Two-Stage Game Models of International Oligopoly

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Abstract
This paper reexamines the import protection as export promotion thesis in a series of two-stage games in which firms choose R&D and/or capacity in the first stage and quantity or price in the second. It is shown that a tariff affects exports in two ways; firstly, if marginal cost is increasing, by raising home sales directly it crowds out exports; secondly by increasing R&D and/or capacity it raises exports indirectly. Prospects for export enhancing protection are compared for the different games and shown to be better in the long run than in the short run.

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1. Introduction

There has long been a view among business people that protection against imports can help firms in their export markets. In particular it is often argued that import protection is one explanation for the export success of Japanese firms since the war\(^1\). What has economic theory to say on the issue? There is of course the traditional "infant industry argument" for protection: the temporary protection of an industry may allow it to grow and take advantage economies of scale either of the static kind or the dynamic learning by doing variety, and so permit it to compete more successfully. In order to apply however, the traditional infant industry argument requires either the existence of capital market imperfections or that positive externalities will be generated by the protected industry.

Krugman (1984) puts forward a game-theoretic variant of the infant industry argument which does not require capital market imperfections or externalities. In his model import protection is in effect a strategic export policy. In persuading foreign firms to produce less it alters the subsequent course of the game in a way that benefits the home firm in all markets.

The purpose of this paper is to examine the issue of import protection as export promotion in a formal two-stage game framework and to isolate those market linkage effects that make it more or less likely.

\(^{1}\) See Yamamura (1986) for an economist's perspective on the role of import protection in the Japanese export success. He argues that Japan's industrial policy and trade policy of the "rapid growth phase" involved, among other things this type of import protection as export promotion. He stresses that during the period, not only was the domestic economy heavily protected, but Japanese firms were able to exploit important economies of scale. This put firms under strong pressure to reduce unit costs, through increased sales, that could only come from exporting more.
In the models I consider firms choose cost reducing R&D or productive capacity non-cooperatively in the first stage of a game and then choose price or output in the second stage. The R&D or capacity level chosen, a commitment made prior to the output or price stage, cannot be subsequently modified. Although there is now a substantial literature on the two-stage game approach to modelling oligopolistic rivalry,\(^1\) apart from notable exceptions such as Venables (1990) and Ben-Zvi and Helpman (1992), most of the papers, including those in the field of strategic trade policy have been concerned with a single market.

For import protection to be export promotion it is essential that markets be in some way linked. Following much previous work on international oligopoly by Brander (1981), Brander and Krugman (1983), Dixit (1984) and Krugman (1984) which adopted a static single-stage game approach, I assume that markets are segmented. This means that arbitrage is ruled out, so that firms in equilibrium can charge different prices in different markets. This is both a plausible assumption and one that permits two-way trade in homogeneous products. In most of these papers, however (though not in Krugman (1984)) attention is restricted to the case of constant marginal cost. When this assumption is combined with that of segmented markets it implies that the game played between firms in one market is completely separate from the game played by the same firms in other markets. If it is further assumed that the number of firms is fixed, markets are unlinked. Import protection will not affect exports at all in that case. In contrast in the models I examine here markets are linked in two ways: Firstly, by assuming that marginal costs are increasing, domestic and export sales become substitutes in supply. Secondly,

when a firm's competitive position is improved in one market it may invest in more R&D or capacity, so affecting its costs, and through this its sales everywhere. Using a two-stage game framework it is possible to view the impact of protection on exports at constant R&D and capacity as a short-run effect, and the total impact of the tariff when the first-stage variable can change, as a long-run effect.

This paper is organised as follows: In section 2 I set up the general model. In section 3 I consider two-stage games in which R&D expenditures are chosen in the first stage and output or price in the second. The implications of tariff protection for exports are considered. In section 4 I look at a special case of the model in which firms choose capacity in the first stage and then face an absolute constraint on total output in the second-stage. I show that the prospects for import protection as export promotion are not encouraging in that case.

2. The General Model

Throughout the paper I consider an imperfectly competitive model in which there are two firms. One of these firms is located in the home country, and I refer to this as the home firm. The other is located abroad and will be called the foreign firm. There are two national markets, referred to as the home and foreign market and the two firms sell in both of these. The firms play a two-stage game choosing R&D or productive capacity in the first stage and output or price in the second. Given the generality of the model, the analysis can soon become rather complicated and results difficult to interpret.

