any increment to profit is infinite. In either of these cases profit is maximized where LRMC = LRMC. But for any finite ϕ or positive r, it is maximized to the left of that equality.

Let us now examine the effects on the system of an increase in the marginal cost of the monopolist, starting from long-run equilibrium at E_0. Both $Q\,Q$ and $S\,S$ in Fig. III. will shift downwards. (The former shift is omitted in the interests of clarity. The shift in $Q^* Q^*$ is of course zero.) The downward shift in $S\,S$ is, from (17) and (21) :

$$\frac{\delta Q}{\delta \theta} = -(1 - \frac{m_2}{n_2}) \left[\frac{\beta}{\{2(1-\alpha)+(2-\alpha)r\}} \right] < 0$$

Q^* can change only over time. So when marginal cost rises, there will be an instantaneous reduction in Q at constant Q^*, a discrete movement from E_0 to E_1 on $S'S'$, the new $S\,S$ curve vertically below E_0. Thereafter there will be a smooth movement along $S'S'$, Q and Q^* falling together as required for stability, until eventually equilibrium is reached at E_2. The short-run fall in Q (from E_0 to E_1) is less than the long-fall to E_2, and Q does not overshoot.

One can gain more insights into the dynamic and comparative-dynamic behaviour of the model if one looks at it in price-quantity space, rather than in $Q - Q^*$ space. Substitute for Q^* from the stable path equation (17) into the demand curve (1). The result is :

$$(23) \quad P = \frac{\gamma}{\beta} - \frac{\alpha}{\beta}\frac{n_2}{m_2}\left(1 - \frac{m_2}{n_2}\right)\bar{Q}^* + \frac{1}{\beta}\frac{n_2}{m_2}\left[\alpha - \frac{m_2}{n_2}\right] Q.$$

This equation is the only relationship between price and quantity which satisfies the stable path. Its slope is

$$(24) \quad \frac{\delta P}{\delta Q} = \frac{1}{\beta}\frac{(\phi+r-\lambda_2)}{(\phi + \frac{1}{2}r)}\left[1 - \frac{(\phi+\frac{1}{2}r)}{(\phi+r)-\lambda_2}\right] > 0$$

(whether addiction is maleficent or beneficent), and is drawn in Fig. IV. as A A. (For the time being, apart from D*D* and DD, ignore the other curves in the picture).

Suppose one starts at the point B in Fig. III, and let the corresponding point in Fig. IV. also be B. The system moves from B down the stable path S^oS^o towards E_o, both Q and Q^* falling. These movements exert conflicting forces on price according to the demand equation

$$P = \frac{\alpha}{\beta} + \frac{\alpha}{\beta}Q^* - \frac{1}{\beta}Q$$

A one unit fall in Q^* (a <u>leftward shift</u> in the short-run demand curve) will cause a <u>fall</u> in price of $\frac{\alpha}{\beta}$. A one unit fall in Q (a <u>movement up</u> the short-run demand curve) will cause a <u>rise</u> in price of $\frac{1}{\beta}$. If Q and Q^* were to fall at the same rate (i.e. if $\frac{\delta Q}{\delta Q^*}$ were unity along $S^o\ S^o$) then the effect of Q on price would dominate $(\frac{\alpha}{\beta} < \frac{1}{\beta})$ and price would rise. But Q and Q^* do not fall at the same rate. A one-unit fall in Q^* along $S^o S^o$ is accompanied, (for stability) by a fall in Q of only

$$\frac{\alpha(\phi + \frac{1}{2} r)}{(\phi + r) - \lambda_2}$$

which is less than α. Hence the upward pressure exerted on price by the fall in Q is dominated by the downward pressure of the fall in Q^* and A A, the stable path in price quantity space, is upward sloping.

The A A curve is a useful tool. Among other things, it shows that the value of the monopoly right is higher, the higher is the arbitrary initial price [5)]. The difference in the present values of the monopoly rights as between any two starting points depends only on the present values of profit in the <u>transition</u> from the starting point to the long-run equilibrium at E_o, since at E_o the present values will be identical. The present value of $(P-\theta)\ Q$ in the transition from the point B to E_o is greater than in the transition from C to E_o. The higher is initial price (the farther up the A A curve we start) the greater the value of the monopoly right.

It also sheds more light on the comparative-dynamics. An upward shift in marginal cost θ will shift the A A curve of Fig.IV upwards to A'A'. If we differentiate (23), the equation of A A, we get

$$(25) \quad \frac{\delta P}{\delta \theta} = - \frac{\alpha}{\beta} \frac{n_2}{m_2} (1 - \frac{m_2}{n_2}) \frac{\delta \bar{Q}^*}{\delta \theta} > 0$$

Since Q^* can only change over time, there will be a discrete jump, shown in Fig. IV, from E_o to E_1, (the intersection of $A'A'$ with the short-run demand curve of the original equilibrium), corresponding to the same discrete jump in Fig. III. Since quantity undershoots in the short-run, price must overshoot, which is only another way of saying that $A\,A$ slopes upwards. From E_1 the market will move smoothly down $A'A'$ to E_2, the short-run demand curve shifting leftwards as Q^* falls. We now turn to apply the model to some problems of policy.

