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THE COEFFICIENT OF TRADE UTILIZATION:
BACK TO THE BALDWIN ENVELOPE

by
Peter Neary
and
James E. Anderson

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*Boston College

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Abstract

We define a distance function appropriate to the quantitative evaluation of trade reform, the coefficient of trade utilization. It is the natural measure for quota reform, and has theoretical and practical advantages over the standard tariff equivalent/consumer surplus method. We illustrate its properties by stating observable and very general measures of the welfare effects of trade reform.
The Coefficient of Trade Utilization: Back to the Baldwin Envelope
James E. Anderson  J. Peter Neary

10/11/88

Quantitative restrictions have become the principal means of trade distortion in developed countries in the last 30 years. The vast literature on the measurement of trade inefficiency has nevertheless focussed almost exclusively on tariffs. Treatment of quotas\(^1\) is basically an afterthought, covered by equivalence. The theoretical foundation for the usual measure is the expenditure function. Its value has the welfare interpretation of a lump sum transfer of income and its arguments are prices. In this essay we propose a more appropriate measure, the coefficient of trade utilization. Its value has the welfare interpretation of a lump sum transfer of quota licenses and its arguments are quantities. The coefficient of trade utilization is in the class of distance function measures (Debreu(1951), Deaton(1979)) extended to close the general equilibrium. It is the natural approach to the contemporary situation in developed countries where quotas are the main ingredients of protection, is simpler to use, and has numerous operational implications for the evaluation of trade reform and of growth in the presence of fixed trade policy. We present some of the principal trade reform results of our companion paper (1988) below.

The Baldwin envelope (1948) of the set of maximum attainable consumption bundles given foreign behavior can be related to distance function concepts appropriate to a trade reform. Baldwin's focus on treating the opportunity to trade as an alternative technology is captured in our treatment. A useful parallel is that the rate of change of the coefficient of trade utilization is conceptually akin to the rate of change of total factor productivity used in growth accounting.

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\(^1\)By quota we shall mean any quantitative restriction, with the exact means of division of the rent being an additional feature to be specified.
The new measure has both theoretical and practical advantages over the standard approach in a quota–ridden world. In practice, investigators analyze discrete changes in policy with a potentially erroneous conversion of the quota to tariff equivalence. Any correct version of welfare measurement must at least implicitly use the coefficient of trade utilization because its derivatives are always the correct shadow prices of quotas, which must be integrated over the interval of change. The conceptual steps leading to the appropriate treatment of quotas with expenditure function methods are slippery and some investigators have fallen. The coefficient of trade utilization also may have a practical element of superiority in reduced information requirements for its rate of change form. This is certainly true when the trade control system approaches being a pure import quota system, with rents retained at home. Reduced information requirements are especially useful in allowing a feasible emphasis on the inefficiency of the detailed misallocation of resources involved in quota systems. Standard methods impose a high degree of aggregation due to the infeasibility of calculating a large number of demand parameters.

Theoretically, the new measure has advantages in interpretation. The standard measure of inefficiency is a compensating variation in income, assumed to be collectible (payable) as a lump sum. This is a convention of such long standing in public finance that it is difficult to remember how artificial it is to assume the presence of an implicit extra instrument, the lump–sum transfer. The representative consumer analysis shared by both approaches is sustained by background transfers between dissimilar consumers; but the coefficient of trade utilization has the virtue of working exclusively with the actual instruments of distortion, hence it implicitly achieves the transfers by assignment of the quota rights. A related advantage is that the coefficient of trade utilization is appropriately scaled by trade; whereas the usual compensating variation is scaled by total expenditure.

\[^1\text{See Anderson (1988) for evidence on the considerable size of such inefficiency.}\]
necessarily implying trivial proportionate changes. Some investigators find it expedient to report the compensation measure scaled by base expenditure in the controlled category; the coefficient of trade utilization is the rigorously based counterpart to this procedure.

The quota reform theorems we state to illustrate uses of the new concept are substantial extensions of quota reform theorems of Corden and Falvey (1985) to allow for international sharing of the rent. They show that if pure import quotas (100% rent retention) are the only means of protection, all quota increases are welfare improving\(^1\). If rent is shared, however, the domestic prices of quota-constrained goods will change, which alters the amount of rent transferred abroad. Our approach shows that nevertheless the Corden–Falvey result goes through under weak separability between the quota goods and the remaining goods. We also give useful expressions for the case where some quotas decrease, and where some non-quota goods have tariffs.

I. Outline of the Paper

The subtitle of this essay is "Back to the Baldwin Envelope", and is the organizing theme relating our work to the earlier literature.

Part II first develops the consumption space analysis where imports and domestic goods are perfect substitutes, the subject of Baldwin's original analysis. Inefficiency can result from domestic distortions which drive production below the domestic frontier, from wedges between the marginal cost of imports and their marginal production cost, or from transfers of quota rent. This paper is concerned only with the latter two, and it is very convenient to shift the analysis to the trade space, since this is the space of the instruments. For the small country case of constant marginal cost of imports, the main focus of the present paper\(^2\), a pure import quota (with 100% rent retention) does not shift a country off the Baldwin frontier, since it does not change the equality of foreign

\(^1\)Neary (1988) also derives results for quota reform in the polar cases of zero and 100% rent retention.

\(^2\)It is easy to develop a "coefficient of trade utilization" approach to the large country case following the methods of this paper. Our companion paper develops partial reform expressions in the large country context.
and domestic prices which are parametric to the Baldwin problem. Pure import quotas are a rarity, however, with the norm being a mixed instrument in which the rent is shared. The empirically very significant inefficiency due to rent transfer is analyzed in terms of the coefficient of rent utilization, the fraction of frontier consumption of the import good which is achieved by the Initial bundle.

The remainder of the paper works in trade space with two additional inefficiencies of distorted trade. For trade in final goods we add a consideration of the exchange inefficiency (operation at the wrong point on the Baldwin frontier). For trade in inputs, quotas result in inefficient production, hence operation off the Baldwin frontier. Either case closes the analysis by endogenizing the domestic prices of the quota-constrained goods. For trade in final goods in Part III we measure the proportionate expansion from an initial bundle to a Meade trade indifference curve in the subspace of controlled trade. Following the results of our companion paper (1988) reviewed in the Appendix we develop the general equilibrium reduced form budget constraint which supports the reduced form trade utility level and derive the shadow price of a quota. It is the unit quota rent retained at home plus two additional terms involving respectively the marginal revenue gained from less rent transfer abroad and the marginal tariff revenue lost from taxed non-quota goods. For a pure import quota system the latter terms are zero. Shadow prices are associated with distance functions, so on this base we build an operational general equilibrium distance measure of trade inefficiency, the coefficient of trade utilization.

