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Real Exchange Rates, Co-Integration and Purchasing Power Parity: Irish Experience in the EMS

by

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Real Exchange Rates, Co-Integration and Purchasing Power Parity:
Irish Experience in the EMS.

1) Introduction.

The volatility of real exchange rate movements during the
last ten to fifteen years has led to widespread skepticism about
the ability of the standard purchasing power parity (PPP) model to
dequately explain co-movements in nominal exchange rates and
relative prices of internationally traded goods. Although there is
general agreement that PPP does not appear to hold in the short
and intermediate runs, there remains considerable disagreement
over the validity of PPP in the longer run. For example, Dornbush
1976) and Aizenman (1986) attempt to rationalise deviations from
parity in terms of commodity markets characterised by slow price
adjustment interacting with flexible asset markets. These sticky
prices, or overshooting, models permit sustained deviations from
parity but typically maintain PPP as a valid long-run hypothesis.
However, Roll (1979) and Alder and Lehman (1983) have developed
theoretical models, based on efficient international capital
markets, which suggest that PPP is violated in the long-run. These
authors also present econometric evidence supporting the
hypothesis that real exchange rates follow a random walk, implying
that shocks have infinitely long-lived effects and that there is
no tendency to revert to parity in the long-run.

In an Irish context, Walsh (1983 and 1988) has documented the
volatility of real exchange movements since the foundation of the
European Monetary System (EMS) in 1979. For example, four years
after Ireland’s decision to participate in the EMS Walsh commented
that the Irish Experience ‘...may be seen as another example of a

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significant and relatively enduring change in real exchange rates, to be added to the list that has already been compiled by Frenkel (1981) ... it implies that the SOE view of inflation does not fit the facts over the short to medium run in Ireland.‘ (Walsh, 1983 p.178) The significance of this statement lies in the fact that it was the SOE view of inflation, or PPP, which provided the theoretical base for Ireland’s decision to participate in the EMS. In the period immediately preceding the formation of the EMS, sterling had been a relatively weak high inflation currency. Hence by breaking the parity link with sterling and fixing the nominal value of the Irish pound within a quasi-fixed exchange rate system based on the D-Mark, Ireland anticipated a decline in domestic inflation as the price level converged towards the EMS average. Further, if sterling continued to depreciate against EMS currencies (UK inflation remained high) then PPP would ensure that Ireland did not lose competitiveness against the UK in the sense that any nominal appreciation against sterling would be compensated for by an offsetting rise in the UK/Irish relative price.

These expectations were, however, unrealised during the initial years. Not only did British and Irish inflation rates remain relatively high, but sterling appreciated in nominal terms relative to participating currencies, with the consequence that Ireland experienced competitive gains vis-à-vis Britain and lost competitiveness within the system. Note that PPP implies that the appreciation of sterling should have been accompanied by a relative decline in the British inflation rate thereby maintaining Ireland’s competitive position against the UK. Further, when the British inflation rate eventually declined it was accompanied by a nominal depreciation of sterling which eroded the initial
competitive gains to the point, in March 1983 and again in August 1986, at which Ireland decided to devalue against the other EMS currencies in order to restore a competitive position against the UK. Hence the apparent failure of nominal exchange rates and relative prices to move in accordance with the predictions of PPP has led to a policy dilemma in the sense that Ireland has to balance its commitment to EMS parities with the objective of maintaining a reasonable level of competitiveness against its most important trading partner, the UK.

However, it is important to note that an adjustable peg policy within the EMS is not necessarily inconsistent with PPP. For example, in the context of a Dornbusch-type sticky price model, the speed of adjustment towards parity may be so slow as to justify direct intervention designed to moderate the extent to which the nominal exchange rate overshoots its long-run equilibrium level. In this case the underlying trend will be towards parity and realignments of the nominal exchange may be interpreted as measures which smooth the adjustment process. On the other hand, if the real exchange rate follows a random walk, as suggested by Alder and Lehman for example, then there will be no tendency towards reversion and realignments of the nominal rate will not necessarily move the real exchange rate towards its parity level.

Hence the purpose of this paper is to utilise time series data on Irish pound real exchanges against sterling, the dollar and the D-Mark to test the hypothesis that the behaviour of these series is inconsistent with the long-run implications of PPP. These implications are outlined in the next section - that is, that PPP implies that a given real exchange rate series must be
generated by a stationary process with a time invariant mean and variance.

(2) Implications of PPP.

I define the real exchange rate as the product of the nominal spot exchange rate and the ratio of foreign to domestic prices. Working in logarithms the real exchange rate may be expressed as:

$$ q_{jt} = s_{jt} + r_{jt} \quad (1) $$

where $s_j$ is the domestic currency price of currency $j$ and $r_j$ is the ratio of the price level in country $j$ to the domestic price level. PPP permits $q_j$ to deviate from parity in the 'short-run', but requires convergence to an equilibrium in the 'long-run'. In a univariate context, this long-run convergence to parity can be interpreted as implying that the generating mechanism for the real exchange rate is a stationary process with a time invariant mean and variance. To illustrate the concept of stationarity assume that the evolution of the real exchange rate can be modeled by the simple AR(1) process:

$$ q_{jt} = \gamma + \beta q_{jt-1} + u_{jt} \quad (2) $$

which solves as:

$$ q_{jt} = \gamma \sum_{0}^{\infty} \beta^i u_{jt-i} \quad (3) $$

where $|\beta| < 1$ and $u_j \sim (0, \sigma^2)$. The mean and variance of $q_j$ are given by:

$$ \text{EC}(q_j) = \gamma (1-\beta)^{-1} \quad \text{VAR}(q_j) = \sigma^2 (1-\beta^2)^{-1} $$

and the covariances between $q_{jt}$ and $q_{jt-k}$ by:

$$ \text{COV}(q_{jt}, q_{jt-k}) = \beta^k \sigma^2 (1-\beta^2)^{-1} $$

regardless of whether PPP is defined in its absolute ($\gamma = 0$) or relative ($\gamma = \text{const.}$) sense, the expected value and variance of $q_j$ are finite and time invariant so that the expectation of $q_j$ at
time \( t+k \) is independent of shocks at time \( t \)\(^1\). Further, although this process has an infinite memory, the covariances between \( q_{jt} \) and \( q_{jt+k} \) will decline with time so that the magnitude of the dependence of \( q_{jt+k} \) on \( u_t \) diminishes with \( k \).

