LABOUR SUPPLY AND COMMODITY DEMANDS:

AN APPLICATION TO IRISH DATA

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Working Paper No. 43

September 1986

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Abstract.

Annual Irish data are used to estimate a model which allows for the joint determination of commodity demands and labour supply. Consumer preferences are modelled by a cost function of the Gorman ploar form which permits exact linear aggregation over individuals with different money wage rates. Separability between goods and leisure is rejected by the data. Labour supply is found to be a positive function of the wage rate.
Labour Supply and Commodity Demands:  
An Application to Irish Data.

1. Introduction.

Over the last ten years there has been considerable interest in the joint modelling of labour supply and commodity demands. Abbot and Ashenfelter (1976) use a variety of models to estimate the joint demands for goods and leisure from annual US time series data. Ashenfelter (1980), using an augmented LES, estimates a model in which the commodity demands of unrationed consumers are combined with those of consumers who are restricted in their choice of leisure hours (labour supply). Blundell and Walker (1982) estimate similarly rationed and unrationed demands derived from a generalisation of the LES suggested by Deaton and Muellbauer (1981). Using UK cross section data Blundell and Walker decisively reject the hypothesis of separability between goods and leisure as does Barnett (1979) using US times series data.

If separability between goods and leisure is not a sustainable hypothesis then parameter estimates derived from a model which either imposes it or which excludes leisure from the preference ordering will be inconsistent and will result in incorrect inferences. Rejection of separability also has important implications for optimal tax theory which requires knowledge of the substitute/complement patterns between goods and leisure.

The primary objective of the present paper is to utilize annual Irish data for the period 1953 to 1983 to estimate a model which allows for the joint determination of commodity demands and labour supply and which permits an explicit test of the separability hypothesis. As such, our objective is somewhat limited. We do not, for example, include demographic factors or distinguish between male and female labour supply decisions and the role of children. Nor do we consider the possibility that some consumers may be rationed in their choice of leisure hours.1 An adequate treatment of

1. For a comprehensive treatment of these issues see Blundell and Walker (1982).
these aspects of household behaviour requires the use of cross-section or panel data. Unfortunately we do not have direct access to cross-section data and Irish panel data does not exist. Being restricted to a time series context we consequently address only those questions which the data may be reasonably expected to deal with.

A further restriction on our approach is the consideration that while individuals may be reasonably assumed to face a common set of commodity prices it is totally implausible to assume that they sell their labour at a common wage. Given that wage rates differ over individual consumers, the use of aggregate time series data requires that commodity demand functions permit exact linear aggregation in money wages. In order to comply with this restriction we model behaviour by a cost, or expenditure function, of the Gorman polar form. This type of cost function is somewhat restrictive in that it implies that preferences are quasi-homothetic. Thus the Engel curves are linear in full income. The requirement of linear aggregation combined with time-series data eliminates more flexible functional forms such as Deaton and Muellbauer's Almost Ideal Demand System (AIDS). Section 2 outlines the model and discusses its properties. The estimates are presented section 3 while Section 4 discusses the paper's main conclusions. An Appendix is included which gives definitions and sources of the data used.

2. Theory.

Consider an individual who is free to allocate a fixed time endowment $T$ between hours worked $h$ and leisure $q_0 = T - h$. Corner solutions are ignored. In addition the consumer allocates money income among the elements of an $n$-commodity vector $q$ at exogenous prices $p$. Full expenditure per period is given by

$$x = wq_0 - p'q$$  \hspace{1cm} (1)$$

where $w$ is the money wage rate or the opportunity cost of one hour of leisure. We also constrain total commodity expenditure per period to equal the sum of labour and
non-labour incomes so that \( p'q = wT + y \), where \( y \) denotes non-labour income. Substitution into (1) gives the full income budget constraint

\[
x = wT + y
\]  
(2)

To derive the consumer's demand system it is first necessary to define a preference ordering over \( q_0 \) and \( q \). Throughout this paper we assume that preferences are intertemporally additive and describe within period preferences by the utility function:

\[
u = \nu(q_0, q)
\]  
(3)

and model the individual as choosing leisure hours and commodities so as to maximise (3) subject to (2) in each period. Alternatively, we may think of the individual as choosing \( (q_0, q) \) in each period so as to minimise the full cost of achieving the utility level given by the solution to the primary problem. Hence we can define the cost (or expenditure) function for this problem as:

\[
c(w,p,u) = \min wq_0 + p'q \text{ s.t. } \nu(q_0, q) = u
\]  
(4)