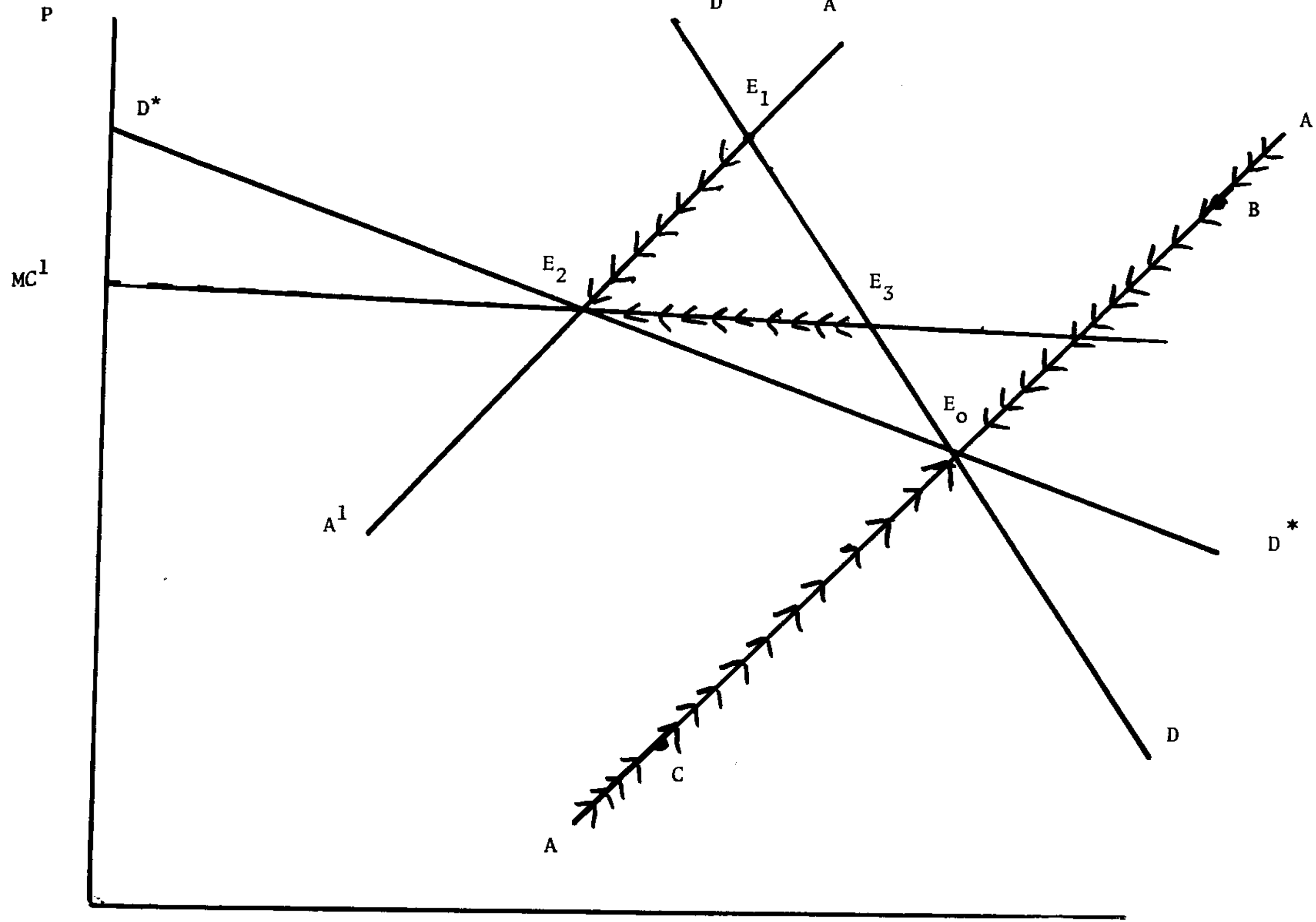

FIG. IV.

IV. APPLICATION: THE HEROIN MARKET AND DRUG-RELATED CRIME

The model can be used to shed light on some questions of policy interest. I give two such applications here. First, will anti-narcotic policies directed at the demand side result in more crime under monopoly than under competition ? Second, will supply side policies result in more crime under monopoly than under competition ? Let crime be a function of expenditure on the drug, (suppressing other arguments in the crime supply function - see Becker (6)) according to :

(26) $D = f(PQ); \quad f' > 0;$

where D is the monetary value of external crime costs.

First, consider a given reduction in the long-run supply of hereoin caused by upward shifts in marginal cost curves (in turn caused by some anti-narcotic policy). Let the reduction in long-run supply be from E_o to E_2 on the long-run demand curve D^*D^* of Fig.IV.[6] In the long-run, expenditure, and therefore crime, will be the same whether monopoly or competition obtains. But what of the transition ? Will transitory external costs in terms of crime be greater or smaller under competition than under monopoly ? The answer depends only on the price-elasticity of demand of the short-run demand curve at E_o.

Under competition, the upward shift in marginal cost required to effect a long-run movement from E_o to E_2 in Fig. IV. is that from the horizontal line through E_o (not drawn) to MC', running through E_2. Such a shift will cause a jump from E_o to E_3, and thereafter a smooth movement along MC' to E_2. Under monopoly the increase in marginal cost is that required to shift AA up to A'A', passing through E_2. There will be a jump from E_o to E_1, followed by a smooth movement down A'A' to E_2.

