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LEARNING BY DOING IN
INTERNATIONAL SUBSIDY GAMES

by

Dermot Leahy

ABSTRACT

A series of two-period, three-stage games with learning by doing is developed. In the first stage firms choose first-period outputs. Then governments choose export subsidies. Finally firms choose second-period outputs. I show (i) firms use first-period output strategically to manipulate export subsidies and the second-period outputs of rivals. (ii) These strategic effects are weakened when experience is diffused and by a third government tariff. (iii) When initial costs are symmetric and home residents partly own the foreign firm home outputs and subsidies exceed their foreign counterparts. These differentials increase in the speed of learning.

Key words: Learning by doing, Export subsidies, Strategic effects.

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LEARNING BY DOING IN INTERNATIONAL SUBSIDY GAMES

1 Introduction

In oligopolistic industries firms and governments will often have an incentive to act in a strategic manner. As defined by Helpman and Krugman (1989) strategic actions are:

"actions that by themselves are not desirable but alter the behaviour of others in ways that work to the strategic player's advantage".1

Learning by doing is similar to investing in capacity or process R&D in that it reduces a firm's future costs of production and so can be used strategically to restrict the expansion of rival firms or deter future competition altogether2. The strategic use of experience by firms against other firms has been examined by Spence (1981) and Fudenberg and Tirole (1983) among others and is one of the main themes of this paper. Another is the strategic use of export subsidies by governments against firms. Brander and Spencer (1985) considered a model in which one home firm and one foreign firm compete in a third market. They examine the case of Cournot competition and find that an export subsidy is optimal. The home government enjoying a first mover advantage precommits itself to grant an export subsidy to the home firm before the firms choose their outputs.

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1 Perhaps the best known example of strategic play in industrial organisation occurs in the so called "entry game" in which an incumbent monopolist is faced with the possibility that another firm enter the market. In that game the incumbent firm may "overinvest" in capacity in order to deter the entry of the other firm. Viewed in isolation the investment in excess capacity reduces the firm's own profits, yet if this persuades the other firm to stay out of the industry it could prove optimal. See for instance Spence (1977) and Dixit (1979,1980).

2 Krugman (1984) was the first to model learning by doing in a strategic trade context.
The subsidy raises home profits by more than the subsidy itself, because the foreign rival is persuaded to produce a lower output than it otherwise would, so profits are shifted from the foreign firm to the home firm and home welfare rises.\footnote{It is worth pointing out that this result is not very robust and depends among other things on the assumption that firms play Cournot. Eaton and Grossman (1986) show that an export tax is usually optimal if firms play Bertrand. See Neary (1988) for an overview of the literature.}

So far I have mentioned strategic play by firms against other firms and by governments against firms. However firms may also play strategically against governments. If firms move before export subsidies are chosen then they may be able to influence them. As de Meza (1986) and Neary (1991) have shown, optimal export subsidies depend on firm costs. If a home export subsidy is justified then it should be higher, the more cost-competitive is the home firm relative to the foreign firm. Therefore a firm may be able to ensure a higher subsidy for itself by producing a lot early, sacrificing short-run profits in order to move down its learning curve. This third strategic interaction, firms against governments, is the relatively least explored\footnote{Though it has been examined in a somewhat different framework by Gruenspecht (1988) among others.}. 

The plan of this paper is as follows: In section 2 I set out the basic model. This is a two-period three-stage game between two firms and two governments. In that model a home and a foreign firm play a two-period Cournot game with learning by doing. As in Brander and Spencer (1985) the two firms are exporting a homogenous product into a third market. The model is similar in form to the Fudenberg and Tirole (1983) model in which firms also use experience as a
strategic variable. However in my model they also have an incentive to play strategically against governments. Since first-period outputs affect second-period costs of production the firms are able to use these outputs to influence the governments' choice of subsidy level. In section 3 I look at an extension of the basic model in which the third-country government imposes a tariff. In section 4 I examine a situation in which residents of one country partly own the firm located in the other country. This has an important effect on optimal subsidies. (It may be optimal for one of the governments to impose an export tax.) In section 5 I look at the possibility that a firm may not appropriate all the benefits of its own learning. An increase in home first-period output may reduce the home and the foreign firm's second-period costs. Section 6 concludes.

The work most resembling this is Gatsios (1989). His model is also a three-stage game with export subsidies and learning by doing. However in his game the government moves first precluding any possibility that the firms can play strategically against governments.

2 The Basic Model

In this section I examine learning by doing in a two-period model in which I assume that there are two firms, a home firm and a foreign firm exporting a homogeneous product into a third market.\(^5\) This third market assumption has the advantage of allowing one to abstract from the home market effects of the

\(^5\)This framework has been often used before. Examples include Brander and Spencer (1985) and Eaton and Grossman (1986) among others.
export subsidy. The game is played as follows: In the first stage firms choose their first-period output levels simultaneously. The first-period production levels then affect second-period marginal costs. In the second stage a home and foreign government choose their export subsidy levels simultaneously in order to maximise their second-period welfare functions. Then in the third stage the firms choose their second-period outputs again simultaneously.

The equilibrium of the game is subgame perfect, a concept due to Selten (1975). In games such as these the players who move at earlier stages of the game correctly forecast the outcomes of subsequent stages when they choose their optimal current action. This means that in order to solve for the equilibrium of the game one must begin by solving the last stage and work backwards to the first stage. The equilibrium optimal actions at the last stage are calculated for all possible combinations of previous actions. The payoffs associated with these are used in determining optimal actions in the second last stage, which are calculated for all feasible previous actions, and so on, back to the start of the game.

