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<th><strong>Title</strong></th>
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Determinants of the Equilibrium
Real Exchange Rate

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Determinants of the Equilibrium Real Exchange Rate

Abstract

This note presents a compact derivation of the determinants of changes in the equilibrium real exchange rate (the inverse of the price index of nontraded goods relative to traded goods), in a small open economy with any number of goods and factors. It is shown that the change in the real exchange rate equals a simple weighted sum of the differences between marginal propensities to consume and to produce individual nontraded goods. Implications of the result are noted for a variety of applied questions, including the effects of foreign aid, the "Dutch Disease" and purchasing power parity comparisons between countries.
DETERMINANTS OF THE EQUILIBRIUM

REAL EXCHANGE RATE

1. Introduction

The importance of the equilibrium real exchange rate, the relative price of traded to nontraded goods consistent with balance-of-payments equilibrium, has long been recognised. However, no simple, compact derivation of the determinants of the real exchange rate appears to be available. Section 2 below attempts to fill this gap and Section 3 notes the implications of the main result for a number of applied questions.

2. Theory

I assume a small open economy producing arbitrary numbers of traded and non-traded goods under competitive conditions. The world prices of traded goods are exogenously given and, without loss of generality, I take one of the traded goods as numeraire so that domestic prices are measured in terms of foreign currency. Since I consider only situations of balance-of-payments equilibrium, it is immaterial whether this is achieved by adjustment of a floating exchange rate or by monetary inflows and outflows under a fixed exchange rate. I assume that there are n nontraded goods, whose prices, denoted by the vector p, are endogenously determined so as to equate domestic demand and supply. The determination of equilibrium may be illustrated in a compact manner by making use of the trade expenditure function. This equals the excess of home expenditure, determined by a standard expenditure function,

1 Oppenheimer (1974) documents early writings on models with nontraded goods and Salter (1959) is the classic modern exposition.

2 By contrast, the building blocks of such a derivation are well known. See, in particular, Caves and Jones (1985, pp. 494-8), Dixit and Norman (1980, chap. 5) and Woodland (1982, chaps. 8 and 10).

3 The detailed properties of this function have been examined by Dixit and Norman (1980, pp. 90-91), who call it the "excess expenditure function" and Woodland (1982, p.170), who calls its negative the "maximum net revenue function."
e(p, u, a), over home income from production, determined by a GNP function, g(p, β), plus any additional income received by the private sector, γ. In symbols:

\[ E(p, u, \phi) = e(p, u, a) - g(p, \beta) - \gamma. \] (1)

Here \( \phi = (\alpha, \beta, \gamma) \) is a vector of shift parameters whose interpretation will vary with the particular shocks under consideration. For example, \( \alpha \) may include the prices of non-numeraire traded goods as well as taste parameters; \( \beta \) may include both factor endowments and technology parameters; and \( \gamma \) may include redistributed tax or tariff revenue as well as direct transfers from abroad. The scalar \( u \) measures home utility, which can be interpreted either as the actual utility of a single aggregate consumer, or as the level of social welfare attained when optimal redistribution is continually carried out by means of lump-sum transfers.

Relaxing this assumption to allow for different individuals in the economy with optimal lump-sum redistribution not carried out would introduce additional complications which, though potentially important in practice, are not central to the applications of the model discussed below. In any case, these considerations have been widely discussed elsewhere.\(^4\)

The great convenience of the trade expenditure function is that, for fixed \( \phi \), its properties resemble those of the usual expenditure function, \( e(p, u, \alpha) \). In particular, it is increasing in \( p \) and \( u \), it is concave in \( p \) and its partial derivatives with respect to \( p \) equal the Hicksian or utility-compensated excess demand functions for nontraded goods:\(^5\)

\[ E_p(p, u, \phi) = e_p(p, u, \alpha) - g_p(p, \beta). \] (2)

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\(^4\) Dixit and Norman (1986) examine the problems which arise in measuring welfare changes when lump-sum redistribution is not feasible, and Taylor (1974) is one of the few papers to consider the positive implications of intranational differences in spending propensities in a model with traded and nontraded goods.

