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WELFARE EFFECTS OF TARIFFS AND INVESTMENT TAXES

by

J. Peter Neary

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WELFARE EFFECTS OF TARIFFS AND INVESTMENT TAXES

Abstract

This paper derives first- and second-best levels of optimal tariffs and taxes on internationally mobile capital in a general model of an open economy. When world prices are fixed (so that non-intervention is optimal), iso-welfare contours in tax-tariff space are shown to be ellipses centred around the origin which are tilted upwards if and only if importables are relatively capital-intensive. Under plausible assumptions, the same is true in the large open economy case, except that the contours are now ellipses centred around the non-zero first-best intervention point.
1. Introduction

In a world with internationally mobile factors of production, commercial policy is intimately bound up with the questions of whether and to what extent factor flows should be restricted. This is true even in a small open economy which cannot influence the world prices at which goods and factors are traded: non-interference with all international transactions is the first-best policy but constraints on the implementation of this policy on one category of transactions will in general imply non-zero levels of optimal second-best regulation of the other category. In a large economy which can influence its terms of goods and factor trade, the interdependence between restrictions on trade and international factor flows is even more pronounced. The first-best policy package may now call for a subsidy to some transactions rather than a tax; and constraints on policy choice may have surprising implications for the optimal values of those instruments whose values may be altered.

All these issues have been extensively considered in the literature. The possibility of welfare gains from restricting foreign investment was pointed out by MacDougall (1960) and the interdependence between tariffs and investment taxes was noted by Pearce and Rowan (1966) and definitively examined in the two-sector Heckscher-Ohlin context by Kemp (1966) and Jones (1967). However, although these writings have spawned an extensive literature,¹ the general principles underlying optimal policy choice when both goods and factors are internationally mobile seem to defy convenient synthesis. A major reason for this must be the concentration of most of the papers cited on the Heckscher-Ohlin case. As is well known, this framework introduces an indeterminacy in the pattern of specialisation and the results obtained appear to

¹ See, for example, Chipman (1972), Jones and Ruffin (1976) and Brecher and Feenstra (1983).
be very sensitive to detailed assumptions about the structure of production. Two exceptions to the general concentration on the Heckscher-Ohlin case may be mentioned. Gehrels (1971) gives a full analysis of optimal restrictions on trade and foreign investment in the case where aggregate supply functions are assumed to be differentiable; and Jones (1979) discusses these issues in the specific-factors model, though without noting that his results apply more generally. Despite the insights obtained from this literature, it seems fair to say that it leads to no simple conclusions about the relationship between the two types of restrictions.

The objective of this paper is to reexamine these issues and to show that the essential features of the interaction between optimal restrictions on trade and factor flows may be summarised in terms of a few simple principles. In particular, provided we concentrate on the differentiable case, so that changes in the pattern of specialisation are ignored, it turns out that most of the results hinge on whether importables use internationally traded factors relatively intensively or not.

In order to derive the results in an easily interpretable way, it is desirable to specify appropriately the underlying behavioural relationships of the model. In particular, considerable insight is obtained by treating goods and internationally mobile factors *symmetrically* but the home and the foreign countries *asymmetrically*. Specifically, I specify domestic behaviour as depending on on the prices of both goods and factors but foreign behaviour as depending on the volume of net trades in goods and factors.2 The fact that the home and foreign countries are specified differently simply reflects the asymmetric nature of the assumptions made about them: the home country is relatively free to choose the values of trade and international capital restrictions, whereas the foreign country responds passively without any retaliation. An addi-

2 By contrast, Kemp and Jones treat goods and factors asymmetrically (expressing welfare as a function of the relative price of imports and the quantity of capital imported or exported), but they treat the home and foreign countries symmetrically (since the same specification is adopted for both countries).
tional technical contribution of the paper is to introduce some new functions which facilitate this asymmetric specification.

The plan of the paper is as follows. Section 2 introduces the specification of the home country and Section 3 considers the relationship between domestic welfare and the values of the two sets of policy instruments when the terms of goods and factor trade cannot be influenced by the home country. Section 4 then turns to the large country case and shows how the pattern of first-best and second-best intervention is identified. Finally, the concluding section presents some general principles which the analysis implies.

2. Equilibrium in the Domestic Economy

Throughout the paper, the setting is that of a competitive economy trading $m$ factors and $n + 1$ final commodities with the rest of the world. One of the final commodities is chosen as numeraire, and its price is suppressed throughout. The traded factors are collectively referred to as "capital", whereas non-traded factors are not considered explicitly, since they play no role in the analysis. World prices are denoted by vectors $r^*$ and $p^*$ and the only departures from Pareto efficiency considered are restrictions on trade and foreign investment by the home country. In particular, undistorted competition is assumed to prevail abroad and the foreign country is assumed not to retaliate to home policies. The home country is thus free to choose the deviations between the domestic and foreign prices of factors and commodities, denoted by an $m$-by-one vector of investment taxes $\rho$ and an $n$-by-one vector of trade taxes $\tau$ respectively:4

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3 The choice of numeraire does not affect the ranking of commodities by the deviation between their domestic and foreign prices at the optimum. However, it does affect the sign of the optimal restriction. See Bond (1987) for further discussion. The algebraic results in the paper hold whether the numeraire good is imported or exported, but in the diagrams, for concreteness, it is assumed to be exported.