The answer to the question therefore depends only on whether the integral over time of expenditure is greater in the transition from E_1 to E_2 than from E_3 to E_2. (The areas under A'A' and MC' do not shed any light on the problem). Let us make the plausible assumption that DD is price-inelastic at E_o. Then total expenditure is greater at E_1 than at E_3.

Fig. V, which measures expenditure (PQ) on the vertical axis and time (t) on the horizontal, illustrates the argument. The curve labelled MM represents the time path of expenditure under monopoly. The monopolist starts at time zero, at the point E_1, with a relatively high total expenditure $PQ(E_1)$. He then moves down A'A', both price and quantity falling continuously, and expenditure asymptoting towards $\bar{P}\bar{Q}^*$, expenditure at E_2. The curve CC illustrates the behaviour of expenditure under competition. The competitive industry will begin at E_3, with a relatively low expenditure $(PQ\ (E_3)$, in Fig. V.) and will move along MC', quantity falling at a constant price $\bar{P}$, and total revenue again asymptoting towards $\bar{P}\bar{Q}^*$ Thus if DD is price-inelastic the MM curve will lie uniformly above the CC curve. The transition from E'_o to E'_2 will involve a greater cost in terms of crime under monopoly than under competition. (If the DD curve is price elastic, then the positions of MM and CC are reversed, as is the conclusion.)

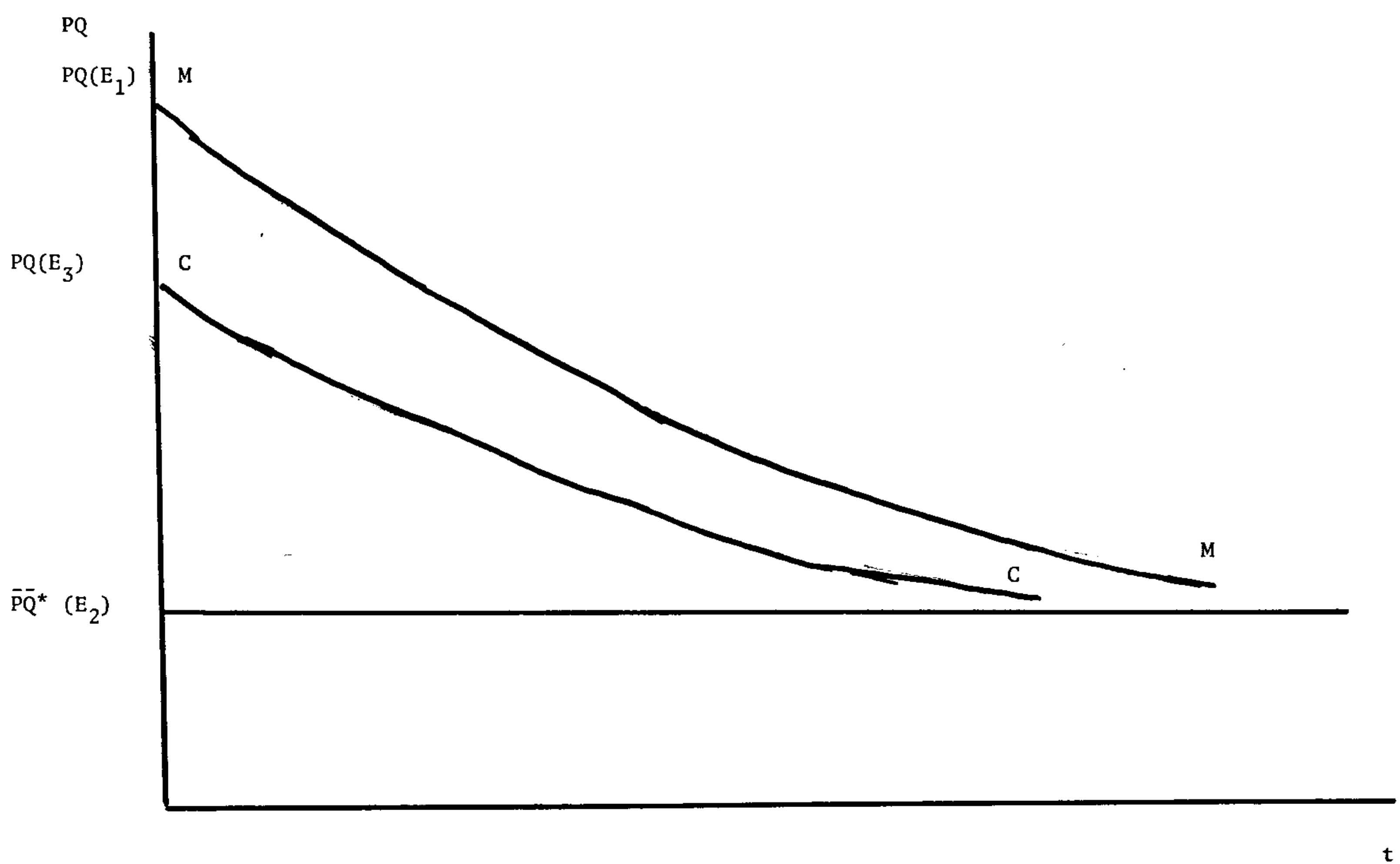

FIG. V.

