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THE TRADE-OFF BETWEEN PRECOMMITMENT AND FLEXIBILITY
IN TRADE UNION WAGE SETTING

by

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ABSTRACT

This paper examines two types of contract structures in a model where a trade union supplies labour to an industry, and sets the wage to maximize welfare. Firms investment is endogenous, and the industry price is stochastic. Under short-term contracts, the union sets the wage after the firms investment is in place, but also after the industry price is known. Under long-term contracts, the wage is chosen before investment and before the industry price is known. With short-term contracts the union has the benefit of ex-post wage flexibility, while under long-term contracts the union has the benefit of advance wage commitment. There arises a trade-off between flexibility and precommitment which may be an important determinant of contract structure. This trade-off is examined in detail.

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SECTION I INTRODUCTION

This paper is an investigation into the implications of alternative contract structures in a labour market characterised by wage-setting trade unions. The particular emphasis of the paper is the occurrence of a trade-off between the benefits of wage precommitment and the costs of wage inflexibility in the design of the optimal contract length. We present a model where a trade union may benefit by precommitting to a wage through a long-term wage contract, but these benefits are limited by the inability to adjust wages to ex-post realizations of labour demand. There arises a natural welfare trade-off between wage precommitment and wage flexibility. This suggests a possible explanation of optimal contract structure, particularly relating to contract length, based on this trade-off.

A number of recent papers have analyzed the strategic issues involved in trade union wage setting (e.g. Grout, 1984, Anderson and Devereux 1988, Van der Ploeg 1987, Calmfors and Horn, 1985). One result that comes out of this literature is that a trade union can benefit by binding itself to a wage in advance of the investment decision of firms. In the models of Anderson and Devereux and Van Der Ploeg, the welfare benefits of precommitment suggest that a trade union will have an incentive to enter into long-term wage contracts which bind its hands in advance. However, if fully contingent contracts cannot be written, then in the presence of uncertainty in total labour demand, it may be beneficial for a union to have a free hand in adjusting wages to ex-post demand disturbances. But in that case the benefits of advance wage commitment are lost. There is a trade-off between the benefits of wage precommitment and the costs of ex-post wage inflexibility. This paper presents an analysis of this trade-off.
We construct a model where a monopoly trade union supplies labour to an industry. Firms employ both labour and capital. The firms' investment decision must be made in advance of production. The trade union chooses a wage to maximize its welfare, subject to the level of employment being on the labour demand curve. If the union can precommit to a wage, this involves choosing the wage before the investment decision. In a deterministic environment this would always (at least weakly) be desirable, since the union takes into account the firms' investment policies in setting wages. However, the firms' production technology is subject to random shifts which affect labour demand. When fully contingent contracts are infeasible, the union will generally benefit if wages are adjusted to the ex-post outcomes for the technology shock, since it can take advantage of high or low shocks by adjusting the wage up or down, when wages directly affect employment. But this involves setting wages after the investment decision has been made, and so eliminates the benefits of wage precommitment.

We interpret the two forms of wage setting in terms of contract structure. If wages are set in advance of the investment decision, but before the outcome for labour demand has been revealed, we describe this as a long-term contract. On the other hand, if wages are set after the investment decision, and also after the labour demand shock, we call this a short-term contract. A contract here is interpreted as a device by which to enforce wage commitment, or bind the trade union.

We explore in detail the characteristics of short-term and long-term contracts in a model where there are both benefits of advance commitment in wages and benefits of ex-post wage flexibility. It is shown that the two contract structures have different implications for average wages, investment, and average employment levels. Wages will always be lower under
long-term contracts and will also be negatively related to the variance of the industry price, for a given mean price. Expected employment will be higher under long-term contracts, but also more variable. Investment will be higher under long-term contracts, and will be positively related to the variance of prices, while under short-term contracts investment is independent of price variance.

We characterize the welfare trade-off by computing expected union utility under long-term and short-term contracts. The two key factors that determine the desirability (in terms of the unions' welfare) of precommitment relative to ex-post wage flexibility are a) the degree of complementarity between labour and capital in production, and b) the variability of the industry price. Under short-term contracts it is shown that expected union utility is increasing in the variance of industry price, while under long-term contracts it is decreasing in this variance. The higher is factor a) the greater the benefits of long-term contracts, while the higher factor b) the more beneficial will be short-term contracts. Interpreting precommitment in terms of contract length, these two factors might then be expected to be important determinants of the long-term relative to short-term wage contracts.

An important hypothesis of the model is the absence of fully contingent contracts. Wages in a long-term contract cannot be conditioned on the realization of the firm's price shock. This parallels the assumptions made in all the previous literature on contract length (see section 2 below). The main motivating reason for this is empirical - beyond COLA clauses union contracts rarely include contingencies for the adjustment of wages during the life of the contract. A serious explanation for the existence of incomplete
contracts is clearly beyond the scope of the present paper.\(^1\)

The next section briefly surveys some theoretical literature on optimal contract length. Section 3 presents a short analysis of the benefits of precommitment in an industry with free entry. Section 4 outlines the general source of the trade-off between the strategic benefits of precommitment and the costs of limited flexibility. Section 5 then restricts the form of the model and explores in detail the characteristics of long-term and short-term contracts in an environment with an uncertain industry price. Section 6 briefly discusses some empirical implications of the model, as well as some possible extensions and qualifications to our arguments.