By Sheppard's Lemma the compensated or Hicksian labour supply and commodity demands are:

\[
h = T - c_0(w,p,u)
\]  
(5)

\[
q_i = c_i(w,p,u), \ i = 1...n.
\]  
(6)

where \( c_0 \) and \( c_i \) are partial derivatives of the cost function with respect to \( w \) and \( p_i \) respectively. Inverting (4) to give the indirect utility function \( u = f(w,p,x) \) and substituting for \( u \) in (5) and (6) gives the uncompensated or Marshallian demands:

\[----------------------\]

2. The assumption of intertemporal additivity is almost universal in empirical demand studies. We note that in an explicit intertemporal model \( y \) equals the change in wealth plus interest income, or savings from labour income. See MaCurdy (1983).
\[ h = T - g_0(w,p,x) \]  
\[ q_i = g_i(w,p,x), \ i = 1\ldots n. \]

With time series data it is desirable to choose a functional form for (4) that allows exact linear aggregation over w and y. Muellbauer (1981) shows that exact linear aggregation over N consumers requires that the 'micro' and 'macro' cost functions be of the Gorman polar form:

\[ c_j(w_j,p,u_j) = s(p) + w_jd(p) + w_j^{k_b(p)(1-k)}u_j ; j = 1\ldots N. \]  
\[ c(w,p,u) = s(p) + wd(p) + w^{k_b(p)(1-k)}u \]  
\[ \frac{\partial s(p)}{\partial p} = \frac{1}{N} \sum a p_j \]

where \((1/N) \sum a(p) \) and \(b(p)\) are homogeneous of degree 1, and \(d(p)\) is homogeneous of degree zero. If leisure and total commodity expenditure are both normal then \(0 < k < 1\).

The cost function (10) yields the following uncompensated demands:

\[ wh = w(T - d(p)) - kM \]
\[ P_i q_i = p_i s(p) + w p_i d_i(p) + p_i (1-k)^{b*(p)} \]

Where \(M = x - a(p) - wd(p)\) and \(b*(p) = b_i(p)/b(p)\). Note that the labour hours supply function does not permit linear aggregation over individuals with differing wage rates. We therefore choose to work with a labour income function and to express commodity demands in expenditure terms.\(^3\)

Apart from yielding a tractable demand system (10) also permits a relatively simple test for separability. Goldman and Uzawa (1964) show that goods and leisure are weakly separable if \(c_{ij} = z c_{iu}\) where \(z\) is a constant for all \(i = 1\ldots n\). In the case of (10) separability holds when \(d_i(p) = 0\) for all \(i\).

\(^3\) Deaton and Mullbauer (1981) show that if the cost function is specified so that \(h\) is linear in \(w\) then the resulting commodity demands are non-linear in \(w\).
3. Estimates

Estimation of the system (11)-(12) requires that we first specify appropriate functional forms for \( a(p) \), \( d(p) \) and \( b(p) \). We choose:

\[
a(p) = \sum_{i} \sum_{j} Y_{ij} (p_i p_j)^{0.5}; \quad Y_{ij} = Y_{ji} \\
\]

\[
d(p) = \alpha_0 \pi p_i \beta_i \quad \text{; } \quad \Sigma \alpha_i = 0 \\
b(p) = \beta_0 \pi p_i \beta_i \quad \text{; } \quad \Sigma \beta_i = 1
\]

(13)

As in the Linear Expenditure system the terms \( a(p) \) and \( d(p) \) may be interpreted as minimum expenditures on goods and leisure respectively. For example the minimum quantity of good \( i \) purchased per period is given by:

\[
q_i = Y_{ii} + \sum_{i' j} Y_{i' j} (p_{i' j} / p_i)^{0.5}
\]

(14)

Hence minimum quantities are specified as general functions of prices rather than as constants as in the LES. We note that alternative specifications for (13) are available but choose the above on grounds of tractability.\(^4\) Using (13) gives the following labour supply and commodity demands:

\[
wh = w(T - \alpha_0 \pi p_i \alpha_i) - k\lambda \\
P_i q_i = Y_{ii} p_i + \sum_{i' j} Y_{i' j} (p_{i' j} / p_i)^{0.5} + (1 - k) \beta_i \lambda
\]

(15)

(16)

Separability between goods and leisure requires that \( \alpha_i = 0 \) for all \( i = 1 \ldots n \) and can be tested using the likelihood ratio test.