To keep the analysis tractable I will assume that firms face linear inverse demand functions:

\[(2.1) \quad p_j = a - b(x_j + y_j) \quad (j = 1, 2),\]

where subscripts refer to the time period; the price of the homogenous product on the third country market is given by \(p\); the exports of the home firm are represented by \(x\) and the exports of the foreign firm are represented by \(y\). Production is subject to "constant instantaneous returns to scale" in the
terminology of Fudenberg and Tirole (1983). However there is a "learning by
doing" effect in that a firm's second-period marginal costs decline in first-period
output. Home first and second-period marginal costs are:

\[ c_1 = c, \quad c_2 = c - \lambda x_1, \]

where \( \lambda \) is the home learning parameter. The foreign firm has similar first- and
second-period marginal costs:

\[ c^*_1 = c^*, \quad c^*_2 = c^* - \lambda^* y_1, \]

(\text{foreign variables will often be starred}). In order to ensure that the second period
costs are positive it must be the case that: \( \lambda < c/x_1 \) and \( \lambda^* < c^*/y_1 \).

Let \( \pi_j \) be the home firm's \( j \)th period profits. The home firm's maximand is the
discounted sum of its first- and second-period profits:

\[ \pi = \pi_1 + \delta \pi_2 = x_1(p_1 - c) + \delta x_1(p_2 - c + \lambda x_1 + s), \]

where \( s \) is the export subsidy introduced by the home government at the start of
the second time period. The discount factor \( \delta \in (0,1] \), is the value in first period
pounds of a pound to be received in the second time period. Fixed costs are
assumed away as they would not play any role in the analysis. Any transport costs
that firms face are included in \( c \) and \( c^* \) respectively.

The foreign firm maximises the following function:

\[ \pi^* = \pi^*_1 + \delta \pi^*_2 = y_1(p_1 - c^*) + \delta y_1(p_2 - c^* + \lambda^* y_1 + s^*), \]

where \( s^* \) is the subsidy that the foreign firm receives from the foreign government.

As explained above the usual way to analyse multi-stage games of this type is to
start at the last stage and work back to the first. In the third stage of the game
the firms' optimisation problem take the following form for the home and foreign firm respectively:

\[(2.6) \quad (i) \quad \max_{x_2} \pi_2 = x_2(p_2 - c + \lambda x_1 + s), \]

\[(ii) \quad \max_{y_2} \pi^*_2 = y_2(p_2 - c^* + \lambda^* y_1 + s^*), \]

The first-order conditions for these maximisation problems are the respective second-period reaction functions (or best-reply functions) for the two firms:

\[(2.7) \quad (i) \quad \partial \pi_2 / \partial x_2 = a - b(2x_2 + y_2) - c + \lambda x_1 + s = 0, \]

\[(ii) \quad \partial \pi^*_2 / \partial x_2 = a - b(x_2 + 2y_2) - c^* + \lambda^* y_1 + s^* = 0. \]

Now solve simultaneously to give the third-stage second-period equilibrium outputs:

\[(2.8) \quad (i) \quad x_2^o = [a - 2c + c^* + 2\lambda x_1 - \lambda^* y_1 + 2s - s^*]/3b, \]

\[(ii) \quad y_2^o = [a - 2c^* + c + 2\lambda^* y_1 - \lambda x_1 + 2s^* - s]/3b. \]

This clearly shows that experience effects provide a link between time periods. An increase in home first-period output reduces the firm's second-period marginal costs: This not only helps second-period home profits directly but also tends to reduce the outputs of the foreign rival.

I turn now to the second stage of the game in which the home and foreign governments choose their optimal export subsidies, given first-period outputs and in anticipation of the second-period cost functions of the two firms. Governments carry out their optimisations at the start of the second period and they maximise the unweighted sum of home firm profits and government revenue:
(2.9)  (i) \[ \max W = \pi_2 \cdot s x_2, \]
        \[ s \]

(ii) \[ \max W^* = \pi^*_2 \cdot s^* y_2. \]

The governments move simultaneously and the first-order conditions are:

(2.10) (i) \[ \frac{dW}{ds} = (\partial \pi_2 / \partial y_2)(dy_2/ds) - s dx_2/ds = 0, \]

(ii) \[ \frac{dW^*}{ds^*} = (\partial \pi^*_2 / \partial x_2)(dx_2/ds^*) - s^* dy_2/ds^* = 0. \]

Note that the terms \( \partial \pi_2 / \partial x_2 \) and \( \partial \pi^*_2 / \partial y_2 \) are both equal to zero from (2.7). Now from (2.1), (2.4) and (2.5) it is possible to show that \( \partial \pi_2 / \partial y_2 = -bx_2 \), and that \( \partial \pi^*_2 / \partial x_2 = -by_2 \). Using (2.8) in these two expressions and then substituting them into (2.10) gives:

(2.11) (i) \[ \frac{dW}{ds} = (a - 2c + c^* + 2\lambda x_1 - \lambda^* y_1 - 4s - s^*)/9b = 0, \]

(ii) \[ \frac{dW^*}{ds^*} = (a + c - 2c^* - \lambda x_1 + 2\lambda^* y_1 - s - 4s^*)/9b = 0. \]

These can be solved simultaneously to give explicit expressions for the equilibrium subsidies:

(2.12) (i) \[ s^o = (a - 3c + 2c^* + 3\lambda x_1 - 2\lambda^* y_1)/5, \]

(ii) \[ s^{*o} = (a + 2c - 3c^* - 2\lambda x_1 + 3\lambda^* y_1)/5. \]

So it is clear from this that if learning occurs \( (\lambda > 0, \lambda^* > 0) \), a government's optimal subsidy increases in its own firm's first-period output and decreases in the first-period output of the rival firm.