4 The numeraire is untaxed. Note that the elements of $\rho$ and $\tau$ need not be positive. As we shall see, some of these values may be negative even in the first-best optimum. If good $i$ is imported then a positive value for $\gamma_i$ represents a tariff and a negative value an import subsidy; whereas if good $i$ is exported a positive value for $\gamma_i$ represents an export subsidy and a negative value an export tax.
\[ r = r^* + \rho, \quad \text{(2.1a)} \]
\[ p = p^* + \tau. \quad \text{(2.1b)} \]

The specification of household behaviour in the home country is conventional. I assume that aggregate welfare can be represented by a scalar utility measure and that the minimum outlay needed to attain a given utility level facing given prices can be represented by an expenditure function, \( e(p, u) \), which is a concave function of prices. A key feature of the model is that factors of production do not yield utility directly.

To model producer behaviour, I adopt a slightly less orthodox specification. Assume that the vector \( \bar{k} \) denotes home *ownership* of capital and that the vector \( k \) denotes *net imports* of capital. The vector \( \bar{k} + k \) therefore denotes the amounts of capital used in production at home. For a given level of foreign investment, competition maximises the value of production in the home country, and the maximised value may be denoted by a GDP function \( g(p, \bar{k} + k) \), which is convex in \( p \) and concave in \( k \). However, the level of foreign investment is not exogenous. Instead, it is carried out to the point where the vector of domestic rentals equals the vector of world rentals plus domestic capital taxes, \( \rho \), as indicated by (2.1a). The domestic rentals in turn are equated by competition to the marginal products of the capital factors in every domestic use, which, from a standard property of the function \( g(p, \bar{k} + k) \), may be written as:\(^6\)

\[ g_k(p, \bar{k} + k) = r. \quad \text{(2.2)} \]

With capital internationally mobile, it proves to be convenient to specify producer behaviour as depending on the variables \( p \) and \( r \) which are exogenous to the domestic production sector.

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5 As explained in the introduction, I wish to exclude the possibility of changes in the pattern of specialisation. Hence I assume that the function \( g(p, k) \) is strictly concave in \( k \) so that the matrix \( g_{kk} \) is non-singular. From (2.5) below, this implies that the aggregate supply functions are differentiable in output prices.

6 Throughout the paper, subscripts denote partial derivatives and a prime denotes the transpose of a vector.
This is achieved by working with the mobile-capital GNP function, which equals the maximum value of gross domestic product less the gross returns to foreign-owned factors:

\[ \tilde{g}(p,r) = \max_k [g(p, k + k) - k'r]. \] (2.3)

This function was introduced in Neary (1985), where its properties were considered in detail. In particular, its derivatives were shown to be related to those of the GDP function at the same initial capital endowment as follows:

\[ \tilde{g}_p = g_p, \quad \tilde{g}_r = -k, \] (2.4)
\[ \tilde{g}_{pp} = g_{pp} - g_{pk}g_{kk}^{-1}g_{kp}, \] (2.5)
\[ \tilde{g}_{rr} = -g_{kk}^{-1}, \] (2.6)
\[ \tilde{g}_{pr} = g_{pk}g_{kk}^{-1}. \] (2.7)

These properties will prove useful below.

Since I wish to work with net imports of the home country, the model is most easily specified in terms of the mobile-capital trade expenditure function, which equals the excess of domestic expenditure over GNP as defined in (2.3), all at domestic prices:

\[ E(p,r,u) = e(p,u) - \tilde{g}(p,r). \] (2.8)

The properties of E follow directly from those of the e and \( \tilde{g} \) functions. In particular, it is homogeneous of degree zero in all commodity prices, it is concave in both p and r and its

---

7 For simplicity, I suppress the dependence of \( \tilde{g} \) on \( \bar{k} \).
8 The trade expenditure function when factors are internationally immobile has been considered under a variety of names by many authors. Textbook expositions are given in Dixit and Norman (1980) and Woodland (1982). To the best of my knowledge the straightforward extension to the case of international factor mobility is new.
9 Commodity prices here include p, r and the price of the numeraire good. Following standard practice, I assume that there is some substitutability in excess demand between the numeraire and other goods, so that the Hessian of E is non-singular. This also ensures that E is jointly concave in p and r.
partial derivatives with respect to these two arguments equal the Hicksian net import demand functions and the excess demand functions for foreign investment respectively:

\[ E_p = e_p - \tilde{g}_p = m(p,r,u), \]
\[ E_r = -\tilde{g}_r = k(p,r). \]  

(2.9)  

(2.10)

Note that, because capital does not yield utility directly, the demand functions for foreign investment are independent of \( u \). The cross derivatives of \( E \) also have a useful interpretation. From (2.9) and (2.7):

\[ E_{pr} = -\tilde{g}_{pr} = -g_{pk}g_{k}^{-1}. \]  

(2.11)

Following Dixit and Norman (1980), the cross derivatives of the GDP function, the elements of the matrix \( g_{pk} \), may be interpreted as general equilibrium measures of relative factor intensity. Since \( g_{kk} \) is negative semi-definite, it follows that the same interpretation applies to the elements of the matrix \( E_{pr} \). For example, in the two-good one-mobile factor case, \( E_{pr} \) is a scalar and is positive if and only if importables are relatively capital-intensive in home production.\(^{10}\)

Since the function \( E \) is the excess of private-sector spending over GNP at domestic prices, it differs from zero in general when international transactions are distorted. Throughout the paper, I assume that any net revenue from tariffs or foreign investment taxes is returned to the household sector as a lump-sum subsidy (and conversely, if trade and capital flows are subsidised on average, the net disbursements are financed by a lump-sum tax). The household’s budget constraint, which is also the condition for balance-of-payments equilibrium, may therefore be written in the following form:\(^{11}\)

\(^{10}\) Since I have assumed that the matrix \( g_{kk} \) is invertible, the two-factor two-good Heckscher-Ohlin model with mobile capital is not subsumed under the two-good, one-mobile-factor case. However, many other interesting models are, including the specific-factors model of Jones (1971) and the three-factor two-good model of Jones and Easton (1983) with international mobility of capital.