If the real exchange can be modelled as a stationary process then it is said to be integrated of order zero, or \( q \sim I(0) \). That is, \( q \) is stationary in its level. If, on the other hand, \( \beta = 1 \) in (2) so that \( q_j \) follows a random walk (with drift if \( \gamma = \text{const.} \)) then the mean and variance will be undefined and the series will be non-stationary with deviations from parity increasing over time. Note that in this case \( q \) is stationary in first differences and is said to be integrated of order one, or \( q \sim I(1) \).\(^2\) Hence an appropriate test of long-run PPP is that \( q_t \sim I(1) \) against the alternative that it is \( I(0) \). PPP requires that the null be rejected.

Note that the requirement that the real exchange rate be a stationary series does not necessarily imply that the constituent series (\( s_t \) and \( r_t \)) must themselves be stationary. If the nominal exchange rate and relative price term are individually \( I(0) \) then it is generally true that linear combinations such as the real exchange rate will also \( I(0) \). However it is possible that \( s_t \) and \( r_t \) may each be \( I(1) \), but there exists a linear combination of these variables which is a stationary \( I(0) \) series. If this is the case then PPP holds in the sense that a shock will cause \( s_t \) and \( r_t \)

\(^1\)Note that this representation of the real exchange is broadly consistent with the exchange rate dynamics implied by 'sticky-price' type models, Dornbusch (1978), which rationalise deviations from parity in terms of flexible asset markets interacting with commodity markets characterised by slow price adjustment.

\(^2\)With \( \beta = 1 \), \( \Delta q_{jt} = u_{jt} \) so that \( E(\Delta q_j) = 0 \) and \( VAR(\Delta q_j) = \sigma^2 \).
to drift apart initially but converge towards a stationary process in the longer-run. A simple example is to treat PPP as an economic theory which suggests a long-run equilibrium relationship of the form:

\[ s_t = \alpha + \beta r_t \]

with \( s_t \) and \( r_t \) integrated of the same order. If \((\alpha, \beta) = (0, -1)\) then the real exchange rate measures the of equilibrium error between \( s_t \) and \( r_t \) which, if PPP is a valid hypothesis, must be I(0). More generally, if \( s_t \) and \( r_t \) are both I(1) but there exists a vector \((\alpha^*, \beta^*)\) such that:

\[ z_t = s_t - \alpha^* - \beta^* r_t \]

(4)
is a stationary I(0) process, then PPP may be said to describe an equilibrium relationship between the nominal exchange rate and relative prices in the sense that these variables may drift apart in the 'short-run' but converge towards a common trend in the 'long-run'. When this is the case then, using the terminology of Engle and Granger (1987), \( s_t \) and \( r_t \) are said to be co-integrated. Section (4) discusses co-integration in more detail. Prior to that Section (3) presents results from univariate tests on the hypothesis that \( q_t \) is I(1).

(3) Univariate Tests for Purchasing Power Parity.

Dickey and Fuller (1981) provide an appropriate test of the hypothesis that a series \( x_t \) is I(1) against the alternative that it is I(0). In the Augmented Dickey-Fuller regression:

\[ \Delta x_t = \alpha + \gamma T + \beta x_{t-1} + \sum_{i=1}^{m} \gamma_i \Delta x_{t-i} + e_t \]

(5)

where \( T \) denotes a time trend and \( m \) is large enough to ensure that the residuals are white noise, \( x_t \) is I(1) without drift if we cannot reject the hypothesis that \( \alpha = \gamma = \beta = 0 \) or \( x_t \) is I(1) with drift if \( \gamma = \beta = 0 \). If, for example, the first hypothesis is
maintained then the appropriate modelling procedure for \( x_t \) is an AR(1) process which is integrated order of order one, or a random walk without drift.  

Table 1 gives the Augmented Dickey-Fuller (ADF) statistics for three real exchange rates — the Irish pound against sterling, the US dollar and the German mark. The statistics \( \hat{\beta}_2 \) and \( \hat{\beta}_3 \) test the hypotheses that the real exchange rate follows a random walk with drift (\( \hat{\beta}_2 \)) and without drift (\( \hat{\beta}_3 \)). Note that under the null hypothesis that \( x_t \) is I(1) these statistics will not have the standard F-distribution. However, critical values are given in Dickey and Fuller (1981). The data are monthly and the sample period is 1980(1) to 1987(12). The nominal exchange rate series are taken from the Central Bank of Ireland Quarterly Bulletin (various issues) and the prices are indices of wholesale prices based on 1980 = 100 taken from the OECD Main Economic Indicators.

