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<th><strong>Title</strong></th>
<th>Two notes about Ponzi schemes</th>
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Two Notes about Ponzi Schemes

Patrick Honohan

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Abstract

A Ponzi scheme is an arrangement whereby a promoter offers an investment opportunity with attractive dividends, but where the only basis for the dividends is the future receipts from new investors.

The first of these two notes explores some of the analytical properties of a Ponzi scheme, addressing in particular the question whether it is possible for a Ponzi scheme to exist if all the participants are rational.

The second note briefly examines the collapse of the PMPA insurance company whose accounts document the evolution of an unsound financial operation displaying many of the characteristics of the Ponzi scheme.
TWO NOTES ABOUT PONZI SCHEMES

by

Patrick Honohan
University College Dublin

Introduction

A Ponzi scheme is an arrangement whereby a promoter offers an investment opportunity with attractive dividends, but where the only basis for the dividends is the future receipts from new investors. Obviously such an arrangement can only continue to pay dividends while it is attracting new investors in sufficient quantities. Accordingly the pure Ponzi scheme is usually thought of as being fraudulent. However there must be many instances where the life of a financial institution has been prolonged by the Ponzi mechanism, employed either innocently by a naive management, or in desperation by the management of an insolvent company trying to bridge the interval of time before a hoped-for return to profitability.

The first of these two notes explores some of the analytical properties of a Ponzi scheme, addressing in particular the question whether it is possible for a Ponzi scheme to exist if all the participants are rational.

Although it is probably true that the typical Ponzi scheme relies on the deception of the unsophisticated and gullible small investor, it could also represent a calculated gamble, as early investors could share the promoter's gains. This is revealed by the analysis of a very simple
model, which also illustrates how the size of the dividend might be chosen by the promoter, and why there will generally be some potential "victims" who escape because the scheme collapses before they have subscribed.

The second note briefly examines a recent financial collapse in Ireland. The PMPA insurance company was not operating a fraud; but its accounts document the evolution of an unsound financial operation displaying many of the characteristics of the Ponzi scheme.
A SIMPLE EPIDEMIC MODEL OF A RATIONAL PONZI SCHEME

In our model, participants subscribe one unit, in return for which they secure a constant dividend flow of \( r \) per unit of time until the scheme ends. The number of participants at any time \( t > 0 \) is given by \( y(t) \), and we assume that the scheme begins with one participant, \( y(0) = 1 \).

Our main assumption is that the flow of new participants \( dy(t)/dt \) obeys the epidemic model:

\[
\frac{dy(t)}{dt} = c \ y(t) \ (1-y(t)/k), \quad \text{for } t > 0; \quad (1)
\]

or,

\[
y(t) = \frac{k}{(1 + (k-1) \exp(-ct))}. \quad (2)
\]

The parameter \( k \) thus gives the total susceptible population and \( c \) measures the speed of diffusion of the epidemic. A Ponzi scheme ends when the net flow of cash is zero; i.e. when the inflow of new participants \( dy(t)/dt \) equals the outflow of dividends \( ry(t) \). (The inflow and outflow are plotted in figure 1, as a function of the number of subscribers and in figure 2 as a function of time). From (1) it is immediate that, at the end of the scheme \( t^* \) a fraction \( \theta \) of the total susceptible population has not subscribed and,

\[
1 - y(t^*)/k = \theta = r/c \quad (3)
\]
Given the parameters $\theta$ and $k$ we may derive an expression for the surplus $s$ of the scheme, i.e. the cumulative inflows less the cumulative outflows from time zero to time $t^*$ (the shaded area in figure 1.):

$$s = (1 - \theta + \theta \ln \theta - (1/k + \theta \ln [(k - 1)/k])\ k \ (4)$$

If $k$ is large an approximation is: $s \approx k(1 - \theta(1 + \ln \theta))$.