To estimate the demand system (15)-(16) we used annual observations on seven classes of non-durable expenditures over the period 1951 to 1983.\(^5\) A full description of the data and its sources are given in the Appendix. Our measures of the wage and labour

\(^4\) See, for example, Blundell and Walker (1982)

\(^5\) Initial results derived from a data set which included durables proved unsatisfactory. We note that it is not uncommon for empirical demand studies to exclude durables. See Blundell and Walker (1982), Anderson and Blundell (1983) and Attfield and Browning (1985). Strictly, durables are assumed to be separable from non-durables and leisure.
supply are average earnings and numbers employed. This is necessitated by the lack of a suitable economy-wide series on hours worked. In order to remove the singularity of the system the labour supply equation was deleted and the remaining seven expenditure functions were estimated using the appropriate maximum likelihood procedures in the SHAZAM package. Parameter estimates are presented in Table 1. As data is not available for the time endowment we treated T as a parameter in the estimation. As a rough check on the estimate for T we require that wT be greater than labour income. The estimate for T meets this condition at all sample points. The model was also estimated with separability imposed and a likelihood ratio test used to test the hypothesis that goods and leisure are separable. The statistic 2DLR, equal to twice the absolute difference \[ \log \frac{L_1}{L_2} \] between the respective likelihood functions, is distributed as chi-square with six degrees of freedom and indicates a decisive rejection of the separability hypothesis with a critical level of approximately zero.6

The parameter estimates appear to be reasonably "sensible". Leisure and total commodity expenditure are both normal as are the individual commodity groups. Various elasticities, computed at the sample means, are presented in Table 2. Both the uncompensated and compensated labour supply wage elasticities are positive indicating an upward sloping supply curve. This result was replicated at thirty out of thirty-one sample points, the exception being 1983 which gave an uncompensated wage elasticity of -0.027. All commodities and leisure are unambiguous net substitutes which implies that all commodities and labour supply are net complements. Leisure is a gross substitute for fuel, housing and other goods but a gross complement the remaining commodity groups. Table 3 presents own-price elasticities based on parameter estimates when separability is imposed on the system. With the exception of services the absolute values of these elasticities are lower than those computed from the unrestricted system suggesting the possibility of a systematic bias in the restricted model.

6. The critical level is the significance level at which the null hypothesis is just rejected.

This paper presented a model of consumer behaviour in which decisions on commodity demands and labour supply are taken jointly. As such it constitutes an advance on previous empirical studies of Irish consumer behaviour by O'Riordan (1975) and McCarthy (1977) which exclude leisure hours from the preference ordering. The paper's principal conclusion is that the data decisively rejects the hypothesis of separability between goods and leisure. The estimates also suggest that the uncompensated labour supply curve is upward sloping rather than backward bending. Although these results appear plausible there are several reasons for treating them with some caution. First separability is an hypothesis about individual preferences so that its rejection in tests based on aggregate data may not have a clear interpretation. Second the functional forms used are rather restrictive. Although separability and additivity are not imposed, homogeneity and symmetry are. We note that this is the price which has to be paid for exact linear aggregation over individuals with different wage rates. Third, the model assumes that individuals are free to allocate their time between work and leisure. A more realistic approach may be one in which some individuals are rationed in their choice of leisure hours. However, as we have shown elsewhere exact linear aggregation over rationed and rationed consumers is only possible under one of the following restrictive assumptions.7 (i) Preferences are weakly separable between goods and leisure. (ii) All rationed individuals are rationed at the same level. (iii) All rationed individuals sell their labour at the same wage. In the case of labour supply the assumptions of a common ration or a common wage are implausible and we wish to test for separability, not to impose it.

Finally, the approach taken in this paper is that dictated by theory. We recognise that certain aspects of the model may be restrictive. However, given the constraints imposed by time series data, alternative approaches are probably no less restrictive.

7. See Murphy and Thom (1986)
<table>
<thead>
<tr>
<th>Category</th>
<th>$x_{1i}$</th>
<th>$x_{12}$</th>
<th>$x_{13}$</th>
<th>$x_{14}$</th>
<th>$x_{15}$</th>
<th>$x_{16}$</th>
<th>$x_{17}$</th>
<th>$\beta_i$</th>
<th>$\alpha_i$</th>
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<tr>
<td>Food</td>
<td>569.2</td>
<td>-195.5</td>
<td>104.1</td>
<td>-152.4</td>
<td>138.9</td>
<td>-309.9</td>
<td>-90.9</td>
<td>0.259</td>
<td>-4.44</td>
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<tr>
<td>Alcohol*</td>
<td>311.1</td>
<td>46.2</td>
<td>-40.9</td>
<td>124.8</td>
<td>-431.0</td>
<td>210.5</td>
<td>0.140</td>
<td>-0.03</td>
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<tr>
<td>Clothing</td>
<td>107.5</td>
<td>40.2</td>
<td>-184.3</td>
<td>80.1</td>
<td>5.2</td>
<td>0.065</td>
<td>4.47</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fuel</td>
<td>154.6</td>
<td>-6.7</td>
<td>-5.5</td>
<td>-21.0</td>
<td>0.066</td>
<td>0.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing</td>
<td>-61.1</td>
<td>179.9</td>
<td>-283.6</td>
<td>0.085</td>
<td>5.83</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Services</td>
<td>105.1</td>
<td>128.1</td>
<td>0.245</td>
<td>-5.11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Oth. Goods</td>
<td>347.2</td>
<td>0.134</td>
<td>-0.78</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

$T = 1962.2 \ (5.9); \ k = 0.536 \ (6.15); \ 2DLR = 79.714$

Figures in brackets are t-statistics

* Including tobacco.
### Table 2. Elasticities.