It will later prove useful to have an expression for the optimal second-period outputs in terms of first-period outputs and parameters alone. This is found by using (2.12) in (2.8), eliminating \( s \) and \( s^* \) in order to obtain:

(2.13) (i) \[ x^o_2(x_1,y_1) = 2[a - 3(c - \lambda x_1) + 2(c^* - \lambda^* y_1)]/5b, \]
(ii) \( y^*_{2}(x_1,y_1) = 2(a + 2(c - \lambda x_1) - 3(c^* - \lambda^* y_1))/5b. \)

Turning now to the first stage of the game and the optimal first-period output choices of the two firms. The home firm faces the following optimisation problem:

\[
(2.14) \quad \max_{x_1} \pi(x_1,y_1,x_2,y_2,s)
\]

Profit maximisation by the home firm therefore implies the first-order condition:

\[
(2.15) \quad \frac{d\pi}{dx_1} = 0 - \frac{\partial\pi_1}{\partial x_1} - \delta \frac{dc_2}{dx_1} \left[ \frac{\partial\pi_2}{\partial c_2} + \frac{\partial\pi_2}{\partial y_2} \frac{\partial y_2}{\partial c_2} \left( 1 - \frac{\partial s}{\partial c_2} \right) + \frac{\partial\pi_2}{\partial y_2} \frac{\partial y_2}{\partial s} \frac{\partial s}{\partial c_2} \right].
\]

The envelope theorem has been invoked in order to eliminate terms in \( \partial\pi/\partial x_2 \).

The right hand side of (2.15) can be decomposed into six terms as follows:

(i) \( \frac{\partial\pi_1}{\partial x_1} = a - b(2x_1 + y_1) - c, \)

This term represents first-period marginal profit and would be set equal to zero in the absence of home learning (\( \lambda = 0 \)).

(ii) \( \delta (dc_2/dx_1)(\partial\pi_2/\partial c_2) = \delta x_2^* \)

This is the direct non-strategic learning by doing effect. The firm produces more than the first-period profit-maximising amount in order to reduce its second-period costs and so raise its profits in that period. This term represents the discounted second-period cost-reducing effect of a marginal increase in first-period production and it clearly depends on the home learning parameter and the level of second-period sales.

(iii) \( \delta (dc_2/dx_1)(\partial\pi_2/\partial y_2)(\partial y_2/\partial c_2) = \delta x_2^* /3. \)

Firms will choose higher output levels when they play strategically than when they do not in order to reduce the second-period output of the rival directly. This reason has already been mentioned by Spence (1981) and
Fudenberg and Tirole (1983).

(iv) \[ \delta(dx_1/dx_1)(d\pi_2/ds)(ds/dc_2) = (3/5)\delta x_2. \]

(where \( d\pi_2/ds = -d\pi_2/dc_2 \))

The home firm will also choose a higher output level to encourage the home government to grant a higher subsidy which will increase the firms second-period profits.

(v) The term:

\[ \delta(dx_1/dx_1)(d\pi_2/dy_2)(dy_2/ds)(ds/dc_2) = \delta x_2/5, \]

(where \( dy_2/ds = -dy_2/dc_2 \))

also reflects the endogeneity of policy. The home firm produces more so as to reduce the second-period output of the rival firm by inducing a higher home subsidy.

(vi) \[ \delta(dx_1/dx_1)(d\pi_2/dy_2)(dy_2/ds^*)(ds^*/dc_2) = (4/15)\delta x_2. \]

In addition an increase in home first-period output works to reduce the second-period output of the rival firm by reducing the rival export subsidy.

Term (i) which is both non-strategic and non-intertemporal, will be negative since the terms (ii) to (vi) which represent the intertemporal effects of first-period output, are all positive. These intertemporal effects can be divided into the strategic effects (iii) to (vi), and the non-strategic intertemporal effect (ii). The strategic effects themselves can be divided into those that work through the endogenous policy variables (iv) to (vi), and one that does not, term (iii).

Making use of (i) to (vi) above it is now straightforward to show that:
\[(2.16) \quad \frac{dx_i}{dx_{1}} = a - b(2x_i + y_i) - c + (12/5)\delta x_2 = 0.\]

Since second-period output is endogenous (2.13) can be used in (2.16). The resulting expression is a more explicit version of the first-stage first-order condition for the home firm:

\[(2.17) \quad (25b + 24\delta \lambda)(a - c) + 48\delta \lambda(c^* - c) - (50b^2 - 72\delta \lambda^2)x_i - (25b^2 + 48\delta \lambda \lambda^*)y_1 = 0,\]

The first-order condition for the foreign firm is similarly:

\[(2.18) \quad (25b + 24\delta \lambda^*)(a - c^*) - 48\delta \lambda^*(c^* - c) - (25b^2 + 48\delta \lambda \lambda^*)x_i - (50b^2 - 72\delta \lambda^2) y_1 = 0.\]

The two equations (2.17) and (2.18) can be solved simultaneously for the equilibrium output levels.

I first consider the case of symmetrical costs \(c = c^*\) and symmetrical learning \(\lambda = \lambda^*\). In that case the equilibrium first period outputs are:

\[(2.19) \quad x_i^* = y_i^* = \frac{(25b + 24\delta \lambda)(a - c)}{(75b^2 - 24\delta \lambda^2)}.\]

It is then possible to obtain the symmetrical equilibrium outputs for the second period by using (2.19) in (2.13):

\[(2.20) \quad x_i^* = y_i^* = \frac{10(3b + \lambda)(a - c)}{(75b^2 - 24\delta \lambda^2)}.\]

**Proposition 1:** In a symmetric equilibrium, first- and second-period outputs are increasing in the learning parameter \(\lambda\) and in the discount factor \(\delta\).
As can easily be seen, second-period output is higher than first-period output in the absence of learning by doing. This is of course due to the fact that the firm is subsidised in the second period but not in the first. It is however possible for first-period output to be higher when there is learning by doing and firms play strategically. The difference between first- and second-period outputs is:

\[(2.21) \quad x_2 - x_1 = \frac{[5b + 2\lambda(5 - 12\delta)](a - c)}{(75b^2 - 24\delta\lambda^2)}\]

When \(\delta > 5/12\), this difference is more likely to be non-positive the larger is \(\lambda\). It can be shown that the government’s optimal export subsidies are also increasing in the learning parameter and the discount factor.

