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COST ASYMMETRIES IN INTERNATIONAL SUBSIDY GAMES:

SHOULD GOVERNMENTS HELP WINNERS OR LOSERS?*

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ABSTRACT

This paper examines the optimality of export subsidies in oligopolistic markets, when home and foreign firms have different costs and there is an opportunity cost to public funds. Subsidies are found to be optimal only for surprisingly *low* values of the shadow price of government funds and, if subsidies are justified, they should be higher the *more* cost-competitive are domestic firms. These results hold under both Cournot competition and Bertrand competition when firms move before governments. The results suggest that recent arguments for export subsidies apply *only* for firms that would be highly profitable even without subsidies.

(99 words)

COST ASYMMETRIES IN INTERNATIONAL SUBSIDY GAMES: SHOULD GOVERNMENTS HELP WINNERS OR LOSERS?

1. Introduction

The recent outpouring of research on imperfect competition in international trade has called into question many of the lessons of traditional trade theory. One of the most striking contrasts between "new" and traditional trade theories concerns the optimality of export subsidies. If markets are competitive, export subsidies should be zero in a small open economy and should be *negative* if the economy has monopoly power in trade. By contrast, Brander and Spencer (1985) were the first to show that, in a Cournot duopoly setting, an export subsidy to a home firm is desirable because it raises the firm's market share and profits at the expense of its foreign competitor.

This surprising departure from conventional wisdom has naturally generated a great deal of controversy, and subsequent work has shown that the case for export subsidies is subject to significant qualifications: for example, Eaton and Grossman (1986) show that, if firms compete on price rather than quantity (so that firms play a Bertrand rather than a Cournot game), then the optimal policy is an export *tax* rather than a subsidy.[1] Nevertheless, the case of Cournot duopoly has such a central place in oligopoly theory that the argument for export subsidies based on it cannot easily be dismissed and, indeed, it has proved to be highly influential in

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[1] See also Cheng (1988). The case for a positive subsidy is also weakened if the foreign government retaliates (Brander and Spencer, 1985), if there is more than one home firm (Dixit, 1984), if subsidised firms compete for scarce factors of production (Dixit and Grossman, 1986) or if entry to the industry is free (Markusen and Venables, 1988). For an overview of this literature, see Neary (1988), where I also show how the Brander-Spencer result may be related to the traditional results of competitive trade theory.

policy debates.[2] However, an issue which has not been adequately explored is, if subsidies are indeed optimal, *which* firms should be more favoured.

In this paper, I explore this issue of the appropriate pattern of subsidies across firms by focussing on two central and hitherto relatively neglected cost asymmetries. The first of these is the asymmetry between *private and social costs*. Most previous studies have assumed that there is no opportunity cost to subsidy payments: an extra dollar earned in profits by the home firm has the same social valuation as an extra dollar in subsidy payments foregone by the home government.[3] But public funds are never in perfectly elastic supply and it is necessary to explore how sensitive is the case for subsidies to the relaxation of this assumption. The second key asymmetry to be considered is that between *home and foreign costs of production*. While this consideration has been mentioned in passing by some authors (including Dixit and Grossman (1986) and Spencer (1986)), its importance has been studied formally only by de Meza (1985), whose results are discussed in Section 4 below. This relative neglect is regrettable, since the difference between home and foreign costs is central to the crucial policy question of whether government assistance should be targeted towards weaker firms (as an infant industry argument would suggest) or towards firms which would be able to compete successfully on international markets even in the absence of subsidies.

The plan of the paper is as follows. In Section 2, I present the simplest linear model of a Cournot duopoly and show how the two cost asymmetries discussed above influence the sign and magnitude of the optimal subsidy. Section 3 extends this analysis to the case of general demands and shows that the results with respect to the distinction between private and

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[2] See, for example, the contributions to Krugman (1986).

[3] Exceptions to this rule are the papers by Gruenspecht (1988) and Neary (1991) which are considered further in Section 5 below. These deal only with the case of Bertrand competition under the assumption that firms move before governments. The implications of a deadweight loss from raising revenue are also discussed briefly in Brander and Spencer (1988), p. 233.

social costs are relatively robust. Section 4 then considers the case where both the home and foreign governments engage in subsidisation and relates the difference in optimal subsidy rates to the difference between home and foreign costs.

The remainder of the paper turns from the Cournot case to consider the Bertrand case of price competition. As already noted, Eaton and Grossman (1986) have shown that optimal policy in this case is an export tax rather than a subsidy. However, this result assumes that governments move before firms, whereas there is substantial anecdotal evidence (see, especially, Carmichael, 1987) that the opposite timing of moves is adopted in practice. This raises two questions: is the optimal policy in the "government moves second" game a positive subsidy or not?; and, does this optimal policy yield a higher level of welfare than either free trade or the optimal "government moves first" policy? These issues are addressed in Section 5, where the roles of both types of cost asymmetry in affecting the answers are explored. Section 6 then examines the case where both home and foreign governments compete in this framework. Finally, the paper's conclusions and some suggestions for further research are noted in Section 7.

