HOUSING STARTS AND SALES EXPECTATIONS
IN THE UK

by

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1. INTRODUCTION

The purpose of this paper is to model housebuilding activity in the UK, with particular emphasis on housing starts. The importance of a deeper understanding of activity in this sector of the economy arises from the fact that investment in private housing represents a significant part of UK total investment. Consequently, the explanation of new housebuilding has been the subject of many studies in the past (see, for example, Hadjimatheou 1976, Whitehead 1974, Mayes 1979 and Thom 1984).

Previous studies have not always provided a full explanation of the inventory decision and production adjustment processes displayed in new housebuilding activity. Further, the studies have also failed to take adequate account of the influence of expected house sales and the manner in which fluctuations in the mortgage market may impinge upon builders' expectations.

A typical approach is to specify construction activity (for example, housing starts or investment in residential construction) as depending upon variables such as expected house sales, construction costs, house prices and interest rates. Expected sales are either proxied by actual sales or substituted out by using mortgage market variables such as borrowing costs and credit availability. An important feature of the present study is that we explicitly model
sales expectations by specifying observed sales as a linear function of real income, real borrowing costs, mortgage availability and an indicator of cyclical economic activity. The estimated parameters of the sales equation are then used to generate a series for sales forecasts.

In section 2 we outline the model employed to explain actual housing starts. Within this section sales expectations are also modelled. Section 3 gives details of the data employed and the steps taken to overcome some limitations on data availability. The main econometric results of the study dealing with the estimation of house sales and house starts are reported in section 4 while a final section summarises the findings and draws some conclusions. An appendix is also included, describing the code names for variables and the relevant statistical sources.
2. THE MODEL

2.1 Housing Starts

Throughout this study we assume that houses can be sold at any stage of the construction process; that is, builders accept sales for completed, partially completed and newly-initiated dwelling units. Given expectations about the demand for and profitability of new housing, the speculative builder's problem is to plan production so that the available stock of units for sale is sufficient to maximise expected profits. Letting $A_t$ denote the number of units available for sale in period $t$ and $N_t$ the end-of-period inventory of unsold units, then:

$$A_t = N_{t-1} + STS_t \quad \quad \quad \quad \quad (1)$$

where $STS_t$ denotes current housing starts per period. If $AP_t$ represents the 'planned' level of $A_t$, then planned starts for period $t$ are given by:

$$STS_t^P = AP_t - N_{t-1} + UF_{t-1} \quad \quad (2)$$

where $UF_{t-1}$ are unfilled orders inherited from the previous time period. Unfilled orders are included in equation (2) because speculative builders are known to accept orders for new houses in advance of the actual start being initiated.
To model the planned level of $A_t$, we employ the choice-theoretic model of US inventory construction behaviour developed by Hayashi and Trapani (1978) which suggests that $A^p_t$ may be approximated by a linear relationship of the form:

$$A^p_t = a_0 + a_1S^e_{t,t-1} + a_2CCR^e_{t,t-1} + a_3FC^e_{t,t-1} \ldots (3)$$

where $S$ denotes sales of houses, CCR construction costs (relative to house prices) and FC the financing costs of holding unsold units. The superscript 'e' on the explanatory variables indicates expectations of these variables, with the expectation for period $t$ being held at the end of the previous period $(t-1)$. Hence, $X^e_{t,t-1}$ is the expectation of $X_t$ formulated at $t-1$. A priori, we would expect $a_1 > 0$ and $a_2, a_3 < 0$. Construction costs and financing costs are included in (3) to account for the possibility that individual builders may not estimate future sales correctly. An underestimate of the demand for new houses may result in builders missing out on profitable sales opportunities (reflected in construction costs relative to house prices) while an overestimate leads to financing costs associated with unsold stocks. Hence, a fall in either CCR or FC is assumed to induce an increase in the planned stock of housing units for sale associated with any given level of expected sales.
Data for the level of unfilled orders are not available in a readily accessible form. However, it seems probable that unfilled orders are directly related to underestimates of actual sales. For example, if in any period demand for new houses exceeds the level that was anticipated, then the planned stock of units for sale will be insufficient so that builders may accept orders without having initiated the housing units (as noted earlier). We therefore approximate unfilled orders by assuming that UF is proportional to positive values of the sales forecast error; that is:

\[ UF_{t-1} = k(S_{t-1} - S_{t-1}^e) \] ................. (4)

where \( 0 \leq k \leq 1 \). It is logical to assume that in periods when the forecast error is negative (that is, when actual sales are overestimated), \( UF = 0 \), so that only positive values of equation (4) are employed in the empirical analysis reported in Section (4).

