A New Approach to Evaluating Trade Reform

by

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Abstract

Dual methods are awkward to use for quota reform analysis. This paper develops a distance function concept (Deaton(1979)) relevant to trade distortions: the Distorted Trade Distance Function. It has the great advantage of being a minimum value function, permitting the use of standard techniques for easy achievement of general positive and normative results in the evaluation of trade reform. Quotas are the predominant means of protection in developed countries and distance function methods are the natural method for their analysis. We also define an operational general equilibrium distance measure, the coefficient of trade utilization.

Our results below state conditions for welfare—improving trade reform which are much less restrictive than those available for tariff reform in the past. Welfare improves under unilateral quota (or quota equivalent) reform for a small country regardless of the division of the licenses if the reform is toward free trade in the average sense. Under mixed tariff and quota regimes, cross—effect terms create a presumption for developed countries that: (i) tariff reforms with fixed quotas are welfare—decreasing, and (ii) quota reforms are still welfare—increasing. For multilateral reform, quota reform is Pareto—superior if the move is toward free trade in an average sense. Moreover, international compensation may be achieved entirely by the assignment of quota licenses, the instruments of distortion.
A New Approach to Evaluating Trade Reform
James E. Anderson and Peter Neary

Dual methods are awkward to use for quota reform analysis because: (1) the solution values of tax equivalents are implicit, and (2) the first and second derivative conditions needed to evaluate changes in taxation are often too complex to evaluate save under strong restrictions, leading to reform statements which are over–sufficient. This paper will develop and apply primal methods which overcome both difficulties.

We define in Part I a new distance function concept (Deaton–Muellbauer (1980), Debreu (1951)) in relevant to trade distortions: the distorted trade distance function. Its arguments are quantities, hence the link with quota policy is direct, and it has the great advantage of being a minimum value function in constrained goods prices. This implies that the quota rent function is also a minimum value function, permitting the use of standard techniques for easy achievement of general positive and normative results. Furthermore, the minimum value property is a new source of non–equivalence of tariffs and quota, because the tariff revenue function is not a minimum value function. The distorted trade distance function in combination with the external budget constraint, yields the general equilibrium reduced form distorted trade utility function, with realized utility as a function of the trade instruments.

The distorted trade utility function is then applied to evaluation of unilateral trade reform in Part II. Both tariffs and quotas may be treated with the method, though for quota equivalents of tariffs alone there is no real advantage over expenditure function methods. Quotas are the predominant means of protection in developed countries and distance function methods are the natural method for their analysis. Our results state conditions for welfare–improving trade reform which are much less restrictive than those available in the past for tariff reform. In particular, the new concept avoids the need for tricks to handle income effects and aggregation over commodities. Welfare improves
under unilateral quota or quota equivalent reform for a small country regardless of the
division of the rent if the reform is toward free trade in the average sense that trade
increases be positively correlated with unit quota rents. Turning to explicit treatment of
tariffs and quotas, the set of welfare-improving tariff reforms is reduced by the fixed
quota effect, with a (perhaps surprising) presumption for developed countries that further
tariff cuts in the presence of VERs are welfare-decreasing. For quota reform in the
presence of tariffs, a tariff revenue effect term reduces the benefit of reform, but the
simple quota reform theorem holds so long as quota and tariff goods are substitutes (in a
new sense defined below) and average ad valorem tax equivalents of quotas exceed
average ad valorem tariffs, a condition which is met in developed countries. Our results
point to the importance getting good information on a key observable variable, the rate
of rent retention, the fraction of quota rents retained at home.

In Part III we use the distorted trade distance function to develop closely allied
conditions for Pareto–superior multilateral trade reform; again it suffices that trade move
upward on average in higher unit quota rent categories. The multilateral reform theorem
has the very important property that it is self financing; international compensation can
be done entirely within the quota system.

In Part IV we develop a general equilibrium distance measure of the inefficiency
of trade policy, the coefficient of trade utilization. It is akin to Debreu's (1951) famous
coefficient of resource utilization and Hicks' (1944) quantity variation consumer's
surplus. We develop operational expressions for welfare-improving trade reform in
terms of it. Our companion paper (1988b) contains numerous extensions and
applications.

A second companion paper (1988a) develops some technical properties of the
functions in more detail, and applies them to the theory of public assistance delivery
(public housing and medicine, subsidized housing and food, etc.).
The method undoubtedly has other applications we have not explored. For example:

1. Pollution problems can be redone using these techniques.
2. Factor mobility issues and the associated quotas on immigration can be redone using the techniques. It seems natural to consider taxes vs. quotas as in Anderson (1988).
3. Quality controls and/or quality shifts in response to trade policy may be naturally and usefully handled with these techniques.
4. Intertemporal trade under credit constraints may possibly be modelled with these techniques.

I. The Distorted Trade Distance Function

Deaton (1979) defines a concept dual to the expenditure function called the distance function and develops its applications to commodity tax theory. His prescription has largely been ignored, but we shall argue that it has a natural home in international trade where so much of government intervention is in the form of quantity controls.

Deaton–Muellbauer (1980) develop the distance function's properties for the case where in the expenditure function the consumer chooses all goods facing fixed prices and in the dual distance function the consumer "chooses" all shadow prices facing fixed quantities.

There are two technical contributions of our work. First, in section I.1, we show how the analysis extends to choice restricted to some commodities, and in the dual to some shadow prices. Second, in section I.2, we extend the analysis to the general equilibrium of a trading economy. Using the techniques of this paper, a very flexible treatment of the general equilibrium of a distorted trading economy is possible.