Part IV develops the case of trade in inputs, where the imported inputs are subject to quotas. The coefficient of trade utilization is the ratio of initial inputs to the inputs needed to achieve the maximal consumption on the Baldwin envelope, available with free trade. It has an entirely parallel development to Part III.

Part V applies the new concept to the topic of trade reform. Section V.1 presents powerful new theorems on the direction of welfare-improving quota reform in terms of
the coefficient of trade utilization.\textsuperscript{1} Section V.2 turns its practical application in terms of operational measures. For the most commonly used case of weak separability, the coefficient of trade utilization measure is operational, and requires less information than what appears to be required by expenditure function methods. The latter are shown to be potentially misleading, with a correct version requiring use of the coefficient of trade utilization in any case.

II. The Baldwin Envelope and Trade Efficiency

Figure 1 depicts the Baldwin envelope of maximal consumption bundles given optimal utilization of both home and foreign production possibilities for the small country case. All other distortions are suppressed and all controlled traded goods are final goods which are perfect substitutes for domestic goods. The Baldwin problem is to maximize the value of consumption at any given consumer price ratio. Maximal consumption possibilities lie on the plane through $E$ with slope equal to minus the foreign price of good 2 in terms of good 1. It is tangent to the domestic production possibilities frontier at $E$, depicting the first-order condition of the Baldwin problem.

\textsuperscript{1}"Back to the Baldwin Envelope" in the other sense of the phrase. For the case of final goods, these are a review of theorems in our companion paper. For imported inputs, the theorems are very similar, but novel in their extension to the case of inputs.
Inefficient trade policy results in operation below the frontier. Alternatively, trade reform can be evaluated in terms of approaching the frontier. For the small country case shown, quotas only restrict the length of the line segment achievable for consumption above or below \( E \), such as at \( \bar{Q} \). Trade remains efficient. For the large country case the foreign offer curve is placed at \( E \) (located at the maximal production income point given the domestic price), and a point like \( \bar{Q} \) is not on the envelope, since it does not equate the slope of the offer curve with the slope at \( E \); the foreign marginal cost of imports varies with \( Q \). A point on the foreign offer between \( \bar{Q} \) and \( E \) will be efficient for the given prices. To find points superior to \( \bar{Q} \) it is necessary to trace out the remainder of the Baldwin envelope, involving different domestic prices and the corresponding optimal settings of trade and domestic production. Fortunately this elaborate structure can be avoided in the small country case, and in the large country case when the market clearance link between domestic prices and trade controls is established. A trade space analysis of productive and exchange inefficiency is then possible.

Evidently a pure import quota cannot be inefficient in the Baldwin sense. But rent sharing involves a lower budget line in Figure 1, or operation below the Baldwin
frontier, such as at Y. Alternatively, trade is below the foreign offer surface in the trade space analysis. We now develop a general formulation for measuring such inefficiency.

II.1. The Coefficient of Rent Utilization

There are two classes of traded goods, Z and Q. We suppose that only the Q's are subject to government control. For simplicity we develop only the small country case of fixed foreign prices, $P^*$ for the Q's and $\pi^*$ for the Z's. In Figure 2 the initial distorted trade consumption bundle lies at C on the Meade trade indifference curve $U^0$.

In the vector case, $P,Q,\pi,Z,\pi^*,P^*$ are vectors with for example $PQ$ denoting the inner (dot) product of the vector P and the vector Q. The quota $Q^0$ gives rise to total rent $[P-P^*]Q^0$, which is split between the two nations with $\omega[P-P^*]Q^0$ going to foreigners and $(1-\omega)[P-P^*]Q^0$ going to home residents. This is consistent with an award of $\omega Q^0$ worth of licenses to foreigners, or with a VER of $Q^0$ supplemented by a vector of home specific tariffs at rate $(1-\omega)[P-P^*]$, which retain $(1-\omega)[P-P^*]Q^0$ of the total rent.

In Figure 2 the budget line supporting $U^0$ at domestic prices has a horizontal intercept at E with value $OE = (1-\omega)[P-P^*]Q^0$ (in terms of Z), while the budget line through C at foreign prices has a horizontal intercept at D with value of $-\omega[P-P^*]Q^0$, representing the transfer to foreigners. Alternative points of consumption may be found by varying Q in the feasible set bounded by OA while keeping Z fixed. The coefficient of rent utilization is $\delta = BA/BC$, the ratio by which the potential consumption at A must be deflated to reach $U^0$. With a pure import quota, 100% of the rent is retained and $\delta$ equals one, and trade is efficient (although exchange may not be, the marginal rate of substitution on the indifference curve through the intersection of $Q^0$ with OA may not equal the slope of OA). A value of $\delta$ greater than one implies inefficiency. So the coefficient of rent utilization serves to measure the inefficiency arising from the gift of rent to foreigners.

More subtly, a quota system usually involves non–competitive allocation among suppliers, and in a multi–country world is equivalent in terms of Figure 2 to picking an
average ray OA which is less favorable than the least cost ray. \( \delta \) can be refined to measure the distance from the least cost ray OA* down to point C, in which case it picks up the (empirically very significant) additional trade inefficiency arising from misallocation among suppliers.

Figure 2. The Coefficient of Rent Utilization

\[
\delta = \frac{BA}{BC} \\
OD = -\omega[P-P^*]Q \\
OE = (1-\omega)[P-P^*]Q
\]

We now formalize the development of \( \delta \). \( Q^0 \) is the vector of controlled trades. \( Z \) is the vector of unconstrained trades, assumed to have zero tariffs for simplicity.\(^1\) \( P^* \) and \( \pi^* \) are the associated fixed foreign price vectors. The trade balance for the small domestic economy is

\[(1) \quad P^*Q^0 + \pi^*Z + \omega[P-P^*]Q^0 = 0.\]

In forming (1), \( \omega \) is taken to be the same for all controlled trades for simplicity. The generalization to differential rent retention (or alternatively, different ad valorem tariffs in each controlled sector) involves composition effects which we ignore to start. In terms of moving in the \( Q \) direction, \( Q^0 \) differs from the point \( Q^R = Q^0\delta \) on the foreign offer surface as the implicit solution to \( \delta \) in

\[(2) \quad P^*Q^0\delta + \pi^*Z = 0.\]

\(^1\)In the present context this does not matter, though it will for the coefficient of trade utilization.
The domestic price expenditure at the initial equilibrium is $PQ^0 + \pi^*Z = (1-\omega)[P-P^*]Q^0$. The initial amount of revenue $G^0 = (1-\omega)[P-P^*]Q^0$ supports utility $U^0$. Now consider an alternative feasible $Q$, such as $Q^R$. The domestic price expenditure at $A$ is $PQ^R + \pi^*Z = (1-\omega)[P-P^*]Q^R$.