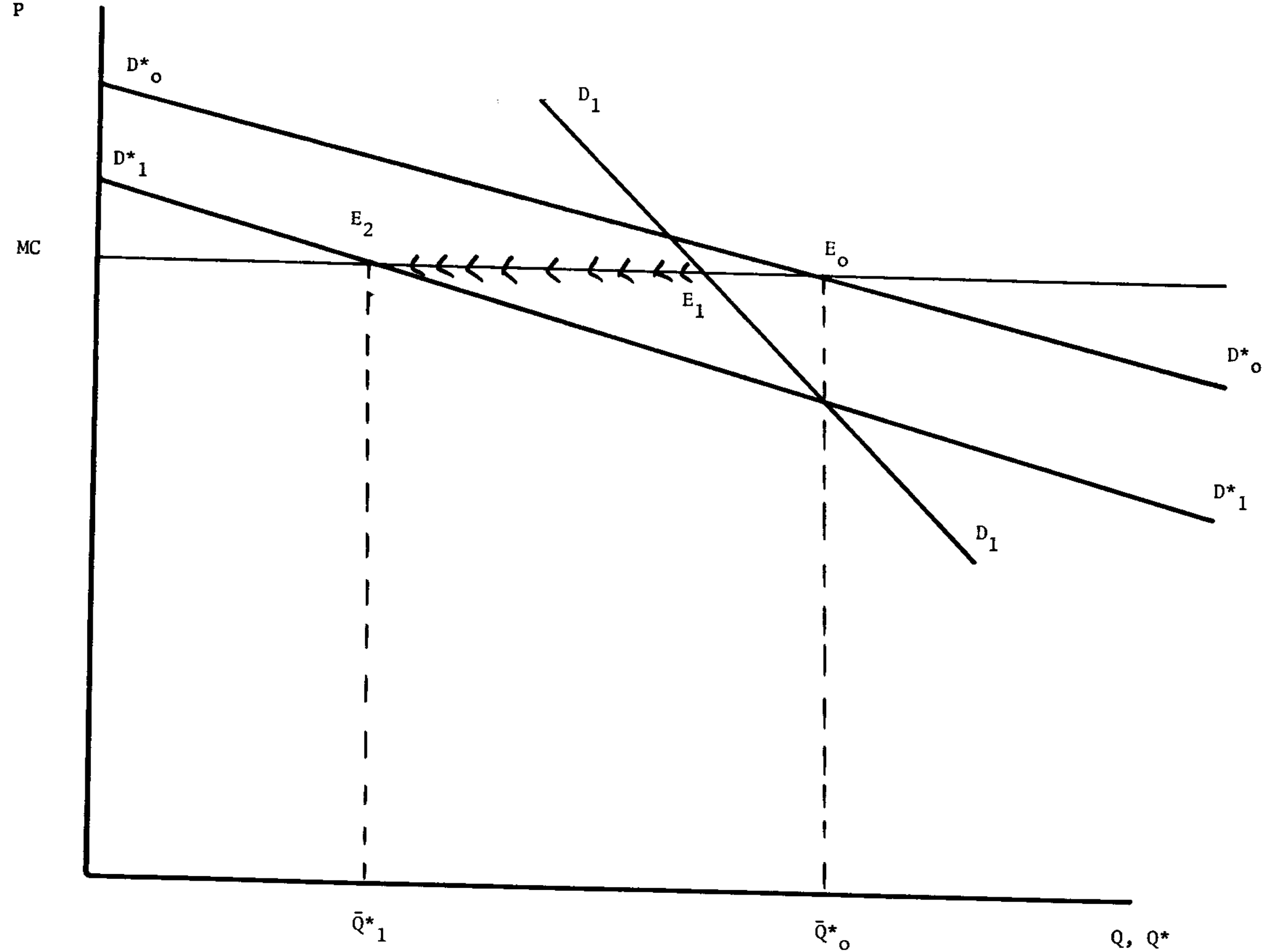

FIG. VI.

Next, let us examine the effects of any given reduction in quantity caused by any demand-side policy directed at reducing the parameter γ. Assume that the policy-maker in a competitive market wishes to reduce consumption from the equilibrium quantity $\bar{Q}^*_o$ to $\bar{Q}^*_1$ in Fig. VI. He manipulates γ to shift the long-run demand curve downwards until it passes through the intersection of LRMC with the vertical line through $\bar{Q}^*_1$. The new long-rung demand curve is $D^*_1 D^*_1$. Q^* is fixed at $\bar{Q}^*_o$ in the short-run. The market will shift discretely to the point E_1, the intersection of MC with the short-run demand curve D_1D_1 passing through $\bar{Q}^*_o$ on the new long-run demand curve $D^*_1D^*_1$. After this discrete jump there is a smooth transition along MC to E_2.

Under monopoly, the D^*D^* curve must also shift downwards. However, the AA curve may shift upwards or downwards according to:

$$(27) \qquad \frac{\delta P}{\delta \gamma} = \frac{1}{\beta} - \frac{1}{\beta}\,\frac{(\phi + r - \lambda_2)}{(\phi + \tfrac{1}{2} r)}\,(1 - \frac{m_2}{n_2})\,\frac{d\bar{Q}^*}{d\gamma} \gtrless 0.$$