2. Cournot Competition with Linear Demand

In this section, I consider the simplest model of Cournot or quantity competition, where a single home firm competes against a single foreign firm for sales in a third country market, whose inverse demand curve takes the simple linear form:

$$(2.1) \qquad p = a - b(x+y).$$

Here x and y are the levels of home and foreign output respectively, assumed to be homogeneous, and the two firms have constant marginal costs equal to c and c^*.[4] The free trade equilibrium is then easily calculated as the

[4] Fixed costs are ignored in the analysis. The effects of fixed costs in generating links between markets or in influencing entry are considered in

intersection of the two firms' reaction functions HH' and FF', as illustrated in Figure 1 at point C.[5]

We now wish to determine the optimal subsidy by the home government. (Until Section 4, the foreign government is assumed not to pursue a subsidy programme.) In order to do so, we must specify a social welfare function which, in the absence of home consumption, depends only on the home firm's profits and the value of subsidy payments. However, I depart from the usual Brander-Spencer framework here in attaching to the value of subsidy payments a weight δ which may exceed unity:

(2.7) $$W = \pi - \delta s x, \qquad \delta \geq 1.$$

The parameter δ may reflect pure distributional considerations, whereby a lower social valuation is attached to corporate profits than to those types of income which must be taxed to finance subsidy payments.[6] Alternatively, it may reflect the deadweight cost of raising taxes elsewhere in the economy, on the assumption that lump-sum taxation is not available.[7] Finally, δ may

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Krugman (1984), Dixit and Kyle (1985), Horstmann and Markusen (1986) and Markusen and Venables (1988).

5 The reaction functions may be calculated from the firms' first-order conditions. For the home firm this is:

(2.2) $$p - c + s - bx = 0.$$

Substituting for p from (2.1) yields:

(2.3) $$HH': \quad 2bx + by = a - c + s,$$

Similar calculations for the foreign firm yield:

(2.4) $$FF': \quad bx + 2by = a - c^*.$$

Solving for x and y yields:

(2.5) $$3bx = a - 2c + c^* + 2s,$$

(2.6) $$3by = a + c - 2c^* - s.$$

6 This would be appropriate, for example, if the home firm was partly foreign-owned, in which case δ equals the reciprocal of the home equity share.

7 This is the interpretation adopted by Gruenspecht (1988). Calculations by Browning (1987) for the U.S. economy, on the assumption that subsidies are

reflect the limited budget available to a public agency charged with allocating subsidies between a number of home firms, each of which is in a similar situation relative to its foreign competitor as the single home firm considered here; this interpretation is formalised in the Appendix. Whatever the reason for an asymmetry between social and private costs, it seems plausible to assume that δ will typically be greater than unity and may be significantly so.

With the welfare function as specified in (2.7), it is straightforward to calculate the relationship between the optimal export subsidy and the level of home output at the optimum. Totally differentiating (2.7) yields:

$$(2.8) \qquad dW = [p-c+(1-\delta)s]dx + xdp + (1-\delta)xds.$$

Using (2.1) to eliminate dp and the home firm's first-order condition (2.2) to simplify the coefficient of dx, this becomes:

$$(2.9) \qquad dW = -\delta s dx - bxdy + (1-\delta)xds.$$

The first term on the right-hand side measures the standard allocative loss from the expansion of home exports while the second measures the Brander-Spencer rent-shifting effect arising from the induced reduction in the foreign firm's output. The third term is new and measures the deadweight loss of raising revenue to finance the subsidy payments. dx and dy can now be eliminated from (2.9) using (2.5) and (2.6) to obtain:

$$(2.10) \qquad s^o = \frac{4-3\delta}{\delta}\,\frac{b}{2}\,x^o.$$

This shows that a positive subsidy is justified only for surprisingly low values of the shadow price of government funds:

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financed by taxes on labour earnings, imply a value for δ between 1.10 and 4.03, with his preferred estimates lying between 1.32 and 1.47. I assume that any change in subsidy payments (whether an increase or decrease) is accommodated by a change in tax revenue and that there is some other component of public spending (e.g., public goods) which must always be financed. This ensures that the value of δ is the same whether s is positive or negative.

Proposition 1: With linear demands in a Cournot duopoly, a positive export subsidy is justified if and only if the shadow price of government funds is less than 1.33.

The fact that the threshold value of δ is 1.33 may be explained as follows. From (2.6), $dy = -ds/3b$. Substituting this into (2.9) shows that, following a one dollar increase in the subsidy, the induced reduction in the foreign firm's output raises welfare by $x/3$ dollars while the additional deadweight loss lowers welfare by $(1-\delta)x$ dollars. The threshold value of δ at which these two effects cancel is 1.33. It is natural that the optimal subsidy should be decreasing in δ: the higher its value, the more the government wishes to tax the home firm rather than to raise its profits at the expense of the foreign firm's. The surprising feature of this result is that an export tax becomes optimal for relatively low values of δ, well within the empirically plausible range, as noted in footnote 7.

Finally, (2.5) may be used to eliminate x^0 from (2.10) to give an explicit expression for the optimal subsidy:

$$(2.11) \qquad s^0 = \frac{4-3\delta}{3\delta-2} \frac{a-2c+c*}{4}$$

This shows clearly the influence of relative production costs:

Proposition 2: If the optimal subsidy is positive in a Cournot duopoly with linear demands, its value is greater the more competitive is the home firm.

This is illustrated in Figure 1, where the line HC'J is the locus of optimal output combinations as c* varies (i.e., as the foreign firm's reaction function FF' moves upwards or downwards). The more competitive the home firm (i.e., the higher is c* and so the lower is the line FF' in Figure 1), the greater the gap between this locus and the locus of free trade output combinations HCH'.