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1 This is the framework adopted in Ben-Zvi and Helpman (1992) and Venables (1990).
in an intuitive way. So in order to permit a unified treatment of R&D and capacity under both Cournot and Bertrand competition some sacrifices in generality must be made. Therefore to keep the analysis relatively transparent I assume that inverse demand curves are linear. One advantage of this is that it is then possible to solve explicitly for equilibrium outputs. The goods produced by the firms are substitutes and the home market inverse demand functions are:

\[ p(x, y) = a - b(x + \epsilon y), \]
\[ q(x, y) = a - b(\epsilon x + y), \]
\[ 0 < \epsilon \leq 1. \]

The home market sales of the home firm are represented by \( x \) and the exports of the foreign firm to the home market are given by \( y \). The home firm sells at a price \( p \) and the foreign firm at a price \( q \). The parameter \( \epsilon \) is a measure of product differentiation. When this has a value of unity the goods are homogeneous and for values less than this they are imperfect substitutes.\(^4\)

The foreign market inverse demands are similarly:

\[ p^*(x^*, y^*) = a - b(x^* + \epsilon y^*), \]
\[ q^*(x^*, y^*) = a - b(\epsilon x^* + y^*), \]

for the home and foreign firms respectively. I will often use a star to represent foreign variables. The home firm exports \( x^* \) at a price \( p^* \) and the foreign firm sells \( y^* \) on the foreign market (its own domestic market) at the price \( q^* \).

When firms choose their R&D and capacity in the first stage they take into account how this will affect the outcome of the second-stage output or price.

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\(^4\) For the case in which price is chosen in the second stage I will assume that \( \epsilon \) is strictly less than unity. This means that firms are producing differentiated products. This is necessary if we are to conduct policy analysis without resorting to mixed-strategy Nash equilibria. Homogeneous products need not be ruled out if firms choose quantity in the second stage.
subgame. I assume that an increase in R&D or capacity reduces second-stage marginal costs. When the marginal cost curve is upward sloping, but not vertical R&D and capacity both shift the curve downwards to the right. If the marginal cost curve is constant in output then capacity choice will have no effect while R&D will shift the curve downwards in a parallel manner. In the situation, in which the marginal cost curve is perfectly vertical (the absolute capacity constraint case) however, it is R&D that will be completely ineffective while an increase in capacity will shift the the marginal cost rightwards (see fig 1).

(Figure 1 about here)

In order to capture the different effects of R&D and capacity on the firm’s second-stage costs, I will assume that the total production cost function takes the following form:

$$C = \gamma(n) - \mu[X - (x + x^*)/2](x + x^*),$$  \hspace{1cm} (23)

where $n$ is the level of home R&D and $X > 0$, represents a measure of the firm's productive capacity. Total production cost is assumed to be quadratic in output and the parameter $\mu \geq 0$, is the constant slope of the resulting marginal cost curve. For given levels of $n$ and $X$, there are decreasing returns to scale. I shall impose the following restrictions on $\gamma(n)$:

$$\gamma(n) > 0, \hspace{1cm} \gamma'(n) < 0, \hspace{1cm} \gamma''(n) > 0.$$  \hspace{1cm} (24)

This restriction on the second derivative, which implies that R&D is cost reducing at a diminishing rate, is used to ensure an interior solution.

The home firm's total costs are made up of total production costs, which are variable in the second stage and the R&D and capacity costs which are fixed in the second stage. Transport costs are assumed to be zero for convenience.

The home firm's profit function is:
\[ \pi = xp(x,y) + x*p(x,y) - C(x,x^*,n,X) - n - K(X), \quad (2.5) \]

where \( K(X) \) is the total capacity cost and \( n \) is the total cost of R&D. The marginal cost of R&D is assumed to be constant and normalised at unity. The foreign firm's production cost function is similarly:

\[ C^* = \{\gamma^*(n^*) - \mu[Y - \mu(y + y^*)/2]\}(y + y^*), \quad (2.6) \]

where \( n^* \) is foreign R&D and \( Y \) is a measure of the foreign firm's total capacity. I will assume that the foreign cost function responds to changes in own R&D in the same way as home costs:

\[ \gamma^*(n^*) > 0, \quad \gamma^*(n^*) < 0, \quad \gamma^*(n^*) > 0. \quad (2.7) \]

In addition the foreign firm faces a tariff \( t \), imposed by the home government.

The profit function of the foreign firm is therefore:

\[ \pi^* = y[q(x,y) - t] + y*q*(x^*,y^*) - C^*(y,y^*,n^*,Y) - n^* - K^*(Y), \quad (2.8) \]

In order to facilitate comparison of the alternative quantity and price equilibria, the second-stage subgame can be modelled using a conjectural variations approach. This approach involves modelling each firm as choosing quantity while conjecturing a particular quantity response \( v \), from the rival. The Cournot quantity conjectural variation is \( v = 0 \), because the firms are taking rival output as given. Alternatively if they are playing Bertrand they take the rival price as given. This can be modelled as firms choosing quantity while conjecturing an adjustment of the rival output sufficient to keep rival price constant. The home firm's "Bertrand quantity conjectural

\[ \text{Note that only Cournot and Bertrand conjectures can be rigorously justified. Due to the fact that firms move simultaneously in the second stage of the game they cannot react to one another. Therefore to conjecture that the rival will react is to make a mistake. Because of this, only the Cournot and Bertrand conjectures will be considered here.} \]
variation" (BQCV) can be obtained by totally differentiating the rival inverse demand function and setting this equal to zero:
\[ \frac{dq}{dx} = 0 = b(\varepsilon dx + dy). \]  
(29)

Rearrangement of this equation yields:
\[ \nu \equiv \left( \frac{dy}{dx} \right)_c = -\varepsilon. \]  
(2.10)

It is straightforward to verify that the home firm's foreign market BQCV, as well as the foreign firm's home and foreign market BQCV are all equal to \(-\varepsilon\).