The ADF statistics for the full sample period, 1980(1) to 1987(12), reject the null hypothesis that the Irish pound/sterling real exchange rate is I(1) at the 5% level but accept it for the real exchange rate against the dollar. Tests on the D-Mark real exchange rate accept the random walk hypothesis at the 5% level but reject it at the 10% significance level. Hence while the sterling rate appears to be a stationary series, as implied by PPP, the other series are non-stationary and at variance with the predictions of PPP. Results for the D-Mark are, however, inconclusive.

\[ ^3 \text{If either } \alpha = \beta = \gamma = 0 \text{ or } \beta = \gamma = 0 \text{ is maintained then both the mean and variance on } x_t \text{ are undefined and the series will display the characteristics of a random walk. For example if } m = 1 \text{ and } \alpha = \beta = \gamma = 0 \text{ then the mean of } x_t \text{ is given by } \sigma^2 \left( 1 - \gamma_1 + \gamma_1 \right)^{-1} \]
(4) Co-Integration and PPP.

The results from the previous section reject the random walk hypothesis for the sterling real exchange rate, but accept it, at the 5% significance level, for Irish pound real exchange rates against the dollar and the D-Mark. However, at least three objections may be raised against these tests. First, their power may be relatively weak against plausible alternatives such as $q_t$ being AR(1) with a autocorrelation coefficient close to, but significantly different from, unity. Second, if the variance of the real exchange rate series is dominated by one of the constituent variables (nominal exchange rate and relative price) which itself follows a random walk then the univariate tests may be biased towards accepting the random walk hypothesis. If, for example, prices are slow to adjust to nominal exchanges rate changes, then short-run changes in real exchange rates will reflect movements in nominal exchange rates. Hence, if the nominal exchange follows a random walk then univariate tests may suggest that the real exchange rate is also a random walk even though the constituent series converge to parity in the long-run. Third, univariate tests impose a $(0,-1)$ constraint on the vector $(\alpha^*, \beta^*)$ in equation (4) which implies that PPP holds only in its absolute form and that the nominal exchange is linear homogeneous with respect to the relative price term. In what follows I treat PPP as a more general proposition which simply suggests the existence of a long-run equilibrium relationship between the nominal exchange rate and the corresponding relative price term, but does not assume that this relationship is homogeneous of degree one.

An alternative approach to testing for long-run PPP which utilizes the individual series on nominal exchange rates and relative prices is based on the concept of co-integration. Engle
and Granger (1987). If PPP is a valid hypothesis then the variables $s_t$ (the nominal exchange rate) and $r_t$ (relative price) cannot drift apart in the long-run. Starting from a position of parity (say, $q_t = 0$) a random shock may cause $s$ and $r$ to diverge, but such deviations must be transient with the equilibrium relationship being restored in the long-run. Put another way, if real exchange rate is generated by a stationary process then $s_t$ and $r_t$ must share a long-run equilibrium relationship.

Engle and Granger (1987) formalise the idea of variables sharing an equilibrium relationship in terms of co-integration between time series. To illustrate, consider two series $x$ and $y$ both of which are I(1). Linear combinations of $x$ and $y$ will, in general, also be I(1). However it is possible that there exists a vector $A$ such that the combination:

$$z_t = x_t - Ay_t$$

is I(d-b) $b > 0$. If $A$ exists then $z_t$ may be interpreted as an equilibrium error and $x_t$ and $y_t$ are said to be co-integrated of order $(d,b)$, or CI$(d,b)$. Hence, if $d=b=1$ the existence of $A$ implies that the equilibrium error is I(0) and the combination (6) is a stationary process. Engle and Granger (1987) also show that when $x_t$ and $y_t$ are co-integrated then there exists an error correction model (ECM) of the form:

$$\Delta x_t = \alpha_1 + \gamma_1 z_{t-1} + \sum_i \gamma_{x_i} \Delta x_{t-1} + \sum_i \mu_{x_i} \Delta y_{t-1} + \epsilon_{xt}$$

$$\Delta y_t = \alpha_2 + \gamma_2 z_{t-1} + \sum_i \gamma_{y_i} \Delta x_{t-1} + \sum_i \mu_{y_i} \Delta y_{t-1} + \epsilon_{yt}$$

where one of $\gamma_1, \gamma_2$ is non-zero. Note that if $x$ and $y$ are both I(1)

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5. That is, both $x$ and $y$ are stationary after differencing $d$ times.
and are also CI(1,1) then every term in (7) will be I(0). If, on the other hand, \( x \) and \( y \) are not CI(1,1) then \( z \) will not be I(0) and will have no place in (7). For example, if \( x \) and \( y \) are CI(1,1) then \( z_t \) represents the deviation from equilibrium in period \( t \), and the ECM determines the proportion of the disequilibrium which is corrected in period \( t+1 \). Hence if \( z_t \) is I(0) with, say, \( EC(z_t) = 0 \) then \( x \) and \( y \) will eventually converge to an equilibrium. However, if \( x \) and \( y \) are not CI(1,1) then they cannot share an equilibrium relationship and the error correction term \( z \) will have no place in (7). It follows that we can accept PPP in the sense that it predicts a long-run equilibrium relationship between \( s_t \) and \( r_t \) if these variables are found to be co-integrated with an I(0) equilibrium error.

Engle and Granger (1987) suggest a two-step estimation procedure to test for co-integration between time series. At the first stage the co-integrating vector \( A \) is estimated and the residuals are tested for a unit root — that is, that they are I(1). Given that the unit root hypothesis can be rejected at step one, the residuals from the prior co-integrating regressions can replace level terms in the error correction model in step two. Hence if the series are co-integrated the error correction terms, residuals from step one, will be I(0) and play a significant role in explaining the dynamics of the system. If, on the other hand, the error correction terms are not I(0) then they will be insignificant in the ECM and the variables in question cannot be said to be co-integrated.