It may seem paradoxical to argue that governments should provide more help to relatively profitable firms than to unprofitable ones. The paradox is resolved by noting that the motive for subsidisation in this model is not to encourage learning by doing but to raise home profits at the expense of foreign competitors. The more competitive is the home firm, the greater the payoff to shifting rents towards it. However, if the shadow price of government funds mandates an export *tax* rather than a subsidy, then the tax should be higher the more competitive the home firm: with revenue-raising now the dominant consideration, a firm should be more highly taxed the more efficient it is as a generator of tax revenue.

3. Cournot Competition with General Demands

Equation (2.11) shows a neat separability between the effects on the optimal subsidy of the two types of cost asymmetry.[8] However, this reflects the assumption of linear demand and so it is necessary to investigate the robustness of the results when a general form is assumed for the demand function. In this section, I continue to assume that the home and foreign firms produce a homogeneous product, but the demand curve they face now takes, instead of (2.1), the general form, $p(x+y)$. Profit maximisation by the home firm therefore implies the first-order condition:

$$(3.1)\qquad \pi_x = xp' + p - c + s = 0.$$

Using this to simplify the total differential of the welfare function (2.7) yields, instead of (2.9), the following:

$$(3.2)\qquad dW = -\delta s dx + xp'dy + (1-\delta)xds.$$

To proceed further, it is necessary to relate changes in output levels to changes in the subsidy. This may be done by totally differentiating (3.1) and the corresponding condition for the foreign firm:

$$(3.3)\qquad \begin{bmatrix} \pi_{xx} & \pi_{xy} \\ \pi^*_{yx} & \pi^*_{yy} \end{bmatrix} \begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} -ds \\ 0 \end{bmatrix}.$$

[8] An implication of this is that, if δ arises from the actions of a budget-constrained public agency, then the sign of the optimal subsidy should be the same for all firms.

Here, the diagonal terms π_{xx} and π^*_{yy} (which equal $2p'+xp''$ and $2p'+yp''$ respectively) are negative from the second-order conditions for profit maximisation; while the off-diagonal terms π_{xy} and π^*_{yx} (which equal $p'+xp''$ and $p'+yp''$ respectively) are negative provided the two goods are *strategic substitutes*, in the terminology of Bulow, Geanakoplos and Klemperer (1985). To assist in interpreting the results, I will assume henceforward that the latter condition holds, though the algebraic derivations do not depend on it. Solving (3.3) yields (as in Brander and Spencer (1985)):

$$(3.4) \qquad \Delta dx = -\pi^*_{yy} ds \qquad \text{and} \qquad \Delta dy = \pi^*_{yx} ds,$$

where Δ is the determinant of the coefficient matrix in (3.3) and is positive from stability considerations:

$$(3.5) \qquad \Delta \equiv \pi_{xx}\pi^*_{yy} - \pi_{xy}\pi^*_{yx} > 0.$$

Substituting from (3.4) into (3.2) yields the following expression for the optimal subsidy:

$$(3.6) \qquad s^o = -\frac{x}{\delta \pi^*_{yy}} [p'\pi^*_{yx} + (1-\delta)\Delta].$$

This shows clearly that higher values of δ make it more likely that the optimal subsidy will be negative. Substituting from (3.5) and the definition of π^*_{yx}, this becomes:

$$(3.7) \qquad s^o = -\frac{xp'}{\delta \pi^*_{yy}} [(4-3\delta)p' + \{(1-\delta)x + (2-\delta)y\}p''].$$

Comparing this with (2.10) shows that, for δ greater than or equal to 2, convex demand ($p''>0$) tends to raise the optimal subsidy above the value it takes in the case of linear demand, and conversely for concave demand ($p''<0$). Finally, the bracketed expression in (3.7) may be manipulated to show that the threshold value of δ, at which the optimal subsidy switches from a positive to a negative value, equals:

$$(3.8) \qquad \bar{\delta} = \frac{4p' + (x+2y)p''}{3p' + (x+y)p''}.$$

From this it is easy to derive:[9]

Proposition 3: Provided the goods are strategic substitutes, the threshold value of δ at which the optimal subsidy switches from a positive to a negative value must be less than 2.

This confirms that the result found in Section 2 for the linear case carries over without substantial modification to the general case: it remains true that the optimal intervention is an export tax rather than a subsidy for surprisingly low values of δ.

4. International Subsidy Games with Cournot Competition

In Section 3 I showed that the influence of an asymmetry between social and private costs derived for the linear case in Section 2 continues to apply when the demand function is non-linear. Unfortunately, it is not possible to say anything concerning the effects of an asymmetry between home and foreign production costs on the optimal subsidy given by (3.7). However, it is possible to consider this issue if we assume that both governments pursue a policy of subsidising their own firm, choosing subsidy levels non-cooperatively before firms choose their output levels. This approach was first explored by Brander and Spencer (1985) and was extended by de Meza (1986). In this section I extend the results of the latter paper to the case where δ exceeds unity.

With a subsidy s* provided by the foreign government, the foreign firm's first-order condition is exactly analogous to that for the home firm, (3.1):

$$(4.1) \qquad \pi^*_y = yp' + p - c^* + s^* = 0.$$

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[9] Proposition 3 may be established as follows:

$$(3.9) \qquad 2 - \bar{\delta} = \frac{2p' + xp''}{3p' + (x+y)p''} = \frac{\pi_{xx}}{\pi_{xx} + \pi^*_{yx}}.$$

A negative value for π^*_{yx} is sufficient but not necessary for 2 to exceed $\bar{\delta}$.