The equilibrium concept employed is that of subgame perfection, originally due to Selten (1975). In two-stage games such as these when the players choose their optimal first-stage action they correctly forecast the outcomes of the second stage of the game. This means that in order to solve for the equilibrium of the game we must begin by solving the second stage and work backwards to the first stage. The equilibrium optimal actions at the second stage are calculated for all possible combinations of previous actions. The payoffs associated with these are used in determining optimal actions in the first stage.

3. R&D Games

In this section I assume that firms choose R&D in the first stage and quantity or price in the second stage. The measures of capacity, X and Y, are assumed to remain constant. Following standard practice I start by analysing the second stage of the game. In the second stage R&D levels are given, and using the conjectural variations approach, the following first-order conditions for profit maximisation by the home and foreign firms are obtained:

\[ (d\pi/dx)|_c = \pi|_c = (\partial\pi/\partial x) + \nu(\partial\pi/\partial y) = 0, \]  
(i)

\[ (d\pi/dx*)|_c = \pi*|_c = (\partial\pi/\partial x*) + \nu(\partial\pi/\partial y*) = 0, \]  
(ii)

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(iii) \( \frac{d\pi^*}{dy} |_{c} = \pi^* |_{c} = (\frac{d\pi^*}{dy}) + \nu(\frac{d\pi^*}{dx}) = 0, \)

(iv) \( \frac{d\pi^*}{dy^*} |_{c} = \pi^* |_{c} = (\frac{d\pi^*}{dy^*}) + \nu(\frac{d\pi^*}{dx^*}) = 0. \)

Imposing (2.1), (2.2), (2.3), and (2.5) on (3.1) yields the first-order conditions in a more explicit form:

(i) \( \pi^* |_{c} = a - \beta x - \phi y - \gamma(n) - \mu(x + x^* - X) = 0, \)

(ii) \( \pi^* |_{c} = a - \beta x^* - \phi y^* - \gamma(n) - \mu(x + x^* - X) = 0, \)

(iii) \( \pi^* |_{c} = a - \beta x - \phi y - \gamma^*(n^*) - \mu(y + y^* - Y) - ty = 0, \)

(iv) \( \pi^* |_{c} = a - \beta x^* - \phi y^* - \gamma^*(n^*) - \mu(y + y^* - Y) = 0, \)

where \( \beta \equiv b(2 + \nu) > 0, \) and \( \phi \equiv be. \) When \( \mu > 0, \) the markets are linked through the cost functions. In that case an increase in sales in one market reduces the marginal profitability of the firm's sales in the other. When \( \mu \) is zero markets are unlinked in the second stage.

The four equation system in (3.2) can be solved for the four second-stage endogenous variables; \( x, y, x^*, \) and \( y^*. \) The linear demand and marginal cost assumptions permits the derivation of explicit expressions for these variables:

(i) \( x^0 = \left\{ [\beta + 2\mu][a - \gamma(n) + \mu X] - \phi[a - \gamma^*(n^*) + \mu Y] \right\} \)

\[ + \frac{t\phi(\beta^2 - \phi^2 + 2\mu(\beta + \mu))/((\beta^2 - \phi^2))}{D}, \]

(ii) \( x^0 = \left\{ [\beta + 2\mu][a - \gamma(n) + \mu X] - \phi[a - \gamma^*(n^*) + \mu Y] \right\} \)

\[ - 2t\mu(\beta + \mu)/(\beta^2 - \phi^2))/D, \]

(iii) \( y^0 = \left\{ [\beta + 2\mu][a - \gamma^*(n^*) + \mu Y] - \phi[a - \gamma (n) + \mu X] \right\} \)

\[ - (\beta + \mu)(\beta^2 - \phi^2 + 2\beta \mu)/(\beta^2 - \phi^2))/D, \]

(iv) \( y^0 = \left\{ [\beta + 2\mu][a - \gamma^*(n^*) + \mu Y] - \phi[a - \gamma (n) + \mu X] \right\} \)

\[ + t\mu(\beta^2 + \phi^2 + 2\beta \mu)/(\beta^2 - \phi^2))/D, \]

where \( D \equiv (\beta + \phi + 2\mu)(\beta - \phi + 2\mu) > 0. \)

Proposition 1: Independently of whether firms are choosing quantity or price, a tariff imposed after firms have chosen their R&D levels will (i) increase the
domestic sales of the home firm, (ii) provided that $\mu > 0$, reduce its exports, (iii) increase total worldwide sales of the home firm, for all values of $\mu$ less than infinity.