\(^{11}\) This is more easily seen to be a budget constraint if it is rewritten, using (2.1a), (2.3) and (2.8), as equating
\[ E(p,r,u) = r'm + r'k. \] \hspace{1cm} (2.12)

This equation, when combined with (2.9) and (2.10), gives \( m+n+1 \) equations which may be solved for the values of the \( m+n+1 \) endogenous variables, \( k, m \) and \( u \), as functions of world prices, \( r^* \) and \( p^* \), and policy variables \( p \) and \( r \).

Finally, it will be convenient at times to reexpress the model in a more compact fashion, which draws attention to the symmetry between trade in goods and factors. I do this by introducing the vector \( M \) to denote the net imports of all commodities, both goods and factors, with \( q, q^* \) and \( t \) denoting corresponding vectors of home prices, world prices and trade taxes:

\[ M = \begin{bmatrix} m \\ k \end{bmatrix}; \quad q = \begin{bmatrix} p \\ r \end{bmatrix}; \quad q^* = \begin{bmatrix} p^* \\ r^* \end{bmatrix}; \quad t = \begin{bmatrix} r \\ p \end{bmatrix}. \] \hspace{1cm} (2.13)

The equilibrium conditions, equations (2.9) to (2.11), may now be expressed in more compact form as follows:

\[ E_q(q,u) = M, \] \hspace{1cm} (2.14)
\[ E(q,u) = t'M. \] \hspace{1cm} (2.15)

3. Welfare Effects of Intervention in a Small Open Economy

It is immediately clear that the first-best policy for an economy which cannot influence the prices of any traded commodities is to avoid any restrictions on goods or factor trade. However, this does not exhaust the range of questions which can be considered in the small open economy case. On the contrary, an extensive literature has developed concerned with

\[ e(p,u) = g(p, x + k) + r'm - k'r^*. \] \hspace{1cm} (2.12a)
questions of “second-best” intervention in this context\textsuperscript{12} and the framework introduced in the last section allows us to synthesise and extend this literature. In any case, since the same considerations continue to influence policy choice in the large open economy case, it is convenient to consider them first in the simplifying context of fixed world prices.

I begin by differentiating (2.15). With fixed world prices (so that $dq = dt$), this simplifies to give the following:

$$dy = t'dM. \quad (3.1)$$

Here I have used $dy$ as a shorthand for $E_q du$, the change in utility or real income measured in expenditure units. Equation (3.1) thus gives the familiar result that welfare is positively related to the volume of commodity imports when the latter are restricted by trade taxes. The only novel feature is that commodities are interpreted here to include both goods and traded factors. Since $M$ is itself endogenous the next step is to differentiate (2.9) and (2.10) and substitute into (3.1). Collecting terms, this yields:

$$(1 - tX^*_t) dy = t'E_{qq} dt. \quad (3.2)$$

The coefficient of $dy$ on the left-hand side is often called the “tariff multiplier", following Jones (1969), because it arises from the spending effects induced by changes in tariff revenue. In the literature on project appraisal the tariff multiplier is usually called the “shadow price of foreign exchange," since it measures the marginal effect on welfare of a unit transfer from abroad of the numeraire good. The term $X^*_t$ is the vector of income effects on domestic demand, but since capital does not yield utility directly this can be written as:

\textsuperscript{12} Naturally, the term “second-best” is to be interpreted throughout with respect only to the set of instruments considered. It goes without saying that allowing for other instruments such as consumption or production taxes could make my “second-best optimal tariffs and capital taxes" third-best or worse. See Brecher (1983) for further discussion of this issue in the Heckscher-Ohlin context.
\[ x_t = \begin{bmatrix} x_{1t} \\ k_{1t} \end{bmatrix} = \begin{bmatrix} E_u^{-1}E_{pu} \\ 0 \end{bmatrix}. \] (3.3)

Hence the tariff multiplier can alternatively be written as \( 1 - r'x_t \). I will assume throughout that this term is positive. A sufficient condition for this is that all goods are non-inferior in demand.

It has also been noted in both the public finance and international trade literatures that a positive value of this multiplier is necessary for local stability of equilibrium. In order to justify the diagrams which I draw below, it is necessary to assume that this restriction holds globally, since otherwise welfare is not a unique function of a given vector of tax and tariff rates.

Turning to the right-hand side of (3.2), setting \( dt \) equal to \( t\alpha \), where \( \alpha \) is a scalar measure of the average height of both tariffs and capital taxes, implies the unsurprising result:

\textit{Proposition 1:} A uniform reduction in both tariff and capital tax rates must raise welfare.

Algebraically, this result follows from the concavity of \( E \) in \( q \) (i.e., in \( p \) and \( r \) together), so that the quadratic form \( r'E_{qq}r \) is negative semi-definite. The economic interpretation is that a uniform reduction in all distortions permits us to treat all traded commodities as a Hicksian composite commodity.