SECTION 2 PREVIOUS LITERATURE ON CONTRACT LENGTH

Much of the recent debate in the trade unions literature has concerned the appropriate contract structure within which to model the interaction between a trade union and a firm. One paradigm is the Efficient Contracts model pioneered by Macdonald and Sclow (1981). In this framework a union and a firm bargain directly over wages and employment, and in general the outcome of the bargain will be on the contract curve. An alternative framework is the monopoly trade union model developed by Oswald (1982) and others. In this framework, it is taken as given that for incentive or other reasons, bargaining over employment levels is not feasible. Instead, the union acting as a monopoly seller of labour, directly presents the firm with a wage demand, and employment is unilaterally determined by the firm. This model is motivated by the empirical observation that actual labour contracts seldom contain provisions on the level of employment. The actual form of the trade-union wage setting decision is affected by a number of features of the

\(^1\)If contracts are complete, with contingencies for every unforseen event, it is, in fact, unclear how contract length would matter at all.
contracting environment, such as the union voting arrangement (Oswald 1984), or the presence of differences between insiders and outsiders in the value of a worker to the firm (e.g. Lindbeck and Snower 1988).

Our paper is not directly concerned with this debate. Rather we employ a form of the monopoly trade union model to investigate a separate issue – that of the trade-off between flexibility and commitment mentioned in the introduction. In the conclusions we discuss the extent to which our results would apply in the alternative contractual environment.

Although a large amount of literature has been produced on trade unions and wage employment contracts in general, there has been relatively little work done on the determinants of optimal contract length. One well known paper is that of Gray (1978), who develops a model of optimal contract length in a macroeconomic environment with Fischer-Taylor type nominal wage contracts. In her paper there is a trade-off between a fixed cost of contract rewriting and the benefits of readjusting nominal wages in an environment where nominal and real disturbances are Weiner processes so that the variance of forecasting errors rises over time. Contract length in a typical industry will be a negative function of industry specific uncertainty and a positive function of direct costs of writing contracts. More recent work by Dye (1985) and Ball (1987) has examined issues in contract length in similar models.

A recent paper by Danziger (1988) takes a different approach to the analysis of contract length. He argues that risk-sharing motives lead unions to desire long-term contracts, in which the wage is smoothed intertemporally, 2Dye’s analysis is much more general than Gray’s however. Ball’s paper incorporates the role of imperfect competition and shows that contract length in general will not be chosen efficiently at the level of the individual firm.
rather than short-term contracts, in which more frequent renegotiation will lead to a more volatile wage profile. This argument is reminiscent of the ideas of implicit contract theory. An important feature of this theory, and Danziger's paper, is that the wage agreed to in a long-term contract plays no allocational role. Employment is not determined along the labour demand curve. If this were the case, then Danziger's conclusions would be changed. This is argued in Ragan (1989), who, in a model similar to the present paper, argues that in general workers will prefer short-term contracts. The similarity between our paper and Ragan (1989) is discussed below.

A common feature of models which endogenize contract length is that an exogenous resource cost of contracting is imposed on the problem at the beginning. Thus one side of the trade-off governing contract length is left unexamined. In this paper we suggest a possible interpretation of the costs of frequent contract renegotiation, not in terms of some exogenous resource costs, but in terms of the losses associated with the absence of commitment in the strategic interaction between union and firms. In our model the cost of short-term contracts is exactly the type of inefficiency highlighted by Grout (1984) and Van der Ploeg (1987). The benefit of clearly identifying these costs is that empirical research on contract length can be better focused. We outline below the type of evidence that one would look for.

SECTION 3 THE BENEFITS OF LONG-TERM CONTRACTS

In this section we present a simple nonstochastic analysis of the benefits from a long-term contract for a union. This is similar to the analysis of Grout (1984) and Van der Ploeg (1987) except that we explicitly embed the union in an environment with a competitive industry. Follow Oswald (1982) in assuming that a large trade union supplies labour to an industry made up of many homogeneous firms. Each firm chooses capital and labour
competitively to maximize profits taking as given a wage, price, and rental rate on capital. Given a market demand curve, entry will guarantee zero profits per firm. Wages are chosen exclusively by the union, and there is no direct bargaining between union and firms on the level of employment.

In order to motivate the two types of contract structures, we need to look at the situation of the representative firm in two positions. The first is the situation where capital and labour are both being chosen, and the second is the situation where capital is in place and the firm chooses the optimal labour supply. From each position we can extract the relevant industry labour demand curve faced by the monopoly trade union.

Let each firm $i$ face the CRS technology $y_i = F(L_i, K_i)^3$. Then each has a unit cost function that can be represented as $c(w, r)$, with standard properties, where $w$ is the wage and $r$ the rate of return to capital. Industry equilibrium is then given by the conditions

$$(1) \quad y = D(p)$$

$$(2) \quad p = c(w, r)$$

where $y$ is industry output and $D(p)$ the industry demand curve. Equations (1) and (2) must hold in equilibrium for any wage and interest rate. However the problem faced by the union will differ as between long-term and short-term wage contracts.

1. Under a long-term wage contract, the union chooses the wage in advance of the industry investment decision and faces the long-run industry labour demand curve, which may be written as $L = c_w(w, r)y$. By standard duality theorems the wage elasticity of labour demand can be written as

$$(3) \quad \eta = (1-v)\sigma + v\varepsilon$$

3. Since the number of firms in an industry with perfect competition and CRS is indeterminate, for economy of notation we drop the $i$ subscript and analyze the situation as if there were a single price taking firm with output $y$. 

8
where $\sigma$ is the elasticity of substitution between labour and capital, $\epsilon$ the price elasticity of product demand, and $v$ labour's share in total costs.