Parts (i) and (ii) of proposition 1 are straightforward from inspection of (3.3(i)) and (3.3(ii)). In order to prove part (iii) combine (3.3(i)) and (3.3(ii)) and differentiate with respect to $t$ to get:

$$\frac{d(x + x^*)}{dt} = \frac{\phi}{D} \geq 0. \quad (3.4)$$

So the tariff has what I call a positive "output creation effect" (OCE), for the home firm, the improvement in its competitive position brought about by the tariff increases its total sales. All of the increase in output occurs in the home market. It is straightforward to show that this OCE is larger under Bertrand competition than when firms play Cournot. There is also what I will call a "crowding out effect" (COE) on exports: The tariff, in bringing about an expansion of the home firm's total level of production, leads to its marginal costs rising in its export market so causing it to reduce sales there. For the tariff to have such a COE it is necessary that exports and domestic sales are substitutes in supply for at least one of the firms (this is the case here for $\mu > 0$). Because of the COE, a tariff introduced between the first and second stages of the game will hurt exports. In this case import protection is not export promotion. From (3.3) the quantity of exports crowded out as a result of the tariff is:

$$- x^* = \frac{2\mu \phi (\beta + \mu)}{D (\beta^2 - \phi^2)}. \quad (3.5)$$

It is straightforward to show that the crowding out effect is larger under price competition than under quantity competition for $\mu > 0$.

It is also possible to use (3.3) to derive:
\[
d(y + y*)/dt = -(\beta + 2\mu)/D \leq 0. \tag{3.6}
\]

The tariff has a negative output creation effect for the foreign firm for \(\mu\) less than infinity. As the foreign firm’s total production falls this leads to a fall in its marginal production cost and a negative crowding out effect on its domestic market sales. Note that if \(\mu\) is zero, the constant marginal cost case, all the crowding out effects disappear.

An increase in home R&D has an output creation effect for the home firm. This OCE is not biased toward one or other of the markets. To see this differentiate (3.3(i)) and (3.3(ii)) to get:

\[
\partial x/\partial n = \partial x*/\partial n = -\gamma'(\beta + 2\mu)/D(\beta^2 - \phi^2) > 0. \tag{3.7}
\]

From (3.3(iii)) and (3.4(iv)) the impact of home R&D on foreign output is found to be:

\[
\partial y/\partial n = \partial y*/\partial n = \gamma'\phi/D(\beta^2 - \phi^2) < 0. \tag{3.8}
\]

Proposition 2: An increase in a firm’s R&D expenditure will lead to an equal increase in that firm’s domestic sales and exports and an equal decrease in the exports and domestic sales of its rival.

Home output expands and foreign output contracts in both markets as a result of the increase in home R&D even when these markets are of unequal size. However, the fact that the home firm’s output expands equally in both markets reflects the assumption that home and foreign market demands are symmetric. If instead the slope parameter \(b\) of the foreign market demands in (2.2) is multiplied through by \(\sigma > 0\) we get:

(i) \[ p^* = a - \sigma b(x^* + ey*), \tag{2.2'} \]
(ii) \[ q^* = a - \sigma b(ex^* + y*), \]

where \(\sigma\) is a measure of the relative size of the home and foreign markets. The bigger is \(\sigma\) the smaller is the foreign market and the faster the inverse
demands fall in sales. It is straightforward to show that $\partial x/\partial n$ is greater than $\partial x^*/\partial n$ and $\partial y/\partial n$ is more negative than $\partial y^*/\partial n$ for $\sigma$ greater than unity.

I turn now to the first stage of the game where firms choose their R&D taking into account how these will affect second-stage variables. R&D affects profits in two ways; directly through its impact on costs and indirectly through its effect on outputs. In the first stage of the game the home and foreign firms face the following optimisation problems respectively:

\[ \text{(i)} \quad \max_{n} \pi(x(n,n^*,t), y(n,n^*,t), x^*(n,n^*,t), y^*(n,n^*,t), n), \quad (3.9) \]

\[ \text{(ii)} \quad \max_{n^*} \pi^*[x(n,n^*,t), y(n,n^*,t), x^*(n,n^*,t), y^*(n,n^*,t), n*,t]. \]

Profit maximisation implies the following first-order conditions:

\[ \text{(3.10)} \]

\[ \text{(i)} \quad \frac{\partial \pi}{\partial n} = \pi_n = \pi_x x + \pi_y y + \pi_{xx} x^2 + \pi_{yy} y^2 - (x + x^*) y' - 1 = 0, \]

\[ \text{(ii)} \quad \frac{\partial \pi^*}{\partial n^*} = \pi^*_n = \pi^*_x x^* + \pi^*_y y^* - (y + y^*) y^* - 1 = 0, \]

It is possible to simplify (3.10) using the second-stage first-order conditions in (3.1) to get:

\[ \text{(i)} \quad (x + x^*) y' + 1 = \pi_y (y^* - y) + \pi_{yx} (x^* - x), \quad (3.11) \]

\[ \text{(ii)} \quad (y + y^*) y^* + 1 = \pi^*_y (y^* - y) + \pi^*_{yx} (x^* - x), \]