Rather more surprising results follow if I now rewrite (3.2) in a form which relates welfare changes to changes in the levels of tariffs and investment taxes separately:

\[ (1 - r'x_t) dy = (r'E_{pp} + \rho'E_{rp}) dr + (r'E_{rr} + \rho'E_{tt}) d\rho. \] (3.4)

---

13 See Dixit (1975), Hatta (1977) and Smith (1980). Vanek (1965) appears to have been the first to point out the association between paradoxical outcomes (such as an increase in a tariff raising welfare in a small open economy) and negative values of this term.
Here, the terms $E_{ij}$ are the appropriate sub-matrices of the matrix $E_{pq}$. Now, for given values of either set of instruments, $\bar{v}$ or $\bar{w}$, we can solve explicitly for the optimal second-best values of the other instruments, $r^0$ or $\rho^0$, by setting the coefficients of $dr$ and $d\rho$ equal to zero in turn:

$$ r^0 = - E_{pp}^{-1} E_{pr} \bar{v}, $$

$$ \rho^0 = - E_{tt}^{-1} E_{tp} \bar{w}. $$

(3.5) 
(3.6)

Because capital does not yield utility directly, (3.6) may be written in terms of production parameters only, using (2.10) and (2.11):

$$ \rho^0 = g_{kp} \bar{w}. $$

(3.7)

It is shown in Neary and Ruane (1988) that the right-hand side of (3.7) equals the difference between the domestic and shadow prices of capital, where the evaluation of the latter ignores changes in factor payments to foreigners.\footnote{The proof is straightforward. Write $E(p,k,u) = e(p,u) - g(p,k + k)$ for the trade expenditure function when capital is internationally immobile, set it equal to $r^t m$ and differentiate to obtain:

$$(1 - \text{t}'k)dy = (r - g_{kp})dk.$$  

(3.8)} In a similar manner, equation (3.5) may be interpreted as the difference between the domestic and shadow prices of importables.\footnote{This is most easily shown by specifying a new trade expenditure function corresponding to the case where importables are quota-constrained but capital is freely mobile internationally:

$$ E(m,r,u) = \min_p [ E(p,r,u) - m'p]. $$

(3.9)} This allows us to state the following:

\textit{Proposition 2:} If the levels of restriction on one category of international transactions cannot be altered, second-best optimal intervention requires that the world prices of the other category of traded commodities be set equal to their domestic shadow prices rather than their domestic market prices.

\footnote{This function has derivative properties analogous to those of $E$ as set out in equations (2.9) to (2.11): $E_m = - \rho$, $E_p = E_r = k$ and $E_{mm} = E_{rp} E_{pp}^{-1}$. By making use of these properties, setting $E(p,r,u)$ equal to $\rho k$ and totally differentiating, the following expression for the shadow price of importables may be derived:

$$ dy = (p + E_{mp})dm = (p + E_{pp}^{-1}E_{rp})dm. $$

(3.10)}
A different route to interpreting (3.5) and (3.6) is to note that they imply that the relationship between the fixed values of one set of instruments and the second-best values of the other set hinges on relative factor intensities. (For $\rho^o$ this is obvious from (3.7); while for $r^o$ it is implied by (2.11) and the fact that $E_{pp}$ is negative semi-definite.) For example, in the two-good one-mobile-factor special case, (3.5) and (3.7) imply:

**Proposition 3:** With only two traded goods and one traded factor, the fixed policy instrument and the second-best optimal instrument have the same sign if and only if importables are relatively capital-intensive.

To see intuitively why this is so, consider the effect of a fixed tax on capital imports. This lowers welfare by reducing capital imports below their optimal level. From (3.1), any policy which offsets this reduction will raise welfare. If importables are relatively capital-intensive, a tariff has such an effect, since it raises the home demand for capital and so encourages a capital inflow. Of course, a tariff also tends to reduce welfare directly by restricting imports of goods, but for a small tariff this effect can be ignored, and so the optimal second-best tariff is positive when imports are relatively capital-intensive.

Returning to (3.4), equations (3.5) and (3.6) can now be used to rewrite it in an illuminating way:

\[
(1 - r^o x) \, dy = (\tau - r^o) \, E_{pp} \, dr + (\rho - \rho^o) \, E_{rr} \, d\rho. \quad (3.11)
\]

Writing $dr$ as $(\tau - r^o) \, d\beta$ and $d\rho$ as $(\rho - \rho^o) \, d\gamma$, where $\beta$ and $\gamma$ are scalars, yields a new result:

**Proposition 4:** If the values of one set of instruments cannot be altered, then welfare will be increased by a uniform proportionate reduction in the distance between the values of the other instruments and their optimal second-best levels.
Note that (unlike Proposition 1) the relative prices of the commodities whose distortions are being altered are not kept constant by such a reform. Note also that Proposition 4 implies that there are many circumstances in which welfare is an increasing function of the instruments which policy-makers are free to alter.

It should be stressed that Proposition 4 and the other results to follow must be interpreted with some caution if the initial values of the instruments which are being altered differ by a finite amount from their optimal second-best values. To see this, I rewrite (3.11) with \( \rho \) held constant, using \( \psi(r, \rho) \) to denote the function implied by (3.5) (i.e., \( \tau^0 = \psi(r^0, \rho) \)):

\[
(1 - r'x_t) dy = \left[ \left\{ \psi(r, \rho) \right\} + \left\{ \psi(r^0, \rho) - \psi(r, \rho) \right\} \right] E_{pp}(r, \rho) dr. \tag{3.12}
\]

Proposition 4 must be interpreted as an approximate result only to the extent that it ignores the difference between the values of \( \psi \) evaluated at the initial and the second-best optimal values of \( r \). Alternatively, it may be interpreted as an exact result if \( \psi \) is evaluated at the initial value of \( r \); this approach guarantees a welfare improvement but has the disadvantage that \( \psi \) must be recalculated at each step. All these complications disappear if the third derivatives of \( E \) vanish, that is, if the Hicksian net import demand functions for goods and factors are linear in prices.