\(^5\) Throughout, terms such as \( E_p \) denote the vector of partial derivatives of \( E \) with respect to \( p \) and terms such as \( E_{pp} \) denote the matrix of second partial derivatives. All vectors are column vectors and a prime denotes a transpose.
The function $E$ is homogeneous of degree zero in the prices of all goods. However, since $p$ denotes the prices of nontraded goods only, the matrix of second partial derivatives of $E$ with respect to $p$ will be non-singular provided there is some substitutability in either demand or supply between traded and nontraded goods. I denote the negative of this matrix by $S$:

$$S = -E_{pp}(p, u, \phi).$$

(3)

Because of the properties of $e$ and $g$, $S$ is symmetric and positive semi-definite.

It is now straightforward to characterise equilibrium. Firstly, balance of payments equilibrium requires that aggregate expenditure equal aggregate income:

$$E(p, u, \phi) = 0.$$

(4)

Secondly, commodity market equilibrium requires that excess demand for each non-traded good be zero:

$$E_p(p, u, \phi) = 0.$$

(5)

The $n+1$ equations (4) and (5) determine the values of the $n+1$ exogenous variables, $p$ and $u$, as functions of the parameters $\phi$.

To solve the model, first totally differentiate (4) to obtain:

$$du = E'_u d\phi = d\phi.$$

(6)

Here, I have used (5) to eliminate a term in $dp$ and have chosen units of measurement for utility such that $E_u = e_u = 1$. I also confine attention to exogenous shocks which can be expressed as changes in a single parameter $\Phi$. As a result, the scalar $d\Phi$ measures the net change in utility arising from changes in all the exogenous variables. Next, totally differentiate (5) to obtain:
\[-Sdp + E_{pu} \, du = -E_{p\phi} \, d\phi = -E_{p\phi} \, d\phi.\] (7)

The two terms on the left-hand side of (7) measure respectively the substitution effects of price changes and the income effects of utility changes on the excess demand for nontraded goods. The right-hand side measures the effects of changes in exogenous variables on excess demand; as in (6), it is convenient to aggregate these effects, with each element of the vector \(E_{p\phi}\) giving the net increase in excess demand for each nontraded good as a result of changes in exogenous variables. Combining (6) and (7):

\[Sdp = (E_{pu} + E_{p\phi}) \, d\phi.\] (8)

The interpretation of the results is facilitated if the price changes are expressed in proportional terms. This may be done by introducing a diagonal matrix \(P\), the \(i^{th}\) element on the principal diagonal of which is the price of the \(i^{th}\) nontraded good. We therefore obtain the following expression for \(\hat{p} ( = P^{-1} \, dp)\), the vector of proportional changes in nontraded goods prices:

\[\hat{p} = \bar{S}^{-1} (\mu - \sigma) \, d\phi.\] (9)

Here \(\bar{S}\) equals PSP, a simple transformation of the substitution matrix defined in (3); \(\mu\) equals \(PE_{pu}\), the vector of marginal propensities to consume nontraded goods; and \(\sigma\) equals \(-PE_{p\phi}\), which may be described as the vector of marginal propensities to produce nontraded goods. Of course, the values of the latter depend on the particular shock under consideration.

The final step is to aggregate the price changes of individual nontraded goods to obtain the change in the real exchange rate. I define the latter as a fixed-weight index number of the relative prices of nontraded goods:

\[\pi \equiv x' \hat{p}.\] (10)
where the elements of the vector $x$ are the base-period consumption (and production) levels of individual nontraded goods. Totally differentiating:

$$\dot{\pi} = \omega' \dot{p}$$  

(11)

where $\omega = Px/\pi$. Combining this with (9) gives the principal result of this note:

$$\dot{\pi} = \omega' S^{-1} (\mu - \sigma) d\Phi.$$  

(12)

3. Applications

Equation (12) may be interpreted as implying that any exogenous shock will raise $\pi$ and so lead to a real appreciation if the demand effects of the shock outweigh the supply effects. The coefficient of (12) is a weighted sum of the differences between the marginal propensities to consume and to produce each nontraded good. The weights equal the elements of the vector $\omega' S^{-1}$ and so reflect both the relative importance of each good in consumption and the difficulty of substituting it for other goods. While there is no guarantee that all these weights must be positive, the fact that $S$ is a positive definite matrix justifies the statement that a real appreciation will occur provided the elements of $\omega$ and $(\mu - \sigma)$ are positively correlated. The significance of this result is best seen by considering a number of applications:

(a) Effects of a Transfer: An incoming transfer, such as foreign aid, is necessarily effected in terms of traded goods. Hence, at initial prices, it does not affect the output of nontraded goods and so all the elements of $\sigma$ are zero. It follows that a transfer is likely to induce a real appreciation, a prediction which has been confirmed empirically by Michaely (1981).