The use of (2.19) in (2.12) yields the following for a symmetric equilibrium:

\[(2.22) \quad s^\circ = s^* = \frac{5b(3b + \lambda)(a - c)}{75b^2 - 24\delta\lambda^2}\]

Intuitively what is happening here is this: The stronger is the learning effect and the more future profits are valued, the more firms will produce in the first period foregoing future profits in order to move down their learning curve. However, the more that firms produce in the first period the lower will second-period costs be and so the firms will receive higher second-period subsidies. Lower second-period costs and higher second-period subsidies combine to give higher second-period outputs.

I turn now to a situation in which firms have different first-period marginal costs, \((c \neq c^*)\) but I continue to assume that they learn at the same rate \((\lambda = \lambda^*)\). The
first-period equilibrium outputs obtained from (2.17) and (2.18) are now:

2.23  (i) \[ x_1^o = \frac{(25b + 248\lambda)(a - c) + E(c^* - c)}{75b^2 - 248\lambda^2} \]

(ii) \[ y_1^o = \frac{(25b + 248\lambda)(a - c^*) - E(c^* - c)}{75b^2 - 248\lambda^2} \]

where \( E = 5b(25b^2 + 168b\delta \lambda + 48\delta \lambda^2)/J(75b^2 - 248\lambda^2) > 0, \)

and \( J = 5b^2 - 248\lambda^2 > 0, \) in order to ensure that the equilibrium is stable.

If and only if the home firm has lower initial marginal costs than the foreign firm, \((c^* - c) > 0,\) it is clear from (2.23) that it has a larger first-period output than the foreign firm. It is also clear that the home firms's output is increasing in the learning parameter and the discount factor. The strategic and the non-strategic incentives that the home firm faces to produce a high first-period output are strengthened by its initial cost advantage.

Second-period outputs are obtained by using (2.23) in (2.13) to give:

2.24  (i) \[ x_2^o = \frac{10(3b + \lambda)(a - c) + F(c^* - c)}{75b^2 - 248\lambda^2} \]

(ii) \[ y_2^o = \frac{10(3b + \lambda)(a - c^*) - F(c^* - c)}{75b^2 - 248\lambda^2} \]

where \( F = 10b(30b^2 + 35b\lambda + 24\delta \lambda^2)/J(75b^2 - 248\lambda^2) > 0. \)

It is interesting to consider how the differences in outputs between the two firms are affected by initial cost differences \((c^* - c),\) and by the presence of learning by doing. The output differences obtained from (2.23) and (2.26) are:

2.25  (i) \[ x_1^o - y_1^o = \frac{(5b + 248\lambda)(c^* - c)}{J}, \]

(ii) \[ x_2^o - y_2^o = \frac{10(b + \lambda)(c^* - c)}{J}. \]
**Proposition 2:** When firms learn at the same rate \((\lambda = \lambda^*)\), but the home firm has an initial cost advantage \((c^* - c > 0)\), the difference between home and foreign output in the first and second period, is increasing in the learning parameter \(\lambda\), the discount factor \(\delta\) and the initial cost gap.

Using (2.2), (2.3) and (2.12) the export subsidy differential is found to be identical to the second-period cost differential\(^6\):

\[
(2.26) \quad s^o - s^{*o} = c^*_2 - c_2.
\]

In order to examine the impact of "learning by doing" on the subsidy differential, decompose the right-hand side of (2.26) to get:

\[
(2.27) \quad s^o - s^{*o} = c^* - c + \lambda(x_1 - y_1).
\]

The subsidy differential depends on the initial cost differential and in addition, because there is learning, it depends on the first period output differential.

The use of (2.25(i)) in (2.27) yields:

\[
(2.28) \quad s^o - s^{*o} = 5b(b + \lambda)(c^* - c)/J.
\]

The subsidy differential is increasing in the learning parameter, the discount factor and the initial cost gap.

I turn now to the case of asymmetrical learning \((\lambda \neq \lambda^*)\). To keep the analysis simple I will assume that only one of the firms (the foreign firm) learns, so that \(\lambda = 0\), and \(\lambda^* > 0\). I further assume that first-period marginal costs are equal. Returning to equations (2.15) and (2.16) it is clear that all the home firm's

\^{6} This result is sensitive to the assumption of linear demands (see Neary (1991))
intertemporal terms disappear, leaving \( \frac{dx}{dx_i} = \frac{\partial \pi_i}{\partial x_i} = 0 \), as the home firm's first-order condition. The home firm simply chooses its first-period sales to maximise its first-period profits. The foreign firm continues to choose more than the first-period profit maximising amount as before. It is obvious that foreign output will be larger than home output. As before first-period outputs are obtained from (2.17) and (2.18) this time imposing \( c = c^* \) and \( \lambda = 0 \):

(2.29)  
(i) \[ x_i^*(0, \lambda^*) = \frac{25b^2 - 245\lambda^*(3\lambda^* + b)}{3bG} (a - c) \]

(ii) \[ y_i^*(0, \lambda^*) = \frac{25b + 485\lambda^*}{3G} (a - c) \]

where \( G = 25b^2 - 485\lambda^* \).