Definition 1: The coefficient of rent utilization is
\begin{equation}
\delta(Q, G^0) = \frac{(P-P^*)Q}{G^0}.
\end{equation}

That is, the value of $\delta$ is the rent at $Q$ normalized by the rent at $Q^0$ which supports $U^0$. $\delta(Q, G^0)$ is a general function, but when $Q=Q^R$, $\delta$ measures the inefficiency of the current allocation relative to the Baldwin envelope. (3) appears to be unnecessarily elaborate as a means of expressing the value of $\delta$, but in fact it has a very useful similarity to the coefficient of trade utilization used in succeeding sections of the paper.

II.2 Applications

Note that $\delta_Q = [P-P^*]$; increases in the quota increase efficiency in proportion to the unit quota rent. The unit rent is properly regarded as the shadow price of the quota. All quota reforms in the half-space above the quota revenue budget line with slopes equal to unit rents are efficiency increasing. For nonuniform rent retention the corresponding derivative expression is
\begin{equation}
\delta_Q = \frac{(I-\Omega)[P-P^*]}{G^0},
\end{equation}

where $I$ is the identity matrix and $\Omega$ is the diagonal matrix of rent shares ($\omega^i$).

Quota reform can be evaluated generally in terms of moving toward the Baldwin envelope. In percentage change form the coefficient of rent utilization moves by:

\footnote{For nonuniform rent retention, the factor $1-\omega$ no longer cancels.}
\[
\bar{\delta} = \sum \theta_i \bar{Q}_i,
\]
where \( \theta_i = \frac{(1-\omega_i)(P_i - P^*)}{\sum(1-\omega_i)(P_i - P^*)} \), the retained rent share of quota \( i \).

It is simple to derive from (3) analogous expressions to (4) for changes in the rent retention shares \( \omega_i \) and for exogenous shifts in \( P^* \) or other factors affecting the initial position. One important implication of this paper is that the measurement of \( 1-\omega_i \) is a very significant empirical issue, since its magnitude is critical to the efficiency (and in the remainder of the paper welfare) effects of tariff and quota reform.

A significant alternative use of (3) takes account of inefficient allocation of quotas by country. The vector \( Q^0 \) may contain country-specific elements which are perfect substitutes for domestic use. Let \( Q_i \) be the \( i \)th country's export to the home country and let \( \sum Q_i = \bar{Q} \). A reallocation of the \( Q_i \)'s subject to the same \( \bar{Q} \) will not affect the common \( P \). Sugar is a good example, with trade for which the \( P^* \)'s are very widely dispersed among supply sources. It is clear that a reallocation of \( Q \) to cheaper sources acts as a reduction in the price index \( P^* Q^R \) in (3), with no effect on \( P \) or \( G^0 \). Then \( \delta \) rises to reflect the additional source of inefficiency. In Figure 2, the refined measure would be \( BC/BA' \) for the point \( A' \) (not drawn) lying on \( OA^* \) due north of \( Q^0 \).

III. The Coefficient of Trade Utilization

Now let us turn to focus on the production-cum-consumption inefficiency in the trade space. The conceptual elements are illustrated in Figure 3. As in Figure 2, \( C \) is the initial trade bundle, associated with Meade trade utility \( U^0 \). \( C \) is created by a quota \( Q^0 \). Total rent in terms of the export good is \( DE \), the difference between the value of \( Q^0 \) at domestic prices, \( BE \), and at foreign prices, \( BD \). The fraction \( \omega \) of total rent is given to foreigners, \( OD \) in Figures 2 and 3. Then \( (1-\omega) \) per cent of total rent is retained at home, \( OE \). At free trade the utility level \( U^* \) is attainable, with imports \( Q^* \), lying on the foreign offer curve.
Now consider the measurement of inefficiency of the trade equilibrium. Distance measures (Hicks' (1944) "quantity-variations") specialized to the trade case involve movements in the $Q$ direction toward a reference utility. How is the distance from $U^0$ to $U^*$ to be measured? As in the income compensation case (Hicks(1944)), there is generally a "compensating" and an "equivalent" quantity-variation measure. For the case shown, these coincide, since the amount of $Q$ which yields $U^0$ or $U^*$ under the budget constraints and the optimizing behavior of agents is unique. The (equivalent variation) factor by which $Q^0$ must be expanded to yield $U^*$ is $1/\Delta'$, where $\Delta'=Q^0/Q^*$; the (compensating variation) factor by which $Q^*$ must be contracted to maintain $U^0$ is $1/\Delta$ where $\Delta = Q^*/Q^0$. In higher dimensions we develop the appropriate generalization of this almost trivial accounting. For more than one import, the compensating and equivalent variation measures are not coincident. Generally the new reference utility will not be at the free trade level, and for trade reform theorems we shall be concerned with local changes in $Q$.

The key ingredient behind all the applications below is the general equilibrium shadow price of a marginal change in a quota. This is derived for the final goods case in our companion paper (1988) and summarized in the Appendix. Shadow prices are
always the derivatives of distance functions, so on this base we can construct the
coefficient of trade utilization. Part IV shows that the same structure goes through for
traded inputs as well.

In general, we solve for Δ, the coefficient of trade utilization, using the distorted
trade utility function derived in the Appendix. It is a reduced form general equilibrium
function relating the realized utility to the trade instruments:

\[ U = v(Q,t;\omega;P^*,\pi^*) \]

where t is the trade tax on the unconstrained group, set at zero for convenience here (the
Appendix and Part V develop the complete version which also allows for differential rent
retention). The key shadow price of the quota is proportional to vQ and is shown to be:

\[ r = (1-\omega)[P-P^*] - \omega P'Q, \]

where ' denotes the transpose of a matrix and P_Q is a matrix expression developed in
the Appendix. Under weak separability, the second term is positive. Also, let v(P^*,\pi^*)
be the general equilibrium reduced form utility for free trade. Δ is a radial expansion or
contraction factor for either the old bundle Q^0 or the new bundle Q^*. Generally, Δ, the
compensating variation form of the coefficient of resource utilization, is implicit in:

**Definition 2** The coefficient of trade utilization is

\[ \Delta(Q',U^0;\omega;P^*,\pi^*) = \{ \Delta | v(Q'/\Delta;\omega;P^*,\pi^*) = U^0 = v(Q^0,\omega;P^*,\pi^*) \} \]