Subtracting this from (3.1) gives a relationship between the subsidy differential and the cost and output differentials between the two firms:

$$(4.2) \qquad s - s^* = c - c^* - p'(x - y).$$

This shows that there are two competing influences on the subsidy differential: at given output levels, a home cost advantage warrants a *lower* home subsidy; but a home cost advantage is likely to imply a higher relative output by the home firm, which in turn warrants a *higher* subsidy. To determine which of these influences dominates when both governments intervene, I make use of the expressions for the optimal subsidies. That for the home subsidy has already been derived in (3.7) and that for the foreign subsidy is similar:

$$(4.3) \qquad s^o_* = -\frac{yp'}{\delta\pi_{xx}}[(4-3\delta)p' + \{(1-\delta)y + (2-\delta)x\}p''].$$

Combining (3.7) and (4.3) and simplifying yields a relationship between the optimal subsidy differential and the output differential:

$$(4.4) \qquad s^o - s^o_* = \frac{\xi}{\mu}(x - y),$$

where:

$$(4.5) \qquad \mu \equiv -\delta\pi_{xx}\pi^*_{yy}/p' = -\delta[xy(p'')^2+2\theta p']/p' > 0.$$

$$(4.6) \qquad \xi \equiv xy(p'')^2 + \theta\{(4-3\delta)p' + (1-\delta)(x+y)p''\}.$$

$$(4.7) \qquad \theta \equiv 2p'+(x+y)p'' < 0.$$

Note that the assumption that θ in (4.7) is negative amounts to assuming that the two goods are strategic substitutes. Equation (4.4) must be supplemented by relating the output differential (which is of course itself endogenous) to the cost differential. By combining (4.4) and (4.2), this is found to be:

$$(4.8) \qquad x - y = -\frac{\mu}{\varphi}(c - c^*),$$

where:

$$(4.9) \qquad \varphi \equiv \theta p' - (1-\delta)\{xy(p'')^2+\theta(\theta+3p')\} \quad > \quad 0.$$

The sign of the coefficient of c - c* in (4.8) is unambiguously negative, confirming that a home cost advantage implies a relatively *higher* home output in the equilibrium of the Nash subsidy game. Finally, combining (4.4) and (4.8) yields the desired result:

$$(4.10) \qquad s^o - s^o_* \quad = \quad -\frac{\xi}{\varphi}\,(c - c*).$$

In this expression, the denominator φ is positive but the numerator ξ, which was defined in (4.6), cannot be signed unambiguously. However, it must be positive in two important special cases. The first of these is when δ is unity: this result was obtained by de Meza (1986). In this case, (4.10) becomes:

$$(4.11) \qquad s^o - s^o_* \quad = \quad -\frac{xy(p'')^2 + \theta p'}{\theta p'}\,(c - c*).$$

This shows that, with δ equal to unity, the subsidy differential exactly equals the production cost differential when demand is linear and exceeds it when demand is non-linear. A second case is when δ exceeds unity but demand is either linear or convex ($p''\geq 0$): then the coefficient of θ in the expression for ξ, (4.6), must be negative whenever the home optimal subsidy (given by (3.7)) is positive.[10] Summarising:

Proposition 4: When a positive subsidy is optimal for the home government in a Nash subsidy game, sufficient conditions for the home subsidy to be relatively higher the more cost-competitive is the home firm are that <u>*either*</u> *(a) the social opportunity cost of public funds δ is close to* <u>*unity;*</u> <u>*or*</u> *(b) demand is non-concave.*

This shows that, in a significant fraction of cases, the result derived in Section 2 for the case where demand is linear and only the home government intervenes continues to hold when demand is non-linear and the two

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[10] The coefficient of θ in (4.6) exceeds the bracketed expression in (3.7) by $-yp''$ and so must be positive whenever s^o is positive *and* p'' is non-negative.

governments play a Nash game in subsidies: the more cost competitive is the home firm, the higher the subsidy which it should receive.

5. Bertrand Competition when Firms Move First

So far, I have assumed that both firms play a Cournot game, choosing output levels in a non-cooperative manner. As Eaton and Grossman (1986) have shown, if all the other assumptions are retained but firms are assumed to play Bertrand, then the optimal policy is an export *tax* rather than a subsidy. However, a role for subsidisation when firms play Bertrand can be salvaged along lines suggested by Carmichael (1987) and Gruenspecht (1988). In this section, I summarise their approach and then investigate the role of cost asymmetries in this framework.

The key feature of the Carmichael-Gruenspecht paradigm is a reversal of the order of moves by firms and governments. Specifically, they assume that governments decide on subsidy levels only *after* firms have set their prices. While this may seem unlikely at first, Carmichael presents persuasive evidence that it characterises the behaviour of the US ExIm Bank; and it also appears to be a plausible description of the behaviour of the export credit agencies of many European countries. Essentially, it amounts to assuming that the government commits itself not to a subsidy *level* but to a subsidy *rule* and that the home firm chooses its price in the knowledge that both its own and its competitor's price will influence the level of the subsidy. Since the equilibrium is perfect, this influence is taken into account by both firms in choosing their prices in the first stage of the game.

To formalise these ideas, I assume that products are differentiated and that demands are linear and symmetric.[11] The home and foreign firms produce output levels x and y, as before, and charge prices p and q respectively. In this section I assume that only the home government offers a subsidy

[11] Products must be differentiated if an equilibrium in pure strategies is to exist while the special form assumed for demand is necessary to allow an explicit solution of the model. See Neary (1991) for further discussion.