Substituting (3) and (4) into equation (2), therefore, gives an expression for planned starts. Once again we follow Hayashi and Trapani (1978) in postulating that UK builders may be influenced by the cost of rescheduling production. Changing the level of starts per period may involve builders in varying the size of the labour force, purchasing land, obtaining planning permits, and resiting equipment, so that
the actual change in starts is given by the familiar adjustment model:

\[ \text{STS}_t - \text{STS}_{t-1} = \lambda (\text{STSP}_t - \text{STS}_{t-1}) \] ............ (5)

where \( \lambda \) is the adjustment coefficient and may take values in the range \( 0 \leq \lambda \leq 1 \).

There are, however, two crucial differences between the model developed in this study and that presented by Hayashi and Trapani (H & T). First, H & T appear to confuse the concepts of the stock available for sale in any period (\( A_t \)) and the end-of-period inventory of unsold units (\( N_t \)). In deriving their basic inventory model H & T correctly define \( A_t \) as the total number of units which a builder has available for sale in each period. But in their inventory adjustment model they implicitly re-define \( A_t \) as being end-of-period unsold stock (see H & T, 1978 p.109 equation 9). Hence, they express the change in inventory as

\[ A_t - A_{t-1} = \text{STS}_t - S_t \] ............... (6)

when in fact the correct expression should be

\[ N_t - N_{t-1} = \text{STS}_t - S_t \] ............... (7)
By their confusion of stock available for sale and end-of-period inventory, H & T are led into a second mistake in that they assume that the cost of adjusting inventory is independent of the cost of adjusting new production; that is, starts. However, it is clear from equation (7) above that changes in end-of-period inventory are due to either current starts or actual sales. While it is reasonable to assume that altering the level of starts involves rescheduling costs, changes which result from current sales are costless to the builders. Thus the adjustment process in our equation (5) reflects the cost of changing both \( N_t \) and, given the level of current sales, end-of-period inventory \( STS_t \).

Substituting for planned starts in (5) gives us the following relationship to explain actual housing starts:

\[
STS_t = a_0 + (1-\lambda)STS_{t-1} - \lambda N_{t-1} + \lambda k(S_{t-1} - S^e_{t-1,t-2}) + \lambda a_1 S^e_{t,t-1} + \lambda a_2 CCR^e_{t,t-1} + \lambda a_3 PCE^e_{t,t-1} \ldots \ldots \ldots (8)
\]

2.2 Sales Expectations

Previous models of house-building activity have proxied expected sales by either current sales (Hayashi and Trapani, 1978) or by lagged sales (Tompkinson, 1979). A major drawback of both of these approaches is that they fail to
take account of the manner in which fluctuations in the level of economic activity and developments in the mortgage market may influence builders' expectations. Rather than simply replace $S_{t,t-1}^e$ by either $S_t$ or $S_{t-1}$ we attempt to model sales and use the resulting forecasts to proxy builders' expectations. To do this we assume that sales of new dwelling units may be explained as a linear function of per capita real income ($Y$), the capital cost of new housing (CCH), a cyclical indicator of economic activity (CI) and the availability of mortgage finance (MA); that is:

$$S_t = b_0 + b_1Y_t + b_2CCH_t + b_3CI_t + b_4MA_t + u_t \ldots \ldots (9)$$

where $U_t$ represents a residual term.

Mortgage availability is included in equation (9) on the generally accepted hypothesis that UK building societies have, in the past, not used the mortgage rate of interest as a market-clearing mechanism but instead have resorted to non-price rationing so that house purchases may be subject to quantity constraints. (see Nellis and Thom, 1983 and 1986 forthcoming). Hence, the measure of mortgage availability which we employ is the value of net new commitments by UK building societies relative to new house prices.