An appropriate sequence of tests for co-integration requires the following steps. First, the Dickey-Fuller test outlined above can be used to test the hypothesis that the individual series on
$s_t$ and $r_t$ are I(1). Second, given that $s_t$ and $r_t$ are both I(1),
Stock (1987) shows that if the series are co-integrated then an
OLS regression of $s_t$ on $r_t$ (or $r_t$ on $s_t$) provides an efficient
estimator of the co-integration vector $A$. Third, the hypothesis of
no co-integration requires that the residuals from the
cointegrating regressions are a non-stationary process. In what
follows I present the results of three tests on the null
hypothesis that $u_t \sim I(1)$, where $u_t$ are the residuals from the
cointegrating regression.

(i) An augmented Dickey-Fuller test based on the OLS regression:

$$\Delta u_t = \beta u_{t-1} + \sum_{i=1}^{m} y_i \Delta u_{t-i} + \varepsilon_t$$  \hspace{1cm} (8)

If $u_t \sim I(1)$ then the OLS estimator for $\beta$ in (8) should be
insignificantly different from zero, with the rejection region
consisting of relatively large negative values for the ratio of $\beta$
to its estimated standard error. However, as Engle and Granger
(1987) point out, the Dickey-Fuller (1981) test statistics are
inappropriate when the co-integrating parameter $A$ is unknown and
has to be estimated. If, for example $u_t$ is estimated from an OLS
regression of $s_t$ on $r_t$ then the Dickey-Fuller statistics will be
biased towards rejecting the null hypothesis that $u_t$ is I(1)
because OLS 'seeks the value of $A$ which is minimizes the residual
variance and therefore is most likely to be stationary' (Engle and
Granger, 1987, p. 265.). Further, as in the standard Dickey-Fuller
case, the test statistic will not have a t-distribution under the
null. Hence Engle and Granger provide critical values for the ratio
of the OLS estimator for $\beta$ to its standard error based on the
assumption that the null is true.

(ii) An alternative test for co-integration, suggested by Sargan
and Bhargava (1983), is that the Durbin-Watson statistic from the co-integrating regression is significantly different from zero. If, for example, the residuals follow a random walk with a first-order autocorrelation coefficient equal to unity then the DW statistic should be approximately zero. Hence low values for the DW favours acceptance of the null hypothesis of no co-integration.

Note that the DW and ADF tests may have low power against plausible alternatives such as a stationary autoregressive process with an autocorrelation coefficient close to unity. Hence the second stage estimation of the ECM which uses the residuals from the prior level regressions provides an additional check on the co-integration hypothesis. The relatively low power of DW and ADF tests is due to the fact that they attempt to distinguish between series that have no random walk component and series that have a random walk component. In the former $EC(u_{t+k})$ is independent of shocks to $u_t$, while a unit shock at time $t$ increases $EC(u_{t+k})$ by one unit in the the case of a random walk. However, as Cochrane (1988) shows, any series which is first difference stationary can be decomposed into a stationary, or temporary component, and a random walk, or permanent, component. In tests for a unit root, such as DW and ADF, the null assumes that the temporary component is zero and tests for the existence of the latter. Consequently these tests will have difficulty in distinguishing between a stationary series and a series in which the random walk component relatively small. If the series is stationary without a random
walk component then it will exhibit complete reversion to a constant expected value. On the other hand, if the series contains a random walk component then it may exhibit partial mean reversion, with the dependence of \( E(u_{t+k}) \) on shocks at time \( t \) depending on the relative importance on the random walk component. The third test on the residuals of the co-integrating regressions, discussed below, attempts to assess the importance of the permanent component in \( u_t \).

(iii) A variance ratio test based on Cochrane (1988). If \( u_t \) follows the random walk model:

\[
  u_t = u_{t-1} + \varepsilon_t
\]

where \( \varepsilon_t \sim (0, \sigma^2) \) the variance of the first difference if simply \( \sigma^2 \), and the variance of \( (u_t - u_{t-k}) \) is \( k\sigma^2 \), so that the ratio:

\[
  VR = (1/k) \text{VAR}(u_t - u_{t-k}) / \text{VAR}(\Delta u_t)
\]

should equal one for all values of \( k \). If, on the other hand, \( u_t \) is a stationary process with a small random walk component, then VR should decline with \( k \). That is, the greater the decline in VR the smaller the random walk, or permanent, component in \( u_t \).

Tables 2 to 5 give the results of co-integration tests of real exchange rates for the Irish pound against sterling, the US dollar and the German mark. Table 2 gives Augmented Dickey-Fuller statistics (1981) on the null hypothesis that each series is I(1) while Table 3 gives the results from ADF and DW tests on the co-integrating regressions. The variance ratio tests and error correction models are given in Tables 4 and 5 respectively. The results can be summarised as follows.

First, the Dickey-Fuller statistics in Table 2 appear to accept the hypotheses that the individual series on nominal change rates \( s_t \) can be modelled as a random walk without \( t \). Note, however, that the data rejects the second hypothesis,
random walk with drift, for the nominal exchange rates against sterling and the dollar at the 10% significance level and is close to rejecting the first hypothesis at the 5% level. Further, whereas the first hypothesis is accepted for the sterling and dollar relative price series, it is clearly rejected by the D-Mark relative price series.

Second, the Sargan-Bhargava DW tests on the co-integrating regressions in Table 3 unambiguously accept the hypothesis of no co-integration for all three real exchange rates. Over the full sample period the DW statistics are insignificantly different from zero in all cases, indicating the residuals from the co-integrating regressions are AR(1) with a unit first order autocorrelation coefficient - that is, a random walk. Third, the ADF tests on the residuals of the co-integrating regressions give weak support for co-integration on Irish/UK and Irish/German data but reject co-integration between the Irish/dollar nominal exchange rate and the ratio of US to Irish prices. Given that there is some conflict between the results of these tests it is relevant to note that the Engle and Granger (1987) simulations suggest that the ADF test has greater power against plausible alternatives.