(international subsidy games are considered in the next section), which is denoted by σ and is related to the *price* charged by the two firms. The demand functions may therefore be written as:[12]

$$(5.1) \qquad x = \alpha - \beta(p-\sigma-q) \qquad \text{and} \qquad y = \alpha + \beta(p-\sigma-q).$$

As before, marginal costs are constant and are denoted by c and c*. For later use, it is convenient to summarise the demand side by the parameter μ, defined as α/β. In addition, I introduce the parameter ϕ, equal to $(c^*-c)/\mu$, as a measure of home competitiveness: it equals the home firm's cost advantage normalised by the size of the market. For later reference, the free trade outcome is easily calculated to be:

$$(5.2) \qquad \bar{p} = c + \mu\left[1 + \frac{\phi}{3}\right] \qquad \bar{q} = c^* + \mu\left[1 - \frac{\phi}{3}\right],$$

$$(5.3) \qquad \bar{x} = \alpha\left[1 + \frac{\phi}{3}\right] \qquad \bar{y} = \alpha\left[1 - \frac{\phi}{3}\right]$$

$$(5.4) \qquad \bar{W} = \bar{\pi} = \alpha\mu\left[1 + \frac{\phi}{3}\right]^2.$$

To examine the implications of ex post choice of subsidies, I consider first the second stage of the game, in which the government chooses the subsidy σ, given the price decisions taken by both firms. The social welfare function is essentially the same as (2.7):

$$(5.5) \qquad W = \pi - \delta\sigma x = (p-c-\delta\sigma)x.$$

Substituting for x from the demand function (5.1) and maximising yields the government's reaction function, relating its optimal subsidy to the prices set by firms:

$$(5.6) \qquad \tilde{\sigma} = -\frac{\mu}{2} + \frac{1}{2}(p-q) + \frac{1}{2\delta}(p-c).$$

This shows that the government reduces the subsidy by fifty per cent of any price increase by the foreign firm and partly (though, provided δ exceeds one,

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[12] An output subsidy makes no sense in this model, since it has no role once prices are set by firms and so is equivalent to a lump-sum transfer to firms. See Neary (1991) for further discussion.

not completely) compensates for any price increase by the home firm.[13]

Consider next what happens in the first stage of the game. Each firm now chooses its own price to maximise its profits, taking its rival's price as given but anticipating the effects of its choice of price on the government subsidy, as implied by the subsidy rule (5.6). The resulting first-order conditions can be written in matrix form as follows:

$$(5.7) \qquad \begin{bmatrix} 2(\delta-1) & -\delta \\ -(\delta-1) & 2\delta \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} \delta\mu-(2-\delta)c \\ 3\delta\mu+c+\delta c* \end{bmatrix}.$$

Solving for the equilibrium values of p and q yields:

$$(5.8) \qquad \tilde{p} = c + \frac{\delta\mu}{3(\delta-1)}(5+\phi),$$

$$(5.9) \qquad \tilde{q} = c* + \frac{\mu}{3}(7-\phi).$$

These may now be substituted into (5.6) to obtain an explicit expression for the optimal ex post subsidy:[14]

$$(5.10) \qquad \tilde{\sigma} = \frac{5(3-\delta)}{6(\delta-1)}\mu\left[1+\frac{\phi}{5}\right].$$

This shows clearly the effects of the two cost asymmetries on the value of the optimal subsidy. First, the sign of the optimal subsidy is determined solely by the value of δ[15] and it is positive if and only if δ is less than 3. This is a much higher value than was found in the Cournot game of earlier sections. Second, the level of the optimal subsidy is once again

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13 As explained in Gruenspecht (1988) and Neary (1991), this game is degenerate when δ equals one. In that case, the government is indifferent between an extra dollar of corporate profits and a dollar less in subsidy disbursements; but, since the firm is not indifferent between these two options, it has an incentive to increase its price without limit. Hence profits and subsidy payments are unbounded and there is no optimal policy.

14 This result is obtained in a different form by Gruenspecht (1988).

15 It may be checked that the value of the home firm's output at the optimum, $\tilde{x}$, equals $\alpha(5+\phi)/6$. Thus the sign of (5.10) depends solely on the value of δ.

positively related to the competitiveness of the home firm: the greater is ϕ, the greater the potential for the home firm to earn extra-profits and so the stronger the case for subsidising it.

Equation (5.10) fully characterises the optimal subsidy given that the government chooses to play the ex post game. However, as pointed out by Gruenspecht (1988) and Neary (1991), it does not establish whether this choice is the best option open to the government. To do this, it is necessary to compare the level of welfare obtained in the ex post game to those obtained *both* in free trade and in the ex ante game. The level of welfare yielded by the optimal subsidy (5.10) may be calculated by substituting from (5.10) into (5.8) and (5.1) and then into (5.5). This yields:

$$(5.11) \qquad \tilde{W} = \alpha\mu \frac{25\delta}{36} \left[1 + \frac{\phi}{5}\right]^2.$$

I first compare this to the level of welfare obtained in free trade, $\bar{W}$, given by (5.4). Equating (5.4) and (5.11) allows me to solve for the threshold value of δ, $\bar{\delta}$, which equates $\bar{W}$ and $\tilde{W}$,[16] and the resulting locus is illustrated in Figure 2. This shows that the optimal ex post subsidy is likely to yield a higher level of welfare than free trade, except when the home firm is highly competitive or when the opportunity cost of funds is relatively low.