(b) The "Dutch Disease": A sector-specific boom has effects identical to those of a transfer if it occurs in an "enclave" traded-good sector, which has no production links with the rest of the economy. (Natural resource sectors are an obvious example.) Such a boom has a
"spending effect" only (in the terminology of Corden and Neary (1982)) and necessarily leads to a real appreciation. Potential welfare rises, although the loss of competitiveness experienced by non-booming traded sectors may pose problems of adjustment, a phenomenon which has come to be known as the "Dutch Disease." The outcome is less clear-cut if the booming sector is integrated with the rest of the economy, so that the boom also has a "resource movement effect." In particular, if the boom, through general-equilibrium interactions, encourages more production of nontraded goods at initial prices (i.e., the elements of \( g_{p} \) and hence of \( \sigma \) are positive) then a real appreciation is not inevitable. Expressed in the terminology of Hicks (1953), the more home-market-biased is the sectoral pattern of economic growth, the less likely is a real appreciation. In the neutral benchmark case of "balanced growth" and homothetic tastes, \( \mu \), equals \( \sigma \), for each nontraded good; hence all relative prices, including the real exchange rate, are unaffected by growth.

(c) International Comparisons of Purchasing Power Parity: The doctrine of purchasing power parity may be expressed as the prediction that price levels should be equal across countries when compared using equilibrium exchange rates. However, Balassa (1964) and Samuelson (1964) argue that deviations from purchasing power parity are to be expected because higher income countries have higher relative productivity in the production of traded goods. This implication follows clearly from (12): \( \pi \) and \( d\Phi \) may be reinterpreted as the differences between a high-income and a low-income country in the real exchange rate and the level of real income respectively and \( \mu \) and \( \sigma \) may be reinterpreted as the differences between the two countries in average propensities to consume and produce nontraded goods at constant prices. If international productivity differences are smaller in the production of nontraded goods (such as services) than in the production of traded goods, the elements of \( \sigma \) will be close to zero and so the higher-income country will have a lower real exchange rate (a higher relative price of services). This effect will be reinforced if non-traded goods are superior in demand (so

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6 See Neary and van Wijnbergen (1986) for further theoretical and empirical elaboration.
that the elements of $\mu$ are positive and relatively large). Although cross-section evidence in favour of this hypothesis is not strong (see Balassa (1964) and Officer (1976)), Hsieh (1982) shows that it is confirmed by time-series tests. His suggestion that this occurs because country-specific factors such as tastes vary relatively little over time is fully in keeping with equation (12).

(d) Changes in the Terms of Trade: Although I have so far assumed that the prices of all traded goods are given, $d\Phi$ may be interpreted as the increase in income arising from a change in the terms of trade. In this application $\mu$ denotes the marginal propensities to consume nontraded goods as before but $\sigma$ must be reinterpreted as the cross-price effects on excess supply of nontraded goods of a terms of trade improvement. These must be negative if all goods are substitutes in consumption and in production, so reinforcing the income effect. This gives the conclusion that, except where complementarity relationships dominate, a terms of trade improvement tends to lead to a real appreciation.

4. Conclusion

The approach adopted in this note shows that a number of results in the literature on small open economies are special cases of a general phenomenon: changes in exogenous variables are more likely to lead to a real appreciation the greater their effect on the demand for and the smaller their effect on the supply of nontraded relative to traded goods. Unfortunately, the paper throws no light on one of the most difficult problems facing all students of the open economy - theorists, applied economists and policy-makers alike - namely, determining how far actual real exchange rates diverge from their equilibrium values.

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7 Dornbusch (1980, chap.6) gives a three-commodity exposition of this result.
References