Fourth, the variance ratio tests, Table 4, suggest that the random walk components of the residuals from the D-Mark co-integrating equations are small relative to the temporary components. In each case 1/k times the variance of the k differences settles down to approximately 10% of the variance of month to month changes suggesting that these series are dominated

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7 As the results presented so far unambiguously reject PPP on Irish/US data, the dollar terms at dropped from the subsequent analysis.
by stationary, or temporary, components. However, whereas the residuals from the OLS regression of the sterling nominal exchange on relative prices also exhibits a significant stationary component, the relative price innovations are consistent with a random walk. That is, the ratio of $1/k$ times the variance of $k$ differences to the variance of first differences appears to be insignificantly different from unity up to $k$ equal to 50 months. Hence, the VR test tends to reject the null of no co-integration on the Irish/German data, but accept it on Irish/UK data.

Table 5 presents the results from error correction models for the series on nominal exchange rates and relative prices for sterling and the D-Mark. In each case the ECM was selected by first estimating an unrestricted vector autoregression with first differences of $s_t$ and $r_t$ regressed on $s_{t-1}$ and $r_{t-1}$ and six lags on $\Delta s_t$ and $\Delta r_t$. The error correction models were then specified to include significant lags on $\Delta s_t$ and $\Delta r_t$ and the lagged residuals from the co-integrating regression - the error correction term. The CHOW, LM and ARCH statistics are diagnostic checks for parameter stability, aurocorrelation and heteroscedasticity respectively, and FEXC is an F-test on the hypothesis that the coefficients on the omitted lags from the vector autoregression are jointly zero. Note, however, that the co-integrating regressions for sterling, Table 3, differ significantly with the choice of dependent variable. For example, normalising the sterling relative price equation on $s_t$ gives a slope coefficient of -2.222 or more than twice the absolute value of the slope coefficient in the nominal exchange rate regression. Doing the same normalisation on the D-Mark relative price regression gives a slope coefficient of -0.896 which is virtually identical to the corresponding coefficient in the alternative co-integrating regression. The
significance of this result is that the equilibrium solution for
the sterling error correction models (Δ terms = 0) will differ
according to which error correction term is used. Hence the ECM’s
for sterling are estimated both ways. Columns 1 and 2 in Table 5
give the ECM’s for the sterling nominal exchange rate and relative
price using the residuals from their respective co-integrating
regressions while columns 3 and 4 use the residuals from the
alternative regression.

The results from the error correction models show some
consistency with the ADF tests in Table 3. When assessing the
significance of the error correction term it is important to
remember that the 't' statistic will be invalid under the null
that z is I(1). Hence, Banerjee et. al. (1986) suggest, on the
basis of a simulation study, a rejection region consisting of 't'
statistics in excess of three. On this criterion the data is
clearly capable of identifying an ECM for the D-Mark nominal
exchange rate, although there is evidence of parameter
instability, but not for the corresponding relative price term,
which can be interpreted as implying that the relative price term
is exogenous even though the variables may be co-integrated in the
sense that an ECM can be found. For example, in the ECM for the
D-Mark exchange rate, column 5 of Table 5, a fall in the
equilibrium error z_{t-1} (real exchange rate overvalued relative to
parity) is associated with a significant positive change in s_t
(depreciation of the nominal exchange rate) but has no impact on
the adjustment of relative prices.

The data also identifies an ECM for the Irish pound/sterling
nominal exchange rate and for the corresponding relative price

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8That is column 1 uses the residuals from s regressed on r, while
column 3 uses the residuals from r regressed on s etc.
term, although the error correction terms would be judged insignificant on the Banerjee et al. 'greater than three rule'. Further, when the sterling error correction terms are reversed, columns 3 and 4, \( z_{t-1} \) becomes insignificant in the nominal exchange rate equation implying that the variation in \( r_t \) which is not attributable to \( s_t \) plays no role in the dynamics of the latter.

Finally, an important result from Engle and Granger (1987) is that variables which are co-integrated must not only obey an ECM, but data generated from the latter must be co-integrated. Hence an additional test on the ECM is to relax the coefficient restrictions implied by the co-integrating regression and to include \( s_{t-1} \) and \( r_{t-1} \) as separate regressors in the dynamic specification. If the variables are co-integrated then intuitively we might expect the equilibrium solution to these unrestricted ECM's to be similar to the estimated co-integrating regression in Table 3. Table 6 gives estimates of unrestricted models\(^9\). The statistics \( F_1 \) and \( F_2 \) are \( F \)-tests on the hypotheses that the coefficients on the constant, \( r_{t-1} \) and \( s_{t-1} \) (\( F_1 \)) and the coefficients on \( s_{t-1} \) and \( r_{t-1} \) (\( F_2 \)) are jointly zero. These restrictions are equivalent to imposing a zero co-integrating vector on the data and, with one exception, they are rejected thereby giving support for the ECM - that is, the significance of the lagged levels indicates that these terms have a role in explaining the dynamics of nominal exchange rates and relative prices.

The bottom of Table 6 gives the implied co-integrating

\(^9\)Note that the regressions in Table 6 include a constant because this is included in the co-integrating regressions. Constant terms in the ECM in Table 5 were insignificantly different from zero.
equations ($\Delta$ terms = 0) for the unrestricted ECM's. Whereas both
the estimated and implied co-integrating regressions for the
D-Mark are similar, and the equations for $r_t$ are approximately
equal to the inverse of the $s_t$ equations, this is not so for the
sterling equations. As a final check on the ECM I computed the
Dickey-Fuller (1981) $\delta_1$ and ADF statistics using the computed
residuals from the co-integrating equations implied by the
unrestricted models\(^{10}\). The ADF statistics accept the hypothesis
that these residuals are I(1) in all cases. However, the
Dickey-Fuller $\delta_1$ statistic rejects the null of a random walk for
the residuals derived from the co-integrating vector for the
D-Mark nominal exchange rate.