Now focus on the position of the firm for a given capital stock. Define the restricted cost function as $C(w,y,K) = wL(y,K)+rK$ where $L(y,K)$ solves $y=F(L,K)$. If $K$ is chosen optimally, clearly $C(w,y,K)=c(w,r)y$ must hold. However, conditional on $K$, we may describe the industry equilibrium as

$$y = D(p)$$

$$p = C_y(w,y,K)$$

The short-run labour demand curve for the industry can be written as $L=C_y(w,y,K)$. Again using standard duality results, the short-run wage elasticity of labour demand for the industry, when evaluated at the optimal capital stock, is given by

$$\eta_s = \frac{c\sigma}{(\sigma v+(1-v)\epsilon)}$$

Now comparing (3) and (5), it is clear that for $\sigma \epsilon < 1$, $\eta_L > \eta_s$. Note that the short-run elasticity tends to be determined by the minimum of the elasticity of substitution and the elasticity of labour demand, while the long-run elasticity is a weighted sum of the two. As $\sigma \to 0$, the short-run labour demand curve becomes completely inelastic, no matter what the value of $\epsilon$, as labour and capital stock must be put together in fixed proportions, and the capital stock is given. In the long run however, entry or exit can take place, so the elasticity of labour demand is finite (equal to $\frac{\epsilon}{v}$). On the other hand, as $\sigma \to \infty$, the short-run elasticity approaches $\epsilon/v$, since the ability to substitute costlessly between factors of production does not help

4This is shown as follows. From the short-run cost function the elasticity of labour demand is $(\partial L/\partial w)(w/L)=(C_yD'/(1-C_yD'))w/L$. Recalling that $C_y=C_y/w$ and $C_v=p=c(w,r)$ at the optimal capital stock, we may write this as $c(w,r)v/(wL(1+[C_{yy}/C_v]v))$. But with CRS, and at the optimal capital stock, it is easy to show that $C_vy/C_y=(1-v)/(v\sigma)$. Substituting gives (5).
if capital is in place and the market demand curve determines the response of labour demand to wages. In this case however, with entry and exit, the long run elasticity is infinite. For \( \sigma=\epsilon \), the two elasticities are equal.

Thus the short run labour demand curve is generically less elastic than the long run labour demand curve, and this difference becomes greater the greater is the absolute difference between \( \sigma \) and \( \epsilon \). Figure 1 makes this clear. The long-run labour demand curve LL is intersected by a family of short-run curves, SS, each of which cuts LL from above.

Now take the unions objective function as \( U=U(w,L) \), where \( U \) is twice differentiable and \( U_w>0 \) and \( U_L>0 \). Under a long-term contract the wage is the solution to

\[
\begin{align*}
\text{Max } U(w,L) \\
\text{w} \\
\text{subject to} \\
(1), (2), \text{ and } L=c(w,r)y
\end{align*}
\]

Thus under a long-term contract, the trade union explicitly takes account of the industry investment decision. The first order condition for the optimal choice of wage is

\[
U_w(w,L)(w/L)-U_L(w,L)((1-v)\sigma+v\epsilon)=0
\]

Alternatively, under a short-term contract the wage is the solution to

\[
\begin{align*}
\text{Max } U(w,L) \\
\text{w} \\
\text{subject to} \\
(1), (3), \text{ and } L=c(w,y,K)
\end{align*}
\]

The first order condition for problem \( P' \) is then

\[
U_w(w,L)(w/L)-U_L(w,L)((\sigma/(1-v)\sigma+v\epsilon)=0
\]

In each case we assume that the second order conditions are fulfilled. Now it is easy to show that union welfare must be at least as great under long-term

\[5\] This is just an application of Marshall's Laws.
contracts as under a short-term contract. To see this note with the aid of Figure 1 that long-term contracts imply tangency between the unions iso-welfare locus and the long-run labour demand schedule. As long as $\sigma = c$, this cannot be an equilibrium for a short-term contract. Starting at point A in the Figure, a union facing the short run labour demand curve SS will always choose a higher wage. But since all equilibria must lie upon the long-run zero profit locus LL, the only feasible equilibrium under a short-term contract must be on a point such as B to the north west of A. This leaves the union worse off.

Comparing points A and B, we see that wages are higher but employment lower under a short-term contract. It is possible to show also that industry output is lower, but investment may be higher or lower depending upon the sign of $c-\sigma$. If $c-\sigma > 0$, the negative output effects of the higher wage on the demand for capital dominate the substitution effects towards capital, and investment is lower. Alternatively if $c-\sigma < 0$, investment is higher.

The implication of this analysis is that in the presence of interactions between investment and employment decisions, there is a strong presumption in favour of long-term union wage contracts, as such contracts enforce commitment. The benefits of long-term contracts depend upon the elasticity of substitution in production, and the elasticity of industry demand.

It might be expected that a higher elasticity of substitution would raise the relative benefits of long-term contracts based on the argument that the union would have more to lose by not precommitting when it faces an industry with greater ability to substitute capital for labour. Interestingly, the analysis above indicates that this reasoning is only correct if $\sigma > c$ initially. Take the case where the elasticity of substitution is relatively low and the market demand elasticity is high. In this case the
long run labour demand elasticity is the weighted sum of the two and tends to
be high, while the short run demand elasticity is close to $\sigma/(1-\nu)$ and is
relatively low. Then a rise in the elasticity of substitution will increase
the short run labour demand elasticity, and reduce the difference between $\eta_L$
and $\eta_S$, and thereby reduce the benefits of long-term contracts!