Keeping this qualification in mind, note that another way of writing (3.4) is to express each of the coefficients on the right-hand side in terms of deviations of the other instrument from its second-best optimal levels. Straightforward derivations yield:

\[
(1 - r'x_t) dy = \left[ \tau' \left( E_{pp} - E_{pr} E_{rp}^{-1} E_{rp} \right) \right] + \psi(r\rho) \left[ E_{pp} \right] E_{rr} \right] \, dr
\]

\[
+ \left[ \rho \left( E_{rr} - E_{rp} E_{pp}^{-1} E_{pr} \right) \right] \left( \tau - \tau^0 \right) E_{pr} \, dp. \tag{3.13}
\]
It can now be shown that the first expression in parentheses in each of the terms on the right-hand side of (3.13) is negative semi-definite.\textsuperscript{16} Consider first the term in the coefficient of \(dr\) in (3.13). From the properties of \(E\) and \(\bar{g}\) noted in Section 2, this may be simplified as follows:

\[
E_{pp} - E_{pt} E_{rr}^{-1} E_{rp} = e_{pp} - g_{pp} + g_{pk} E_{kk}^{-1} g_{kp} - g_{pk} E_{kk} E_{kk}^{-1} E_{kp},
\]

\[= E_{pp}.\]  

(3.14)

(3.15)

Here I have used \(E_{pp}\) to denote \(e_{pp} - g_{pp}\), the matrix of price derivatives of the immobile capital trade expenditure function.\textsuperscript{17} As required, it is negative semi-definite. A similar series of substitutions shows that the first expression in parentheses in the coefficient of \(d\rho\) in (3.13) is the inverse of the sum of two negative semi-definite matrices:

\[
E_{rr} - E_{rp} E_{pp}^{-1} E_{pr} = (g_{kk} + g_{kp} E_{pp} E_{pk})^{-1}.
\]

(3.16)

After these changes, (3.13) may be rewritten in the following form:

\[
(1 - \tau x)dy = [\tau E_{pp} + (\rho - \rho^0) E_{rp}] dr + [\rho^0 (g_{kk} + g_{kp} E_{pp} E_{pk})^{-1} + (\tau - \tau^0) E_{pr}] d\rho.\]  

(3.17)

This yields a number of new results. Considering first the coefficient of \(dr\) in (3.17), note that the matrix \(E_{pp}\) differs from \(E_{pp}\) by a negative semi-definite matrix: heuristically, international capital mobility increases the price-output responsiveness of the economy.\textsuperscript{18} In Neary and Ruane (1988) this result was applied to the measurement of the cost of protection: with no capital taxes in operation (\(\rho = 0\)), it was noted that international capital mobility raises the cost of protection. (The welfare cost of a uniform increase in tariffs is \(\tau E_{pp}\) when capital is internationally immobile and \(\tau^0 E_{pp}\) when it is internationally mobile.) Equation (3.17) allows

\textsuperscript{16} The fact that these expressions are negative semi-definite may be deduced immediately by making use of a standard property of partitioned inverses. Since \(E_{pp}\) is negative semi-definite, so is its inverse, but the two expressions in (3.13) are the inverses of the diagonal sub-matrices in the inverse and so are themselves negative semi-definite.

\textsuperscript{17} This function was defined in footnote 14.

\textsuperscript{18} As with Proposition 4, this result is exact only if the two matrices are evaluated at the same point. See Neary (1985).
us to generalise this result to the case where capital taxation is in force. In particular, for the single-import single-mobile-capital case, we may conclude the following:

**Proposition 5**: International capital mobility raises the cost of tariff protection if and only if *either* importables are capital-intensive and the rate of capital taxation is *below* its second-best optimal level (i.e., $E_{rp} > 0$ and $\rho < \rho^0$) *or* the converse is true.

A similar interpretation may be applied to the coefficient of $d_\rho$ in (3.17). The term $\tilde{E}_{pp}$ may be interpreted as the price responsiveness of import demand when foreign investment is quota-constrained. Similarly, the term $(\beta_{kt} + \beta_{rp} \tilde{E}_{pp}^{-1})$ in the coefficient of $d_\rho$ equals the rental responsiveness of home demand for foreign investment when imports are quota-constrained. (See Neary (1986).) This yields a result which is symmetric to Proposition 5:

**Proposition 6**: The welfare cost of a tax on international capital movements is greater when imports of goods are tariff-restricted than when they are quota-constrained if and only if *either* importables are capital-intensive and the tariff is below its (second-best) optimal level (i.e., $E_{rp} > 0$ and $\tau < \tau^0$) *or* the converse is true.

Equation (3.17) can also be interpreted as implying a number of envelope-type results. Thus, the coefficient of $d_\tau$ shows that if capital is always optimally taxed (so that $\rho$ equals $\rho^0$), then the relationship between welfare and the tariff rate is exactly the same as when capital is internationally immobile. Similarly, from the coefficient of $d_\rho$, the relationship between welfare and the rate of capital taxation is the same whether imports are quota-constrained or subject to the second-best optimal tariff.
These results have a particularly illuminating representation in the one-importable one-mobile-factor case, as Figures 1 and 2 illustrate. Each of these diagrams shows illustrative iso-welfare contours in the space of the policy instruments. Since \( E \) is concave in \( p \) and \( r \) (provided the tariff multiplier is positive, as I assume), we would expect these contours to enclose convex sets and this is approximately true in general. The key difference between the two diagrams is the assumption about relative factor intensities. In Figure 1, importables are relatively capital-intensive and as a result the contours have an upward tilt. Capital and importables are substitutes in the trade expenditure function in this case (\( E_{pt} \) is positive) and this property underlies a number of recent results in the literature. Note one important implication in particular: if tariffs cannot be removed, then welfare will be increased by a tax on capital imports (e.g., if the tariff rate \( \tau \) in the diagram cannot be removed, then welfare is maximised by imposing a capital import tax to move the economy to point \( E \)). By contrast, in Figure 2 the contours have a downward tilt because importables are relatively labour intensive.