Since $\frac{d\bar{Q}^*}{d\gamma} > 0$, from equation (19), i.e. a fall in γ will reduce steady state consumption, and that force exerts upward pressure on the AA curve. However, from equations (5) and (19) we have:

$$(28) \qquad \frac{d\bar{P}}{d\gamma} = \frac{1}{\beta} + \frac{(1-\alpha)}{\beta}\,\frac{d\bar{Q}}{d\gamma} > 0$$

a fall in γ must reduce steady-state price. Let us assume first of all that the short-run movement in P is in the same direction as that of the long-run change, i.e. that the AA curve shifts downwards. The condition for this is:

$$(29) \qquad \left[\frac{2(1-\alpha)(\phi+r)+\alpha r}{\beta(\phi+r)}\right] - \alpha\left[\frac{\phi+r-\lambda_2}{\alpha(\phi+\frac{1}{2}r)} - 1\right] > 0$$

(The downward shift in AA cannot of course exceed the downward shift in D*D*, otherwise $\bar{Q}^*$ would rise). The policy-maker manipulates γ so as to choose the long-run $D^*_2D^*_2$ and A^2A^2 curves which intersect on the vertical line through $\bar{Q}^*_1$ at the point E_4 in Fig. VII. Such a point must exist since $\bar{Q}^*$ is continuous in γ - see (21). Q^* is fixed in the short-run, and the market will jump to E_3, the intersection of A^2A^2 with the short-run demand curve passing through $\bar{Q}^*_0$ on $D^*_2D^*_2$. The system will then move smoothly down A^2A^2 to E_4.

Under competition, price is constant (equal to marginal cost) as between the two long-run equilibria. Under monopoly, it must, as we have seen, fall. Hence the $D^*_2D^*_2$ curve of Fig. VII lies below the $D^*_1D^*_1$ curve of Fig. VI. Moreover, the D_2D_2 curve of Fig. VII lies uniformly below the D_1D_1 curve of Fig. VI. Provided the price-elasticity of demand on D_1D_1 at E_1 is less than unity, then expenditure at E_3 is less than at E_1. Draw an imaginary horizontal line through E_3 and extend it to cut D_1D_1. At that intersection, if demand is price inelastic at E_1, expenditure will be less than at E_1, and clearly greater than at E_3. Hence expenditure at E_3 is less than at E_1, and declines continuously to E_4, at which it is clearly less than at E_2. Under monopoly (provided AA shifts downwards) supply-side policies result in less drug-related crime, both long-run and transitory, than under competition.

If the AA curve shifts upwards, then the short-run response is ambiguous. In that case it is conceivable that short-run price might rise and that total expenditure might for a time be greater under monopoly than under competition. However, long-run price must unambiguously fall under monopoly, and it is constant under competition at the same output. Hence the integral over all time of the value of drug-related crime must be less under monopoly. From a comparison of Figs. VI and VII with Fig. IV, it is obvious that, for a given reduction in quantity, demand-directed policies result in less crime than supply-directed policies, not only in the long-run but also in the transition from one equilibrium to another, whether monopoly or competition prevails.