This expression is valid only where the scheme has a positive surplus i.e. if and only if

$$\theta < 1 - 1/k = 1 \ (5)$$

It is desirable to assume some dependence of the total susceptible population $k$ on the possible return to an investor. Clearly the progress of a scheme to $t^*$ will involve some of the later investors losing money. But for some schemes early participants will obtain enough in dividends before the crash to justify the investment. Rational investors with full information will not sustain a scheme. But it is characteristic of Ponzi schemes that information is imperfect. In our case we may suppose that participants do not know how many other participants have already subscribed. They know the promised return $r$, the size of the potential population $k$ and the diffusion rate $c$; but they do not know how long the scheme has been running. The decision to participate is based on the limited information available.

As a simple illustrative example of the effect of an endogenous potential population $k$, we may take the case
where population depends linearly on the maximum possible profit, and in particular where

\[ k = 1 + a \pi \]  \hspace{1cm} (6)

with \( \pi \) the maximum possible profit for a participant, i.e. the profit obtained by someone who joins at time \( t=0 \),

\[ \pi = rt^* - 1 = 0 \ln \left( \frac{(k - 1)(1 - 0)/0}{1} \right) - 1. \]  \hspace{1cm} (7)

Equations (7) and (8) define a positive population size \( k \) only for values of the population response \( a \) greater than 35.9, and of the scheme dividend parameter \( 8 \ (= r/c; \ chosen \ by \ the \ promoter \ and \ satisfying \ (5)) \) within a range shown in Table 1.

Given a population response characteristic \( a \), the promoter will presumably choose the surplus maximising dividend ratio \( 8 \). Equations (4), (6) and (7) allow us to compute this optimum ratio which is also shown in Table 1.
The essential characteristic of a Ponzi scheme is that dividends are paid out of the cash inflow from new investors. If the flow of new investors is sufficiently great, and if there is some time lag before the payment of dividends, early investors may be remunerated for a prolonged period before the flow of new investment falls below the sums required for payment of dividends.

In 1967 the PMPA insurance company was a negligible part of Irish motor insurance, accounting for less than a half of one per cent. of premium income. In the following ten years the company experienced an astonishing growth, taking ten per cent of the premium income by 1971, over one quarter by 1975 and 37.5 per cent. by 1978. In 1980 the company's market share peaked at 39.7 per cent. and it was reported that three out of every five private motorists insured with the PMPA. After 1980 the decline in market share was rapid. By 1982 it had slipped below 32 per cent. and there was a further decline in 1983. The decline in premium income dramatically exposed weaknesses in the company's finances. These weaknesses had not previously been evident because of what proved to be over-optimistic accounting. Evidently, the company had been charging premiums that were far too low, and this resulted in a balance sheet deficiency which was later calculated at around 1200 million, equivalent to about three years' premium income. The company had managed to survive without a cash crisis while its market share was growing. Effectively any shortfall in the funds available from the investment of any given year's premium income to meet
claims arising out of the policies written in that year was met by the larger premium income of a subsequent year. In short, the operations of the PMPA resemble those of a Ponzi scheme, though in the case of the PMPA fraud was not involved.

The finances of the PMPA provide an unusual opportunity to explore the mechanics of a Ponzi scheme. Figure 3 shows premium income and claims paid each expressed in real (1975) pounds. The similarities with figure 2 may be noted. The rapid growth in premium income up to 1980 and its decline after 1981 are clearly seen. The series for claims paid lies below the premium series until 1982: that is as one would expect for a growing motor insurance company, with claims in respect of one year settled in a subsequent year. By the same token one should not be surprised to find that when the premium series began to turn down, the claims series continued to rise for some time.