The use of (2.5), (2.8) and (3.3) in (3.11) yields:

\[ \text{(i)} \quad (x + x^*) y' + 1 = - (x + x^*) y'[(\phi/D)(\phi + v(\beta + 2\mu)), \quad (3.12) \]

\[ \text{(ii)} \quad (y + y^*) y^* + 1 = - (y + y^*) y^*[(\phi/D)(\phi + v(\beta + 2\mu)). \]

The terms on the left-hand side of (3.12(i)) and (3.12(ii)) are the effects of R&D on costs at constant outputs. If there was no rival firm these would be
set equal to zero, firms choosing R&D levels to minimise costs. The terms on the right-hand side of the two equations are the strategic effects of home and foreign R&D spending respectively. It can be checked that these are positive in the case of second-stage Cournot competition but negative when firms play Bertrand in the second stage. The presence of a Cournot competitor means that the firms spend more than the cost-minimising amount on R&D in order to exploit its strategic effect. For the home firm the strategic effect in the case of second-stage Cournot competition, is the effect that it has in reducing foreign sales in both markets and so shifting profits to the home firm. By investing in more R&D in the first stage, the firm signals that it will produce more in the second stage. In this linear demands case in which quantities are strategic substitutes (in the terminology of Bulow et al (1985)) this will reduce the rival's output. To use the taxonomy of business strategies developed by Fudenberg and Tirole (1984), the firms adopt "top dog" strategies.

In the case of Bertrand second-stage competition the right-hand side of (3.11(i)) and (3.11(ii)) are negative. R&D expenditures are kept below their cost-minimising level. The intuition behind this result is that more home R&D causes home prices to fall in the second stage and, because prices are strategic complements (in this linear demands case), rival prices also fall, which cuts into the demand for home output and hence profits. The "puppy dog" strategy of "underinvestment" in R&D is called for.

It is useful to rewrite the first-stage, first-order conditions in the form:

\[(i) \quad x' = -B(x + x^*)\gamma' - 1 = 0, \quad (3.13)\]

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6 This can easily be checked; When the goods produced by the two firms are unrelated the cross effects in demand disappear and \(\varepsilon\) equals zero. This implies that \(\phi\) is zero and the terms on the right-hand side of (3.10) vanish.

7 See chapter 8 in Tirole (1988) for an overview of this literature.
(ii) \[ \pi_{\text{st}}^* = -B(y + y^*)y^* - 1 = 0, \]
where \( B \equiv (\beta + 2\mu)(\beta + 2\mu + v\phi)/D > 0. \)

The second-order conditions for the home and foreign firms are:

(i) \[ \pi_{\text{ht}} = -B[(x_h + x^*_{\text{s}})y^* + (x + x^*)y^{**}] < 0, \] (3.14)
(ii) \[ \pi^{*}_{\text{st}t} = -B[(y_{\text{st}} + y^*_{\text{s}})y^{**} + (y + y^*)y^{**}] < 0. \]

For the second-order conditions to hold it is necessary that the overall term in parentheses be positive. The terms in \( y^* \) and \( y^{**} \) are negative and so must be dominated by the positive terms in \( y^{**} \) and \( y^{**} \). The cross effects of R&D on marginal profits for the home and foreign firm are:

(i) \[ \pi_{\text{st}} = -B(x_{\text{st}} + x^*_{\text{s}})y^*, \] (3.15)
(ii) \[ \pi^{*}_{\text{st}t} = -B(y_{\text{st}} + y^*_{\text{s}})y^{**}, \]
which are both negative for all values of \( \mu \) less than infinity. This implies that reaction functions are negatively sloped in R&D space; R&D expenditures are strategic substitutes.  

Now I will assume that the home government can credibly commit itself to a tariff before the firms choose their R&D levels. It is now possible to examine the total effect of a tariff on the equilibrium of the game. The tariff affects output both directly, and indirectly via changes in the levels of R&D chosen. In order to look at the effect of the tariff on the optimal levels of home and foreign R&D it is first necessary to examine how the tariff affects the firms' marginal profitability of R&D. Differentiating (3.13) with respect to \( t \) and substituting from (3.4) and (3.6) gives:

(i) \[ \eta_{\text{t}} = -B\phi y'/D \geq 0, \] (3.16)
(ii) \[ \pi^{*}_{\text{st}t} = B(\beta + 2\mu)y^*/D \leq 0. \]

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8 Note that this is the case irrespective of whether firms play Cournot or Bertrand in the second stage.
From this it is clear that the tariff raises the marginal profitability of home R&D and reduces the marginal profitability of foreign R&D. Proceed by totally differentiating the first-order conditions given in (3.13) and rearrange in matrix form to give:

\[
\begin{bmatrix}
\pi_{nx} & \pi_{n*}\pi_{n*} \\
\pi_{n*}^{*n} & \pi_{n*}^{*n*}
\end{bmatrix}
\begin{bmatrix}
\frac{dn}{dt} \\
\frac{dn*}{dt}
\end{bmatrix} = \begin{bmatrix}
-\pi_n \\
-\pi_{n*}
\end{bmatrix}
\tag{3.17}
\]

The Routh-Hurwitz condition for reaction function stability is that the determinant \( A = \pi_{\text{nt}}\pi_{\text{nt}}^{\text{nt}} - \pi_{\text{nt}}^*\pi_{\text{nt}}^{\text{nt}} > 0 \). This implies that the own effects of R&D on marginal profits dominate the cross effects. If this condition holds globally it implies that the equilibrium is unique.