Next, I must compare the level of welfare obtained in the ex post game with that which the government would attain if it decided on the level of subsidy *before* rather than *after* firms set their prices. This amounts to calculating the level of welfare in a standard Brander-Spencer-Eaton-Grossman game, with the minor difference that the government's policy instrument is a price rather than an output subsidy.[17] Consider first the second stage of

[16] The explicit expression for $\bar{\delta}$ is: $4(7+\phi)^2/(5+\phi)^2$. It is easily checked that this is increasing in ϕ and equals 1.44 (36/25) when ϕ equals zero. Note that, from (5.3), ϕ must lie between -3 and 3 if both firms are to operate in free trade, so only this range is illustrated in the diagram.

[17] I show in Neary (1991) that output and price subsidies are equivalent in the ex ante game. As already noted, this is not true in the ex post

this game, in which firms choose prices given a particular subsidy level. The solution for prices may be calculated as:

$$(5.12)\qquad p^{o} = \mu + \frac{1}{3}(2c-c^{*}+\sigma) \qquad \text{and} \qquad q^{o} = \mu - \frac{1}{3}(c-2c^{*}+\sigma).$$

Now consider the first stage of this game, in which the government chooses the optimal value of σ, in the anticipation that firms' reactions will be as given by (5.12). This yields the following:

$$(5.13)\qquad \sigma^{o} = -\frac{3}{2}\,\frac{3\delta-2}{3\delta-1}\,\mu\left[1+\frac{\phi}{3}\right].$$

We know from Eaton and Grossman that this must be negative for δ equal to one. Since higher values of δ merely increase the government's interest in taxing the home firm, a positive subsidy is never optimal in the ex ante game. As for the level of welfare attained, this may be calculated by substituting from (5.13) into (5.12) and (5.1) and then into (5.5):

$$(5.14)\qquad W^{o} = \alpha\mu\,\frac{9\delta^{2}}{4(3\delta-1)}\left[1+\frac{\phi}{3}\right]^{2}.$$

The combinations of δ and ϕ which equate W^{o} and $\widetilde{W}$ are illustrated by the second locus, labelled δ^{o}, in Figure 2.[18] This shows that, within the relevant parameter range, the ex ante game yields a higher level of welfare in all cases where the home firm has a competitive advantage. However, moving leftwards in the diagram, the ex post game is more likely to yield a higher level of welfare than either free trade or the ex ante game. Thus, the desirability of choosing the ex post game depends *negatively* on the home firm's cost competitiveness. In particular, there is a region, labelled A2, in which the ex post game yields the highest level of welfare *and* δ is less than 3, so that the optimal ex post subsidy is positive. Hence, when the home firm has a relative cost disadvantage, it may be optimal for the

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game, since an output subsidy is then redundant.

[18] The expression for δ^{o} is: $-(5+\phi)^{2}/6(1+4\phi+\phi^{2})$. This has an infinite asymptote when ϕ equals -0.268 and is negative for higher values of ϕ.

government to establish an ex post subsidy regime, rather than to impose an ex ante export tax, as the analysis of Eaton and Grossman would imply. Of course, it is still true that the *level* of the optimal ex post subsidy is positively related to the home firm's relative competitiveness.

6. International Ex Post Subsidy Games with Bertrand Competition

So far, I have assumed that the foreign government does not assist its own firm in the ex post game. If it does, then the net relative price to consumers in the demand functions (5.1) becomes $(p-\sigma-q+\sigma*)$. I now assume that the governments play a Nash game in subsidies in the second stage of the game. The home government therefore maximises (5.5), taking p, q and $\sigma*$ as given. Instead of (5.6) this yields:

$$(6.1) \qquad \tilde{\sigma} = -\frac{\mu}{2} + \frac{1}{2}(p-q) + \frac{1}{2\delta}(p-c) + \frac{\sigma*}{2}.$$

Similarly, the foreign government's optimal ex post choice of subsidy yields:

$$(6.2) \qquad \tilde{\sigma}* = -\frac{\mu}{2} - \frac{1}{2}(p-q) + \frac{1}{2\delta}(q-c*) + \frac{\sigma}{2}.$$

Thus, each government compensates its home firm for fifty per cent of any additional subsidy paid by the other government. Subtracting (6.2) from (6.1) allows me to solve for the difference between the two governments' optimal subsidies, given the prices chosen by firms:

$$(6.3) \qquad \tilde{\sigma} - \tilde{\sigma}* = \frac{2\delta+1}{3\delta}(p-q) + \frac{c*-c}{3\delta}.$$

Now, consider the first stage of the game, in which each firm chooses prices anticipating the behaviour of both governments as given by (6.3). The resulting first-order conditions are now:

$$(6.4) \qquad \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} c + \mu(3\delta+\phi)/(\delta-1) \\ c* + \mu(3\delta-\phi)/(\delta-1) \end{bmatrix},$$

which may be solved for the equilibrium prices:

$$(6.5) \qquad \tilde{p} = c + \frac{3\delta}{\delta-1}\mu\left[1 + \frac{\phi}{9}\right] \quad \text{and} \quad \tilde{q} = c* + \frac{3\delta}{\delta-1}\mu\left[1 - \frac{\phi}{9}\right].$$

Finally, substituting into (6.1) and (6.2) we may solve for the home government's optimal subsidy:

$$(6.6) \qquad \tilde{\sigma} = \frac{4-\delta}{\delta-1}\,\mu\left[1+\frac{\phi}{9}\right].$$

This shows that the threshold value of δ at which the optimal ex post subsidy becomes negative is now higher; thus, there is a broader range of values of the social opportunity cost of funds consistent with a positive subsidy. However, with respect to the implications of differences in international competitiveness, the same result as before holds: whenever the optimal subsidy is positive, it should be higher the greater the competitiveness of the home firm.