The real question to ask of such data in order to assess whether the insurance company is solvent is: does the claims series lie sufficiently far below the premium series. As is now obvious, in the case of the PMPA it did not, as subsequently became clear. However the analysis is complicated by such factors as the high and variable inflation rates (potentially affecting the real value of claims settled long after the policy was written), the low, often negative, real interest rates (affecting the value of the invested fund), and the long and variable time lag between the receipt of premium and payment of the relevant claims.
In principle, it should be possible to get a better picture of the emerging insolvency of the PMPA during the 1970s and early 1980s by examining the accountancy data on the estimated cost of claims incurred in each year rather than simply the cash claims actually paid in that year [1]. However, up to 1982 the company made provision for claims which proved grossly inadequate. Therefore the cost of claims incurred data, which was derived from those provisions, is unsuitable for the purpose of discerning at what point the company's finances became unsound. Therefore we are forced to rely on the cash figures. These are of course also available for the other companies in the market, and it is instructive to plot the PMPA's figures as a percentage of the total. By doing so (cf. figure 4) we can hope to eliminate some of the difficulties in interpreting the cash figures. In effect we can see from figure 4 how the PMPA's cash figures differed from those of solvent companies.

Immediately we are struck by the fact that PMPA's claims as a percentage of the total began to exceed its share of premium income as early as 1979, before its market share peaked. By 1980 PMPA was paying out almost 47% of the value of all claims paid by the industry, even though its share of premium income was always below 40%. In the year of collapse 1983 PMPA was still paying over 44% of the industry total while its share of premium income had dropped below 29%.

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1. And likewise one would normally look at earned premium income rather than simply cash premium income.
But we should not jump to the conclusion that the PMPA's weakness dated back only as far as 1979. It is noteworthy that (apart from one year, 1975) the company's share of claims was very close to its share of premium income in all years before 1979. For a growing company gaining market share so rapidly, one would have expected that the lag in paying claims would ensure that claims paid were a much smaller share of the total in any given year than the share of premium income. As it is, the claims paid curve is approximately the same as the premium curve shifted less than six months to the right. That would be consistent with a situation where a viable company was receiving premiums at the beginning of a year and paying claims instantly as they arose in the course of a year. Since claims are actually paid much later than this, we must deduce that, for the PMPA, the premium income was inadequate from the very start in the late 1960s. Only the dip in 1975 casts any doubt on this proposition.

Note that we are not erring by neglecting the role of invested funds in this analysis. By expressing claims and premiums as a percentage of the total market we are able to abstract from investment earnings to the extent that the PMPA's investment earnings per pound invested was not appreciably different from that of other companies. (Actually the investment performance of the PMPA may not have been as good as the average, though this was only a contributory factor to the collapse. After the company's insolvency the Administrator sold off some loss-making investments, but these formed only a small fraction of the deficiency).
When PMPA's insolvency had become clear beyond doubt, the Government decided to avoid the disruption likely to be caused by failure. They arranged the financing of an High Court administration of the company's affairs as going concern. This required the enactment of emergency legislation. The company's cash deficiency was to be met by an annual levy on all insurance policies written in the State. It was estimated that this levy would remain in effect at least until the end of the century.

In the classic Ponzi scheme, the investors lose their money when the promoter absconds. In the case of the PMPA the promoter did not abscond, but continued to operate when the cash flow had become negative. Nor did the "investors" (i.e., the policyholders) lose, as the Government rescued them from the collapse. Perhaps it was anticipation of such a rescue which allowed this Ponzi scheme to become the biggest in Irish history.
Table 1: Feasible and surplus maximising values of the scheme parameter $\theta$.

<table>
<thead>
<tr>
<th>Population response</th>
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<th>Surplus maximising $\theta$</th>
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<td>Lower bound</td>
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Table 2: PMFA Premium Income and Claims Paid

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*Before 1977: Claims paid and outstanding.
Figure 1: Inflows and outflows of a Ponzi scheme
(cf. Eqn. 2)
Figure 2: Evolution of Pond 1 Scheme flows over time (cf. Eqn 2)
FMPA 1967-85

Notes: Before 1977 C.P.+O. = Claims Paid and Outstanding
Chart: Share of Market 1967-85

Note: Before 1972 C.P.R.O.