It should be noted that our definition of mortgage availability excludes lending for house purchase by other financial institutions on the grounds that it is building
societies who have generally been regarded as the main perpetrators of mortgage rationing. However, since 1981 two important developments have radically altered the nature of the UK mortgage market. First, the main UK banks have become much more active in the supply of finance for house purchase. For example, banks only accounted for 7 per cent of total net advances in 1980, but for 24.4 per cent and 35.8 per cent in 1981 and 1982 respectively. In 1983 this proportion fell to 25.2 per cent and subsequently to 13.8 per cent in 1984. Secondly, the UK building society movement has tended to develop a more competitive environment in recent years with a significant relaxation of the recommended interest rate system. Nowadays, building societies often vary their lending and deposit rates individually rather than waiting for a recommendation from the Building Societies Association (BSA).

These developments suggest that the role of mortgage availability in influencing new house sales should be considerably weaker since 1981. Further, in the context of a more competitive environment the role played by the other right hand side variables in equation (9) may also have altered since that period. For example, if purchases are subject to borrowing constraints, then a change in the mortgage interest rate may have little influence on the demand for housing either because mortgages are relatively scarce in the case of an interest rate cut, or because
building societies may have to satisfy or moderate a backlog of demand in the case of an interest rate rise. In other words, the slope coefficients of (9) may have changed considerably over the past few years. The method used to estimate new house sales attempts to incorporate these effects explicitly.

The process we employ to measure sales expectations is first to estimate equation (9) in order to obtain values for the coefficients $b_i$ and then to proxy expectations of the explanatory variables by four-quarter weighted averages. Sales expectations can then be generated from:

$$S_{t,t-1}^e = b'_0 + b'_1Y_{t,t-1}^e + b'_2CCH_{t,t-1}^e + b'_3CI_{t,t-1}^e + b'_4MA_{t,t-1}^e \quad \ldots \ldots \ldots \ldots (10)$$

where $b'_i (i = 0, 1, \ldots, 4)$ are the ordinary least squares (OLS) estimates of the $b_i$ coefficients and, for example,

$$Y_{t,t-1}^e = c_1Y_{t-1}^e + \ldots + c_4Y_{t-4}^e \text{ etc.}.$$

The estimates for equations (8) and (9) are provided in Section 4. The next section, however, gives some further details on data availability and limitations.
3. THE DATA

Estimation of the model described above is hindered by some limitations on data availability. This is especially true of sales information and the end-of-period inventory of unsold units for which no official series are published. We approximate quarterly sales by the number of mortgages for new houses provided by building societies, insurance companies, local authorities and banks*

End-of-period inventory \( N_t \) is given by \( (V_t - PH_t \cdot S_t) / PH_t \) where \( V_t \) is the officially published series on value-of-output (see appendix for details), \( PH_t \) is the average price of new houses measured in current pounds and \( S_t \) is the number of housing units sold. Given our assumption that units may be sold at any stage in the production process, then \( N_t \) is a reasonable approximation of the number of units unsold at the end of each period.

Construction costs CCR are defined as the ratio of the index of construction costs scaled by an index of new house prices.**

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* New house purchases financed by ready cash are excluded, but they account for only an insignificant proportion of total new sales.
** See Mayes (1979 p.81) for a discussion on the difficulties of constructing a more comprehensive cost index which includes the price of land. Other details may be found in Whitehead (1974).
Financing costs FC are defined as \( R_t \cdot PH_t/P_t \) where \( R_t \) is the cost of bank borrowing and \( P_t \) is a general price index. These financing costs therefore measure the opportunity cost of holding unsold housing units in real pounds.

In equation (9), \( Y_t \) is the official series on real per capita income and the indicator of cyclical economic activity (CI) is the level of unemployment per quarter (excluding school leavers).