The use of (2.29) in (2.13) yields:

(2.30)  
(i) \[ x_2^*(0, \lambda^*) = \frac{2(15b^2 - 10b\lambda^* - 485\lambda^*^2)}{3bG} (a - c) \]

(ii) \[ y_2^*(0, \lambda^*) = \frac{10(b + \lambda^*)}{G} (a - c) \]

It is immediately clear from (2.29) and (2.30) that the foreign first- and second-period sales are increasing in the learning parameter, and it is straightforward to show that both home outputs fall in \( \lambda^* \).

3 The Third Government Imposes A Tariff

I turn now to a variant of the basic model in which the third-country government, becoming a player in the game, is able to impose a tariff. How will this affect the behaviour of the home and the foreign firm, and the home and foreign government?

It is now well known that there exists an optimal rent-extracting tariff for the
third-country government in models such as this. The approach adopted here most closely resembles that used in Brander and Spencer (1985). I look at a case in which one home firm and one foreign firm, each receiving an export subsidy from their respective governments, are selling into a third market where they face an import tariff. I assume that the tariff and the two export subsidies are being chosen simultaneously. The order of play is now as follows: In the first stage the home and the foreign firms choose their first-period outputs simultaneously. Then the home and the foreign governments choose their export subsidies and the third government chooses its tariff simultaneously in the second stage. Finally, in the third stage, the home and the foreign firms choose their second-period output levels, again simultaneously. To keep the analysis simple I will assume symmetrical costs and learning.

In the third stage of the game the firms now face the marginal costs $c_2 + t$, where $t$ is the specific tariff. As a result the third-stage equilibrium outputs now depend on the tariff and subsidies in the following way:

$$ (3.1) \begin{align*}
(i) \quad x_2^* &= (a - c - t + \lambda(2x_1 - y_1) + 2s - s^*)/3b, \\
(ii) \quad y_2^* &= (a - c - t + \lambda(2y_1 - x_1) + 2s^* - s)/3b.
\end{align*} $$

I will assume that the third-country government maximises the following welfare function:

$$ (3.2) \quad \max_{t} W^{**} = u(x_2 + y_2) - (x_2 + y_2)p_2 + (x_2 + y_2)t, $$

where $u(x_2 + y_2)$ is the utility that consumers in the third country get from

---

consuming the imperfectly competitive good and $W^{**}$ represents the third country welfare. The difference $u - (x_2 + y_2)p_2$ is the third-country consumer surplus. I assume that the government maximises the unweighted sum of this consumer surplus and the tariff revenue. The first order condition for welfare maximisation implies that:

$$ (3.3) \quad \frac{dW^{**}}{dt} = [x_2 + y_2][1 - (dp_2/dt)] + t[(dx_2/dt) + (dy_2/dt)] = 0, $$

[Note that $u'(x_2 + y_2) = p_2$]

From (2.1) it is clear that $dp_2/dt = -b[(dx_2/dt) + (dy_2/dt)]$. The use of this and (3.1) in (3.3) yields the following:

$$ (3.4) \quad \frac{dW^{**}}{dt} = (2(a - c) + \lambda(x_1 + y_1) + s + s^* - 2t)/9b = 0. $$

This is the third government's reaction function in implicit form: as can be seen the optimal tariff is increasing in the home and foreign export subsidies. The increase is considerably less than countervailing however.

The home and the foreign governments maximise the welfare functions given in (2.9) and this gives first-order conditions identical in form to those in (2.10). The only difference now is that second-period outputs are represented by the equations in (3.1) rather than (2.8). This means that the home and the foreign governments first-order conditions for welfare maximisation can be written as:

$$ (3.5) \quad \begin{align*} (i) \quad \frac{dW}{ds} &= (a - c - t + \lambda(2x_1 - y_1) - 4s - s^*]/9b = 0, \\ (ii) \quad \frac{dW^*}{ds^*} &= (a - c - t - \lambda(x_1 - 2y_1) - s - 4s^*]/9b = 0. \end{align*} $$

It is now possible to rearrange (3.4) and (3.5) in matrix form:
(3.6) \[
\begin{bmatrix}
4 & 1 & 1 \\
1 & 4 & 1 \\
-1 & -1 & 8
\end{bmatrix}
\begin{bmatrix}
s \\
s^* \\
t
\end{bmatrix}
= 
\begin{bmatrix}
\bar{a} - c + \lambda(2x_1 - y_i) \\
\bar{a} - c - \lambda(x_1 - 2y_i) \\
2(a - c) + \lambda(x_1 + y_i)
\end{bmatrix}
\]

This yields solutions for the equilibrium export subsidies and tariff.

(3.7) (i) \( s^* = [a - c + \lambda(4x_1 - 3y_i)]/7, \)

(ii) \( s^{**} = [a - c + \lambda(4y_1 - 3x_i)]/7, \)

(iii) \( t^o = [2(a - c) + \lambda(x_1 + y_i)]/7 = s^o + s^{**}. \)

From (3.7) it is clear that an increase in the home firm's first-period output leads to an increase in the home export subsidy, a fall in the foreign export subsidy and an increase in the optimal tariff. From (3.4) and (3.7) it is clear that the total effect of first-period output on the tariff is the sum of three effects: Firstly the increase in home first-period output leads to a fall in the home firm's second-period production costs and thus to a direct increase in the optimal rent extracting tariff. This is represented by: \( \partial t/\partial x_1 = \lambda/8. \) Secondly the increase in the home subsidy brought about by the increase in \( x_1 \) leads to a partially countervailing increase in the tariff: \( (\partial t/\partial s)(ds/dx_1) = \lambda/14. \) Thirdly the fall in the foreign subsidy brought about by an increase in \( x_1 \) works against an increase in the tariff: \( (\partial t/\partial s^*)(ds^*/dx_1) = -3\lambda/56. \)

It is worth examining the net taxes, (tariff minus subsidy) that are encountered by firms. These can be represented as:

(3.8) (i) \( \theta = t^o - s^o = [a - c - \lambda(3x_1 - 4y_i)]/7 = s^* \)

(ii) \( \theta^* = t^o - s^{**} = [a - c + \lambda(4x_1 - 3y_i)]/7 = s. \)

It can be seen that the net tax faced by the firm decreases in its own first-period
output and increases in the first-period output of its rival.