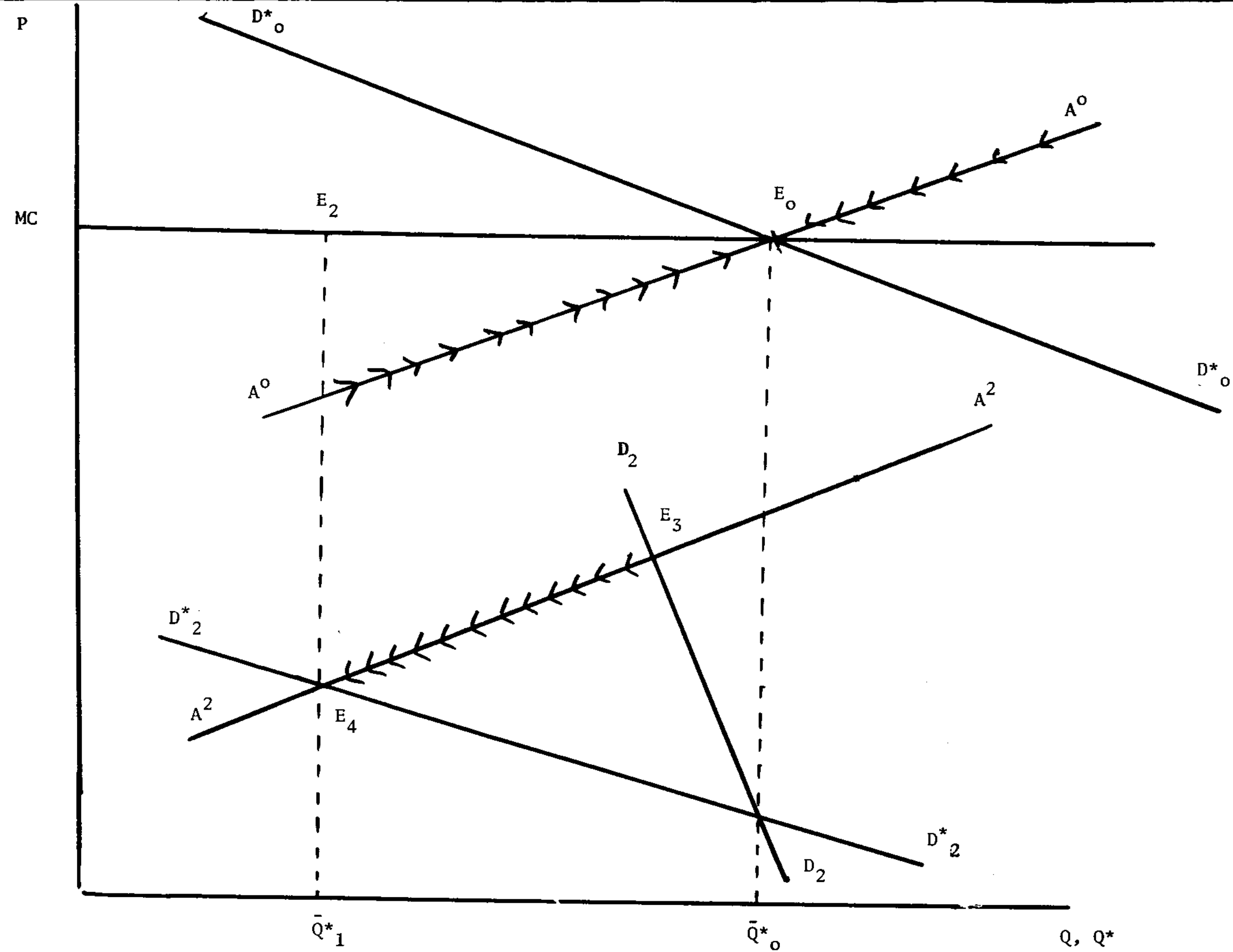

FIG. VII.

V. BENEFICIAL ADDICTION

Fig. VIII shows the phase - diagram. Since α is negative QQ slopes downwards, as does S^oS^o, which is flatter than QQ. Again because α is negative, the monopolists' long-run equilibrium will, except if r is zero or ϕ infinite, occur to the right of the textbook equilibrium, i.e. he will produce an output greater than that at which LRMR equals LRMC. If he were producing at that equality he could always at a trivial loss of long-run profit, substantially increase his short -run profit by dropping his price along the short-run demand curve (which is flatter than the long-run demand curve), thereby keeping price for a time above that obtaining along the long-run curve. His equilibrium will occur where the gain in short-run profit from this activity is just offset by the present value of the loss in long-run profit. Will the beneficent monopolist ever produce as much as the competitive market ? No. The competitive market will produce where marginal cost equals price :

$$(30) \qquad \bar{Q}^* = \frac{(\frac{\gamma}{\beta} - \theta)}{\frac{1}{\beta}(1-\alpha)}$$

The monopolist will produce where :

$$(31) \qquad \bar{Q}^* = \frac{(\frac{\gamma}{\beta} - \theta)}{[\frac{1}{\beta}(1-\alpha)+\frac{1}{\beta}(\phi+r-\alpha\phi)\ ]}$$