4. Optimal Intervention in a Large Open Economy

Naturally, the conclusions of the last section require considerable qualification if the domestic economy is able to influence relative prices in the world economy. To see how this alters the analysis, once again totally differentiate (2.15) but without holding world prices constant. Making use of the fact that \( dq = dq^* + dt \), this allows me to relate home welfare to changes in the volume and terms of trade:

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19 Similar diagrams have been considered by Foster and Sonnenschein (1970), though they did not examine their properties in detail. Bond (1986) also presents similar diagrams in his discussion of the one-good, two-mobile-factor model of Jones, Coelho and Easton (1986), though his are drawn in the space of factor trades rather than (as here) in instrument space.

20 The assumption that the second-best instruments defined by (3.5) and (3.6) exist ensures that the contours are concave towards the origin in the neighbourhood of the loci \( r^0 \) and \( \rho^0 \). More generally, concavity of the contours can be shown to be true provided terms in third and higher-order derivatives are ignored.

21 See, for example, Brecher and Findlay (1983) and Casas (1985). These papers consider the specific factors model with internationally mobile capital used only in one sector, which implies that that sector is capital-intensive in the general equilibrium sense.
\[ dy = r'dM - M'dq^*. \]  \( (4.1) \)

So far, this derivation is standard. However, it turns out to be most convenient to diverge from common practice at this point by expressing foreign demands not in terms of import demand functions but rather in terms of inverse import demand functions.\(^{22}\) These express the world price vector, \( q^* \), as a function of the net import vector offered for trade by the rest of the world, \( M^* \). As we shall see, this approach has the great advantage that the results of Section 3 for the small open economy emerge as special cases of those for the large open economy.\(^{23}\) Assuming that the inverse import demand functions are differentiable, their differentials may be written as follows:

\[ dq^* = q^* M'dM^*. \]  \( (4.2) \)

Hence, since foreign net imports \( M^* \) plus home net imports \( M \) must sum to zero in balanced trade, \( (4.1) \) and \( (4.2) \) combine to give:

\[ dy = (r' + M'q^*_M) dM. \]  \( (4.3) \)

This immediately gives standard expressions for first-best optimal restrictions on trade:

\[ (r^{oo})' = -M'q^*_M. \]  \( (4.4) \)

which in turn may be disaggregated to give the first-best optimal tariffs and capital taxes:

\[ (r^{oo})' = -m'p^*_m - k'r^*_m, \]  \( (4.5) \)

and

\[ (\rho^{oo})' = -m'p^*_k - k'x^*_k. \]  \( (4.6) \)

\(^{22}\) Dixit (1986) also adopts this approach.

\(^{23}\) By contrast, the usual treatment proceeds in two different directions from \( (4.1) \) depending on whether the small or large open economy case is being considered.
It is clear that the pattern of optimal tariffs and capital taxes depends crucially on the properties of the inverse import demand functions. To get some feel for the likely pattern of optimal intervention, consider the single-import single-mobile-capital case. The own-price effects are then negative, and, if they dominate, both the optimal tariff and optimal capital tax are positive. However, negative values for either are possible if the cross-price effects $r^*_m$ and $p^*_k$ are sufficiently negative. One case where there is a presumption that both instruments take positive values is when foreign income effects are negligible. Total revenue from restrictions on trade and capital flows is now positive, so that on average the optimal values of the instruments must be positive.

What if tariffs and capital taxes are not at their optimal levels? Note first that (4.3) may be rewritten as follows:

$$dy = i'dM,$$  \hfill (4.7)

where:

$$i = t - t^0,$$  \hfill (4.8)

the vector of deviations of actual tariff levels from their optimal levels. Equation (4.7) is clearly an extension of (3.1) to the large open economy, and it has a similar interpretation: if tariffs and investment taxes are above their optimal levels, any policy which increases import volumes will raise welfare.

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24 These properties are particularly simple in the case of gross substitutes. (See Cheng (1985).) More generally, they could be related to the properties of the underlying expenditure and GNP functions in the foreign country by developing either a restricted trade expenditure function or a restricted trade distance function. (See Deaton (1979) and Anderson (1987).) This seems an important direction for further research.

25 From (4.4), $(t^0)'M = -M'q'M$, which, with no income effects abroad, is minus a quadratic form in a negative definite matrix. Feenstra (1986) and Bond (1987) derive this result for the multi-commodity optimal tariff case.
The next step, as in the small open economy case, is to eliminate $dM$ from (4.7). Differentiating (2.14) and making use of (4.2) yields the following general equilibrium import demand equation:

$$dM = M_d dt + M_I dy,$$  \hspace{1cm} (4.9)

where:

$$M_I = (I + E_{qq} q^*_M)^{-1} E_{qq},$$  \hspace{1cm} (4.10)

and:

$$M_I = (I + E_{qq} q^*_M)^{-1} X_I.$$  \hspace{1cm} (4.11)

(Here I is the identity matrix of order $(m+n)$.) Finally, substituting from (4.9) into (4.7) and collecting terms yields:

$$(1 - i'M_d)dy = i'M_d dt.$$  \hspace{1cm} (4.12)

The formal resemblance between (4.12) and equation (3.2) in the last section is striking and it would be very desirable to interpret (4.12) in the same manner. To investigate whether this is possible, I consider the individual terms in (4.12) in turn.