The definition of the capital costs of new housing (CCH) is similar to that used by Atkinson and King (1990), and is defined as:

\[
CCH_t = [MR_t(1-TR_t)L_t + AR_t(1-TR_t)(PH_t-L_t)]/P_t \quad \ldots (11)
\]

where \( MR_t = \) nominal mortgage interest rate \( TR_t = \) standard rate of income tax \( L_t = \) average mortgage loan size \( PH_t = \) average price of new houses \( AR_t = \) the opportunity cost of housing equity \((PH_t - L_t)\) \( P_t = \) a general price index.

Expression (11) can be simplified by setting \( MR_t = AR_t \), which yields:

\[
CCH_t = MR_t(1-TR_t) \cdot PH_t/P_t \quad \ldots (12)
\]

Equation (12) gives the capital cost of new housing in real pounds.
Finally, all expected variables, other than sales, are proxied by four-quarter weighted averages as noted in section (2) above.

4. RESULTS

4.1 House Sales

It follows from the discussion in Section 2 on the role played by mortgage availability that a relationship such as that shown in equation (9) is unlikely to display parameter stability over the entire sample period. A Chow test based on two sub-periods - 1971Q3-1980Q4 and 1981Q1-1983Q2 - gave an F statistic of 2.42 which is significant at the 95 per cent level. As a consequence of this result two additional F-tests were undertaken*: first, a test for differential intercepts produced $F(1,46) = 2.22$ which is insignificant at the 95 percent level; and secondly, a test for differential slope coefficients gave $F(4,42) = 4.46$ which is significant at the 99 per cent level.

These results suggest that equation (9) might be re-specified in vector form as:

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* See Johnston (1984), p.218 for technical details of the procedures employed.
\[
\begin{bmatrix}
S_1 \\
S_2
\end{bmatrix} = 
\begin{bmatrix}
1 & X_1 & 0 \\
1 & 0 & X_2
\end{bmatrix} 
\begin{bmatrix}
b_0 \\
B_1 \\
B_2
\end{bmatrix} + u \ldots (13)
\]

where \(S_1\) and \(S_2\) are vectors of observations on sales in the first and second sub-periods, \(X_1\) is a matrix of observations on \(Y, \text{CCII, CI and MA}\) over 1971Q3 - 1980Q4 and \(X_2\) is the equivalent matrix for 1981Q1 - 1983Q2. \(B_1\) and \(B_2\) are vectors of slope coefficients and \(b_0\) is the common intercept. The OLS estimates of (13) are given in Table 1 below. The most dramatic feature of these results is that the estimated coefficient on mortgage availability is highly significant in the first period but insignificantly different from zero in the second period. Further, the absolute values of the other slope coefficients are all increased in the second period indicating that a change in the mortgage interest rate, for example, has a much greater impact on house sales in the latter period. The estimation of (13) therefore supports the hypothesis that the intervention of the banks in the mortgage market, together
with the gradual deregulation of the building society movement, has had a significant effect on new house sales.*

The series for expected sales is therefore computed using the estimates presented in Table 1 but with the coefficient on MA set equal to zero for the period 1981Q1 - 1983Q2.

*This result is also consistent with those reported by Thom (1985) on the insignificance of mortgage availability in determining housing starts in the United States since the introduction of deregulation in late 1979.
### Table 1

**ESTIMATING HOUSE SALES**

**Dependent Variable** $S_t$

<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>0.049</td>
<td>0.152</td>
</tr>
<tr>
<td></td>
<td>(2.22)</td>
<td>(2.98)</td>
</tr>
<tr>
<td>$CCH$</td>
<td>- 0.453</td>
<td>- 2.51</td>
</tr>
<tr>
<td></td>
<td>(2.13)</td>
<td>(3.41)</td>
</tr>
<tr>
<td>$CI$</td>
<td>- 0.006</td>
<td>- 0.021</td>
</tr>
<tr>
<td></td>
<td>(3.61)</td>
<td>(2.01)</td>
</tr>
<tr>
<td>$MA$</td>
<td>666.91</td>
<td>292.47</td>
</tr>
<tr>
<td></td>
<td>(5.51)</td>
<td>(0.76)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.091</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td></td>
<td>0.85</td>
</tr>
<tr>
<td>D.W</td>
<td></td>
<td>1.90</td>
</tr>
<tr>
<td>S.E.E.</td>
<td></td>
<td>2.72</td>
</tr>
</tbody>
</table>

* $t$-statistics in parenthesis, $\bar{R}^2$ = adjusted coefficient of determination; D.W = Durbin-Watson statistic, S.E.E. = standard error of the equation.
4.2 House Starts

Equation (8) was estimated by the technique of ordinary least squares over the period 1970Q1-1984Q2. The results are shown below in Table 2*. Three seasonal dummies and an extra dummy for the three day working week in 1973/74 were also added to the estimating equation but these are not reported in the interests of brevity.