We assume v_Δ = -vQQ/Δ^2 <0, which means vQQ>0. That is, the aggregate shadow
value of quotas must be positive. The assumption is required for Δ to be defined, since
the global version of the implicit function theorem cannot be applied to (7) if the derivative
v_Δ is not one-signed.\(^1\) The condition is analogous to the familiar condition in the
analysis of trade distorted by taxes that the aggregate revenue be positive, and reduces to
it in the case of pure import quotas, but it is obviously more restrictive. A rise in Δ then

---

\(^1\)Sufficiency places some restrictions on the substitutability of the Q's and Z's.
measures a welfare increase, and $\Delta > 1$ when $Q' = Q^*$. The **equivalent variation** form is implicit in:

\[
(7') \quad \Delta'(Q^0, U; \omega; P^*, \pi^*) = \{\Delta' \mid v(Q^0/\Delta'; \omega; P^*, \pi^*) = U'\}.
\]

A rise in $\Delta'$ again measures improvement in the utilization of the trade opportunity. $\Delta' < 1$ when $U > v(Q^0, \cdot)$. $\Delta'$ is not generally defined when $U'$ is the free trade level, since the maximal $U^*$ is achieved uniquely with $Q^*$, which is not generally a radial expansion of $Q^0$.

Evidently a single coefficient of trade utilization function yields the values of $\Delta$ or $\Delta'$, with the difference lying in the function being evaluated at different points. Henceforth we will refer to the function as $\Delta(Q, U; \omega; P^*, \pi^*)$. $\Delta$ is a quantity index closely related to those analyzed in the productivity literature. We list some properties of $\Delta$ in $Q$ for reference; see our companion paper for details:

(i) $\Delta$ is homogeneous of degree one and concave in $Q$.

(ii) $\Delta'$, the equivalent variation form of $\Delta$, is a quota-metric utility function.

Calculation of $\Delta$ for evaluation of the initial point compared with free trade requires solution of a general equilibrium model to obtain $Q^*$, defined by $v_Q(Q^*, \cdot) = 0$. Either form can also be used, however, to compare local changes in the efficiency of trade policy, as when $Q'$ is the reformed bundle. The compensating variation form is especially easy to compute in this case, as we develop in Part V.

IV. Imported inputs

While it is obvious to an economist that taxes on imported inputs are inefficient, trade policy does in fact often have such controls. For example, in 1988 the US VER controls on imported DRAM chips led to production delays for computer manufacturers.

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1 For $Q' = Q^*$, $\Delta$ is defined, so the difficulty at the free trade level disappears.
and high spot market premia for DRAM chips. The coefficient of trade utilization will be
extended in this Part to the case of imported inputs. Inefficiency occurs for a small
country even with 100% rent retention, due to the production inefficiency, while there is
no exchange inefficiency.

Figure 4 illustrates the conceptual issues for the case of one imported input $Q$
and one (composite) output $Y$. The production function is $OH$, and $QM$ trades for $Y$
at the fixed world price $P^*/\pi^*$, with the free trade solution being production at $C$, with
$GA$ of $Y$ trading for $GC$ of $QM$. $OA$ is left for domestic consumption. The feasible set
of consumption bundles is on the line segment $OA$, with $A$ the maximal point. A quota
$QM^0$ results in production at $C'$, with $C'B'$ of $Y$ traded for $A'B' = QM^0$. With 100% of
the revenue $AE$ assumed to be retained at home to simplify the diagram, consumption
is at $A'$. The coefficient of trade utilization is $\Delta = CG/CD$ the proportion of efficient
input use actual to input use.$^2$

$^1$It is straightforward to incorporate $\omega>0$, and we do so in the formal development.

$^2$OA/OA is the output measure of inefficiency. The productivity literature has a well-developed
examination of the distinction between the two. In the case shown they are not the same. The choice of
approach is formally arbitrary, but in the trade case it is natural to focus on a measure in the space of the
instruments. For evaluating trade reform, for example, the link between the instruments and the measure is
direct rather than through the entire factor demand system.

Diezert (1985) develops a distance-type (he does not use this terminology) output-based measure
of productive inefficiency in an open economy. In his model, the quantities under "control" are the domestic
outputs, which is less useful for the purpose of evaluating trade control than a measure focussed on the trade
quantities.
We now develop general expressions for $\Delta$, first constructing some necessary building blocks. The development is entirely parallel to that for the final goods case reviewed in the Appendix. We first derive expressions for optimal expenditure or revenue associated with quantity-constrained and "free" categories as functions of the trade control instruments, the quotas. Then these are substituted into the general equilibrium budget constraint to obtain the reduced form consumption as a function of the quotas. Finally, we define the coefficient of trade utilization as the distance in quota space from an arbitrary quota vector to the quota needed to sustain a target consumption level.

To proceed formally, we define the maximal domestic product function as:

\[
R(P,\pi) = \max_{Y,Q_M} \{\pi^* Y - PQ_M \mid T(Y,Q_M) \leq 0\},
\]

where $T(Y,Q_M) \leq 0$ is the domestic production possibilities frontier. $R$ is the domestic value-added. Any imported primary factor services appear as elements of $Q_M$. $R$ is the standard dual function in general equilibrium production theory (see for example Dixit and Norman (1980)). It is convex in prices $P,\pi^*$; with first derivatives $R_P = -Q_M$ and $R_{\pi^*} = Y$. Next we define the gross revenue function:
(9) \[ \tilde{R}(Q_M, \pi^*) = \min_P R(P, \pi^*) + P Q_M \]

(9) is a well behaved minimization problem, since $R$ is convex in $P$. By (8) we know $\tilde{R}$ equals the gross value of output, $\pi^* Y$, where $Y$ is at a maximum and $P$ is at a minimum, given $(\pi^*, Q_M)$. $\tilde{R}$ has first derivatives $\tilde{R}_Q = \tilde{P}$ by (9) ($\tilde{P}$=the value of marginal product) and $\tilde{R}_{\pi^*} = Y$ by (8). $\tilde{R}$ is concave in $Q_M$ by the minimum in $P$ property of (9).

Now we are ready to define $\Delta$. The net value of consumption is $C = \pi^* (Y + Z)$, where exports $\pi^* Z < 0$ must be given up to pay for imports $Q_M$. The external payments constraint is:

(10) \[ P^* Q_M + \pi^* Z + \omega [P - P^*] Q_M = 0. \]

Using (9)–(10)

(11) \[ C = \pi^* (Y + Z) = \tilde{R}(Q_M, \pi^*) - (1 - \omega) P^* Q_M - \omega P Q_M. \]

The first order condition for efficient use of imported inputs $Q_M$ is

\[ -R_P(P, \pi) = Q_M. \]

This in inverted to form the domestic price of inputs function:

\[ P = P(Q_M, \pi). \]

We now define the distorted trade consumption function:

(12) \[ C(Q_M; \omega, \pi^*) = \tilde{R}(Q_M, \pi^*) - (1 - \omega) P^* Q_M + \omega P_M(Q_M, \pi^*) Q_M. \]

The critical property of this function is the implied shadow price of quotas:

\[ r_M = C_{Q_M} = (1 - \omega) [P - P^*] - \omega P Q_M(Q_M, \pi) Q_M. \]

Note that it is entirely parallel to (6), the shadow price of quotas for the final goods case.