#### Commodities

<table>
<thead>
<tr>
<th></th>
<th>Own Price Uncomp.</th>
<th>Comp.</th>
<th>Wage Rate Uncomp.</th>
<th>Comp.</th>
<th>Non-Labour Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>-.383</td>
<td>-.262</td>
<td>.695</td>
<td>.514</td>
<td>.171</td>
</tr>
<tr>
<td>Alcohol*</td>
<td>-.477</td>
<td>-.409</td>
<td>.683</td>
<td>.509</td>
<td>.164</td>
</tr>
<tr>
<td>Clothing</td>
<td>-.324</td>
<td>-.292</td>
<td>.643</td>
<td>.537</td>
<td>.137</td>
</tr>
<tr>
<td>Fuel</td>
<td>-.401</td>
<td>-.371</td>
<td>.981</td>
<td>.783</td>
<td>.234</td>
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<tr>
<td>Housing</td>
<td>-.795</td>
<td>-.756</td>
<td>1.334</td>
<td>1.031</td>
<td>.286</td>
</tr>
<tr>
<td>Services</td>
<td>-.753</td>
<td>-.693</td>
<td>.620</td>
<td>.461</td>
<td>.149</td>
</tr>
<tr>
<td>Oth.Goods</td>
<td>-.415</td>
<td>-.301</td>
<td>1.247</td>
<td>.939</td>
<td>.316</td>
</tr>
</tbody>
</table>

#### Labour Supply Elasticities.

<table>
<thead>
<tr>
<th></th>
<th>Uncomp.</th>
<th>Comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>.262</td>
<td>-.177</td>
</tr>
<tr>
<td>Alcohol*</td>
<td>.188</td>
<td>-.101</td>
</tr>
<tr>
<td>Clothing</td>
<td>.008</td>
<td>-.055</td>
</tr>
<tr>
<td>Fuel</td>
<td>-.009</td>
<td>-.477</td>
</tr>
<tr>
<td>Housing</td>
<td>-.035</td>
<td>-.075</td>
</tr>
<tr>
<td>Services</td>
<td>.022</td>
<td>-.097</td>
</tr>
<tr>
<td>Oth.Goods</td>
<td>-.057</td>
<td>-.161</td>
</tr>
<tr>
<td>Wage Rate</td>
<td>.185</td>
<td>.721</td>
</tr>
</tbody>
</table>

* Including tobacco.*
Table 3. Elasticities — Separability.

<table>
<thead>
<tr>
<th>Own Price</th>
<th>Uncomp.</th>
<th>Comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>-.284</td>
<td>-.162</td>
</tr>
<tr>
<td>Alcohol*</td>
<td>-.394</td>
<td>-.327</td>
</tr>
<tr>
<td>Clothing</td>
<td>-.211</td>
<td>-.177</td>
</tr>
<tr>
<td>Fuel</td>
<td>-.373</td>
<td>-.341</td>
</tr>
<tr>
<td>Housing</td>
<td>-.236</td>
<td>-.186</td>
</tr>
<tr>
<td>Services</td>
<td>-.979</td>
<td>-.908</td>
</tr>
<tr>
<td>Oth. Goods</td>
<td>-.187</td>
<td>-.079</td>
</tr>
<tr>
<td>Lab. Supply</td>
<td>.210</td>
<td>.752</td>
</tr>
</tbody>
</table>

* Including tobacco.
References


- 8 -


Data.

The data consist of annual Irish data for the period 1951-1983. The series on commodity expenditures are taken from various issues of National Income and Expenditure. Prices are computed as the ratio of expenditures at current prices to expenditures at constant (1975) prices.

Time-series of economy-wide data on hours is unavailable for Ireland. This did not present any problems for estimation as the labour supply equation is expressed in income terms and is deleted to satisfy the adding-up constraint. To compute the labour supply wage elasticity, we use the labour income elasticity:

\[ E_{(wh),w} = (1-k)(T-d(p))(w/wh) \]

For hours the elasticity is:

\[ E_{h,w} = -1 + E_{(wh),w} \]

The wage rate is computed as the net of tax non-agricultural wage bill divided by non-agricultural employment. Non-labour income is obtained as a residual from the budget constraint. That is: \( y = p'q - wh \), where \( wh \) is the total wage bill. As a series on hours is unavailable, labour supply is measured as numbers employed and the wage rate as annual earnings.
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University College Dublin  

Working Papers