From (3.17) it is possible to derive expressions for the impact of the tariff on home and foreign R&D:

\[
\begin{align*}
(i) & \quad A\frac{dn}{dt} = \pi_{\text{nt}}^*\pi_{\text{nt}} - \pi_{\text{nt}}^*\pi_{\text{nt}}^{\text{nt}} > 0, \\
(ii) & \quad A\frac{dn^*}{dt} = \pi_{\text{nt}}^*\pi_{\text{nt}}^* - \pi_{\text{nt}}^*\pi_{\text{nt}}^{\text{nt}} < 0.
\end{align*}
\tag{3.18}
\]

**Proposition 3:** A tariff raises home R&D and reduces foreign R&D independently of whether firms play Cournot or Bertrand in the second stage of the game.

Since home sales in both markets are increasing in home R&D and falling in foreign R&D, the tariff has a further positive home output creation effect, and negative foreign output creation effect, that operate via the induced changes in R&D levels.

Differentiation of \( x(n,n^*,t) \), \( x^*(n,n^*,t) \), \( y(n,n^*,t) \) and \( y^*(n,n^*,t) \) yields the following results:
(i) \[ \frac{dx}{dt} = x_2 \left( \frac{dn}{dt} \right) + x_4 \left( \frac{dn^*/dt}{dt} \right) + x_5 > 0, \]

(ii) \[ \frac{dx^*/dt}{dt} = x_2 \left( \frac{dn}{dt} \right) + x_4 \left( \frac{dn^*/dt}{dt} \right) + x_5^*, \]

(iii) \[ \frac{dy}{dt} = y_3 \left( \frac{dn}{dt} \right) + y_4 \left( \frac{dn^*/dt}{dt} \right) + y_5 < 0, \]

(iv) \[ \frac{dy^*/dt}{dt} = y_3 \left( \frac{dn}{dt} \right) + y_4 \left( \frac{dn^*/dt}{dt} \right) + y_5^*. \]

Proposition 4: A tariff imposed before firms play a two-stage game, choosing R&D in the first stage and output or price for segmented markets in the second stage, will lead to an increase in the home firm's domestic sales and will have an ambiguous effect on home exports.

The slope of the marginal cost function plays a crucial role in determining whether or not tariff protection promotes exports in this model. In the important special case of \( \mu = 0 \), constant marginal costs, the tariff must be export promoting as the term \( x_5^* \) disappears. I have discussed this special case in detail elsewhere (see Leahy (1991)). In the further special case that arises as \( \mu \) goes to infinity all the R&D derivatives vanish, (in fact in this absolute capacity constraint case investment in R&D is futile) as do other output creation effects. Only crowding out effects remain and the tariff unambiguously hurts exports.

4. Capacity games

In this section, as in section 2, I will assume that the home and foreign firms play a two-stage game choosing price or quantity for segmented markets in the second stage, but the first-stage strategic variable is now total capacity rather than R&D. As before the firms produce commodities which are substitutes for one another with inverse demands given in equation (2.1). It is clear from (3.3) that, if \( \mu \) is zero \( X \) and \( Y \) will have no effect on the second-stage output levels. It is also straightforward to show that when \( \mu \) lies
between zero and infinity \( X \) and \( Y \) affect outputs in the same direction as \( n \) and \( n^* \) affect them. Therefore capacity and R&D choice can be analysed in essentially the same way when \( \mu \) lies in that particular range. For these reasons I will restrict attention to the special case that arises as \( \mu \) goes to infinity, that of an absolute capacity constraint.

This section draws on the work of Venables (1990), who analysed in detail, two-stage games in which capacity is chosen first. I will examine the implications for market linkage and the Krugman (1984) result "import protection is export promotion".

In the first stage of the game the home and foreign firms choose their total capacities \( X \) and \( Y \) respectively. In the second stage they choose either the sales, or prices for each market. As usual the second stage is solved first. In the second stage with capacity given output can be produced at zero marginal cost up to the capacity constraint, when marginal costs become infinite. The outcome is a special case of that analysed in section 3 as \( \mu \) goes to infinity. In this case (3.2) yields:

\[
\begin{align*}
x + x^* &= X, \quad \text{and} \\
y + y^* &= Y.
\end{align*}
\]

Provided that perceived marginal revenue net of any taxes is always positive the capacity constraint will hold with equality in equilibrium. As can be seen from (4.1) the firms cannot choose outputs for each market independently now. All they can do now in the second stage is allocate sales between markets.