The maximal value of consumption is

\[ C^* = \pi^* (Y^* + Z^*) = R(P^*, \pi^*), \]

since (10) at free trade is $P^* Q_M^* + \pi^* Z^* = 0$.

Definition 3 The coefficient of trade utilization is implicit in:
\( \Delta(Q_M, C^0; \omega; P^*, \pi^*) = \{ \Delta \mid C(Q_M/\Delta; \omega; \pi^*) = C^0 \} \).

The equivalent variation form is:

\( \Delta'(Q_M^0, C'; \omega; P^*, \pi^*) = \{ \Delta' \mid C' = C(Q_M^0/\Delta'; \omega; P^*, \pi^*) \} \).

We assume \( \tau_M Q_M > 0 \), so that \( \Delta \) is defined, since the implicit function theorem requires

\[ C_{\Delta'} = -C_Q Q_M/\Delta^2 = -\tau_M Q_M/\Delta^2 < 0. \]

If \( C' = C^* \), the free trade level, the equivalent variation form is not defined, as in the case of final goods quotas. (13) is defined everywhere, and is the form used below for local evaluation of quota reform. With sufficient information the maximal domestic product function or the gross revenue function, equations (13) or (14) are operational globally.

V. Applications

The coefficient of trade utilization will now be applied. In section V.1 it yields theorems on partial quota reform which are much more general than any available in the past. In section V.2, we discuss the operationality of the new concept.

V.1. Evaluation of Trade Reform

Typical trade negotiations concern partial moves toward free trade with possible backsliding in some categories. The regressive motion is pronounced at periodic renegotiation of VERs, such as the Multifiber Arrangement. The standard gradual reform theorems in the literature concern tariff reform, and are highly restrictive. Welfare improves if all taxes are cut equiproportionately, or if the highest taxes are cut first provided that all goods are substitutes.\(^1\) This meager harvest can be greatly extended for the case of quotas when their full implications are utilized.

We present here some of the reform results of our companion paper. Quite generally, quota increases are locally welfare improving if they move in the half space above the initial shadow budget value of the quotas: \( rQ > G^0 \), where \( r \) is the shadow price.

\(^1\)The compensated excess demand system has positive off-diagonal derivatives.
of the quota. Special cases allow ready calculation of \( r \) and very useful theorems on partial reform. We show that all quota increases are welfare improving under weak separability and uniform rent retention, since \( r \) is positive in this case. In fact \( r \) has a particularly simple form which is operational, so the full space of welfare increasing reforms is readily characterized. One implication is that some regression is permitted in the reform (some quotas may decrease), so long as the rent on quota goods rises and the foreign price value of quota goods rises. Outside these cases we must either do more work to calculate \( r \), or restrict the domain of reforms.

For restricted domain reforms, making no restriction on substitutability or tariffs, all uniform quota increases are welfare improving quite generally (assuming the total shadow value of quotas is positive)\(^1\). This is a counterpart to the uniform–radial–cut–in–taxes rule, but is much more general in that a number of embedded distortions are left alone. With differential rent retention and no tariffs, the space of guaranteed–welfare–improving reforms widens to any convex combination of \( Q \) and \( \Omega Q \) where \( \Omega Q \) is the total of licenses given to foreigners. Our companion paper also has useful results on quota reform in the presence of tariffs on the non–quota group and tariff reform in the presence of quotas. Using the coefficient of trade utilization, we put these propositions into a constructive form easily calculated by trade negotiators.

How does (7) translate into an operational measure for evaluation of trade reform? A change in \( Q \) accompanied by a change in \( \Delta \) so as to hold \( U \) constant at \( U^0 \) yields a compensating variation measure. If the coefficient of trade utilization rises due to trade reform, the reform is welfare improving, with the calculated increase in \( \Delta \) being a surplus in the form of a proportionate expansion in consumption of the constrained categories. In the equivalent variation version, where the reform shifts \( U \) to \( U' \), \( \Delta' \) is a quota–metric utility function.

\(^1\)This is not an innocuous assumption, but required for definition of \( \Delta \).
Using the implicit function theorem applied to (7), a change in $Q_i$ alters $\Delta$ by

$$\Delta Q_i = -\frac{\nu Q_i}{\nu \Delta} = -\frac{r_i \Delta}{-\Delta \Delta^2} = \Delta \frac{r_i}{rQ}.$$  \hfill (15)

(15) implies that the elasticity of $\Delta$ with respect to a change in quota $i$ is the shadow value share of quota $i$, $\theta_i = \frac{r_i Q_i}{rQ}$. In rate of change form a quota reform alters $\Delta$ by

$$\dot{\Delta} = \sum \theta_i \dot{Q}_i,$$  \hfill (16)

where a circumflex denotes a percentage change. The case of imported inputs is essentially identical. Note that the local change form of $\Delta$ (holding $C$ constant at $C^0$ by shifting $\Delta$ as $Q_M$ changes) from (13) is:

$$\Delta Q/\Delta = \frac{-C_Q}{C_\Delta} = \frac{r}{rQ_M}.$$  \hfill (17)

This is identical to (15) in the final goods case. Then by similar steps:

$$\Delta = \sum \theta_{Mi} \dot{Q}_{Mi} \geq 0,$$

Then we can state:

**Theorem 1** (a) Under a quota system with 100% rent retention, all quota reforms raise welfare at the rate of increase of quota rent.

(b) With uniform rent retention and weak separability, all quota reforms raise welfare if rent rises (condition (a)) and if the external value of $Q$ rises, $P^*dQ > 0$.

**Proof:** In the Appendix we show that in case (a) $r = P - P^*$; in case (b)

$$r = (1-\omega)[P - P^*] - \omega P^* Q Q$$

which we show under weak separability is

$$r = (1-\omega)[P - P^*] - \frac{\omega}{\varepsilon} P,$$

where $\varepsilon$ is the aggregate compensated import demand elasticity for $Q$ goods, necessarily negative.