It can be seen from Table 2 that all explanatory variables have their expected signs and are statistically significant. Further, Durbin's H-statistic, which is the appropriate diagnostic for first-order autocorrelation in the presence of a lagged dependent variable, has a satisfactory value. These results can be solved for the following parameter values:

\[
\begin{align*}
\lambda &= 0.616 \\
k &= 0.958 \\
a_1 &= 0.844 \\
a_2 &= -51.266 \\
a_3 &= -255.519
\end{align*}
\]

*The equation was estimated subject to the restriction that the coefficients on STSt-1 and Nt-1 sum to unity as shown by equation (8).
TABLE 2

ESTIMATING HOUSING STARTS

Dependent Variable STS$_t$

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Estimated Coefficients</th>
<th>t-ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>STS$_{t-1}$</td>
<td>0.384</td>
<td>3.43</td>
</tr>
<tr>
<td>N$_{t-1}$</td>
<td>-0.616</td>
<td>5.53</td>
</tr>
<tr>
<td>S$<em>{t-1}$ - $S^e</em>{t-1,t-2}$</td>
<td>0.590</td>
<td>1.97</td>
</tr>
<tr>
<td>$S^e_{t,t-1}$</td>
<td>0.520</td>
<td>2.53</td>
</tr>
<tr>
<td>CCR$_{t,t-1}$</td>
<td>-31.580</td>
<td>1.88</td>
</tr>
<tr>
<td>FC$_{t,t-1}$</td>
<td>-157.400</td>
<td>2.87</td>
</tr>
<tr>
<td>Constant</td>
<td>71.343</td>
<td>2.64</td>
</tr>
</tbody>
</table>

$R^2 = 0.84$

D.W = 2.05

Durbin's H-statistic = -0.35

S.E.E. = 4.49
These results appear to make intuitive sense. The estimate for $\lambda$ (0.616) implies a construction cycle of approximately four quarters. A rise in expected sales of 1,000 for example, increases starts by 520 in the current period and by 844 after one year. It is of some interest to compare these results with those obtained by Hayashi and Trapani (1978) using quarterly US data. Their estimates suggest that if sales are expected to increase by 1,000 then starts are increased by approximately 3,000 in the current period and, given their separation of inventory and production processes, by 24,000 in the long-run.

Table 3 presents elasticities of housing starts with respect to expected sales ($S^e$), construction costs (CCHK), financing costs (FC) and the four right-hand-side variables in equation (9), namely Y, CCH, CI and MA. The elasticity with respect to CCH, for example, should be interpreted as the percentage response of housing starts to an expected one per cent change in construction costs (scaled by the average price of new houses). Elasticities are reported for three separate years: 1972, 1980 and 1983.
### TABLE 3

ELASTICITY ESTIMATES*

**Elasticity of **$S_{t}$** with respect to:**

<table>
<thead>
<tr>
<th>Year</th>
<th>$S^e_t$</th>
<th>$Y$</th>
<th>CCH</th>
<th>CI</th>
<th>MA</th>
<th>CCR</th>
<th>FC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1972</td>
<td>0.417</td>
<td>-0.044</td>
<td>-0.044</td>
<td>0.251</td>
<td>-0.760</td>
<td>-0.108</td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>0.571</td>
<td>-0.262</td>
<td>-0.161</td>
<td>0.212</td>
<td>-1.295</td>
<td>-1.007</td>
<td></td>
</tr>
<tr>
<td>1983</td>
<td>0.335</td>
<td>1.328</td>
<td>-0.565</td>
<td>-0.435</td>
<td>-0.844</td>
<td>-0.535</td>
<td></td>
</tr>
</tbody>
</table>

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*NOTES:*

Elasticities with respect to $S^e_t$, CCR and FC are computed as

$\lambda_1(S^e_{t,t-1}/STS_t)$, $\lambda_2(CCR^e_{t,t-1}/STS_t)$ and $\lambda_3(FC^e_{t,t-1}/STS_t)$ where the expected levels of CCR and FC are four-quarter weighted averages and $S^e_{t,t-1}$ is as described in section 2.