Considering first the coefficient of dy, it is a generalisation to the large open economy of the tariff multiplier introduced in Section 3 (the coefficient of dy in (3.2)) and it clearly reduces to the latter when world prices are fixed, so that the elements of $q^*_M$ are zero. It is also clear that it reduces to unity when the home country imposes optimal tariffs and investment taxes ($i = 0$). This accords with the interpretation of $(1 - i'M_d)^{-1}$ as the shadow price of foreign exchange: when optimal policies are in effect, a unit transfer of the numeraire good raises real income in terms of the numeraire by exactly one unit. When policies are not at their optimal levels, the sign of this term is indeterminate but, as in the small open economy case, a heuristic stability argument can be used to justify a positive sign. In particular, if prices are assumed to
adjust instantaneously and equilibrium is attained by a redistribution of lump-sum income between the two countries until trade is balanced, it can be shown that a positive value for \((1 - \sum M_i)^{-1}\) is necessary and sufficient for local stability.\(^{26}\) As in Section 3, in order to justify diagrammatic analysis, it is necessary to assume in addition that this term is globally as well as locally positive, and I make this assumption from now on.

Turning next to the term \(M_i\), it represents the responsiveness of home import demand to changes in home tariffs at a given level of home utility, but allowing for the effects of the tariff changes on foreign income. Because of income effects in the foreign country, there is no guarantee that this matrix need be symmetric, far less that it be negative semi-definite. Nevertheless, in a wide variety of circumstances, especially when foreign demands are relatively elastic, it is reasonable to assume that these income effects do not dominate the expansion.\(^{27}\) In order to proceed further, I therefore assume the following:

**Assumption:** The matrix \(M_i\) is negative semi-definite.

If this assumption does not hold then home import demand for at least one composite commodity (possibly including both traded goods and factors) is an increasing function of its own tariff rate even when home real income is held constant. The power of this assumption lies in the fact that, when combined with the assumption already made that the coefficient of \(y\) is positive, it implies that equation (4.12) may be interpreted in exactly the same manner as (3.2). Geometrically, it defines a family of iso-utility contours, which are ellipses in the space of the home country's policy instruments. The difference between (4.12) and (3.2) is that the first-

\(^{26}\) If instead, equilibrium is assumed to be brought about by the adjustment of international prices, the stability condition involves restrictions on a matrix which is a multi-market generalisation of the Marshall-Lerner condition. If stability is required for all possible adjustment speeds of prices and lump-sum incomes, then once again a positive value for \((1 - \sum M_i)^{-1}\) is implied at stable equilibria.

\(^{27}\) One interesting special case where this is true is where the home and foreign countries have identical tastes and technology and where foreign income effects can be ignored. The matrix \(q_M^*\) then equals \(E_{qq}^{-1}\) and \(M_i\) reduces to \(E_{qM}/2\).
best optimal intervention point around which the ellipses are centred is now the vector $t^{oo}$ rather than the origin. This allows a particularly interesting interpretation, since from (4.4), the values of the first-best instruments depend directly only on parameters relating to the foreign country. By contrast, the shape of the iso-welfare contours themselves depend on both home and foreign parameters, from (4.10). To see the implications of the assumptions, I can first of all state a series of propositions which are exactly analogous to those for the small open economy case in Section 3. Firstly, setting $dt$ equal to $idx$ in (4.12) implies:

**Proposition 7**: A uniform proportionate movement in both tariffs and capital taxes towards their first-best optimal levels $t^{oo}$ must raise welfare.\(^{28}\)

What if one set of instruments cannot be adjusted? In that case, I can again define second-best optimal levels of intervention and show that movements of the alterable instruments towards these levels must raise welfare. Thus, rewriting (4.12) in a form similar to (3.4):

$$
(1 - \hat{r}'M_1) dy = (\hat{r}'m, + \hat{\rho}'k_\rho) dr + (\hat{r}'m_p + \hat{\rho}'k_p) d\rho,
$$

(4.13)

where the new terms in the expression are the appropriate sub-matrices of the matrix $M_1$. As before, this yields:

$$
\hat{r}^0 = - m_r^{-1} m_p \bar{r},
$$

(4.14)

$$
\hat{\rho}^0 = - k^{-1}_p k_r \bar{\tau},
$$

(4.15)

where $\bar{\rho}$ and $\bar{\tau}$ are the fixed values of the other instrument. By exact analogy with (3.11), this allows (4.13) to be rewritten as follows:

$$
(1 - \hat{r}'M_1) dy = (\hat{r} - \hat{r}^0)' m, dr + (\hat{\rho} - \hat{\rho}^0)' k_p d\rho.
$$

(4.16)

---

\(^{28}\) This result resembles one of Dixit (1987), Section 4, who derives conditions under which equiproportionate movements in tariffs towards a quasi-optimal level is locally Pareto-improving in a many-person economy.
Writing \(dr = (r - r_o) d\gamma\) and \(dp = (p - p_o) d\gamma\), where \(r\) and \(p\) are scalars, allows us to interpret Proposition 4 as applying to the large economy as well as to the small open economy with no modification whatsoever. In exactly the same manner, propositions analogous to Propositions 5, 6 and 7 may be stated for the large open economy case. I leave the details to the interested reader.