A reform is welfare improving if it raises $r dQ$, which in case (a) means raising the quota rent, $P dQ > P^*dQ$. In case (b)

$$rdQ = (1-\omega)(P dQ - P^* dQ) + \omega \frac{1}{\varepsilon} P dQ.$$
This is positive if both terms are positive. If in addition to \( PdQ > P_dQ \), positive rent increase, we stipulate \( P_dQ > 0 \) we are sure that \( PdQ > 0 \).

Then both terms are positive. ||

Note that Theorem 1 generalizes the Corden–Falvey result that all northeast moves are welfare-improving to the case where some quota decreases are permitted. All quota reforms in the half-space above the initial quota rent budget line are welfare improving:

\[
(P - P^*)Q \geq G^0 = (P - P^*)Q^0
\]

For the pure import quota case, (16) implies \( \Delta \) moves proportionally to the quota rent. Theorem 1(b) generalizes Corden–Falvey to allow rent-sharing.

Now we consider non-uniform rent retention, consistent with non-uniform tariffs in the quota-constrained group. The shadow price of quotas becomes:

\[
r = [I - \Omega][P - P^*] - P'Q\OmegaQ\]

where \( \Omega \) is the diagonal matrix with the \( \omega_i \)'s on the diagonal, differing across quota categories and \( I \) is the identity matrix. (16) is still the formula for \( \Delta \) with (19) used for the prices. The implications for quota reform are severe. The new rent transfer term in the shadow price has essentially the same properties which are so troublesome to tax reform propositions, and it requires strong restrictions on either preferences or the admissible reforms to overcome it. It is possible for a shadow price to be negative, so that some quota increases could be welfare-decreasing. In the tax reform literature this has led to the uniform radial cut rule of equiproporionate cuts in taxes. This has an analogy for quota reform: an increase in all quotas in proportion to \( Q \) will guarantee that the rent transfer effect is positive: \( -\Omega Q P'Q(\Omega Q) > 0 \) since \( PQ \) is negative semidefinite, as shown in the Appendix. This is the exact counterpart to the reason for the uniform radial cut rule in the tariff case. We have also assumed, in order to define \( \Delta \), that the aggregate shadow value of all quotas is positive. Then trivially, a uniform proportionate increase in all quotas raises \( \Delta \) in the same proportion, which is welfare
increasing by assumption. Then it is also true that any quota increase which is a convex combination of the two rules will guarantee a rise in welfare:
\[ \hat{Q}_i = g(\lambda + (1-\lambda)\omega_i) \] for \( 1 \leq \lambda \leq 0, \ g > 0, \) and all \( Q_i. \)
This rule describes a cone of welfare improving quota reforms in Q space.

Then we have shown:

**Theorem 2** Any quota reform which increases the quota in higher transfer categories by more than the average and by less than the rate of transfer is welfare-improving.

Intuitively, the condition in Theorem 2 is needed because a below (above) average increase in a high (low) transfer category could lead to a rise in the total rent transfer. Of course, the condition is over-sufficient. For the case where the \( \omega_i \)'s are the same the rule collapses to allowing only uniform proportional expansions, but in then

Theorem 1 applies.

(16) measures the improvement in the productive efficiency of trade (movement toward the Baldwin envelope) combined with the improvement in the exchange efficiency (moving to the right point on the envelope). The coefficient of rent utilization measures the productive efficiency of trade, equation (4) in rate of change form. The improvement in (16) can be decomposed into the productive efficiency and exchange efficiency components in

\[ \hat{\lambda} = \frac{rQ}{(1-\Omega)(P-P^*)Q} + \left( \sum_{i=1}^{N} \frac{(-P_i\Omega_iQ_i)}{rQ} \hat{Q}_i \right). \]

For the pure import tariff case \( \hat{\lambda} = \hat{\delta} \) as the second order term vanishes. The interpretation is that an initial equilibrium at a point like A on Figure 2 still has a binding quota, with less than the free trade \( Q. \) A relaxation of the quota is feasible by traveling out along the foreign offer curve OA; the percentage improvement in the coefficient of trade utilization is simply the percentage improvement in the Q direction.
V.2 Practical Implications

In this paper an appropriate welfare measure is rigorously constructed. It has the intuitively appealing property of being a measure of the efficiency of utilization of the trade opportunity measured in the space of distorted trade. Its rate of change is conceptually identical to the familiar rate of change of total factor productivity. Due to the concavity of the underlying distance function, the useful discrete approximation theorems of that literature are immediately applicable (Caves, Christiansen, and Diewert (1982)). We now show it is readily operational, and contrast it with the typical erroneous tariff equivalent approach.

For weak separability and uniform rent retention, note from (A.10') that the shadow price

$$ r = (1-\omega)[P-P^*] - \frac{\omega}{\varepsilon}P $$

is fairly simple to calculate in principle. All that is needed is a single import demand elasticity, the calculation of $\omega$, and data on $P-P^*$. For extension to embedded tariffs on $Z$ the shadow value of quotas becomes under weak separability and uniform rent retention from (A.10'):

$$ r = (1-\omega)[P-P^*] - \frac{\omega}{\varepsilon}P - \tau P $$

This again is fairly simple to calculate. When homogeneity of the export aggregate function $h(Z)$ is not plausible, the exact $\bar{\tau}$ requires detailed information on the structure of excess demand, but it can be replaced by $\hat{\tau}$, the highest tariff in the $Z$ group, to form $\bar{\tau}$. Reforms passing the test $\bar{\tau}dQ>0$ will pass the test $\tau dQ>0$.

The most difficult problem is with differential rent retention; and here the analyst must obtain the inverse price derivative matrix $P_Q = E_{P^*}^{-1}$, discussed in the Appendix. But this is no worse than the usual case of welfare analysis, requiring elasticity estimates. We conclude that (16) is operational, and should provide useful guidance for trade negotiations.
In practice of course, observation of the unit rents $P-P^*$ is far from routine, and differential rents over detailed categories are important and defy easy finessing by aggregation. The analytic foundation of (16) emphasizes that such differentials are of the essence in measuring inefficiency, and thus future evaluations of trade reform should spend much effort on gathering such information. In contrast the good news is that the usual effort spent on obtaining appropriate import demand elasticities is not needed for pure import quotas and may not be needed if the rent transfer term is small.\(^1\) We emphasize that the very simple and “partial equilibrium” appearing formula in (16) is in fact a rigorously based general equilibrium construct.

It is also straightforward to use the coefficient of trade utilization to evaluate tariff changes. This can be done either in the space of quota–ridden quantities, or in an augmented space which includes the “quota equivalents” of tariffs. The former is straightforward and carried out in our companion paper. The latter can always be used \textit{ex post,} which is frequently the concern of analysts interpreting the past.