I.1 Distance and Related Functions

For ease of presentation we first develop the case where all trade prices or quantities are under control. X is an arbitrary point in the trade space, \( U^0 \) is an arbitrary Meade trade indifference curves. The analog to the Deaton–Muellbauer distance function
is obtained by moving along a ray like XC in Figure 1. Implicitly, \( U(X/D)=U^0 \) defines the distance function \( D \). Evidently, \( D \) is undefined for points \( X \) lying below the convex hull formed by the tangency of a ray from the origin with \( U^0 \). This rules out a bundle \( X \) relative to a reference \( U^0 \) such that the line through \( X \) tangent to \( U^0 \) implies negative expenditure. Since trade must balance in external prices (the external price budget line passes through the origin) this implies positive trade distortion revenue. The feasibility requirement for the definition of distance is the familiar sufficient condition for gains from trade that distortion revenue be positive. Thus the trade distance function is defined only when there are gains from trade.

Figure 1. The Trade Distance Function

For point \( X \) in the feasible set, \( D \) is found by a uniform expansion (contraction) of imports (exports), with the factor of expansion (contraction) being \( pX/E(p,U^0) \), where \( E(p,U^0) \) is the trade expenditure function evaluated at the shadow price vector \( p \) for given \( U^0 \). Throughout the paper, expressions like \( pX \) will stand for inner product.
When necessary for clarity, a \( \end{equation} \) will denote transpose in a matrix multiplication expression, in which case an untransposed vector \( p \) is understood to be a column vector.

The definition appears to allow point \( F \) as well as point \( C \), but positive net trade expenditure, \( E(p, U^0) > 0 \), is implied by combining feasibility with a minimum value in \( p \) property of the explicit form of the distance function:

**Definition 1** The Trade Distance Function is

\[
D(X, U^0) = \min_{pX} \{ pX \mid E(p, U^0) = 1 \}
\]

where \( E(p, U^0) \) is the trade expenditure function, used as above to normalize the prices:

\[
E(p, U^0) = \min_{X} \{ pX \mid D(X, U^0) = 1 \}.
\]

For the usual trade distortions case, only some products are restricted. Let \( Q \) be the amount of trade in the restricted product group, with foreign price \( P^* \) and domestic price \( P \). For the unrestricted group, \( Z \) is the trade quantity, \( \pi \) is its domestic price = \( \pi^* + t \) where \( t \) is the specific tax and \( \pi^* \) the free trade price. Recalling the normalization in (1), \( P^* \), \( \pi^* + t \), are understood to be normalized by \( E(P, \pi, U^0) = 1 \).

First we consider consumer choice in the unrestricted product group.

**Definition 2**

The distorted trade expenditure function is:

\[
\tilde{E}(Q, \pi, U^0) = \min_{Z} \{ \pi Z \mid U(Q, Z) = U^0 \}
\]

Alternatively,

\[
\tilde{E}(Q, \pi, U^0) = \max_{P} E(P, \pi, U^0) - PQ
\]

(4) is a well behaved maximization problem, since \( E \) is concave in \( P \). Mechanically, the first order conditions solve for the price \( \tilde{P} \) which equates demand \( E_{P}(P, \pi, U^0) \) with \( Q \), the supply. The first derivative properties of \( \tilde{E} \) are straightforward from (6):

---

1 There is a slight awkwardness in that distance is undefined at free trade, since \( E(p, U^0) = 0 \). Since our interest is in distorted trade, this need not concern us. See also Stern (1986) for a related discussion of Deaton's concept when the consumption bundle includes negative inputs (leisure).
\[ \tilde{E}_\pi = Z \]
\[ \tilde{E}_Q = -\tilde{P}. \]

The first property follows from Shepard's lemma. The second is less familiar with intuition that a relaxation of the quota by one unit reduces expenditure on the market group by \( \pi Z_Q \), which gives the marginal willingness to pay for quota ridden goods. \( \tilde{E} \) is concave in \( \pi \) and convex in \( Q \), by its minimum in \( Z \) and maximum in \( P \) property.

Next, we consider expenditure on the restricted group when the market group faces fixed price \( \pi \). The main technical innovation of this paper involves considering the minimum shadow value of expenditure on the controlled group when market goods are purchased at price \( \pi \).

**Definition 3**

The **Distorted Trade Distance Function (DTDF)** is

\[ (6) \quad \tilde{D}(Q,\pi,U^0) = \max_Z D(Q,Z,U^0) - \pi Z. \]

(6) is a well behaved maximization problem, since \( D(Q,Z,U^0) \) is concave in \( Z \).

\( \tilde{D}(Q,\pi,U^0) \) is concave in \( Q \) by the minimum value property of \( D \) in \( P \), and convex in \( \pi \), from the maximum value property of \( \tilde{D} \) in \( Z \). Its first derivatives are

\[ (7) \quad \tilde{D}_Q = P, \text{ and} \]
\[ (8) \quad \tilde{D}_\pi = -\tilde{Z}, \text{ where } \tilde{Z} \text{ is the maximizing quantity of } Z. \]

(7) follows from the envelope theorem and implied \( \tilde{D} \) is homogeneous of degree one in \( Q \). This will be essential in our applications below. Recalling (1), the definition of \( D(Q,Z,U^0) \), it is clear that the implied first order conditions yield \( \tilde{D} = PQ/E(P,\pi,U^0) = PQ \), the shadow value of the controlled goods under the normalization \( E=1 \). That is why we label it the distorted trade distance function\(^1\).