In principle, as in the previous section, this result should be supplemented by a comparison of the welfare levels attainable by the home government under each of the three regimes of free trade, optimal ex post subsidy and optimal ex ante subsidy. However, there are in fact nine regimes when the actions of the foreign government are taken into account and a full analysis would calculate the best response of each government to each choice of regime by the other. Examining how the Nash equilibrium of this three-stage game varies as a function of the underlying parameters ϕ, δ and δ^* is complex. In any case, since it is not the principal focus of the paper it has not been attempted. It is sufficient to note that, once again, the analysis of this section confirms that, whenever the optimal home subsidy is positive, its value is positively related to ϕ, the relative cost competitiveness of the home firm.

7. Concluding Remarks

In summary, this paper has focused on two key asymmetries, between social and private costs and between home and foreign production costs, and has examined their implications for the sign and magnitude of the optimal export subsidy. The thrust of the paper's findings is that, firstly, the optimal subsidy is likely to become negative at surprisingly *low* values of the social opportunity cost of funds; and, secondly, it is likely to be higher in absolute value the *more* competitive is the home firm. These results hold in striking form in the simple model of Section 2, with Cournot behaviour and linear demands: in this case, for example, the threshold value of the social opportunity cost of funds is only one and one third, well within the empirically plausible range. However, with suitable qualifications, both results continue to hold under a variety of assumptions about the nature of consumer demand and the timing of decisions by firms and governments. At the very least, these findings would seem to justify a reevaluation of the policy implications of recent writings in strategic trade policy.

One focus for such a reevaluation would be on the term "profit shifting" which has been widely used to characterise the motive for export subsidisation emanating from the strategic trade policy literature. An important contribution of this literature is the recognition that, when markets are imperfectly competitive, increased profits by home firms (if necessary at the expense of foreign firms) are a legitimate target of public policy. However, the results of this paper suggest that the "profit shifting" argument should be qualified in three respects. Firstly, if raising home profits requires a subsidy, then the opportunity cost to the exchequer should be taken into account. Secondly, if the subsidy programme passes this test, it should take account of differences in the "comparative advantage in earning extra profits" across the group of firms to be subsidised. In contrast to the intuition derived from an infant industry argument, this consideration suggests that policy should be targeted towards home firms which possess a cost advantage over their foreign rivals. Finally, while profit shifting may not always raise home welfare, it is always to the advantage of home firms and their shareholders. For example, in Neary (1991) I showed that both firms always make higher profits when subsidies are provided on an ex post basis, as in Sections 5 and 6, than when the optimal policy is chosen on the standard ex ante basis. This provides a further reason why "profit shifting" is likely to be adopted more frequently than an economy-wide welfare criterion would justify.

APPENDIX

Optimal Choice of Subsidies by a Public Agency

In the text, the shadow price of government funds, δ, is taken to be exogenous. In this Appendix, I show how it can be interpreted as arising from optimal behaviour by a government agency charged with allocating a fixed budget over n domestic firms. I assume that each firm resembles the home firm considered in the text: it is the sole domestic competitor in a market in which it competes for foreign sales against a single foreign rival. I also assume that the firms' costs are independent of each other: in particular, firms do not compete for scarce factors of production as they do in the model of Dixit and Grossman (1986). The agency's welfare function therefore equals the total value of corporate profits less total subsidy payments:

$$(A.1) \qquad W = \Sigma \pi_i - \Sigma s_i x_i,$$

where the summation is over all n firms. This is to be maximised subject to the constraint that the agency's disbursements not exceed its budget R. Forming the Lagrangian yields:

$$(A.2) \qquad \mathbf{L} = \Sigma(\pi_i - s_i x_i) + \lambda(R - \Sigma s_i x_i).$$

$$(A.3) \qquad = \Sigma[\pi_i - (1+\lambda)s_i x_i] + \lambda R,$$

where λ is the Lagrange multiplier on the constraint. Comparing this with (2.7), it is clear that maximising net corporate profits subject to a binding budget constraint is equivalent to maximising the welfare function (2.7) in each market. The only difference is that λ (which equals δ-1) is determined simultaneously as part of the optimal solution along with the optimal subsidy to each firm.