SECTION 4 CONTRACTS WITH UNCERTAINTY

In this section we introduce uncertainty in the model and show how it
alters the presumption in favour of long-term contracts. First the strategic
environment is described. Again we follow Oswald (1982) in focusing on a
monopoly trade union who supplies labour to a given industry. Firms in the
industry act competitively, taking price wage and rate of return on capital
as given. Firms employ labour and capital and the union sets the wage.
Firms have an uncertain revenue stream, and must invest before the state of
the world is known.\(^6\)

The payoffs for the representative firm are

$$Ew = \int_\delta^3 (R(K, L, \delta) - wL - rK)dF(\delta)$$

where the revenue function $R$ is differentiable and strictly concave and
increasing in $K$ and $L$, capital and labour respectively. Normalize so that
the number of firms in the industry is one. Let $\delta$ be a random variable with
finite support $[\delta, \bar{\delta}]$ and let $F(\delta)$ be the distribution governing $\delta$, so that
$F(\delta)=0$ and $F(\bar{\delta})=1$. The variables $w$ and $r$ denote wage and rental rate on
capital respectively.

$K$ is chosen before $\delta$ is revealed, but employment is determined after $\delta$ is

\(^6\)Since a full analysis of the industry equilibrium under uncertainty is
beyond the scope of this paper, we now restrict the analysis to a situation
where the number of firms is fixed, and the industry price is given. In
order to find a determinate solution we must assume some form of decreasing
returns. Concavity of the revenue function plays this role.
known. This implies that profit maximizing firms will set

\( \int R_\gamma(K,L,\delta)dF(\delta) = r \)

On the other hand, employment is determined after the \( \delta \) shock has been revealed, so for any given wage

\( R_\gamma(K,L,\delta) = w \)

for each \( \delta \). We assume that a solution \( K(w,r) \) and \( L(w,r,\delta) = L(\delta,w,K(w,r)) \) exists for all combinations of \( w, r \) and \( \delta \).

The payoffs for the union are given by

\( EU = \int f \delta \overline{U}_w(w,L)dF(\delta) \)

\( U \) is assumed differentiable, concave, and strictly increasing in its arguments.

Under a long-term contract the union sets the wage taking both (9) and (10) as a constraint, but before the outcome for \( \delta \) is revealed. Thus a long-term contract allows for wage precommitment in the sense that the union can take account of the industry investment decision, but wages cannot be conditioned on the ex-post labour demand shock.

Under a short-term contract the union sets the wage taking only (10) as the relevant constraint on labour demand, but after the outcome for \( \delta \) is revealed. Thus short-term contracts rule out wage precommitment, but allow for the flexibility to condition wages on the ex-post shock.

Necessary conditions for a long-term contract are derived by recognizing that (9) and (10) can be written implicitly \( K = K(w,r) \) and \( L = L(\delta,w,K) \). Therefore the wage under a long-term contract is the solution to

\[ \int \left( U_w(w,L(\delta,w,K(w,r))) + U_L(w,L(\delta,w,K(w,r))) \right) \left( L_\delta(\delta,w,K) + L_\gamma(\delta,w,K)K_w(w,r) \right) dF(\delta) = 0 \]

Alternatively under a short-term contract the wage is determined by

\[ U_w(w,L(\delta,w,K)) + U_L(w,L(\delta,w,K))L_w(\delta,w,K) = 0 \]
We now wish to characterize the distinction between short-term and long-term contracts. Unfortunately, at this level of generality it is difficult to precisely compare the two contract structures either for their positive effects on wages, employment and the capital stock or for their welfare effects on expected trade union utility. However, it is helpful to look at two polar cases. First take the case where the distribution $F(\delta)$ is degenerate; $F(\delta)=0$ for $\delta<\bar{\delta}$, and $F(\delta)=1$. Then there is no uncertainty at all and we are back in the environment of section 2. It is then clear that long-term contracts always dominate short-term contracts since

$$\max_w U(w, L(\delta, w, K(w, r))) \geq \max_w U(w, L(\delta, w, K))$$

for all $K$ such that $K=K(w, r)$

At the other extreme, take the case where there is no interaction between labour and capital in the revenue function, so that $R_\kappa=0$. Labour demand can then be written from (10) as $L=L(\delta, w)$ Then short-term contracts must unambiguously welfare dominate for the union. This follows because

$$\int \max_w U(w, L(\delta, w)) dF(\delta) \geq \max \int U(w, L(\delta, w)) dF(\delta)$$

Note that this holds no matter what the union's degree of risk aversion. Wages fixed in advance of the revelation of $\delta$ imply a smooth income stream for employed workers, but more variability in total employment. It is always

"7" To show this, note that

$$\max_w U(w, L(\delta, w)) \geq U(w, L(\delta, w))$$

for all $\delta \in [\underline{\delta}, \bar{\delta}]$ and $w>0$.

Then

$$\int \max_w U(w, L(\delta, w)) dF(\delta) \geq \int U(w, L(\delta, w)) dF(\delta)$$

for all $w>0$.

Taking the maximum of both sides with respect to $w$ leaves the direction of the inequality unchanged, and leads to (since the left-hand side is independent of $w$) the above condition.
better for the union to take advantage of the ex-post trade-off between employment and wage. Mathematically this inequality follows from the fact that the maximum operator is a convex function. Economically it follows from the fact that the union cannot do any worse by having the ability to adjust the wage to ex-post labour demand shocks, in the absence of any substitution between labour and capital in the revenue function.

