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FAT-TAILED PROBLEMS IN RISK MANAGEMENT

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EXTREME RETURNS ARE IMPORTANT
Derivative traders face important challenges in managing their risk and none more so than futures traders. To illustrate, what have the following dates in common: 19/10/1987 and 11/9/2001? A further question – what binds 15/1/94 and 27/2/95? The answer is that they all can be identified as extreme trading events resulting in large levels of investor losses. The first dates relate to all market movements with the 1987 market crash and the fallout from September 11, and the second relates to losses in futures trading with Metallgesellschaft and Barings.

Market movements in excess of ten per cent occurring over relatively short trading intervals bind the dates together. Key to all scenarios is the modelling of market risk and the profile of financial returns. Market risk details losses resulting from financial returns and can involve inadequate capital covering extreme movements leading to investor default. Futures traders must ensure that they have adequate capital reserves to protect against these extreme price movements.

Both size and frequency of large futures trading losses are such that accurate modelling of market risk has become a key issue in risk management. Large losses result in investor default and can be due to extreme market price movements located in the tail of a distribution of outcomes. By definition these are considered to be rare events occurring infrequently. Concerning these extreme events the futures trader is interested in modelling the exceptional and (unfortunately) the catastrophic.

The standard approach to modelling any market movement and its implication is to assume a statistical distribution such as the commonly used normal distribution. Examples include mean-variance portfolio optimisation and in the context of risk management, Value at Risk (VaR), and Excess Shortfall measures for single or aggregated periods. An alternative modelling procedure that has attracted favourable comment recently is the use of extreme value theory that models tail values only.

How well the normal distribution, or any approach, fits the tail futures returns determines its level of success in risk management and hedging applications.

Let us do a comparison of the approaches by first detailing their attributes and then illustrating their operations in practice for VaR estimation. The analysis will indicate the approach that should be adopted by futures traders in measuring extreme market risk.

The approaches popularity in modelling extreme futures returns is due to their many shared advantages. First there is the modelling simplicity with extreme value theory relying on the tail index, and normality on the first two distribution moments the mean and variance.

Second, both allow for modelling low frequency outcomes by detailing out-of-sample probabilities. These low probability risk estimates may be outside the data range for analysis by a futures trader but are confined to boundaries where default risk would occur.

Also, risk estimates can easily be scaled from high frequency estimates to low frequency estimates, say from daily to weekly estimates. Normality uses the square
root of time scaling law whereas extreme value theory suggests applying the $\alpha$-root scaling law.

**MODELLING WITH EXTREME VALUE THEORY**

Recently the use of extreme value theory has been advocated in providing accurate tail risk measures. The approach minimises model risk by explicitly modelling the tail of the distribution only. Common or average risk outcomes are ignored at the expense of tail values. The estimation procedure uses order statistics analysing quantiles from the distribution of financial returns. Extreme value theory provides right answers only for extreme tail situations associated with low probability and high quantile events. Thus it is accurately able to model the frequency and size of extreme futures returns.

Also, exact distributional assumptions are avoided by allowing for a fat-tailed limit law, the fréchet distribution, encompassing a family of fat-tailed distributions. Fat-tailed distributions exhibit regular variation at infinity giving identical tail features. Futures returns are said to exhibit this fat-tailed property although there is no uniformity as to the scale of the property indicating a benefit of applying an all-encompassing distributional framework. Each tail is modelled separately allowing for distinct analysis of long and short trading positions.

Furthermore, as stated extreme value estimates can be scaled by a simple but efficient, $\alpha$-root scaling law. Advantageously no further estimation at different frequencies is required and estimation at the highest frequencies provides the most accurate tail estimates benefiting the analysis of futures traders.

However, using extreme value theory is not a panacea for all the measurement issues facing futures traders in their analysis of extreme returns. Primarily there is ambiguity regarding certain statistical assumptions such as the properties of finite-sample extreme value tail estimators, and these feed directly into the related probability and quantile risk estimates. Also, not all risk, for example credit risk, can be modelled best by the fat-tailed Fréchet distribution although this issue does not arise in analysing futures returns.