**II.2 General Equilibrium**

\(^1\text{Under a renormalization by } P \epsilon P(P,\pi,U^0), \text{ the shadow expenditure on the constrained goods, } \tilde{D} \text{ is a true distance function, giving the factor by which } Q \text{ must be expanded or contracted to reach } U^0, \text{ incorporating the implied change in } Z(Q,\pi,U^0).}
We have developed expressions for domestic expenditure on market goods, $\bar{E}$, and quota constrained goods, $\bar{D}$ as a function of the exogenous variables $Q$ and $\pi$. We now impose the general equilibrium constraint of balanced trade to develop the new approach to unilateral trade reform. Suppose that the fraction of quota revenue retained by the home country is $1-\omega$ (consistent with awarding the fraction $\omega$ of all quota licenses to foreigners, or with a tariff on quota-controlled imports at specific rate $(1-\omega)[P-P^*]$).\(^1\) Tariff revenue $tZ$, if any, is retained at home.\(^2\) In terms of external prices the budget constraint is:

$$\pi^*Z + P^*Q + \omega[P-P^*]Q \leq 0,$$

where the third term represents the transfer to foreigners via the transfer to foreigners of the value of quota licenses. This is alternatively

$$\pi Z + tZ + (1-\omega)P^*Q + \omega PQ \leq 0.$$

Now consider a trading equilibrium. (9') holds with equality and we can substitute into

$$\bar{E}(Q,\pi,U^0) = \pi Z \text{ and } \bar{D}(Q,\pi,U^0)=PQ \text{ in equilibrium.}$$

Also, $Z=\bar{E}_\pi(Q,\pi,U^0)$. The result is the (reduced form) balance of payments function:

$$b(Q,t,U^0) = \bar{E}(Q,\pi,U^0) - t\bar{E}_\pi(Q,\pi,U^0) + (1-\omega)P^*Q + \omega \bar{D}(Q,\pi,U^0) = 0.$$

In b(,), the dependence on $P^*,\pi^*$ is understood but suppressed since we make no use of it in this paper. Understanding that $\bar{D}_{QQ}=-\bar{E}_{QQ}$, we may note that the curvature

\(^1\)This readily generalizes to allow a different $\omega_i$ for each controlled good i. We simplify to a common $\omega$ to focus on the elements of the problem peculiar to quotas. In the general case, aside from an inessential linking of the rent for quota i to a specific $\omega_i$ rather than a common $\omega$, there is a composition effect of changes in quotas which must be carried around in the reform evaluation. It has the form $PQ[\Omega-\Omega]Q$, where $\Omega$ is the diagonal matrix with the $\omega$'s on the diagonal, and $\bar{\Omega}$ is the diagonal matrix with the average $\omega$ on the diagonal. Homogeneity implies $PQ\bar{\Omega}=0$, so this is a covariance expression which we must assume to be small to strictly justify the approach of the text. Note that the rent retention composition effect has the same form as the well-known tariff composition effect arising from differential taxation. For a formal treatment, see equation (25) below. Differential $\omega$'s arise from differential taxation (or its equivalent in license sharing) so the composition effect is really an aspect of the inefficiency of differential taxation rather than an aspect of quota inefficiency. This justifies our neglecting it for analytical purposes.

\(^2\)In practice, quota controlled trade is usually accompanied by a tariff, so some rent retention does occur. The main issue, as this paper shows, is how much retention occurs, but there is a possible significance to its composition.
properties of \( b(\cdot) \) in \( Q \) depend in the small country case (constant \( P^* \)) with no tariffs strictly on the curvature properties of \( \tilde{D}(\cdot) \) in \( Q \). Thus \( \tilde{D} \) essentially runs the analysis of welfare reform of quotas, justifying our focus on it\(^1\).

For welfare analysis we implicitly define the (reduced form) utility as the solution to (10) with equality:

**Definition 4: The Distorted Trade Utility Function** is

\[
(11) \quad v(Q, \pi, U) = \{ U^0 \mid \tilde{E}(Q, \pi, U) - t\tilde{E}_\pi(Q, \pi, U^0) + (1-\omega)P^*Q + \omega\tilde{D}(Q, \pi, U^0) = 0 \}
\]

Evidently the implicit function theorem applied to (10) yields the properties of \( v(\cdot) \):

\[
(12) \quad v_Q = -\frac{b_Q}{b_U},
\]

\[
(13) \quad v_\pi = -\frac{b_\pi}{b_U}.
\]

We now turn to the properties of \( b(\cdot) \). Differentiating \( b'(\cdot) \):

\[
(14) \quad b_U = \tilde{E}_U - t\tilde{E}_\pi U + \omega\tilde{D}_U
\]

\[
(15) \quad b_Q = \tilde{E}_Q - t\tilde{E}_\pi Q + (1-\omega)P^* + \omega\tilde{D}_Q
\]

\[
(16) \quad b_\pi = -t\tilde{E}_\pi \pi + \omega\tilde{D}_\pi.
\]

Now we develop (14)–(16) using the properties of \( \tilde{E} \) and \( \tilde{D} \). Collecting the derivatives evaluated at equilibrium:

\[
\tilde{E}_\pi U = Z_U
\]

\[
\tilde{E}_Q = -P
\]

\[
\tilde{D}_Q = P
\]

\[
\tilde{E}_\pi Q = Z_Q
\]

\[
\tilde{E}_\pi \pi = Z_\pi
\]

\[
\tilde{D}_\pi = -Z.
\]

First we consider (14), which is the inverse of the shadow price of foreign exchange (the amount of utility created by the gift of a unit of foreign exchange). Note that \( \tilde{E}_U = \)

\footnote{Its use in multilateral reform provides another justification.}
$-\tilde{D}_U$, which follows from the fact that under the normalization defining $D$ and thus $\tilde{D}$, $\tilde{E}$ + $\tilde{D}$ = 1. Then (14) becomes:

$$b_U = (1-\omega)\tilde{E}_U - tZ_U.$$  

$\tilde{E}_U > 0$ if the market group as a whole is normal, which we assume. We shall also normally assume that (17), and hence its inverse, the shadow price of foreign exchange, $\mu = 1/b_U$ is positive. But to emphasize the importance of rent retention we note that (17) implies at $\omega = 1$, $Z_U > 0$:

**Theorem 1** Under a pure VER system, with normality in the group of tariff–ridden goods, the shadow price of foreign exchange is negative.