As in Section 3, it is illuminating to illustrate the results in the special case where only one imported good and one imported factor are subject to trade restrictions. Some, though by no means all, of the possibilities in this case are illustrated in the six panels of Figure 3. In panels (a), (b) and (c), the first-best optimal values for the tariff and the tax on foreign capital are both positive (and, for simplicity, they are drawn at the same levels). However, the shape of the iso-welfare contours differs between the three panels, and so the implications for optimal second-best intervention are very different. In panel (a), the terms \(m\) and \(k\) are positive, so that the ellipse has an upward tilt. I shall refer to this case as that where, on average, importables are capital-intensive both at home and abroad. Starting from the origin, the point of zero intervention, it is clear that an increase in either the tariff or the rate of capital taxation raises welfare. The same is true in panel (b), where importables are labour-intensive at home and abroad, so that the ellipse has a downward tilt. The difference is that now the optimal second-best tariff when capital cannot be taxed lies above the first-best optimal level. (i.e., point A lies above point F in panel (b) whereas it lies below it in panel (a)). Panel (c) resembles panel (a) except that the ellipse now has a steeper tilt, so that the locus of second-best optimal tariffs, \(r^*\), now passes below rather than above the origin. It follows that the second-best optimal trade intervention if capital cannot be taxed is an import subsidy rather than a tariff.

The three remaining panels illustrate the case where foreign demand is such that the first-best policy is a tariff combined with a subsidy to foreign capital. In both panels (d) and (e), a movement away from the origin towards the first-best point raises welfare. The differ-
ence between these two cases is that in panel (d) it is optimal to "overshoot", that is to raise each instrument above its first-best optimal level, if the value of the other instrument is con-
strained to equal zero. Finally, panel (f) illustrates a case where, if capital cannot be taxed, then once again imports should be subsidised (at point A) even though the first-best policy in-
volves a positive tariff.

5. Conclusion

In this paper I have presented a general framework for analysing the simultaneous choice of optimal restrictions on trade and international factor flows. My approach contrasts with existing studies of this issue in its treatment of the two key asymmetries in the model. On the one hand, I have deemphasised the asymmetry between internationally mobile goods and fac-
tors. Since there is no difference between them in tradeability, the only important difference is whether or not they yield utility directly; and even this difference is of secondary importance for many purposes. On the other hand, my approach has adopted very different specifications for the home and foreign countries. The key distinction arises not from differences in technology or in the pattern of specialisation but rather from whether they are active or passive participants in the regulation of international transactions.

The paper has presented a novel derivation of the first-best pattern of optimal inter-
vention. As is well known from earlier work, first-best intervention in the small open economy case requires that all tariffs and investment taxes be zero. If the economy faces less than infi-
nitely elastic supplies from the rest of the world, then first-best intervention requires non-zero levels of the different instruments, with their optimal levels determined solely by the param-
eters of the foreign country's excess demands. There is a presumption that both trade and capital flows will be taxed, but the case where some transactions are subsidised at the optimum cannot be ruled out.
These results are reasonably well known from previous work, especially that of Kemp (1966) and Jones (1967). Apart from illustrating them in a novel way, the principal contribution of this paper is to demonstrate what can be said about the relationship between welfare and the available policy instruments when the values of some of the latter are constrained. In particular, it has been shown that the pattern of optimal second-best tariffs and investment taxes hinges on whether or not importables are relatively capital-intensive, in the general equilibrium sense of whether or not an increase in the rental elicits an increase in the output of importables. If importables are capital-intensive, then contours of constant welfare are upward-tilted ellipses in the space of the tariff and the investment tax; the implication is that restrictions on either type of international transaction have counteracting effects on welfare. Thus, if an exogenous constraint sets the rate of capital taxation at a positive level, the optimal second-best restriction on trade is a tariff. The reverse holds if importables are relatively labour-intensive: constant-welfare ellipses are now downward-sloping and (for example) exogenously set levels of capital taxation imply that imports should be subsidised at the second-best optimum. A key feature of these results is that, under plausible assumptions, they have been shown to hold in essentially the same fashion in both small and large economies. Thus, in the small open economy case, the iso-welfare ellipses centre around the point of zero intervention, whereas in the large open economy case they centre around the point corresponding to the first-best tariffs and investment taxes.

It was noted in the introduction that this paper differs from most earlier treatments of the choice of optimal tariffs and investment taxes in not dealing with the Heckscher-Ohlin model. A disadvantage of this is that I have had to assume that a particular pattern of production and trade prevails for all values of the policy instruments considered. However, this drawback necessarily arises in any study which uses calculus tools to determine the optimal values of policy instruments. In the papers of Kemp (1966) and Jones (1967), for example, a detailed consideration was required of the circumstances in which one or other country specialised in
production, and different rules for optimal intervention were calculated in each case. Assuming a particular pattern of specialisation, as I have done here, is therefore no less general an approach. Presumably a full analysis of optimal intervention which takes account of policy-induced changes in the pattern of specialisation will require simulations of computable models.

Finally, the fact that I have treated traded goods and factors in a totally symmetric fashion throughout suggests that the techniques introduced in this paper could be applied to the analysis of policy choice in models where factors are internationally immobile but more than one category of imports is distinguished. This application can indeed be carried out, which links the results of the present paper with the recent work on optimal tariffs in multi-good models by Feenstra (1986), Itoh and Kiyono (1987) and Bond (1987).
References


Figure 1: Iso-Welfare Contour in the Small Open Economy Case when Importables are Relatively Capital-Intensive
Figure 2: Iso-Welfare Contour in the Small Open Economy Case when Importables are Relatively Labour-Intensive
Figure 3: Iso-Welfare Contours in the Large Open Economy Case