I now wish to compare the intertemporal strategic incentives faced by a firm in this model with those it faces in the basic model where there is no tariff. Firstly a marginal increase in a firm's first-period output in a model with a tariff will not bring about as big an improvement in the net subsidy that the firm receives as it does in the basic model. This is apparent from a comparison of (2.12) and (3.8). However, it is now the case that a marginal increase in a firm's first-period output will hurt the rival firm more than in the model without a tariff. This is clear from the following equations, which are obtained by using (3.7) in (3.1):

\[(3.9) \quad \begin{align*}
    (i) \quad & x_i^\circ(x_1,y_1) = 2[a - c + \lambda(4x_1 - 3y_i)]/7b, \\
    (ii) \quad & y_2^\circ(x_1,y_1) = 2[a - c + \lambda(4y_1 - 3x_i)]/7b.
\end{align*}\]

When (3.9) is compared with (2.13) it is clear that a marginal increase in own first-period output has a more negative effect on rival output in this model than in the basic model. This reflects the fact that an increase in home first-period output now forces the foreign firm to contend with a larger tariff in addition to a higher home subsidy. So there are now additional strategic effects to be taken into account by firms when they are choosing the level of first-period sales.

In the first period the home firm faces the following profit maximisation problem:

\[(3.10) \quad \max_{x_1} \pi(x_1,y_1,x_2,y_2,s,t).\]

The only difference between this and (2.14) is that the tariff is an argument in the
profit function. It is also the case now that the second-period outputs depend on
the tariff. The first-order condition for a maximum implies:

\[
\frac{d\pi}{dx_1} = 0 - \frac{\partial \pi_1}{\partial x_1} +
\]

\[
\delta \frac{dc_2}{dx_1} \left[ \left( \frac{\partial \pi_2}{\partial c_2} + \frac{\partial \pi_2}{\partial y_2} \frac{\partial y_2}{\partial c_2} \right) \left( 1 - \frac{\partial \pi_2}{\partial c_2} \right) + \frac{\partial \pi_2}{\partial y_2} \frac{\partial y_2}{\partial c_2} + \left( \frac{\partial \pi_2}{\partial c_2} - \frac{\partial \pi_2}{\partial y_2} \frac{\partial y_2}{\partial c_2} \right) \frac{\partial \pi_2}{\partial c_2} \right].
\]

I will adopt the same numbering of terms as that used in (2.15). A comparison of
(2.15) and (3.11) reveals two additional intertemporal-strategic terms on the RHS
of the latter. Using (3.7(iii)) the first new term is:

(vii) \( \delta (dc_2/dx_1)(\partial \pi_2/\partial t)(\partial t/\partial c_2) = -\delta \lambda x_2/7, \) (where \( \partial \pi_2/\partial t = \partial \pi_2/\partial c_2 \))

which is negative. The home firm will choose a lower first-period output due
to this term in order to keep the tariff that it faces low.

From (3.1) and (3.7) the second new term is found to be:

(viii) \( \delta (dc_2/dx_1)(\partial \pi_2/\partial y_2)(\partial y_2/\partial t)(\partial t/\partial c_2) = \delta \lambda x_2/21, \)

(\text{where } \partial y_2/\partial t = -\partial y_2/\partial c_2)

which is positive. This implies that the home firm will produce a higher
output level when it plays strategically in order to reduce the second-period
output of the foreign firm by inducing a higher tariff.

The terms (i) to (iii) in (2.15) take the same form in (3.11). However using (3.1)
and (3.7) the effects that rely on policy endogeneity are altered as follows:

Term (iv) becomes:

(iv') \( \delta (dc_2/dx_1)(\partial \pi_2/\partial s)(\partial s/\partial c_2) = (4/7)\delta \lambda x_2, \)

and (v) becomes:
(v') \( \delta(dx/dx_1)(\partial \pi_2/\partial y_2)(\partial y_2/\partial s)(\partial s/\partial c_2) = (4/21)\delta \lambda x_2^* \),

These are both smaller than the corresponding effects in the basic model. The anticipation of an increase in the tariff reduces the positive effect of a cost reduction on the optimal home subsidy. The impact of home first-period output on foreign second-period output which works through the foreign export subsidy is now:

(vi') \( \delta(dx/dx_1)(\partial \pi_2/\partial y_2)(\partial y_2/\partial s^*)(\partial s^*/\partial c_2) = (6/21)\delta \lambda x_2^* \).

The use of (3.1) and (3.7) in (3.11) gives:

(3.12) \( d\pi/dx_1 = a - 2bx_1 - by_1 - c + (16/7)\delta \lambda x_2^*(x_1,y_1) = 0 \),

It is now possible to eliminate \( x_2^* \) in (3.12) using (3.9(i)) and to restate the first-order condition in the following way:

(3.13) \( (49b + 32\delta \lambda)(a - c) - (98b^2 - 128\delta \lambda^2)x_1 - (49b^2 + 96\delta \lambda^2)y_1 = 0 \).

Since the assumption that there are symmetrical costs and symmetrical learning has been imposed the equilibrium first-period outputs are:

(3.14) \( x_1^* = y_1^* = (a - c)(49b + 32\delta \lambda)/K \),

where \( K \equiv 147b^2 - 32\delta \lambda^2 \).