Using (4.1) the firms' profit functions can be written as:

\[
\begin{align*}
(i) \quad \pi &= xp(x,y) + (X - x)p^*(X - x, Y - y) - K(X), \\
(ii) \quad \pi^* &= y[q(x,y) - t] + (Y - y)q^*(X - x, Y - y) - K^*(Y).
\end{align*}
\]
In the second stage the firms face the following optimisation problems:

\[
\begin{align*}
(i) \quad & \max_x \pi(x,y,X,Y), \\
(ii) \quad & \max_y \pi^*(x,y,X,Y),
\end{align*}
\]  
(4.3)

which yield the first-order conditions:

\[
\begin{align*}
(i) \quad & p + x(dp/dx)|_t - p^* + (X - x)(dp^*/dx)|_t = 0, \\
(ii) \quad & q + y(dq/dy)|_t - q^* + (Y - y)(dq^*/dy)|_t = t,
\end{align*}
\]  
(4.4)

where \((dp/dx)|_t = \partial p/\partial x + v\partial p/\partial y\), and \((dp^*/dx)|_t = \partial p^*/\partial x - v\partial p^*/\partial y^*\). As before \(v = 0\) in the Cournot case and \(v = -\epsilon\) in the case of Bertrand second-stage behaviour.

Before proceeding I now digress to consider a technical problem. Note that in the Bertrand case if one unit of the home firm's output is diverted from the foreign market to the home market, then the home firm expects that \(-\epsilon\) units of the foreign good will be similarly diverted into the home market (since \(-\epsilon\) is negative the firm expects output to be diverted out of its own domestic market). This expectation is equivalent to conjecturing that the rival's export price is constant. The home firm also expects that when it takes one unit of output out of the foreign market that the foreign firm's sales there will increase by \(\epsilon\) enough to ensure that \(q^*\) is constant and that the rival's capacity constraint continues to be met with equality. Firms conjecture that the rival's capacity constraint will continue to hold with equality, and the technical problems that would arise if it expected the rival to ration its consumers are avoided. In Venables (1990), this is only guaranteed in the neighbourhood of a symmetric equilibrium because of the assumption of general demands, while in this linearised version Bertrand quantity conjectures are independent of the level of sales and so it always holds.
Now it is possible to use (2.1) in (4.4) and solve for the second stage outputs. These can also be obtained directly from (3.3), by using l'Hopital's rule and taking the limit as $\mu$ tends to infinity:

$$x = \frac{X + \phi t/(\beta^2 - \phi^2)}{2}, \quad \text{assuming } X > \phi t/(\beta^2 - \phi^2), \quad (4.5)$$

$$y = \frac{Y - \beta t/(\beta^2 - \phi^2)}{2}, \quad \text{assuming } Y > \beta t/(\beta^2 - \phi^2),$$

It is then easy to derive expressions for foreign market sales by combining (4.1) and (4.5):

$$x^* = \frac{X - \phi t/(\beta^2 - \phi^2)}{2}, \quad \text{assuming } X > \phi t/(\beta^2 - \phi^2), \quad (4.6)$$

$$y^* = \frac{Y + \beta t/(\beta^2 - \phi^2)}{2}, \quad \text{assuming } Y > \beta t/(\beta^2 - \phi^2),$$

It is clear from inspection of (4.5) and (4.6) that with linear demands and $\mu$ infinite capacity has no strategic effects, that is to say it does not affect the distribution of the rival's sales across markets. The second-stage comparative static derivatives for a change in the tariff, are found to be:

$$x_i = -x^*_i = \frac{\phi}{2(\beta^2 - \phi^2)} > 0, \quad (4.7)$$

$$y_i = -y^*_i = -\frac{\beta}{2(\beta^2 - \phi^2)} < 0.$$

It is clear that the increase in home market sales of the home firm brought about by the tariff crowds out an equal amount of export sales.

I turn now to the first stage of the game and the case in which a tariff is imposed before capacity is chosen. The tariff will now have an effect on the total capacity installed as well as the relative attractiveness of the home and foreign markets.

---

9 See Venables (1990) for a discussion of the strategic effects that arise when demands are non linear.
The home firm chooses $X$ in the knowledge that this will affect the second-stage levels of $x$ and $x^*$, but not the levels of $y$ and $y^*$. The home firm’s first-order condition is thus:

$$\frac{d\pi}{dX} = (\partial \pi / \partial x)x_1 + (\partial \pi / \partial x^*)x_1^* + \partial\pi / \partial X = 0,$$

which can be rewritten as:

$$\frac{d\pi}{dX} = [(x_1 + x_1^* + p + p^*)/2] - k' = 0. \tag{4.9}$$

The use of (2.1) and (4.1) in (4.9) yields:

$$a - k' - b[x + (\epsilon/2)y] = 0. \tag{4.10}$$

Similarly the foreign firm’s first-order condition for choice of capacity can be written as:

$$a - k^* - b[y + (\epsilon/2)x] - t/2 = 0. \tag{4.11}$$

Note that the cross effects of capacity are negative; capacities are strategic substitutes.