A negative shadow price of foreign exchange implies that transfers are immiserizing, can be shown to lead to numerous other immiserizing–type paradoxes, and can be linked to instability of the international equilibrium. The paradoxical result is intuitively explained by the fact that quantity constrained trade is so badly distorted that the transfer of foreign leads to gains on the rent account which are immediately reverse–transferred to foreigners, while the standard tariff distortion term acts as always to create a loss. In the absence of quotas, or with $\omega = 0$, the standard "tariff multiplier" is

$$\frac{\tilde{E}_U}{b_U} = \frac{1}{1 - t} \frac{Z_U}{\tilde{E}_U} = \frac{1}{1 - t Z_I}.$$  

It does indeed require pathological distortions to cause a negative tariff multiplier, but (17) and Theorem 1 serve to emphasize that a negative shadow price of foreign exchange requires only sufficiently small rent retention.

Turning to the policy derivatives, $b_Q$ and $b_t$, we obtain:

$$b_Q = -(1-\omega)(P-P^*) - tZQ$$  

$$b_t = -\omega Z - tZ_T$$

We are now in a position to evaluate trade reform using (12)–(13) and (17)–(19).

**II. Unilateral Trade Reform**
The most obvious use of the new technique is to evaluate trade reforms. In section I.1, we evaluate pure quota or quota-equivalent reform. In sections II.2 and II.3 we consider explicit treatment of tariffs looking respectively at the cases of tariff reform in the presence of quotas and quota reform in the presence of tariffs. Section II.4 develops the extension to the large open economy case.

II.1. Quota Reform (or Quota Equivalent Reform)

It is well known that a unilateral tariff reduction need not improve the welfare of a small country. In contrast, if a quota is effective, an increase in any quota or quotas in the absence of tariffs is always welfare-improving. More generally, any reform either of quotas or quota equivalents of tariffs which is trade increasing in an average sense to be developed below is welfare-improving.

Consider any trade reform which results in a small change in restricted imports or exports dQ. (This can be interpreted to include moving the quota equivalents of tariffs, so that there are no exogenously fixed t's). Utility changes by:

\[(20) \quad v_Q dQ = \mu (1-\omega) (P-P^*) dQ\]

since by (12) and (18),

\[(21) \quad v_Q = -b_Q / b_U = \mu (1-\omega) (P-P^*) \geq 0.\]

(21) implies that the general equilibrium shadow price of a unit of the quota constrained goods is \((1-\omega)(P-P^*)\). We can immediately state:

**Theorem 2** Any trade reform which tends on average to increase trade of more restricted (more highly taxed) goods is welfare improving.

Note that the condition of the theorem is necessary and sufficient, so it provides a complete characterization of the directions of welfare-improving reform.

Theorem 2 covers both quota and tariff changes in the form of quota equivalents. For tariffs, of course, the Q's are endogenous and the P's exogenous, so the problem of the income and substitution effects which complicates tariff reform re-emerges in the determination of dQ. Under a tariff reform it is possible for some highly taxed elements
of the restricted group to fall in quantity, hence decreasing trade revenue and thus welfare. Theorem 2 assumes that for tariff changes, any such effect is overcome, implicitly requiring finding the "right" tariff changes. For quota changes this problem is removed by the direct control of quantities.

One special case of the theorem has an especially neat interpretation. Suppose that the trade reform is constrained to have zero balance of trade implications, as is often a side constraint in actual trade negotiations. Then \( P^*dQ = 0 \). Multiply the elements of \( dQ \) by \( P^* \) and divide the elements of \([P - P^*]\) by \( P^* \):

\[
[P - P^*]^t dQ = [P - P^*][P^*]^{-1} \hat{P}^* dQ,
\]

where \( \hat{P}^* \) denotes a diagonal matrix with \( P^* \) along the diagonal. Then the inner product of the vector of ad valorem tariff equivalents with the changes in the foreign exchange value of trade is a covariance, since one of the vectors has zero mean. Formally:

**Corollary:** The condition for welfare improvement with balanced trade impacts is that the correlation of trade value changes and the new ad valorem tariff equivalents be non-negative.

The logic of Theorem 2 implies a welfare-improving reform result can also be written entirely in terms of trade quantities. Let \( Q^1 = Q^0 + dQ \). Let the "free trade" quantity vector conditional on \( U^0 \) be \( Q^* \) such that \( P(Q^*, U^0) = P^*1 \). The product in (20) can be further decomposed using the mean value theorem and evaluating \( P_Q \) at some \( Q^2 \) point intermediate between \( Q^1 \) and \( Q^* \): \( Q^2 = \alpha Q^1 + (1 - \alpha)Q^* \) for \( 1 \geq \alpha \geq 0 \). Then:

\[
(22) \quad v_Q dQ = (1 - \omega)(Q^1 - Q^*)[P_Q(Q^2)]dQ.
\]