This analysis outlines the general nature of the alternative benefits of precommitment versus flexibility in trade union wage setting. However, to explore the nature of the trade-off, it is necessary to restrict the analysis still further. To this end in the next section we make more specific assumptions about the form of the revenue function and union welfare function.

SECTION 5 THE TRADE-OFF BETWEEN PRECOMMITMENT AND FLEXIBILITY

Let us now assume that firms in the industry face a given price, which is the positive random variable $p$, with support $[p,ar{p}]$, and distribution $F(p)^8$. Define the expected price as $\bar{p} = \int_{0}^{\infty} p F(p) dp$. Also define $\hat{p}$ as follows:

$$\hat{p} = \frac{\int (1/p) dF(p)}{\int dF(p)}$$

By Jensen's inequality, $0 < \hat{p} < \bar{p}$. A mean preserving spread in $p$ leaves $\hat{p}$ unchanged and reduces $\bar{p}$.

We take the revenue function for the firm to be

$$R(p,L,K) = p(aL^K - bL^2 - cK^2 + dL + eK)$$

where $a, b, c, d, e > 0$, and $a^2 < bc^9$. Again a firm will choose the level of investment to maximize expected profits taking as given the rate of return on investment.
capital, while employment will be determined after the realization of p.

As in all instances of quadratic approximation, it is important to be careful that expected profits in equilibrium are non-negative. To ensure this some additional parametric restrictions will be imposed presently.

Let the trade union have the following utilitarian objective function.

\[ U = (w - w_0) L \]

where \( w_0 > 0 \) is a positive reservation wage for trade union members. The assumption of union risk neutrality is a very convenient one for exact derivation of the union's wage choice, but as the previous section makes clear, it is not necessary at all for the general nature of the trade-off. 5-A

**Short-term Contracts**

For each contract type, the conditional labour demand curve, derived from the analogue to (10) above, is written as

\[ L = \frac{(d + aK - w/p)}{b} \]

(12)\[ L = \frac{(d + aK - w/p)}{b} \]

In the short-term contract, the union maximizes \((w - w_0)L\) subject to (12). This yields a solution for the optimal wage \( w^* \).

\[ w^* = \frac{1}{2}(p(d + aK) + w_0) \]

(13)\[ w^* = \frac{1}{2}(p(d + aK) + w_0) \]

Note for future reference that the mean wage is independent of the variability of the price, but depends only on the mean price.

The investment problem facing the firm is

\[ \max \int (p(aL - b^2 L^2 + c K^2 + dL + eK) - wL - rK) \, dF(p) \]

\[ \frac{K}{K} \]

Implicitly firms must form rational expectations about the distribution of \( w^* \) given (13). The first order condition for this is

\[ \int (p(aL - cKA + e) \, dF(p) = r \]

Substituting for (12) and (13) gives a solution for \( K \) as\(^{10}\)

\[ b(\bar{e} - r) + a(\bar{p} - w_0) > 0. \]

\(^{10}\)To ensure that \( K > 0 \) it is required that \( b(\bar{e} - r) + a(\bar{p} - w_0) > 0. \)
\( K^e = \frac{2b(\tilde{x} - r) + a(\tilde{p} - w_0)}{\tilde{p}(2bc - a^2)} \)  

The capital stock under a short-term contract is increasing in \( \tilde{p} \) and decreasing in both \( w_0 \) and \( r \). Substituting (15) back into (12) leads to the equilibrium employment level, conditional on \( p \), under a short-term contract. This is

\[ L^e = \left[ \frac{2(ab(\tilde{x} - r) + bc\tilde{p}d) - a^2w_0}{\tilde{p}(2bc - a^2)} \right] \frac{W_0}{\tilde{p}} \left[ \frac{1}{2b} \right] \]

Taking expectations of this gives the mean level of employment under a short-term contract

\[ \bar{L}^e = \left[ \frac{2(ab(\tilde{x} - r) + bc\tilde{p}d) - a^2w_0}{\tilde{p}(2bc - a^2)} \right] \frac{W_0}{\tilde{p}} \left[ \frac{1}{2b} \right] \]

Equation (17) establishes that an increase in the variance of the industry price, holding the mean constant, will reduce expected employment given \( w_0 > 0 \). The mechanism behind this is easy to see from equation (12). Employment is negatively related to the firm's real product wage \( w/p \), so therefore expected employment is negatively related to the expected product wage. But the wage that unions set maintains a constant real product wage only if \( w_0 = 0 \). With \( w_0 > 0 \), the expected value of the real product wage, \( w/p \) is higher, the higher is the variance of the price.

Now using (12), (13) and (15) we compute expected union utility to be

\[ EU^e = (1/4b)(\tilde{p}(d+aK^e)^2 - 2w_0(d+aK^e) + (w_0^2/\tilde{p})) \]

Since \( K^e \) depends only on the expected price, (18) implies that union utility is increasing in the variance of the price. This result holds despite the fact that expected employment is a decreasing function of price variance, and the expected wage and capital stock are independent of price variance. For a
welfare maximizing union, which may choose a wage optimally given the outcome
for the price, utility is convex in the price.

This point can be made in another way by looking back at the union
utility function. From this we can use the covariance decomposition \( \text{Exy} = \text{ExE}y + \text{Cov}(xy) \) to write

\[
(19) \quad \text{EU} = E(w-w_0)EL + \text{Cov}(w-w_0, L)
\]

An increase in price variance leads to a fall in expected employment, as we
have seen. This reduces the first expression on the right hand side of (19).
But under short-term contracts, unions have the flexibility to adjust wages
upwards in times of high prices and high labour demand, and downwards in
times of low prices and low labour demand. Thus the covariance term in (19)
will be positive, increasing in the variance of prices. This gives a precise
statement of the gains from flexibility.