The implicit capacity reaction functions (4.10) and (4.11) can be solved simultaneously for the equilibrium levels of $X$ and $Y$. It is then possible to derive the following comparative static derivatives

$$X_t = \epsilon/b(4 - \epsilon^2) + M > 0, \tag{4.12}$$

$$Y_t = -2/b(4 - \epsilon^2) + M < 0,$$

where $M = 4[K'' + K^*'' + (K''K^*)/b]$. $M$ is positive in the case of increasing marginal capacity cost and negative when both firms have decreasing marginal capacity costs. It is of ambiguous sign when one firm has declining and the other firm has increasing marginal capacity costs.

From (4.12) it is clear that the tariff has a positive capacity creation effect for the home firm and a negative one for the foreign firm. The tariff by raising the capacity installed by the home firm, causes it to increase output (equally) in both markets since \(\partial x/\partial X = \partial x^*/\partial X = 1/2\), from (4.5) and (4.6).
Proceed by totally differentiating: \(x(X(t), t), x_*(X(t), t), y(Y(t), t)\) and \(y_*(Y(t), t)\).

In the case of second-stage Cournot competition this yields:

\[
\begin{align*}
\frac{dx}{dt} &= x_i + x_i^2x_i = 2[2\phi(4 - \epsilon^2) + \epsilon M]/S > 0, \\
\frac{dx_*/dt} &= x_i^* + x_i^*x_i = -2M\epsilon/S, \\
\frac{dy}{dt} &= y_i + y_i^2y_i = -4[2b(4 - \epsilon^2) + M]/S < 0, \\
\frac{dy_*/dt} &= y_i^* + y_i^*y_i = 4M/S,
\end{align*}
\]

where \(S = 4[b(4 - \epsilon^2) + M][b(4 - \epsilon^2)] > 0\).

For the case of second-stage Bertrand behaviour:

\[
\begin{align*}
\frac{dx}{dt} &= x_i + x_i^2x_i = 2[2\phi(2 - \epsilon^2)(4 - \epsilon^2) + \epsilon M]/U > 0, \\
\frac{dx_*/dt} &= x_i^* + x_i^*x_i = -2\epsilon[M + \phi\epsilon(4 - \epsilon^2)]/U, \\
\frac{dy}{dt} &= y_i + y_i^2y_i = -2[(2 - \epsilon^2)M + b(4 - \epsilon^2)(4 - 3\epsilon^2)]/U < 0, \\
\frac{dy_*/dt} &= y_i^* + y_i^*y_i = 2[(2 - \epsilon^2)M + \phi\epsilon(4 - \epsilon^2)]/U,
\end{align*}
\]

where \(U = 4[b(4 - \epsilon^2)(1 - \epsilon^2)][b(4 - \epsilon^2) + M]\),

Proposition 5: When \(\mu\) is infinite and firms choose capacity in the first stage and output or price in the second stage, then for import protection to be export promotion it is necessary but not sufficient that at least one of the firms has declining marginal capacity cost.

For import protection to be export promotion it is necessary that \(M\) is negative and this is only possible when at least one of the firm's marginal capacity costs is declining. For the interesting special case in which firms have constant marginal capacity costs \(M = 0\), the following results are obtained:

Proposition 6: When firms have constant capacity costs a tariff will raise the home firm's domestic sales and (i) leave its exports unaffected when firms play
Cournot in the second stage, and (ii) reduce its exports when firms play Bertrand in the second stage.

When capacity costs are non decreasing \((M \geq 0)\), the output creation effect which increases exports, is not strong enough to outweigh the crowding out effect of the tariff. This crowding out effect is stronger when firms hold the more aggressive Bertrand conjectures in the second stage. This makes export expansion even less likely in the Bertrand case than in the Cournot case.

5. Concluding Remarks

In this paper I have examined the "import protection as export promotion" thesis within the framework of a formal two-stage game. I have shown that a tariff affects exports in two ways: firstly by raising home sales directly it leads to some crowding-out of exports; secondly by increasing R&D and/or capacity it increases exports indirectly. The first effect always works against, and the second effect always works in favour of import protection as export promotion. I have shown in the case of constant marginal cost that the crowding-out effect vanishes. In that case exports will rise if R&D is chosen in the first stage. In contrast if the marginal production cost curve is vertical then R&D is completely ineffective. I also demonstrated that when the marginal cost curve is vertical and firms choose capacities in the first stage, the tariff cannot raise exports if the marginal costs of capacity installation are non decreasing.

It is possible to see the effects of the tariff on exports at constant R&D and capacity as occurring in the short run, while the total effect of the tariff when the first-stage variable can change can be thought of as occurring in the long run. In the short run the tariff will tend to reduce exports as firms
respond by selling more in the market that has become relatively more attractive. In the long run however, the protected firm will engage in more R&D or install more capacity than its rival. If looked at in this way import protection as export promotion only occurs in the long run. Tables 1 to 3 summarise the main results of the paper.

The issue of retaliation was ignored throughout this paper. It is straightforward to extend the analysis to a situation in which both governments employ a tariff. I have examined the implications of this elsewhere. In Leahy (1991) I have shown that this weakens the prospects for an export-increasing tariff.
References