The matrix \( [P_Q] \) is negative semidefinite, hence the quadratic form on the right hand side reduces to:

\[
v_Q dQ = (1 - \omega) \Sigma (Q^1_j - Q^* j) \lambda_i dQ_i, \text{ where } \lambda_i \leq 0.
\]

---

1 An analogous expression follows if the utility level is set at \( U^* \), the free trade level.
The \( \lambda \)'s are the characteristic roots of \([P_Q]\). If we normalize in by \(-\Sigma \lambda(1-\omega)\), the weights \(w_i = \lambda_i / \Sigma \lambda(1-\omega)\) are like probabilities, non-negative with a unit sum. Then:

\[
dv = \Sigma w_i (Q^*_i - Q_i) dQ_i.
\]

We expect the average quantity changes \(\Sigma w_i (Q^*_i - Q_i)\) and \(\Sigma w_i dQ_i\) to be positive; this can be taken to define a trade reform. Then by (23), using the algebra of correlation:

**Theorem 3:** Any trade reform in which trade changes are positively correlated with their distance from free trade levels is welfare improving.

Theorem 3 gives an alternative interpretation of the intuitive sense that a welfare improving trade reform should approach free trade in an average sense. Evidently the role of the \(w_i\), the relative characteristic roots of the Antonelli matrix is to emphasize which categories \(i\) are most restrictive (raise prices the most) when distorted; increases in permitted trade in these categories are given more weight in the sufficient condition for welfare improvement.

Theorems 2 and 3 are far more general than any which can be stated for tariff reform. To account for the difference, it is necessary to develop the implications of the analysis for the non-equivalence of tariffs and quotas. Intuitively, at any point \(Q^0\) (and given the other exogenous variables) the quota has a tariff equivalent \(P(Q^0, U^0)\). But for changes in \(Q\), the quota rent function \([P(Q,\pi, U^0) - P^*]Q = \hat{D}(Q,\pi, U^0) - P^*Q\) is a minimum value function while the tariff revenue function \(tZ(Q,\pi, U^0)\) is not. The tariff and quota revenue functions are thus not duals, there is non-equivalence through the general equilibrium budget constraint, and equivalence breaks down. Due to the minimum value property of the distorted trade distance function, simpler characterizations of welfare-improving trade reform are possible. The standard results on gradual tariff reform (Hatta (1977)) use the rule of uniform proportional cuts in all taxes, hence Hicksian aggregation to a single commodity, to finesse the problem of cross-price substitution effects on tariff revenue. The standard results are sufficient conditions which
do not include many cases of welfare improving reform; in contrast, the present approach locally characterizes all welfare improving quota reforms.

II.2. Tariff Reform with Quotas

For the case of a tariff reform in the presence of quotas the new term arising from interaction with quotas acts to decrease the set of welfare improving reforms, for practical purposes making further cuts in tariffs for developed countries welfare-decreasing.

Let $Z$ be the excess demand for all non-quota goods, with $\pi$ the vector of their domestic prices. From (19) and (16):

\[(24) \quad dv = dt'v_t = -\mu dt'b_t = \mu dt'(\omega Z' + Z'_\pi)\]

On the right hand side of (24) the second, "tariff distortion" term has a negative influence under the right conditions, such as uniform radial increases in taxes, but the first term is always positive for increases in taxes on imports of $Z$. The interpretation of (24) is facilitated if we convert it to a share/elasticity form:

\[(25) \quad dv = dt\tilde{Z}\tilde{Z}^{-1}v_t = \mu dt\tilde{Z}\left(\omega + \tilde{Z}^{-1}Z'_\pi\hat{\Pi}\tilde{\pi}\right) = \mu dt\tilde{Z}(\omega + \Phi \tau),\]

where $\tilde{\omega}$ is the column vector of $\omega$'s (the same for all sectors), $\tau$ is the ad valorem tariff vector, $\tilde{Z}$ is the diagonal matrix with the $Z$'s on the diagonal, $\hat{\Pi}$ is the diagonal matrix with $\pi$'s on the diagonal, and $\Phi$ is the matrix of price elasticities of compensated excess demand for the market group $Z$. Consider the size of each element of the column vector $\Phi \tau$. If all tariff-ridden goods are constrained—Hicks substitutes, the off-diagonal terms of $\Phi$ are positive, hence a lower bound for the second term is, for a change in $t_i$, $\Phi_{ii}t_i < 0$. Alternatively, if all tariffs in ad valorem form were the same, $\tau = \bar{\tau}$, and cut equiproporportionately, the second term would be $\bar{\Phi} \bar{\tau}$, where $\bar{\Phi}$ is the aggregate import.

\[1\text{For exports of } Z \text{ a rise in the tax lowers the domestic price, } Z \text{ is less than zero, hence the effect is also positive, "wrong-signed".}\]
price elasticity of demand for aggregate taxed imports. From (24), (25) and its implications:

**Theorem 4**  
(a) The presence of quotas with less than 100% rent retention reduces the set of welfare-raising tariff reforms. Furthermore:

(b) If all tariff-ridden good are constrained—Hicks substitutes, a cut in a tariff is welfare reducing whenever \( \omega \geq |\Phi_i \tau_i| \).

(c) when tariffs are uniform, a uniform cut in tariffs is welfare reducing if \( \omega \geq |\phi_i| \).

The intuition of Theorem 4 (a) is that reducing a tariff puts upward pressure on quota constrained prices, and this effect tends unambiguously to reduce welfare so long as there is less than 100% rent retention (\( \omega > 0 \)). It is tempting to conclude that the quota rent effect is always larger than the potentially wrong-signed tariff revenue effect, being a first-order term rather than a second-order term, but this is only a rule of thumb.