The use of (3.14) in (3.9) gives:

(3.15) \( x_2^* = y_2^* = 14(a - c)(3b + \lambda)/K \)

Then subsidies, tariffs and net taxes as a function of the learning parameter and discount factor are obtained by using (3.14) in (3.7) and (3.8) giving:

(3.16) \( s^* = s^{**} = \theta = \theta^* = 7b(a - c)(3b + \lambda)/K \),

\( t^* = 14b(a - c)(3b + \lambda)/K \).
**Proposition 3**: In the symmetric equilibrium with a tariff, outputs in the first and the second periods, subsidies, the tariff and net taxes are all increasing in the learning parameter and the discount factor.

A comparison of (3.14) and (3.15) with (2.19) and (2.20) reveals that, for given values of $\delta$ and $\lambda$, both first- and second-period outputs are lower in the equilibrium with a tariff than in the one without a tariff. In the basic model firms enjoyed subsidisation now they face a net tax in the second period. This persuades them to sell less in period two. The intention to sell less in the second period and the knowledge that an increase in first-period output will bring about a smaller improvement in the net subsidy (fall in the net tax) than it did in the basic model, combine to ensure that the first-period output is now lower.

## 4 International Equity Markets

So far I have assumed that all the profits made by a firm located in a particular country accrue to residents of that country, that is I have been assuming that there is no international rent diffusion. In reality equity markets are internationally linked and this will have implications for the strategic interactions of governments and firms. In this section I examine a variant of the basic (no tariff) model in which residents of the home country can own shares in the foreign firm. The approach adopted here is similar to that used by Lee (1990,1991).

As usual I start by examining the third stage and work back to the first. In the third stage firms take subsidies and first-period outputs as given. The resulting equilibrium second-period outputs are represented in (2.8).
In the second stage governments choose export subsidies (or export taxes) in order to maximise their national welfare. The home government maximises:

\[
(4.1) \quad \max_{s} W = \pi_{2} + h\pi_{2}^* - sx_{2},
\]

where \( h \in [0,1] \), is the proportion of the "foreign" firm owned by residents of the home country. All profits of firms are assumed to be redistributed to shareholders in the form of dividends.

Lee (1990) also looked at the situation in which a percentage of the "home" firm is owned by foreigners. However with the addition of a prior stage to the game the algebra would become very unwieldy and difficult to interpret in an intuitive way. For this reason I restrict attention to the case in which international cross ownership goes only one way. For the same reason I ignore the possibility of endogenous determination of the extent of foreign ownership by ruling out international equity arbitrage.\(^8\)

The foreign firm maximises the following welfare function:

\[
(4.2) \quad \max_{s^*} W^* = (1 - h)\pi_{2}^* - s^*y_{2}.
\]

The home and foreign first-order conditions for welfare maximisation are:

\[
(4.3) \quad (i) \quad dW/ds = (\partial\pi_{2}/\partial y_{2})(dy_{2}/ds) + h(\partial\pi_{2}^*/\partial x_{2})(dx_{2}/ds) - s(dx_{2}/ds) = 0,
\]

\[
(ii) \quad dW^*/ds^* = (1 - h)(\partial\pi_{2}^*/\partial x_{2})(dx_{2}/ds^*) - hy_{2}^* - s^*(dy_{2}/ds^*) = 0,
\]

where \( \partial\pi_{2}/\partial x_{2} = \partial\pi_{2}^*/\partial y_{2} = 0 \) from (2.7).

---

\(^8\) See Lee (1991) for a discussion of these issues.
Proceeding in the same manner as in section 2, using (2.1), (2.4), (2.5) and (2.8) in (4.3) yields:

(4.4) \( (i) \quad \frac{dW}{ds} = \frac{\left( Z - 2(2 - h)s - (1 + 4h)s^* \right)}{9b} = 0, \)

(\( ii \) \quad \frac{dW^*}{ds^*} = \frac{(Z^* - (1 - 4h)s - 4(1 + 2h)s^*)}{9b} = 0, \)

where:

(4.5) \( (i) \quad Z \equiv (1 - 2h)(a - c) + \lambda[2(1 + h)x_i - (1 + 4h)y_i], \)

and \( (ii) \quad Z^* \equiv (1 - 4h)[a - c + \lambda(2y_i - x_i)], \)

The equations in (4.4) can be multiplied through by 9b and rearranged in matrix form to give:

(4.6) \[
\begin{bmatrix}
2(2 - h) & 1 + 4h \\
1 - 4h & 4(1 + 2h)
\end{bmatrix}
\begin{bmatrix}
s \\
s^*
\end{bmatrix} = \begin{bmatrix}
Z \\
Z^*
\end{bmatrix}.
\]

The determinant of the coefficient matrix is found to be \(3(5 + 8h).\) The equilibrium subsidies obtained from (4.6) are:

(4.7) \( (i) \quad s^* = \frac{(a - c + \lambda[(3 + 8h)x_i - (2 + 8h)y_i])}{(5 + 8h)}, \)

(\( ii \) \quad s^{**} = \frac{(a - c + \lambda(3y_i - 2x_i))}{(1 - 4h)/(5 + 8h)}. \)

It is clear from (4.7) that the foreign government will employ an export tax if \( h > 1/4 \) and an export subsidy for \( h < 1/4.\) The intuition is straightforward: The lower the proportion of foreign firm rent that accrues to foreign residents the less point there is in shifting rent to their firm and the more there is to be gained from extracting the foreign firm's own rent using a tax. The home governments optimal policy always involves an export subsidy in this model, since \( x_i \geq y_i \) [see Appendix].