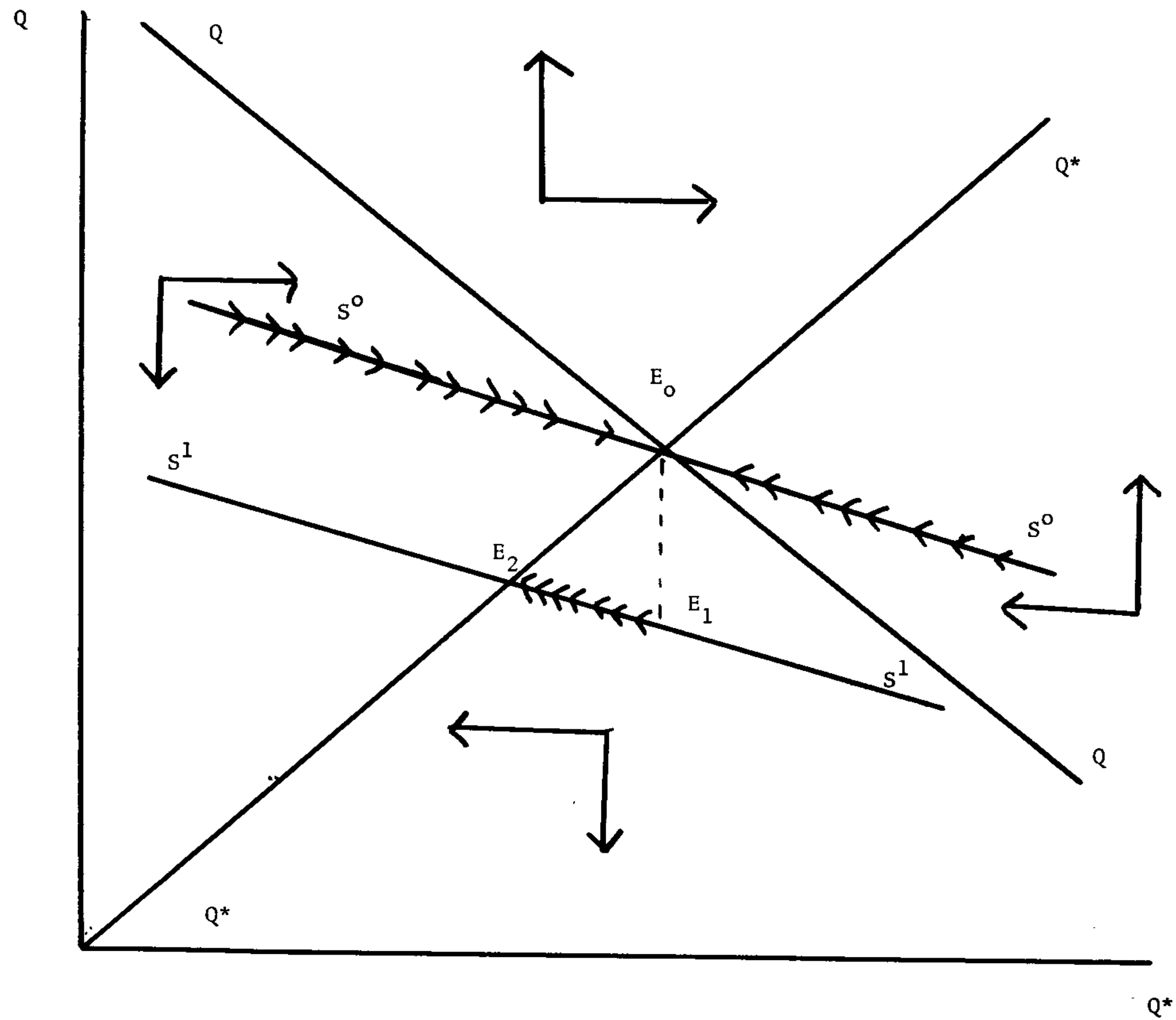

FIG. VIII.

I.e. he will never make long-run absolute losses. However, if Q is sufficiently small he will produce where marginal revenue is negative, as equation (19) shows.

A rise in marginal cost θ will shift S^oS^o down at $\bar{Q}^*$ by:

$$(32) \qquad \frac{\delta Q}{\delta \theta} = -\left(1 - \frac{m_2}{n_2}\right) \left\{ \frac{\beta}{(2-\alpha)} \right\} < 0$$

and there will be a discrete movement from E_0 to E_1 on S^1S^1. Q falls below its new long-run equilibrium level and climbs back up S^1S^1 to E_2.

The slope of the AA curve in price quantity space is upward. Suppose we are at E_1 on S^1S^1, heading for E_2. Q is rising (a movement down the short-run demand curve). Q^* is falling causing a rightward shift of the short-run demand curve ($\alpha < 0$). The slope of AA is :

$$(33) \qquad \frac{\delta P}{\delta Q} = -\frac{1}{\beta}\left(1 - \frac{\phi + r - \lambda_2}{(\phi + 2r)}\right) > 0$$

We are at E_1 on S^1S^1, heading for E^2. Q is rising, we are moving <u>down</u> the short-run demand curves and this exerts a downward pressure on price. Q^* is falling, and since α is negative this exerts an <u>upward</u> pressure on price (this shifts the short-run demand curves rightward). As in the maleficent case the influence of Q^* dominates and price rises. Fig.IX illustrates the AA curve