Parts (b) and (c) of Theorem 4 make this rule of thumb sensible for developed economies like the US, where VERs are important (for example, tariffs on quota-constrained goods recapture some of the rent in textiles, but \( \omega \) would appear to be at least 50%), while average tariffs on non-quota-constrained goods are very low, less than 5%. Thus there is a presumption that further tariff cuts in developed countries are welfare decreasing.

II.3. Quota Reform with Tariffs

Now we consider the symmetric case of quota reform in the presence of tariffs. Using (18):

\[
(26) \quad v_Q = -\mu b_Q = \mu (1-\omega)(P-P^*) + \mu Z_{\tilde{Q}}t.
\]
The first term is the same as in Theorem 1 and is always positive. The second term is new and must be evaluated. Let us multiply and divide the elements of the inner product by $\pi_i$. The ith element of the sum is $\tau_i \hat{\pi}_i Z_{Qj}$, for some quota $j$. The weighted average tariff for quota $j$ is

$$\bar{\tau}_j = \frac{\sum \hat{\pi}_i Z_{Qj}}{\sum \hat{\pi}_i Z_{Qj}}$$

If all $Z$'s are substitutes for $Q_j$ in the sense that $Z_Q$ has negative elements, the weights are all positive. For the case of "trade preferences" weakly separable with respect to the partition between taxed imports and other goods, it can be shown that $\bar{\tau}_j$ is the trade weighted average tariff, constant over $Q_j$; see our companion paper (1988a) for details\(^1\).

Now recall that $\hat{E}(Q, \pi, U^0) = \pi Z(Q, \pi, U^0)$, hence $\pi' Z_{Qj} = -P_j$. (26) becomes:

$$\hat{P}^{-1} v_t = \mu \left( (1-\omega) \rho - \bar{\tau} \right)$$

where $\bar{\tau}$ is the weighted average tariff vector (with equal elements in the weakly separable case), and $\rho$ is the ad valorem tariff equivalent of the quota rent vector (on the domestic price basis). The additional term $-\bar{\tau}$ acts in the "wrong" direction, due to quota reform causing a switch in spending away from tariff ridden goods. Nevertheless we can state:

**Theorem 5** (a) A marginal quota reform is welfare improving whenever the rate of retained rent exceeds the weighted average tariff rate, provided that quota and tariff goods are constrained substitutes.

(b) all small quota reforms in a direction above the plane with slopes formed by ratios of marginal values $(1-\omega) p_i - \bar{\tau}_i$ are welfare improving.

Noting that for developed countries, trade weighted average tariffs are very low, that some rent retention occurs, and that weak separability is almost always used in empirical work on the evaluation of trade reform, the impact of (27) is that the condition of Theorem 5(a) is met. Then essentially the conditions for using a local variant of

\(^1\)Intuitively, this follows from noting that with separability, the proportional expenditure allocation within the taxed group is invariant to changes in $Q$, so all elements $\pi_i Z_j$ change in proportion as $Q_j$ changes.
Theorem 1 obtain, and there is a presumption in contemporary developed countries that all quota increases are welfare-improving.

Note that (27) combined with Theorem 1 implies that with a pure VER system, $\omega=1$, $\mu=0$, and $(1-\omega)\rho - \tilde{\tau} < 0$ under the constrained substitutes assumption, hence a relaxation of a quota is welfare-improving. As $\omega$ falls from 1, both $\mu$ and $(1-\omega)\rho - \tilde{\tau}$ rise, and there can be a range of $\omega$ values for which quota reform is welfare reducing. This is one of the many paradoxes which can be generated under cases approaching the pure VER.

II.4. Large Country Case

Now we turn to the extension of the small country results to the large country case. The new difficulty is that $P^*,\pi^*$ are dependent on $Q$. At the optimal quota, the effects via $P^*,\pi^*$ vanish via the envelope theorem. Here we explore restrictions on the trade reform such that welfare is guaranteed to rise from arbitrary settings.

First suppose that $\pi^*=\pi$ is invariant to $Q$ (weak separability of foreign and domestic trade utility functions with respect to the partition between $Q$ and other goods suffices), but $P^*$ is not. If $P^*$ is convex in $Q$ in the world reduced form, the restricted economy is operating in the "right" (non-Giffen) portion of the foreign offer curve surface, as a brief consideration of the standard 2 good offer curve diagram makes clear.

Domestic price $P$ is a reduced form function of $Q$ and $\pi=P^*+t$. Welfare is:

$$v(Q,t) = (U^0 \mid \bar{E}(Q,\pi,U^0) - \bar{E}(Q,\pi,U^0) + (1-\omega)P^*(Q,t)Q + \omega\tilde{D}(Q,\pi,U^0) = 0)$$

Then by obvious steps, a version of Theorem 2 is available for $t=0$:

**Theorem 2'** For $\pi^*$ constant, a reform is welfare-improving if it on average increases trade of goods for which marginal benefit $P$ exceeds marginal cost $P^* + (P^*_Q)^T Q$.

Theorem 2' implies Theorem 2 at $P^*(\pi,Q)=0$, and implies the optimal tariff when $P=P^* + (P^*_Q)^T Q$. 

Now suppose that \( \pi \) can vary as a function of \( Q \). The effect is to add a component to the marginal cost of imports, \((*_{Q})^{T}Z\). Theorem 2' is easily amended. Mixed tariff and quota reform can also be handled as a routine exercise using the steps of Theorems 4 and 5 amended to properly measure the marginal cost of imports.