We conclude by contrasting the coefficient of trade utilization approach with a typical erroneous “tariff equivalent” approach to evaluation of reform in a single quota. In Figure 6 the quota is raised from $Q^0$ to $Q^1$, keeping other quotas constant. The tariff equivalent to $Q^0$ is $t^0$ and to $Q^1$ is $t^1$.

\^1\footnote{In our companion paper we explore plausible conditions where it is small.}
The "usual" measure (e.g., see Cline et al., 1978)) would be a gain in efficiency of triangle ABE, interpreted as the compensating variation in income required to maintain utility $U^0$, net of the loss of "revenue" $t^0AEt^1$. That is, integrate the derivative of the expenditure function with respect to $P$ from $P^*+t^0(Q^0)$ to $P^*+t^1(Q^1)$, and deduct the rent change $Q^0[t^0-t^1]$. The correct measure is trapezoid ABQ^1Q^0 if the quota rent is 100% retained. This involves integrating the derivative of the coefficient of trade utilization from $Q^0$ to $Q^1$. A careful investigator using standard methods realizes that the added tariff equivalent revenue must be incorporated and so also obtains $ABQ^1Q^0$.

Notice however that this involves two separate calculations, hence is arguably more complex. The coefficient of trade utilization evaluates $ABQ^1Q^0$ relative to initial trade revenue. It is of course possible to obtain $ABQ^1Q^0$ and interpret it as a compensating variation in income, thus achieving a correct expenditure function measure. But note that the correct measure inevitably must use the coefficient of trade utilization function in the process. Thinking in terms of $\Delta$ is more direct and thus it does not invite the conceptual error of Figure 6 by working directly with quotas. Note also that an accurate value of the slope $P_Q$ is less important to the correct measure. ABE is proportional to $P_Q$, but $ABQ^1Q^0$ is not, and the influence of it is very small as $t^1$ is large.
The potential for error is even more dramatic if the rent is shared. The correct analysis evaluates by integrating under the shadow price of quotas function

\[ r(Q) = (1 - \omega)[P(Q) - P^*] - \omega P_Q Q, \]

with measure GHQ^1Q^0 in Figure 6. For a pure VER, \( r(Q) = -P_Q Q \), positively sloped through the origin. This assumes \( \omega \) is constant with respect to \( Q \) for simplicity. (In fact \( \omega \) will change unless the tax instrument is a domestic base constant ad valorem tariff, or the licenses are shared with \( \omega \% \) going to foreigners). A correct expenditure function approach involves, instead of a single measure, calculating the consumer's surplus \( r^0 AB t^1 \), working out that all of ABE is gain, and apportioning correctly the revenue loss \( r^0 AE t^1 \) and the revenue gain \( EBQ^1Q^0 \). Evidently the relative simplicity of the direct approach grows with the complexity of the underlying structure.

VI. Conclusion

Robert Baldwin's envelope taught us to view the trade opportunity as a technology, with the corollary that inefficient trade is similar to inefficient production. Debreu and Deaton taught us to measure inefficiency in quantity space. This essay marries Baldwin's notion of efficiency in production and trade to the measurement technique of Debreu and Deaton. We define a trade-space concept pointing us back to the Baldwin envelope in two senses of the phrase.

The coefficient of trade utilization provides the basis for a natural accounting system for contemporary trade policy, which is substantially in the form of quotas. Operational measures of the welfare effects of quota reform are developed in terms of its rate of change, substantially generalizing earlier work on trade reform. We believe that further development and application of the coefficient of trade utilization is promising.
Appendix. The Distorted Trade Utility Function

We begin with the trade expenditure function $E(P, U^0)$:

$$E(p, U^0) = \min \{ pX \mid U(X) = U^0 \}.$$  

where $U$ is a Meade trade utility function in the vector of trades $X$ and $p$ is the price vector.

For the usual trade distortions case, only some products are restricted. Let $Q$ be the amount of trade in the restricted product group, with foreign price $P^*$ and domestic price $P$. For the unrestricted group, $Z$ is the trade quantity, $\pi$ is its domestic price = $\pi^* + t$ where $t$ is the specific tax and $\pi^*$ the free trade price. First we must consider the unrestricted product group.

**Definition A.1**

The distorted trade expenditure function is:

$$\bar{E}(Q, \pi, U^0) = \min \{ \pi Z \mid U(Q, Z) = U^0 \}$$

Alternatively,

$$(A.3) \quad \bar{E}(Q, \pi, U^0) = \max_P E(P, \pi, U^0) - PQ$$

$(A.3)$ is a well behaved maximization problem, since $E$ is concave in $P$, as is well known. Mechanically, the first order conditions solve for the price $\bar{P}$ which equates demand $E_P(P, \pi, U^0)$ with $Q$, the supply. The first derivative properties of $\bar{E}$ are straightforward from $(A.3)$:

$$(A.4) \quad \bar{E}_\pi(Q, \pi, U^0) = Z$$

$$(A.5) \quad \bar{E}_Q(Q, \pi, U^0) = -P$$

The first property follows from Shepard's lemma. The second is less familiar with the intuition that a relaxation of the quota by one unit reduces expenditure on the market group by $\pi Z Q$, which gives the marginal willingness to pay for quota ridden goods. $\bar{E}$ is concave in $\pi$ and convex in $Q$, by its minimum in $Z$ and maximum in $P$ property. Then the important derivative matrix $P_Q$ is negative semi– definite:
\( y^t P_Q y < 0 \) for any vector \( y \).

Now consider the internal and external payments constraints facing the small economy. Suppose that the fraction of quota revenue captured by the home country is \( 1 - \omega \) (consistent with awarding the fraction \( \omega \) of all quota licenses to foreigners, or with a tariff on quota-controlled imports at specific rate \( (1 - \omega)P - P^* \)). Tariff revenue \( tZ \), if any, is captured at home\(^1\).

In terms of external prices the budget constraint is:
\[
(A.6) \quad b = \pi^*Z + P^*Q + \omega Q[\pi^* - P^*] \leq 0,
\]
where the third term represents the transfer to foreigners via the gift of quota licenses, or other revenue transfer. This is alternatively
\[
(A.7) \quad b = (1 - \omega)P^*Q + \omega P^*Q + \pi Z - tZ \leq 0.
\]

Now we substitute into (A.7) \( -\tilde{E}_Q(Q, \pi, U^0)Q = PQ, tZ = t\tilde{E}_\pi(Q, \pi, U^0) \) and \( \tilde{E}(Q, \pi, U^0) = \pi Z \) in equilibrium; i.e., use the equilibrium level of \( U \) for \( U^0 \). The result is:
\[
\begin{align*}
\tilde{E}(Q, \pi, U^0) &= \pi Z - t\tilde{E}_\pi(Q, \pi, U^0) = 0, \\
b &= -\omega\tilde{E}_Q(Q, \pi, U^0)Q + (1 - \omega)P^*Q \\
&\quad + \tilde{E}(Q, \pi, U^0) - t\tilde{E}_\pi(Q, \pi, U^0) = 0,
\end{align*}
\]
where \( \pi = \pi^* + t \). For the differential rent retention case the first two terms on the right hand side of (A.8) become
\[
\tilde{E}'_Q(Q, \pi, U^0)\Omega Q + P^* [I - \Omega] Q
\]
where \( \Omega \) is the diagonal matrix with the rent transfer shares \( \omega_i \) on the diagonal, \( I \) is the identity matrix, and ' denotes transpose as needed for clarity.