Further, it is of interest to note that the lagged sterling
nominal exchange rate has a significant coefficient in the ECM for
$\Delta s_t$ while the relative price term is insignificant. Not only does
this reflect the result in column 3 of Table 5, but it also points
to the possibility that the sterling exchange rate is a stationary
autoregressive series\(^{11}\). If this is the case then, given that $r_t$ is
I(1), the sterling exchange rate and the ratio of UK to Irish
wholesale prices cannot be co-integrated. Results from the
unrestricted D-Mark ECM's, on the other hand, are consistent with
Table 5 in that both level terms are significant in the exchange
rate equation but insignificant in the relative price equation.
Hence the German/Irish price ratio appears to be weakly exogenous
even though it may be co-integrated with the D-Mark nominal

\(^{10}\) $\delta_1$ is a test on $(\alpha, \beta) = (0,1)$ in $x_t = \alpha + \beta x_{t-1}$. Critical values
are: 8.70(1%), 4.71(5%) and 3.86(10%).

\(^{11}\) Recall that the Dickey-Fuller tests in Table 2 were marginal at
the 5% significance level. Also if $s$ is I(0) and dominates the
variance of $q$, then the tests in Section (3) are likely to suggest
that $q$ is also I(0).
exchange rate.

(5) Summary and Conclusions.

The major conclusions arising from this paper may be summarised as follows. The univariate tests in Section 3 support PPP between Ireland and the UK, but reject the hypothesis on Irish/US and Irish/German data, although the latter accepts the PPP hypothesis at the 10% significance level. However, a major defect of these univariate tests is that they impose a (0,-1) constraint on the equilibrium relationship between the nominal exchange rate and relative prices. In terms of the co-integration approach discussed in Section 4, this is equivalent to imposing a prior restriction on the co-integrating vector which, of course, may not exist. When this constraint is relaxed, and PPP is defined more generally as a theory which simply suggests a long-run equilibrium relationship between the nominal exchange rate and the corresponding relative price term, the co-integration analysis fails to find a unique long-run relationship between the Irish/UK series. Further, the error correction estimates in Tables 5 and 6 indicate that the adjustment process for the sterling nominal exchange rate is determined by the lagged level of that series but is independent of the lagged relative price term. One interpretation of this result is that the series are integrated of different orders, with the nominal exchange rate being stationary. If this is the case, the UK series cannot be co-integrated and PPP will not hold in the long-run.

Co-integration tests on Irish/German data, on the other hand, appear to be more supportive of PPP. ADF and variance ratio tests on the residuals from the co-integrating regressions are consistent with the hypothesis that these series have significant stationary components. The data is also capable of identifying an
appropriate error correction model and the estimates are reasonably robust against the direction of the test. That is, the equilibrium solution is similar regardless of whether the error correction model is unconstrained or restricted by the prior level co-integrating regression.

Hence from an Irish perspective the EMS has been a success in the sense that co-movements in the Irish Pound/D-Mark nominal exchange rate and the corresponding relative price term appear to be consistent with the predictions of PPP, namely long-run convergence to an equilibrium parity value. However, the failure to find a similar relationship between the sterling nominal exchange rate and the ratio of UK to Irish wholesale prices, suggests that PPP does not hold between Ireland and its most important trading partner. Further, results from the variance ratio tests and error correction models, suggest that the sterling nominal exchange rate is a stationary series while the corresponding price ratio is non-stationary. If this is the case then we must take considerable care in assessing the long-run implications for the real exchange rate, and hence competitiveness against the UK, of shocks to the nominal exchange rate such as Irish realignments within the EMS.
Table 1. Augmented Dickey-Fuller Statistics Equation 5.

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sterling</td>
<td>5.629</td>
<td>8.329</td>
</tr>
<tr>
<td>Dollar</td>
<td>3.151</td>
<td>4.707</td>
</tr>
<tr>
<td>D-Mark</td>
<td>4.545</td>
<td>6.429</td>
</tr>
</tbody>
</table>

$\delta_2$ is the Dickey-Fuller (1981) statistic for $H_o: \alpha = \gamma = \beta = 0$ in equation (5). Critical values are 6.50(1%) 4.88(5%) and 4.16(10%).

$\delta_3$ is the Dickey-Fuller (1981) statistic for $H_o: \gamma = \beta = 0$. Critical values are 8.73(1%) 7.44(5%) and 5.47(10%).

Table 2. Augmented Dickey-Fuller Statistics

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_2$</th>
<th>$t_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sterling</td>
<td>4.228</td>
<td>6.290</td>
<td>1.952</td>
<td>2.927</td>
</tr>
<tr>
<td>Dollar</td>
<td>4.222</td>
<td>5.053</td>
<td>4.314</td>
<td>1.921</td>
</tr>
<tr>
<td>D-Mark</td>
<td>4.380</td>
<td>2.933</td>
<td>8.079</td>
<td>4.900</td>
</tr>
</tbody>
</table>

$\delta_2$ is the Dickey-Fuller (1981) statistic for $H_o: \alpha = \gamma = \beta = 0$ in equation (5). Critical values are 6.50(1%) 4.88(5%) and 4.16(10%).