III. Multilateral Trade Reform

A significant application of the DTDF is to multilateral trade reform. Suppose that lump-sum compensation is available internationally so that the Pareto criterion is relevant. Hatta and Fukushima (1979) have applied the standard gradual reform techniques to show that a uniform proportionate cut in tariffs is Pareto-superior in a two good model with normality. This result can be very significantly generalized along the lines of Theorem 1: any reform which results on average in trade increases in high rent categories is Pareto-superior. The result extends to any number of goods, normality is not assumed, a complete characterization of Pareto-superior reforms is possible, and finally, compensation is self-financing: the specification of bilateral quotas and assignment of ownership of the licenses alone can ensure that initial utility is maintained for each country, or that a rise in utility is attained when there is an increase in efficiency.

Let \( \pi \) be the vector of prices for goods not subject to tariffs or quotas. Let \( Q^i_j \) be the vector of imports of country i from country j (hence \( Q^i_j = -Q^j_i \)). For the world as a whole, \( \Sigma Q^i_j = 0 \). Let \( \tilde{D}^i \) be the DTDF of country i. Suppose that we evaluate at initial equilibrium. Then a rise in \( \tilde{D}^i \) implies that some surplus revenue can be deducted from country i's trade account while still permitting maintenance of \( U^0_i \). We define \( \tilde{D} = \Sigma \tilde{D}^i \), and note that a rise in \( \tilde{D} \) implies that on balance revenue can be redistributed so that each country i can maintain \( U^0_i \) with no net revenue requirement change, while freeing some surplus for increasing the utility of at least one country. A Pareto-superior allocation of quotas arises when \( \tilde{D} \) rises due to the reallocation, since some \( Q \) is freed to
raise at least one nation's utility while maintaining all others'. Moreover, the reform is self-financing since the transfers necessary to maintain the net revenue requirements and the increase in utility of at least one country can be achieved through shifting the quota rents alone. This might involve offset to third party terms of trade effects by, for example, assignment to Japan of licenses for export of Hong Kong shirts to the US.

Let $Q$ denote the vector of all bilateral controlled trades $(Q^{ij})$. 

$$\tilde{D} = \Sigma_i \tilde{D}^i(\Sigma_j Q^{ij}, \pi(Q), U^{ij})$$

(33)

The vector $\pi$ is a function of the $Q$'s and $U$'s in the reduced form, but fortunately its properties are not needed. A change in $Q^{ij}=-Q^{ji}$ changes $\tilde{D}$ by

$$\partial \tilde{D}/\partial Q^{hl} = [P^h - P^l], \text{ since:}$$

$$\partial \tilde{D}^i/\partial Q^{hl}_k = P^h_k - \tilde{Z}^i \partial \pi/\partial Q^{hl}_k \text{ for } h=i,$$

$$\partial \tilde{D}^i/\partial Q^{ij}_k = -\tilde{Z}^i \partial \pi/\partial Q^{hl}_k \text{ for } h \neq i, \text{ and}$$

$$\Sigma_i \tilde{Z}^i \partial \pi/\partial Q^{hl}_k = 0, \text{ since at initial equilibrium, } \tilde{Z}^i = Z^i \text{ and } \Sigma Z^i=0.$$ 

Then a Pareto–superior allocation arises whenever:

$$[P^l-P^i]^i dQ^{ij} > 0.$$ 

(35)

Condition (35) is essentially in the form of the standard gains from trade criterion: the reform will be mutually beneficial if it follows comparative advantage by increasing flows with high relative premia. We have established:

**Theorem 6** A Pareto–superior trade reform occurs whenever on average trade increases in high rent categories.

We now develop the self–financing property of the reform. The rise in $\tilde{D}$ comes through the value of the incremental quota licenses (see (35)). Therefore it can be transferred through the ownership of those licenses, or equivalently through the transfer of the government revenue created by the auction of the licenses. No auxiliary lump sum tax and transfer power is needed to complete the reform. Note further that the local
condition for Pareto-superior reform is necessary hence it is complete.: all Pareto-superior reforms are covered. Sufficiency, on the other hand requires examining the details of each DTDF to find the reallocation of quota rents which offsets the move in π which negatively impacts some countries while aiding others.

One instance of Theorem 6 of special practical significance is the reallocation of VER permits from high cost to low cost suppliers, or among close substitutes, while leaving total imports to protected countries unchanged.

The intuition of Theorem 6 is readily seen in the standard Edgeworth exchange box diagram. The reform proposed is a move from \( Q^0 \) to \( Q^1 \). The DTDFs for the two countries rise in the case drawn. The criterion (35) allows any point \( Q^2 \) in the box \( AQ^0B \) southeast of \( Q^0 \) provided that the rays from the origins \( O \) and \( O^* \) to \( U_0 \) and \( U_0^* \) have associated prices such that (35) is met. For example, a point \( Q^2 \) on \( OH \) at its intersection with \( AQ^0 \) is permitted. At that point, enough surplus revenue \( \tilde{D} \) exists so that revenue sufficient to shift the consumption to \( U_0 \) or above can be taken from the foreign country via an assignment of ownership of quota licenses while still leaving the foreign country better off.

Figure 2. Pareto-superior Multilateral Trade Reform
P is the absolute gradient on OH at U^0 and P* is the absolute gradient on O*F at U*_0. The reform is Pareto–superior so long as \[ P(Q^1) - P^*(Q^1) \geq Q^1 - Q^0 \geq 0, \] as in the case shown. This line of graphical argument extends the reasoning of Theorem 6 to the discrete case. It can be formalized using the concavity of \( \tilde{D} \), hence \( \tilde{D} \) in the Q's along with the mean value theorem for a concave function.