5-B Long-term Contracts

Under a long-term contract, the wage cannot respond to the ex-post
market price. To derive the optimal wage for the union under a long-term
contract we first need to derive the expected level of employment and
investment, conditional on the wage.

Again, employment is given by (12). The capital stock is given by the
solution to (14), given (12), and for a given wage. This gives the solution

\[
(20) \quad K = \frac{a(\tilde{p}d-w) + b(\tilde{p}e-r)}{\tilde{p}(bc-a^2)}
\]

Using (20) in (12) gives

\[
(21) \quad L = \left[ d+ \frac{-a^2w + ab(\tilde{p}e-r)}{\tilde{p}(bc-a^2)} - \frac{w}{\tilde{p}} \right] \frac{1}{p}
\]

The wage is then derived from the problem

\[
\max_{w} \int (w-w_0)LdF(p)
\]
subject to (21). The solution to this problem is

$$w^* = \left( \frac{pd + w_0}{2} \right) + \frac{ab(p - r)p}{2(p(bc-a^2) + a^2p)}$$

Now the wage is increasing in \( \hat{p} \), or decreasing in price variability. When wages cannot be conditioned on \( \hat{p} \) ex-post, in order to set an optimal wage, the union must calculate the expected value of labour demand. For a given wage under the long-term contract, higher price variability decreases expected labour demand in the way we outlined above. In order to offset the effect on expected union utility, the union reduces the optimal wage.

Calculating the expected employment level we have

$$f^* = \left[ \frac{pd + w_0(p(bc-a^2) + a^2)}{(bc-a^2)p} + \frac{ab(p - r)}{(bc-a^2)p} \right]$$

What is the effect of a rise in the variance of the price on this? Differentiating (23) with respect to \( \hat{p} \), we have

$$\frac{df^*}{dp} = \text{Sign} (-\hat{p}^2a^2 \hat{p}w_0(bc-a^2))$$

The sign of the expression on the right hand side is ambiguous. Thus a rise in price variability may raise or lower mean employment. Higher price variability for a given wage reduces mean employment. But the wage chosen by the trade union will itself fall in response to higher price variability. The effect of this is two-fold. First there are direct effects on the expected real wage \( E(w/p) \). Secondly there are the effects of a lower wage on the capital stock, through the expression (21). It is easy to show from (22) that the net effect of a rise in price variability on the expected real product wage is positive. This alone would tend to reduce expected employment. But the effect of higher price variability on the capital stock may be so great as to offset this. If \( a=0 \) though, there is no interaction
between factors of production, and the direct effects must dominate.

Expected union utility under long-term contracts just equals the product of the wage less the reservation wage and the expected level of employment. In contrast to the short-term contract, union utility must be increasing in \( \hat{p} \). This follows from use of the envelope theorem. Expected union utility at an optimum is given by

\[
\text{EU}^*(w^*, \hat{p}) = (\tilde{w}^* - w_0) \tilde{w}^* = (\tilde{w}^* - w_0) \left[ \frac{d - a \tilde{w}^* + ab \tilde{p} (e - r)}{p (bc - a^2)} - \frac{w^*}{\hat{p}} \right] \frac{1}{\hat{p}}
\]

By use of the envelope condition \( \partial \text{EU}^*/\partial w^* = 0 \), so that

\[
\frac{\partial \text{EU}^*}{\partial \hat{p}} = (\tilde{w}^* - w_0) \frac{w^*}{\hat{p}^2} \frac{1}{b} > 0
\]

Therefore a rise in price variability (fall in \( \hat{p} \)), unambiguously reduces expected union utility. With advance wage setting, to a first order the only effect of higher price variability on expected utility of the union is the direct effects on expected labour demand, for a given wage. In particular the covariance term in the covariance decomposition expression above is zero, indicating the absence of 'flexibility'.

5-C Comparison of Short-Term and Long-Term Contracts

We now wish to compare the wage, employment levels, capital stock, and expected union utility under short-term and long-term contracts. As a result of the welfare consequences, we then discuss empirical implications for determinants of optimal contract length.

Wage Comparisons

A comparison of (13) and (21) reveals that the mean wage is unambiguously higher under a short-term contract. There are two factors involved in this. The first is that outlined in section 2: for the variable \( a > 0 \), in the case of perfect certainty the union will choose a higher wage.
when wage-setting does not take the investment decision into account. This is easily confirmed by noting that for \( \bar{p} = \hat{p} \), by (13), (15) and (21), \( w^* > w' \) for \( a > 0 \). However, in addition, with uncertain labour demand, a mean preserving spread in prices will reduce the wage under a long-term contract but leave the mean wage under a short-term contract unchanged. Thus the wage gap between short-term and long-term contracts must rise with higher price variability. When \( a = 0 \), there is no interaction between labour and capital in production, and in the nonstochastic case the two wages would be the same. But again with higher price variability the long-term contract wage declines.

Employment Comparisons

By (16) and (22) we can establish that

\[
\bar{c} - \bar{c}_* = \frac{a^2 [a(\bar{p}e - r) - c(\bar{p}d - w_0)]}{2\bar{p}(bc - a^2)(2bc - a^2)} + \frac{a^2d(\bar{p} - \hat{p})(2bc - a^2)}{2\bar{p}(bc - a^2)(2bc - a^2)}
\]

The expression \([a(\bar{p}e - r) - c(\bar{p}d - w_0)]\) must be positive if employment is to be positive at the mean price. Moreover, \( \bar{p} - \hat{p} \) is always non-negative. Thus expected employment is always greater with a long-term contract, as long as there some interaction between labour and capital in production. Note that the gap between the two employment levels increases with the extent of price variability. As we noted above, price variability affects expected employment under both contract types, but reduces expected employment unambiguously only under a short-term contract, since under a long-term contract, the wage will adjust downwards with price variability.