**NORMALITY PERFORMANCE POOR**

There is no doubt that normality is right sometimes underpinning its popularity. This is applicable for example if one assumes market homogeneity with participants being price takers involved in small trades. Taking a large enough sample and assuming the law of large number holds implies that average outcome effects hold and the normal distribution is a good approximation.

However more often futures markets are characterised as being heterogeneous giving rise to different trading levels and the possibilities of small and importantly large price movements that are not best described by averages. Hence, the ability of normality to predict the large financial losses and the associated fat-tails is questionable. These fat-tails imply that averaging is not appropriate even for infinite scenarios.

Moreover, the normal distribution relies on averages by assuming that asymptotically under the central limit theorem the average outcome converges to a normal
distribution. By modelling only two outcomes, mean and volatility (assumed constant), normality predicts the size and regularity of occurrences. This goes against the main focus of modelling extreme events as they occur rarely and do not conform to the average outcome.

Furthermore, some very restrictive assumptions characterise the normal distribution and these are not adhered to by financial returns. These include amongst many, uncorrelated constant volatility but no one believes this as evidenced by the 2003 Nobel Prize Committee’s citation for Robert Engle’s award in economics for his contribution in analysing economic time series with time-varying volatility (ARCH)’. Also, the assumption of symmetric returns does not necessarily hold suggesting separate analysis of long and short trading returns.

Practically, the normal distribution results in inadequate estimation of futures tail behaviour. The weakness can be summarised by model risk that underestimates the size and frequency of large futures returns. For instance, a five-sigma event – one that is greater than five times the standard deviation from the mean - has an almost infinite waiting period under normality but in reality may occur with relatively frequent repetition.

**ILLUSTRATION OF APPROACHES**

Let us illustrate some of these properties with an application to the FTSE250 futures contract using five years of daily data. This futures trades on the London International Financial Futures Exchange (LIFFE). At a 99.5 per cent level the VaR using extreme value theory is 2.68 per cent compared to 1.63 per cent assuming normality. For model verification, the VaR should be violated 7 times for the sample analysed at a rate of 0.5%. Our extreme value estimates are very close to this with 6 violations at a rate of 0.46% proving to be a very accurate modelling procedure. In comparison assuming normality leads to a severe underestimation of the VaRs with 16 violations at a rate of 1.23%. These VaRs would be associated with inadequate capital reserves not capable of dealing with extreme price movements that occur in futures trading.

As stated both modelling procedures allow for out-of-sample estimation giving the trader the advantage of being able to infer loss levels outside the data period analysed. However the underestimation problem for normality is accentuated for very low probability levels due to the fat-tailed characteristic of futures returns. For the FTSE250 the VaRs using extreme value theory for a ten-year trading period is 7.78 per cent in comparison to 2.12 per cent assuming normality, not even reaching the true 99.5 per cent estimates. Thus for lower probabilities or more extreme quantile estimates normality suffers from even greater degrees of model risk.

Using the respective scaling laws gives the futures trader simple ways of measuring risk measures at different frequencies such as going from daily to weekly estimates. For instance the weekly extreme value VaR at the 99.5 per cent confidence level is 6.87 per cent compared to 3.64 per cent underpinned by normality. There should be 1 violation as is recorded by the extreme value approach in comparison to 2 violations predicted by normality. This inaccuracy of normality is due to the inadequacy of the daily estimates.
CONCLUSIONS
Futures traders need to manage extreme risk to avoid default. Accurate modelling of market risk drives risk management and the use of hedging tools. Inaccurate tail modelling gives rise to inappropriate actions and large levels of investor losses. The commonly used assumption of modelling futures returns with normal estimates whilst attractive leads to inaccurate risk measures. In contrast, extreme value theory, focusing on tail movements only, provides a more robust description of extreme futures returns and should be adopted by traders in analysing market risk. Large extreme returns occur rarely and do not conform to the average outcome and should be modelled accordingly.

1Alternative approaches, although not as popular in modelling extreme futures returns, do exist such as historical simulation and Monte Carlo methods and are not discussed here for brevity purposes.