For welfare analysis we implicitly define the (reduced form) utility as:

\textbf{Definition A.2: The Distorted Trade Utility Function is}
\[
(A.8) \quad v(Q, \omega, t; P^*, \pi^*) = \{U^0 | -\omega\tilde{E}_Q(Q, \pi, U^0)Q + (1 - \omega)P^*Q \\
&\quad + \tilde{E}(Q, \pi, U^0) - t\tilde{E}_\pi(Q, \pi, U^0) = 0 \} \quad \text{where} \quad \pi = \pi^* + t.
\]

\(^1\)No borrowing or other transfer is permitted for simplicity. If it is introduced, the constraining value becomes \( A \) rather than 0.
References


Anderson, J.E. and J. P. Neary (1988) "A New Approach to Evaluating Trade Reform".


Evidently the implicit function theorem yields the properties of \( v(\cdot) \). For the present purpose we note:

\[
(A.9) \quad v_Q = -\frac{-\omega \tilde{E}'_Q Q - \omega E_Q + (1-\omega)P^* + \tilde{E}_Q - tE_{\pi}Q}{-\omega E_{QU}Q + E_U - tZ_U} \nonumber ,
\]

\[
= \frac{(1-\omega)[P-P^*] - \omega P^* Q + tZ_Q}{-\omega E_{QU}Q + E_U - tZ_U} \quad \text{using (A.5)}^1.
\]

The denominator of (A.9) is the shadow price of foreign exchange\(^2\). Then the general equilibrium shadow price of a quota is defined as the numerator, generally:

\[
(A.10) \quad r = [P-P^*][1-\Omega]Q - P'QQ + tZ_Q.
\]

The evaluation of quota reform proceeds by imposing special cases for evaluating \( r \). For a pure import quota system \( r = P-P^* \), the unit quota rent. Other cases need at least some structure restricting preferences.

An important special case used in the text is weak separability:

\[
U(Q,Z) = \phi(f(Q),h(Z)).
\]

Under this assumption, we can be sure that \( P'Q < 0 \). This follows from noting that the expenditure function under weak separability is written

\[
E(P,\pi,U^0) = E(\phi(P),\eta(\pi),U^0).
\]

The derivative matrix \( P_Q = P'Q \) is \((E_{PP})^{-1}\). The homogeneity properties of \( E \) guarantee:

\[
P = - (E_{PP})^{-1}E_{P\pi}\pi.
\]

Using \( E_{PP} = E_{P\pi}P = Z_P', \) this is

\[
(A.11) \quad P = - (E_{PP})^{-1}[Z_P']\pi.
\]

---

1 We assume it is positive. \( E_U \) is positive, and equals \( E_U \) by definition of \( \tilde{E} \) in (A.3). The term \(-tZ_U \) is negative and the term \( -E_{QU}Q = E_{U}Q = P'_UQ \) is ordinarily positive, both assuming normality. Thus the two distortion terms offset each other. It is conventional in tariff analysis to assume \(-tZ_U \) does not become large enough so that the distortion effects of a gift of foreign exchange leading to added utility produce a greater loss than the transfer. By extension this now covers \( P'Q - tZ_U \). In this case, that implies the shadow price of foreign exchange is positive.

2 \( \tilde{E}_U \) is positive, and equals \( E_U \) by definition of \( \tilde{E} \) in (A.3). The term \(-tZ_U \) is negative and the term \( -E_{QU}Q = E_{U}Q = P'_UQ \) is ordinarily positive, both assuming normality. Thus the two distortion terms offset each other. It is conventional in tariff analysis to assume \(-tZ_U \) does not become large enough so that the distortion effects of a gift of foreign exchange leading to added utility produce a greater loss than the transfer. By extension this now covers \( P'Q - tZ_U \). In this case, that implies the shadow price of foreign exchange is positive.
Now note that \( Q = \phi \phi \) and that \( Z'P = \phi Z' > 0 \) since the \( Z \)'s and \( Q \)'s are substitutes under separability. Substituting into (A.11)

\[
P = -PQ \frac{Z'P}{E_P}.
\]

The critical term is now:

(A.12) \[
P'Q = -P \frac{E_P}{Z'P} < 0.
\]

The factor \( \frac{E_P}{Z'P} \) in (A.12) turns out to be the inverse of the aggregate own price elasticity of demand for the \( Q \) group, \( 1/e \), necessarily negative. From the envelope theorem for the expenditure function \( E(\phi(P),\eta(\pi),U^0) \), \( E_\phi = f(Q) = F(\phi,\eta,U^0) \), the import aggregate. Also, the standard homogeneity properties imply \( F_\phi + Z'P = 0 \). This means \( -Z'P = F_\phi \phi \) so \( -\frac{E_P}{Z'P} = \frac{\phi F_\phi}{F_\phi} = \frac{1}{\epsilon} \).

The shadow price of a quota for uniform rent retention is then:

(A.10') \[
r = (1-\omega)(P-P^*) - \frac{\omega P}{\epsilon}.
\]

Separability also implies that the tariff term \( tZQ \) in (A.10) is a weighted average tariff times minus \( P \). Note that \( ZQ(f(Q),\pi,U^0) = Zt^0 \). Then recalling \( \pi ZQ = -P \):

(A.13) \[
tZQ = -\Sigma \frac{\pi_i Z_{if}}{\Sigma \pi_i Z_{if}} P = -\bar{\tau}P,
\]

where \( \bar{\tau} = t_i/P_i \). The weights are positive (since all \( Z_{if} \) are negative) so \( \bar{\tau} \) is a proper average. If demand for the \( Z \)'s is homothetic, the weights are the import values. The shadow price of a quota for uniform rent retention and tariffs is now:

(A.10'') \[
r = (1-\omega)(P-P^*) - \frac{\omega}{\epsilon} - \bar{\tau}P.
\]
Working Papers 1988


Peter Neary: "Trade Liberalisation and Shadow Prices in the Presence of Tariffs and Quotas," April, 1989.