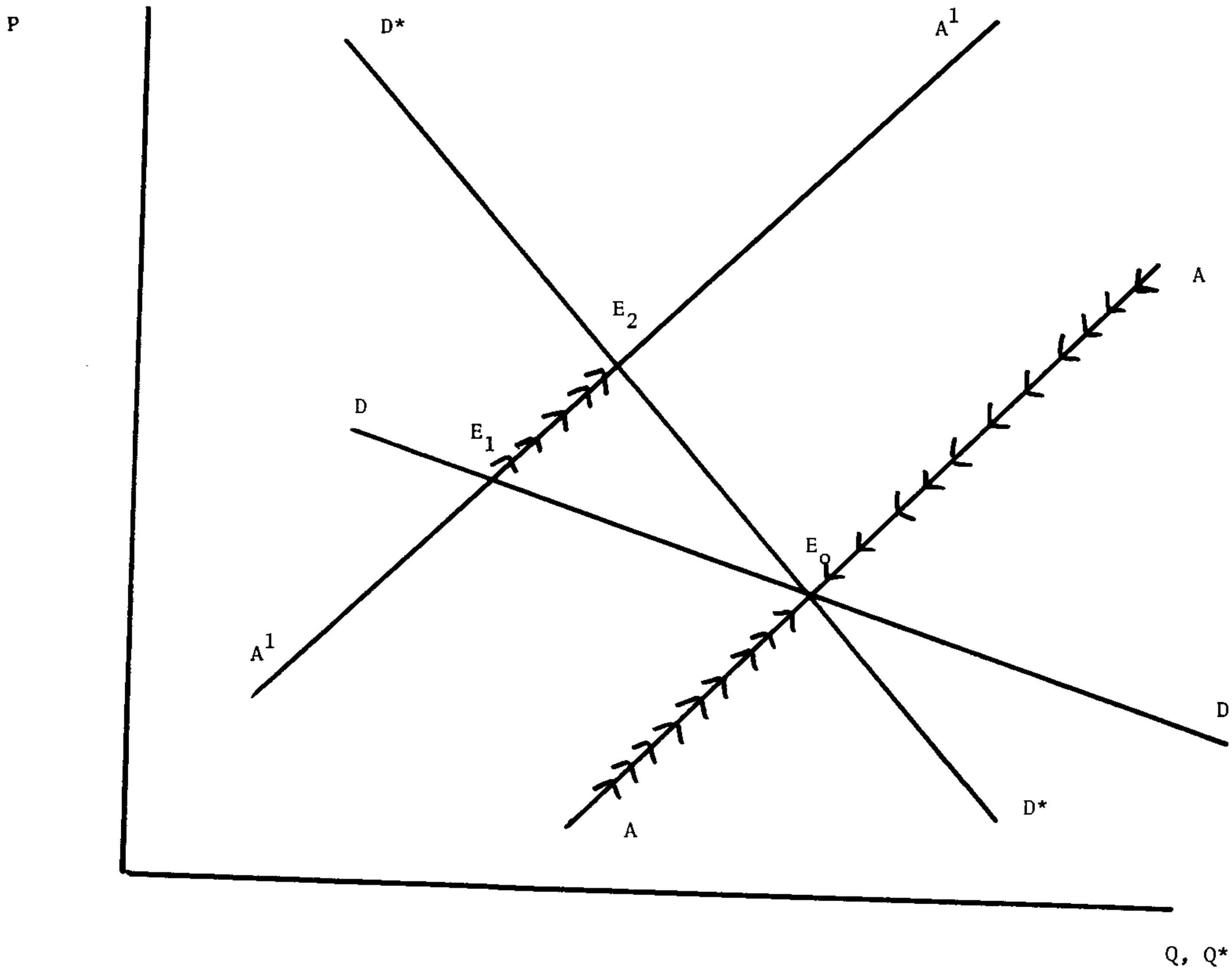

FIG. IX.

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A rise in θ will shift the AA curve upwards to A^1A^1. There will be a discrete jump to E_1. Since quantity overshoots, price undershoots, and thereafter a smooth movement up A^1A^1 to E_2 occurs. Again, the higher the initial price the higher the value of the monopl right. The reader is left to draw the remaining inferences.

APPENDIX: ROOTS, SOLUTION, AND STABLE PATH.

The roots of the system of equations (14) and (2) are obtained from the characteristic equation:

(i) $$\begin{vmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{vmatrix} = \lambda^2 - tr(A) + det(A).$$

Hence

(ii) $$\lambda_1, \lambda_2 = \tfrac{1}{2} \{ tr(A) \pm \sqrt{tr(A)^2 - 4 \, det(A)}$$

$$= \tfrac{1}{2} \{r \pm \sqrt{r^2 - 4\phi[\phi(\alpha-1) + r (\tfrac{\alpha}{2} -1)]}$$

Hence $\lambda_1 > 0$, $\lambda_2 < 0$.

The solutions of (13) and (2) are:

(iii) $$Q_{(t)} = m_1 e^{\lambda_1 t} + m_2 e^{\lambda_2 t} + \bar{Q}$$

and

(iv) $$Q^*_{(t)} = n_1 e^{\lambda_1 t} + n_2 e^{\lambda_2 t} + \bar{Q}^*.$$

where the m_i and n_i reflect the initial conditions. Unless the initial conditions are so chosen that $m_1 = n_1 = 0$, then the terms in the positive root λ_1 will eventually dominate and the system will shoot off. Setting m_1 and n_1 at zero, dividing (iii) by (iv), and remembering that $\bar{Q}^* = \bar{Q}$, then we may solve for $Q_{(t)}$ as a function of $Q^*_{(t)}$

(v) $$Q_{(t)} = (1 - \frac{m_2}{n_2}) \bar{Q}^* + \frac{m_2}{n_2} Q^*$$

We can solve for $\frac{m_2}{n_2}$ from the characteristic vector :

$$\begin{array}{l} \text{(vi)} \\ \\ \text{(vii)} \end{array} \quad \begin{bmatrix} a_{11} - \lambda_2 & a_{12} \\ & \\ a_{21} & a_{22} - \lambda_2 \end{bmatrix} \begin{bmatrix} m_2 \\ \\ n_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}$$

From (vi) :

(viii) $$\frac{m_2}{n_2} = -\frac{a_{12}}{a_{11} - \lambda_2} = \frac{\alpha(\phi + \frac{1}{2} r)}{(\phi + r) - \lambda_2}$$

FOOTNOTES

(1) After the work reported here had been done, I discovered the paper of Brennan, Buchanan and Lee (B.B.L.), whose idea is similar to mine and upon whose paper I comment elsewhere. B.B.L. try to solve a dynamic problem employing the method of comparative statics. In consequence all of their results but one are contradicted in this paper. I indicate in footnotes where our results differ.

(2) The constraint is :

$$\lambda\phi(Q - Q^*) \quad - \quad \lambda \dot{Q}^*$$

Integration by parts yields :

$$\int -\lambda \dot{Q}^* \, dt \quad = \quad [\lambda Q^*] \quad + \int \dot{\lambda} Q^* \, dt$$

which is then substituted into the constraint, the first term on the right-hand side being ignored.

(3) B.B.L. claim (Proposition 3, p. 541) that :

$$\frac{d\hat{P}}{dP_o} < 0$$

where $\hat{P}$ is long-run profit-maximizing price and P_0 any arbitrary initial price, $\hat{P}$ is a continuously differentiable function of P_0, of which there is an infinity. Hence B.B.L. claim, for a partial equilibrium Marshallian model, that there exists <u>in general</u> an infinity of equilibria. Our result contradicts this extraordinary claim.

(4) Uniqueness and stability of the monopolist's time-path render otiose B.B.L.'s conjectures (pp. 542 et. seq.) about oscillating price-strategies.

(5) This is a contradiction of B.B.L.'s Proposition 2, (p.540).

(6) To enable us to make the comparisons of this section, we must assume that the monopolist and the competitive market are, in the initial long-run equilibrium, producing the same output under identical demand conditions. Hence we must make the artificial assumption that marginal costs differ sufficiently to induce them so to do.

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