The elasticities with respect to $Y$, CCH, CI and MA are computed as

$\lambda_1(S^e_{t,t-1}/Y^e_{t,t-1})$, $\lambda_2(S^e_{t,t-1}/CCH)$, $\lambda_3(S^e_{t,t-1}/CI)$ and $\lambda_4(S^e_{t,t-1}/MA)$ etc.

The estimates are based on the average values of the four quarters in each year.
5. SUMMARY AND CONCLUSIONS

The main purpose of this study has been to model UK housing starts over the period 1970Q1-1984Q2 with explicit incorporation of the inventory decision and production adjustment processes displayed in new housebuilding activity. In the paper we recognise the need for research in this area to take account of two key determining factors, namely (a) builders' expectations of new house sales and (b) the manner in which fluctuations in the level of economic activity and developments in the mortgage market may influence these expectations. In contrast with previous studies which have proxied expected sales by either current or lagged observations we attempt to model actual sales and use the resulting forecasts to generate builders' expectations over the estimating period.

Estimation of the sales model supports the hypothesis that the intervention of the commercial banks in the UK mortgage market, together with the gradual deregulation of the building society movement, has had a significant effect on new house sales. It is argued, therefore, that the cost of mortgage finance, rather than its availability, has come to have a much greater impact on new house sales since early 1981. This is in support of earlier research undertaken by the authors.

Finally the results of estimating the model of housing starts suggests that the speed and impact of adjustment
within the UK construction industry in response to a change in expected sales is fairly modest compared to the US experience as reported by Hayashi and Trapani (1978). Our results indicate a response of starts to changes in sales expectations in the UK which is close to unity in the long-run.
REFERENCES

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MAYES, D (1979) *The Property Boom*, Martin Robertson, Oxford


APPENDIX: DATA DEFINITIONS AND SOURCES

STS  Number (thousands) of private sector housing starts.
     Source: Economic Trends (ET)

S   Sales of new houses proxied by the number (thousands) of mortgages granted by Building Societies, Insurance Companies, Local Authorities and Banks.
     Source: Housing & Constructions Statistics (HCS)

V   Value of output by contractors building new private housing (thousands).
     Source HCS

PH  Average price of new dwellings on which building society grant mortgages
     Source: Compendium of Building Society Statistics (CBSS)

PH80 Index of PH based on 1980 = 100

CC  Index of the cost of new house building. Computed as a weighted average of the costs of labour and materials. Based on 1980 = 100.
     Source: HCS

R   Cost of bank borrowing (proxied by London Clearing Banks base rate from 20 August 1981; before August 1981, minimum lending rate and bank rate.

P   Retail Price Index, 1980 = 100.
     Source: ET

Y   Real per capital income at 1980 prices.
     Source: ET

     Source: CBSS

TR  Standard rate of income tax.
     Source: ET

CI  Cyclical Indicator of economic activity. Proxied by numbers unemployed (thousands) excluding school leavers.
     Source: ET

MA  Mortgage availability measured by building society commitments on new dwellings (£m) divided by PH.
     Source: ET
CCH  Capital costs of new housing, proxied by MR(1-TR)PH/P

N  End of period inventory, proxied by (V-PH.S)/PH

CCR  Construction costs scaled by house prices.
     CC/PH60

FC  Financing costs, proxied by R.PH/P

Except for STS_t, N_{t-1}, S_t and S^{e}_{t-1}, all variables are computed as four-quarter weighted averages; that is:

$$X^{e}_{t,t-1} = \sum_{i=1}^{4} c_{i} X_{t-i}$$

where $c_1 = 0.4$, $c_2 = 0.3$, $c_3 = 0.2$ and $c_4 = 0.1$