$\delta_3$ is the Dickey-Fuller (1981) statistic for $H_o: \gamma = \beta = 0$ in equation (5). Critical values are 8.73(1%) 7.44(5%) and 5.47(10%).
### Table 3: DW and ADF Tests On Co-integrating Regressions

Sample Period: 1980(1)-1987(12)

<table>
<thead>
<tr>
<th>Currency</th>
<th>Regression</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sterling</td>
<td>$s_t = 0.093 - 1.032r_t$</td>
<td>DW = 0.205 ADF = -3.001</td>
</tr>
<tr>
<td></td>
<td>$r_t = 0.004 - 0.450s_t$</td>
<td>DW = 0.135 ADF = -2.380</td>
</tr>
<tr>
<td>Dollar</td>
<td>$s_t = -0.605 - 1.408r_t$</td>
<td>DW = 0.080 ADF = -0.426</td>
</tr>
<tr>
<td></td>
<td>$r_t = -0.325 - 0.390s_t$</td>
<td>DW = 0.038 ADF = 0.625</td>
</tr>
<tr>
<td>D-Mark</td>
<td>$s_t = -1.325 - 0.847r_t$</td>
<td>DW = 0.293 ADF = -2.971</td>
</tr>
<tr>
<td></td>
<td>$r_t = -1.503 - 1.115s_t$</td>
<td>DW = 0.289 ADF = -3.207</td>
</tr>
</tbody>
</table>

Critical values for DW are 0.511(1%), 0.386(5%) and 0.322(10%).

Critical values for ADF $(H_0: \beta = 0$ in Eq. 7) are 3.77(1%), 3.17(5%) and 2.84(10%). ADF is a test on $\beta = 0$ in the regression:

$$u_t = \beta u_{t-1} + \sum_{i=1}^{4} \beta_i \Delta u_{t-i}.$$
### Table 4. Variance Ratio Tests.

Residuals From Co-integrating Regressions.

<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Sterling</th>
<th></th>
<th></th>
<th>D-Mark</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s_t )</td>
<td>( r_t )</td>
<td>( s_t )</td>
<td>( r_t )</td>
<td>( s_t )</td>
<td>( r_t )</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>1.13</td>
<td>1.01</td>
<td>1.01</td>
<td>(0.04)</td>
<td>(0.03)</td>
</tr>
<tr>
<td>3</td>
<td>1.39</td>
<td>1.17</td>
<td>0.90</td>
<td>0.89</td>
<td>(0.08)</td>
<td>(0.05)</td>
</tr>
<tr>
<td>4</td>
<td>1.45</td>
<td>1.15</td>
<td>0.74</td>
<td>0.72</td>
<td>(0.11)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>5</td>
<td>1.43</td>
<td>1.09</td>
<td>0.62</td>
<td>0.60</td>
<td>(0.14)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>6</td>
<td>1.36</td>
<td>1.03</td>
<td>0.66</td>
<td>0.63</td>
<td>(0.15)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>7</td>
<td>1.28</td>
<td>0.98</td>
<td>0.69</td>
<td>0.68</td>
<td>(0.16)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>8</td>
<td>1.23</td>
<td>0.97</td>
<td>0.71</td>
<td>0.68</td>
<td>(0.17)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>9</td>
<td>1.18</td>
<td>0.95</td>
<td>0.67</td>
<td>0.64</td>
<td>(0.18)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>10</td>
<td>1.11</td>
<td>0.91</td>
<td>0.65</td>
<td>0.62</td>
<td>(0.17)</td>
<td>(0.11)</td>
</tr>
</tbody>
</table>

20
| 20       | 0.41    | 0.79 | 0.61 | 0.58     | (0.04) | (0.17) | (0.11) | (0.09) |
| 30       | 0.35    | 0.87 | 0.39 | 0.37     | (0.05) | (0.32) | (0.06) | (0.06) |
| 40       | 0.29    | 0.84 | 0.10 | 0.09     | (0.05) | (0.40) | (0.01) | (0.01) |
| 50       | 0.30    | 0.91 | 0.11 | 0.12     | (0.06) | (0.58) | (0.01) | (0.01) |

Note: Figures in parentheses are Bartlett standard errors estimated as \((4k/T)^{1/2} \text{var}(u_t-u_{t-k})/\text{var}(Δu_t)\). The variance of k differences is estimated as:

\[
\Sigma_{k+1} \sum_{t=1}^{T} (u_t - u_{t-k}) - \sum_{t=1}^{T} (u_t - u_{t-k})/(T-k) \text{ with a small sample correction of } T/(T-k+1) \text{ for degrees of freedom. See Cochrane (1988).}
\]
<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Sterling+</th>
<th>Sterling++</th>
<th>D-Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta s_t$</td>
<td>$\Delta r_t$</td>
<td>$\Delta s_t$</td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>$z_{t-1}$</td>
<td>-0.116</td>
<td>-0.081</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(2.197)</td>
<td>(2.651)</td>
<td>(0.270)</td>
</tr>
<tr>
<td>$\Delta s_{t-1}$</td>
<td>0.448</td>
<td>0.362</td>
<td>0.414</td>
</tr>
<tr>
<td></td>
<td>(4.059)</td>
<td>(3.237)</td>
<td>(4.619)</td>
</tr>
<tr>
<td>$\Delta s_{t-3}$</td>
<td>-0.084</td>
<td>0.068</td>
<td>-0.314</td>
</tr>
<tr>
<td></td>
<td>(1.889)</td>
<td>(1.474)</td>
<td>(2.866)</td>
</tr>
<tr>
<td>$\Delta r_{t-4}$</td>
<td></td>
<td></td>
<td>-0.436</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.658)</td>
</tr>
<tr>
<td>$\Delta r_{t-6}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(1.008)</td>
</tr>
<tr>
<td>R$^2$</td>
<td>0.158</td>
<td>0.167</td>
<td>0.108</td>
</tr>
<tr>
<td>DW</td>
<td>2.110</td>
<td>2.013</td>
<td>2.017</td>
</tr>
<tr>
<td>SEE</td>
<td>0.019</td>
<td>0.008</td>
<td>0.020</td>
</tr>
<tr>
<td>CHOW</td>
<td>0.437</td>
<td>0.942</td>
<td>0.120</td>
</tr>
<tr>
<td>LMK12</td>
<td>6.952</td>
<td>8.792</td>
<td>9.582</td>
</tr>
<tr>
<td>FEXC</td>
<td>0.644</td>
<td>0.733</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $z$ is the residual from the co-integrating regression.  
+ Uses residuals from appropriate co-integrating regression - i.e. $s_t$ on $r_t$ in column 1 and $r_t$ on $s_t$ in column 2.  
++ Uses residuals from alternative co-integrating regression.  - i.e. $r_t$ on $s_t$ in column 3 and $s_t$ on $r_t$ in column 4.  
( ) are t-statistics.  CHOW (parerater stability), LM (Lagrange multiplier test for serial correlation) and ARCH (autoregressive-conditional heteroscedasticity test) are given as $\chi^2$.  FEXC is a F-test on the hypothesis that excluded lags on $\Delta s_t$ and $\Delta r_t$ in the unrestricted VAR are jointly zero.  * indicates significance at the 5% level.
<table>
<thead>
<tr>
<th>Dep. Var.</th>
<th>Sterling $\Delta t$</th>
<th>$r_t$</th>
<th>D-Mark $\Delta s_t$</th>
<th>$\Delta r_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Con.</td>
<td>.021</td>
<td>.033</td>
<td>-.154</td>
<td>-.040</td>
</tr>
<tr>
<td></td>
<td>(2.986)</td>
<td>(1.198)</td>
<td>(2.987)</td>
<td>(0.729)</td>
</tr>
<tr>
<td>$s_{t-1}$</td>
<td>-.132</td>
<td>-.057</td>
<td>-.115</td>
<td>-.025</td>
</tr>
<tr>
<td></td>
<td>(2.550)</td>
<td>(2.709)</td>
<td>(2.988)</td>
<td>(0.670)</td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>-.024</td>
<td>-.083</td>
<td>-.099</td>
<td>-.027</td>
</tr>
<tr>
<td></td>
<td>(0.326)</td>
<td>(2.729)</td>
<td>(3.209)</td>
<td>(0.811)</td>
</tr>
<tr>
<td>$\Delta s_t$</td>
<td>.424</td>
<td></td>
<td>.382</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3.888)</td>
<td></td>
<td>(3.874)</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{t-3}$</td>
<td></td>
<td>-.066</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.437)</td>
<td></td>
<td>(1.816)</td>
<td></td>
</tr>
<tr>
<td>$\Delta s_{t-4}$</td>
<td></td>
<td></td>
<td>-.166</td>
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<tr>
<td></td>
<td>(1.862)</td>
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<td></td>
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<td>$\Delta s_{t-5}$</td>
<td></td>
<td></td>
<td>.273</td>
<td></td>
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<tr>
<td></td>
<td>(1.935)</td>
<td></td>
<td>(2.347)</td>
<td></td>
</tr>
<tr>
<td>$\Delta r_{t-3}$</td>
<td></td>
<td>-.507</td>
<td></td>
<td>-.247</td>
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<tr>
<td></td>
<td>(1.935)</td>
<td></td>
<td>(2.347)</td>
<td></td>
</tr>
<tr>
<td>$\Delta r_{t-5}$</td>
<td></td>
<td></td>
<td>.202</td>
<td>-.320</td>
</tr>
<tr>
<td></td>
<td>(2.899)</td>
<td></td>
<td>(3.053)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>.182</td>
<td>.159</td>
<td>.314</td>
<td>.080</td>
</tr>
<tr>
<td>DW</td>
<td>2.096</td>
<td>2.066</td>
<td>1.927</td>
<td>2.413</td>
</tr>
<tr>
<td>SEE</td>
<td>.019</td>
<td>.009</td>
<td>.008</td>
<td>.009</td>
</tr>
<tr>
<td>CHOW</td>
<td>1.548</td>
<td>3.170</td>
<td>34.993*</td>
<td>2.295</td>
</tr>
<tr>
<td>LMK(12)</td>
<td>7.689</td>
<td>7.456</td>
<td>8.901</td>
<td>19.135*</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>4.332</td>
<td>18.672</td>
<td>2.897</td>
<td>11.909</td>
</tr>
<tr>
<td>F1</td>
<td>3.246*</td>
<td>2.952*</td>
<td>4.014*</td>
<td>7.824*</td>
</tr>
<tr>
<td>F2</td>
<td>4.887*</td>
<td>4.412*</td>
<td>5.205*</td>
<td>0.524</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>3.595</td>
<td>1.590</td>
<td>5.869*</td>
<td>2.966</td>
</tr>
<tr>
<td>ADF</td>
<td>-2.483</td>
<td>-2.238</td>
<td>-2.365</td>
<td>-2.129</td>
</tr>
</tbody>
</table>

Notes: F1 is an F-test on the hypothesis that the constant and the coefficients on $s_{t-1}$ and $r_{t-1}$ are jointly zero. F2 is an F-test on the hypothesis that the coefficients on $s_{t-1}$ and $r_{t-1}$ are jointly zero. * indicates significance at the 5% level.

Implied co-integrating equations are:
Sterling: $s_t = .159 - .185 r_t$; $r_t = .036 - .686 s_t$
D-Mark: $s_t = 1.339 - .861 s_t$; $r_t = -1.481 - .926 s_t$

25
References