Another important comparison is the variance of employment. Under a short-term contract this is

\[
(w_0^2/4b^2)E[1/p - 1/\hat{p}]
\]

while under the long-term contract the variance is

\[
(w'^2/b^2)E[1/p - 1/\hat{p}]
\]
But since \( w^* > w_0/2 \), the variance of employment must be greater in a long-term contract. This is quite intuitive. Under a short-term contract the component of the wage that is fixed (state-independent) is just the \( w_0 \) term. Under the long-term contract the wage is completely state independent. Thus, in the latter environment, employment reacts by more to a surprisingly high or low draw for \( p \).

**Investment Comparisons**

Using (15) and (20) it is easy to show that in the nonstochastic case (\( \hat{p} = p \)), the difference between the two capital stocks is

\[
\frac{a^2[a(\hat{p}e-r)+c(\hat{p}d-w_0)]}{2pc(bc-a^2)(2bc-a^2)} > 0
\]

For familiar reasons, the capital stock is higher under long-term contracts for \( a > 0 \). However as price variability rises, the equilibrium capital stock under long-term contracts rises (c.f. (20)) since the equilibrium wage falls, while under short-term contracts, the equilibrium capital stock is independent of the variance in prices. Thus the long-term capital stock is always greater, and the gap rises with greater price variability.

**Union Welfare Comparisons**

How does union welfare differ between the two contract types? The two special cases of section 3 are immediate. With \( a=0 \) and \( \hat{p} \neq \hat{p} \) expected utility is:

\[
(1/4b)(\hat{p}d^2-2w_0d+(w_0^2/p))
\]

under short-term contracts and

\[
(1/4b)(\hat{p}d^2-2w_0d+(w_0^2/p))
\]

under long-term contracts. Thus short-term contracts clearly dominate. On the other hand with \( \hat{p} = \hat{p} \) and \( a > 0 \) long-term contracts must dominate\(^\text{11}\). In \( 11 \)This is straightforward to show by substitution in the expected utility expressions given above, but in any case follows from section 3.
general however the utility comparison is ambiguous. Starting from a position of no uncertainty a rise in the variance of the price will reduce expected welfare under a long-term contract and increase that under short-term contracts.

As an algebraic comparison of expected utilities is not very revealing, we resort to a numerical assessment of the welfare under the two contract forms. The critical parameters are the value of \( \alpha \) and the standard distribution of the industry price. In order to compute the expressions for expected utility in each case, we use the Taylor series approximation

\[
\hat{p}^{-1} = \hat{p}^{-1} + \frac{\text{Var}(p)}{2\hat{p}^3}
\]

Given a mean price, we vary the standard deviation to produce different values of \( \hat{p} \) for computation. Table 1 contains the information from the computation. The following values are chosen: \( b=c=d=e=2, \ r=0.1, \ \omega_0=1, \ \hat{p}=4, \) and a range of values of \( \text{Var}(p) \) and \( \alpha \). In the Table we include the values of the capital stock, expected wages, and expected employment for the two contract types, as well as the expected utility values.

Note that expected employment under long-term contracts tends to fall with increased price variability for low values of \( \alpha \), but the opposite occurs for high values of \( \alpha \). This is to be expected from the condition (24) above. In addition, for low values of \( \alpha \), short-term contracts will dominate long-term contracts in welfare terms. This is just the case where the benefits of 'flexibility' are great and the value of commitment is not high. However, for high values of \( \alpha \), the benefits of wage commitment become much greater, and tend to dominate the costs of reduced wage flexibility.
Alternatively, for a given value of \( a \), increases in the standard deviation of the industry price will raise the utility value of short-term as opposed to long-term contracts\(^{12}\).

SECTION 6 DISCUSSION AND CONCLUSIONS

To the extent that contract length is chosen optimally under union-dominated industries, our analysis gives some guidance as to the explanatory variables in an empirical explanation of contract length. The variable \( 'a' \) from the previous section is monotonically related to the elasticity of substitution between labour and capital in production. Thus the net benefits of long-term versus short-term contracts based on the model of the previous section will depend upon the size of the industry elasticity of substitution relative the degree of variability of industry prices. In industries with very flexible techniques of production, and with relatively stable demand characteristics, the former will be high and the latter low. Thus we should expect to see long-term contracts dominate. On the other hand, with relatively little factor substitution and high price variability, shorter contracts should be observed. This again is on the assumption that the industry price is fairly unresponsive to output movements however. From the analysis of Section 2 we saw that with a high market demand elasticity a rise in the elasticity of substitution can reduce the relative benefits of the long-term contract.