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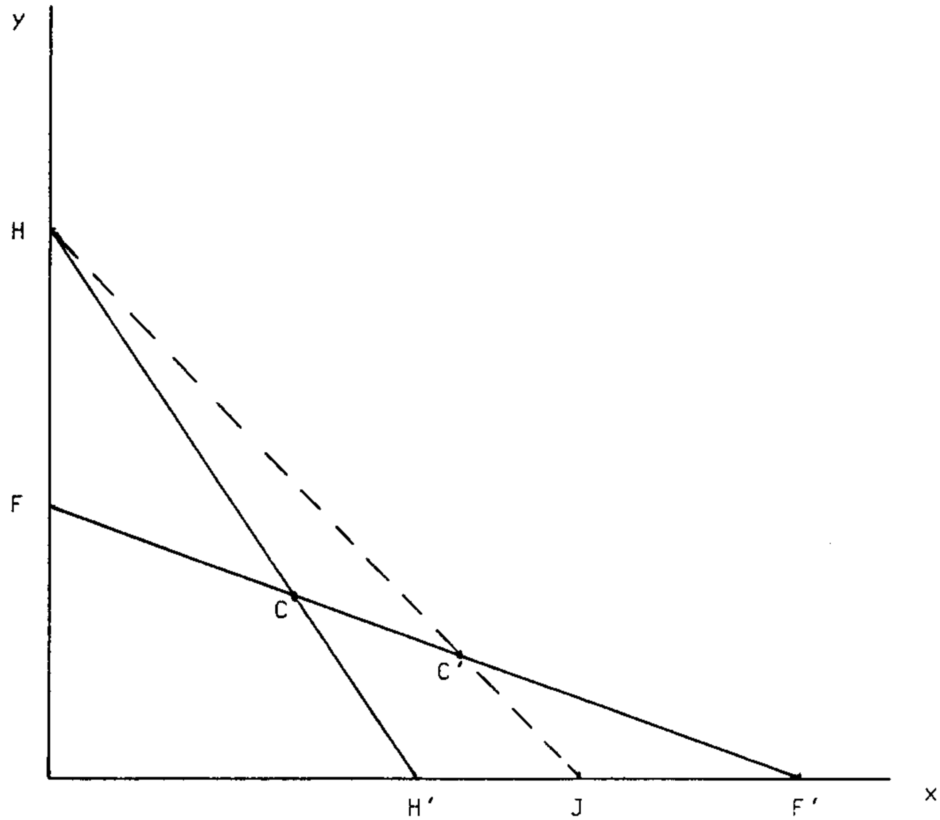

Figure 1: Effects of Changes in the Foreign Firm's Costs on the Free Trade (C) and Optimal Home Subsidy (C') Equilibria

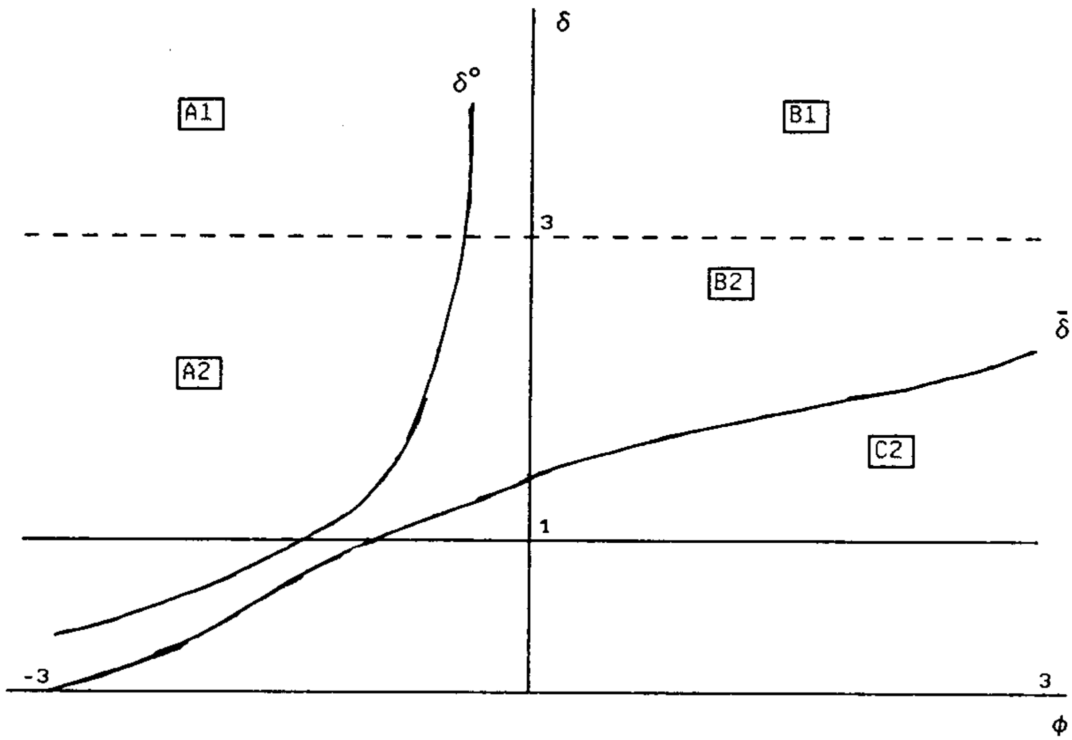

Figure 2: Effects of the Social Opportunity Cost of Funds (δ) and the Home Firm's Relative Competitiveness (ϕ) on the Ranking of Welfare Levels in Free Trade ($\bar{W}$) and in the Ex Ante (W°) and Ex Post ($\tilde{W}$) Games

δ° equates W° and $\tilde{W}$; $\bar{\delta}$ equates $\bar{W}$ and $\tilde{W}$

Region A: $\tilde{W} > W^{\circ} > \bar{W}$

Region B: $W^{\circ} > \tilde{W} > \bar{W}$

Region C: $W^{\circ} > \bar{W} > \tilde{W}$

Sub-Regions A1 and B1: $\tilde{\sigma} < 0$

Sub-Regions A2, B2 and C2: $\tilde{\sigma} > 0$