**IV. A Distance Measure of Inefficiency**

The distance methods used above for evaluation of quota reform suggest an accounting method for measuring the inefficiency of trade. Figure 3 illustrates. U* is attainable with free trade. QF is found at F, due north of Q^0. The distance from Q^0 to QF measures the inefficiency of trade policy in Q space. Let \( Q^0 / \Delta^* = Q^F \) define \( \Delta^* \), which we call the coefficient of trade utilization, BC/BF in Figure 3.

**Figure 3. The Coefficient of Trade Utilization**

\[ \Delta^* = BC/BF \]

From the analysis of Part I, it is clear that OD = \( \omega[P-P^*]Q \), the transfer to foreigners. OE = \( (1-\omega)[P-P^*]Q \), the rent retained at home. Changes in Q shift the budget line CE by moving the intercept E, which we have shown essentially runs the analysis.
What does $\Delta^*$ signify? It is a scalar measure like the familiar income compensation measure, measuring by what factor the focus variable must be altered to maintain the reference utility. In this case, the focus variable is the vector of controlled trades. For quantitative restrictions on trade, distance is a more natural measure than the lump sum transfer of income used in the standard expenditure function approach, because it is entirely based on the instruments of distortion. We thus have found an appropriate use for the other, quantity-based pair of Hicks' (1944) four consumer surpluses.

How is this to be made operational? Measuring a general equilibrium distance in $Q$ space, as opposed to the "partial equilibrium" distorted trade distance function of Part I involves incorporating the general equilibrium budget constraint. This is done in the distorted trade utility function. Then distance can be measured from an arbitrary bundle $Q$ to an arbitrary distorted trade utility $v(\cdot) = U$. paralleling Hicks, we have "compensating" and "equivalent variation" measures of the total trade inefficiency. The "equivalent variation" measure is

**Definition 5** The Coefficient of Trade Utilization is $\Delta^*$ such that:

$$v(Q^0/\Delta^*, \pi) = U^*.$$  

This has a close relation to Debreu's coefficient of resource utilization for efficient production, hence its name. Defining $\Delta^*$ in this way implies that a value less than one is inefficient and the maximum value is one.

Because we deal in trade indifference frontiers there is also a "compensating variation" measure of the coefficient of trade utilization, which is:

$$v(Q^*/\Delta, \pi) = U^0,$$

where $Q^*$ is the free trade quota bundle and $U^0$ is the initial utility. In this case $\Delta > 1$ for distorted trade and a fall in $\Delta$ indicates welfare improvement from the initial position, with the free trade $Q$ yielding the minimal value of $\Delta$. Geometrically, on Figure 3 $\Delta$ would be the ratio of $Q^*$ to the level of $Q$ found on $U^0$ due south of $Q^*$. This version,
as usual, is easier to make operational. Clearly, $\Delta$ and $\Delta^*$ are the same function evaluated at different points, so we suppress separate treatment and focus on $\Delta^*$. Also, we need not exclusively concentrate on initial and free trade positions.

From its implicit solution in (36) $\Delta^*$ is homogeneous of degree one in $Q$, and using (20) has derivatives proportional to $P-P^*$. But we can go further. The distance function of Part I has an implicit definition similar to (36) and an explicit interpretation as a minimum value function. We now turn to the same extension.

The general equilibrium shadow price of a quota in the absence of tariffs is, by (20), $(1-\omega)[P-P^*]$. By analogy with the construction of distance in Part I, this suggests a distance measure in which the shadow value of the quota bundle is minimized. What is the normalization factor, the denominator of the distance measure? An "expenditure function" is suggested by the observation that $v(Q,\pi)$ is supported in $Q$ space by a distorted trade revenue budget line with value $(1-\omega)[P-P^*]Q$. The $Q$ bundle can be regarded as the outcome of the "expenditure"–minimizing selection of $Q$ subject to given prices $(1-\omega)[P-P^*]$ and given $U$. Let $R$ be the minimum value of revenue:

$$R((1-\omega)[P-P^*],\pi,U) = \min_Q \{ (1-\omega)[P-P^*]Q \mid v(Q,\pi) = U \}$$

Recalling (9'), adding and subtracting $PQ$ and rearranging, we obtain at equilibrium:

$$\pi Z + PQ - (1-\omega)[P-P^*]Q = 0.$$  

The normalization rule is $E(P,\pi,U^0) = 1 = \pi Z + PQ$, hence $R((1-\omega)[P-P^*],\pi,U) = 1$.

Then the explicit definition of $\Delta^*$ is dual to (38):

**Definition 5' The Coefficient of Trade Utilization is:**

$$\Delta^*(Q,\pi,U) = \min_{P-P^*} \{ (1-\omega)[P-P^*]Q \mid R((1-\omega)[P-P^*],\pi,U) = 1 \}.$$  

It is clear from (36) or (39) that $\Delta^*$ is a standard quantity index like those explored in the productivity literature. It has numerous operational implications which we
explore in our companion paper (1988b). We will conclude by noting one, reflecting Theorem 2.

A round of trade negotiations typically requires some method for measuring the degree of liberalization. By (39), using the homogeneity of degree 1 property of \( \Delta^* \) we have

\[
\frac{d\Delta^*}{\Delta^*} = \Sigma_i \theta_i \frac{dQ_i}{Q_i},
\]

where \( \theta_i \) is the quota rent share for quota \( i \), \( \frac{[P_i-P^*]Q_i}{[P-P^*]Q} \). (40) is such a simple observable expression that its rigorous general equilibrium foundations must be emphasized. Calculation of (40) could be routinely performed as a way of keeping score on the back of an envelope.
References


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