What empirical evidence is there on the determinants of contract length? A survey paper by Cristofides (1985) discusses some international differences in the average duration of labour contracts. The most striking piece of\(^{12}\) In a recent paper, Ragan (1989) argues that trade unions will always desire to write short-term contracts for a similar reason to that outlined above, i.e. the benefits of flexibility. He does not investigate the value of commitment in long-term contracts, assuming away all investment effects.
cross country evidence seems to be the differences between contract duration in the U.S. and Canada, and those of other countries such as the UK, Japan, and West Germany. In the former two countries, about three quarters of all contracts are for two years or longer, while in the latter, one year contracts are the norm. For instance, Bils (1990) reports that for a sample of 2684 US contracts in manufacturing from 1957 to 1985, constructed by Wayne Vroman,13 fifty percent are for three years or longer, while only fifteen percent are for a year or less. On the other hand, Gregory, Lobban, and Thomson (1985) report that for the CBI databank on British pay settlements between 1979 and 1983, over ninety percent of settlements were for twelve months. Furthermore, any contracts that were longer than twelve months tended to be one off affairs, and the bargaining unit usually reverted to the twelve month mode after that.

It would be very difficult to explain such major differences in labour market institutions across countries with a simple unicausal theory. It may be that these two separate contract structures could each exist for extraneous reasons while having no other implications. In fact, history may play the most important role in a full explanation. It is notable that the differences extend not just to contract length, but also to the structures of bargaining. In Japan for instance, wages and benefits are determined in separate agreements, with the determination of benefits being made much more infrequently, while wages are determined by highly centralized annual agreements. West German wages are also determined much more centrally than those in North America. The UK is something of a special case to the extent that wage contracts have no binding legal status, so, as first expounded by Grout (1984), the commitment value of these contracts is ambiguous.

Given the complexity of institutional histories across countries then, our model might be more easily applied to an investigation of the differences between contract duration across industries within a country rather than differences between average duration between countries. Most of the empirical literature however, looks at the time series determinants of contract length for a given sample of industrial contracts. Cristofides and Wilton (1983) examine a series of about 1500 union contracts for Canada for the years 1965 to 1975, and find a significantly negative effect of price level uncertainty on contract length. This is also borne out by Cristofides (1985), who notes that average union contract length in Canada fell during the 1974-75 period, even those which were covered by COLA clauses.

For the United States, the evidence is more mixed. Ehrenberg, Danziger and San (1984), and Cecchetti (1987), for different contracts, and different time periods, find little evidence of any relationship between measures of inflation or inflation uncertainty and contract length. Similar results are reported in Bils (1990). However, Vroman (1988) obtains quite different results, reporting quite a sharp negative response of contract length to inflation uncertainty. For the UK, Gregory, Lobban, and Thomson (1985) report that there is no significant variation of any kind in the duration of wage contracts for their data.

Bils (1990) examines the cross-industry determinants of contract length for US data. He reports that, in contrast to the theories of Gray (1978), contract length in the US seems to be significantly positively affected by measures of uncertainty. In a regression of contract length on employment variability, the coefficient is positive and significant. Although there is a (perhaps acute) simultaneity problem here, as explained in the following paragraph, Bils uses an instrumental variables approach to circumvent this.
and obtains very similar results. Without knowledge of the degree of substitutability between capital and labour at the industry level, however, it is difficult to assess how these results relate to the predictions of our model.

Note that in assessing the explanatory power of the model, it is very important to have an estimate of the measure of exogenous variability facing the industry, and not to use, for instance, the variability of employment as a proxy for the former. The reason is that as a by-product of contract structures the variability of employment under a long-term contract will tend to be higher anyway, as is shown in section 4. Thus it is quite possible for the underlying variability of the industry price to be higher in industry A relative to industry B, but employment variability in the former is lower because short-term contracts are optimally chosen in this environment. Thus focusing on employment variability can mistakenly lead us to get the exact opposite conclusion to the one suggested by this paper.

Some qualifications and extensions to the model of the paper suggest themselves. First we have chosen an environment where all of the workers in an industry are unionised, and also one in which hiring probabilities are the same for each union member. In that case the union objectives quite naturally reflect the concern with both employment and wages. If the wage setting in a trade union were dominated by insiders, as in the models of Lindbeck and Snower (1986) and others, the character of our results may be quite different. In that case the insiders would not be concerned with flexibility at all because employment variability is of no concern to them. A similar qualification is appropriate if union hiring is based upon seniority as in Oswald (1984).

14 We thank a referee for pointing this out to us.
An additional issue we have neglected is the possibility of direct bargaining between union and firm in the manner of Macdonald and Solow (1981). A large literature exists on the issue of which model of a trade union is appropriate. In the monopoly trade union model, the implicit assumption is that the union is supplying labour to a large group of firms and there is no direct bargaining. In a more symmetric situation, with one firm and one union, the bargaining model would seem to be more appropriate. In that case, the firms' profits in each contract situation would be an important determinant of optimal contract length as well.\(^{15}\)

Note that an important part of our analysis lies in the feature of the Monopoly trade union model that leads the union determined wage to have an allocational role - wages directly determine employment, as firms are on their labour demand curve. In fact this gives rise to the benefits of flexibility in our stochastic model. In the more general bargaining environment, wages may be determined separately from employment. In that case the benefits from adjusting wages to ex-post information may be eliminated. For this reason, it is not at all clear that the results above will apply to the more general bargaining model. Nevertheless, most of the previous literature on contract length applies to environments where wages do in fact play an allocational role (see section 2). In addition, the monopoly trade union model, or its variant, the 'right to manage' model of Nickell and Andrews (1983), has been extensively studied in the theoretical and empirical literature, and may be arguably applied to actual real world labour market situations.

\(^{15}\)It is easy to show from the model above that the firm will always have an incentive to operate under a situation of long-term contracts. This is for two reasons; (i) the wage is lower, and thus expected profits are higher, and (ii) because the wage is state independent, the covariance between wage and employment is zero, which also leads to higher expected profits.
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