---

\(^9\) Note that the term in chain brackets in (4.7(ii)) must be assumed strictly positive in order to ensure that foreign second-period output is positive.
The use of (4.7) in (2.8) gives:

\[(4.8) \quad (i) \quad x_2(x_1, y_1) = 2[(1 + 2h)(a - c) + \lambda[(3 + 4h)x_1 - 2(1 + h)y_1]/b(5 + 8h),

(ii) \quad y_2(x_1, y_1) = 2[a - c + \lambda(3y_1 - 2x_1)]/b(5 + 8h).\]

The first-stage first-order condition of the home firm is given in (2.15), as in the basic model. Some of the intertemporal-strategic effects however have different interpretations\(^{10}\). It is now the case that:

(a) \[(4.9) \quad \delta(dc/dx_1)(\partial\pi_2/\partial s)(\partial s/\partial c_2) = [(3 + 8h)/(5 + 8h)]\delta x_2^o > 0.\]

As in the basic model the home firm will play strategically against its own government in order to improve the subsidy it receives and through this its second-period profits.

(b) \[(4.9) \quad \delta(dc/dx_1)(\partial\pi_2/\partial y_2)(\partial y_2/\partial s)(\partial s/\partial c_2) = [(3 + 8h)/3(5 + 8h)]\delta x_2^o > 0.\]

The effect of the first-period home output on second-period foreign output via the home export subsidy, is increasing in h.

(c) \[(4.9) \quad \delta(dc/dx_1)(\partial\pi_2/\partial y_2)(\partial y_2/\partial s^*)(\partial s^*/\partial c_2) = [4(1 - 4h)/3(5 + 8h)]\delta x_2^o.\]

Higher home first-period output will reduce the foreign export subsidy if \(h < 1/4\) and reduce their export tax if \(h > 1/4\). When home ownership of the foreign firm is sufficiently large the foreign government will respond by reducing its export tax hurting the home firm in the second period. This effect would then work to keep home first period output down.

---

\(^{10}\) Note that the terms in (2.15) numbered (i) to (iii) are unchanged.
The total effect of first-period output on home discounted second-period profit via changes in second-period foreign output is:

\[
\frac{\delta \frac{dc_2}{dx_1} \frac{\partial \pi_2}{\partial y_2} \left( 1 - \frac{\partial s}{\partial c_2} \right)}{\frac{\partial y_2}{\partial c_2}} + \frac{\partial y_2}{\partial s} \frac{\partial s^*}{\partial c_2} \right) = \frac{4}{(5 + 8h)} \delta \lambda x_2^0 > 0
\]

This effect is unambiguously positive. An increase in home first-period sales will reduce foreign second-period output, and so raise second-period profit even when \( s^* < 0 \).

However it is easy too see that this effect is weaker than its counterpart in the basic model for \( h > 0 \).

Now use (4.9) and (4.10) in (2.15) and eliminate \( x_2 \) using (4.8). This finally yields:

\[
(4.11) \quad \alpha_{11} x_1 + \alpha_{12} y_1 = \beta (a - c),
\]

where:

\[
(4.12) \quad \beta = b + \{8 \delta \lambda (3 + 4h)(1 + 2h)/(5 + 8h)^2\},
\]

\[
\alpha_{11} = 2b^2 - \{8 \delta \lambda^2 (3 + 4h)^2/(5 + 8h)^2\},
\]

\[
\alpha_{12} = b^2 + \{16 \delta \lambda^2 (3 + 4h)(1 + h)/(5 + 8h)^2\}.
\]

Turning now to the foreign firm, its first-order condition can be written as:

\[
(4.13) \quad \frac{d\pi^*}{dy_1} = 0 - \frac{\partial \pi^*_1}{\partial y_1} + \delta \frac{dc_2}{dy_1} \left[ \frac{\partial \pi^*_2}{\partial c_2} + \frac{\partial \pi^*_2}{\partial x_2} \frac{\partial x_2}{\partial c_2} \right] 1 - \frac{\partial s^*}{\partial c_2} + \frac{\partial \pi^*_2}{\partial x_2} \frac{\partial x_2}{\partial c_2} \frac{\partial s}{\partial c_2} \right]
\]

Decomposing the RHS of (4.13), it is clearly the case that:

\[
(4.14) \quad \partial \pi^*_1/\partial y_1 + \delta (dc^*_2/\partial y_1) (\partial \pi^*_2/\partial c^*_2) = a - b(x_1 + 2y_1) - c + \delta \lambda y_2,
\]

which is the non-strategic terms combined, takes the same form in this model as it does in the basic model of section 2.
The direct strategic effect of foreign first-period output on home second-period output is \((\partial x_2/\partial c_2^*)(\partial c_2^*/\partial y_1) = -\lambda/3b\) as in the basic model. First-period foreign output also affects second-period home sales via its effect on the home export subsidy. This effect is captured by:

\[
(\partial c_2^*/\partial y_1)(\partial x_2/\partial s)(\partial s/\partial c_2) = -2\lambda(2 + 8h)/3b(5 + 8h) < 0,
\]

which is more negative than its counterpart in the basic model. The impact of first-period foreign output on second-period home output which operates through the foreign export subsidy or tax is given by:

\[
(\partial c_2^*/\partial y_1)(\partial x_2/\partial s^*)(\partial s^*/\partial c_2^*) = -\lambda(1 - 4h)/b(5 + 8h),
\]

which falls in \(h\). If \(h\) is greater than 1/4 the foreign government will use an export tax and a marginal increase in \(y_1\) will lead to an increase in this tax and through this to an increase in home second-period output. This will work against a high first-period output for the foreign firm. It is now straightforward to obtain an expression for the total impact of foreign first-period production on the second-period output of the home firm:

\[
(4.15) \quad \frac{dc_2^*}{dy_1} \left[ \frac{\partial x_2}{\partial c_2^*} \left( 1 - \frac{\partial s^*}{\partial c_2^*} \right) + \frac{\partial x_2}{\partial s} \frac{\partial s^*}{\partial c_2^*} \right] = -\lambda \left[ \frac{4(1 + h)}{b(5 + 8h)} \right]
\]

This continues to be negative for all \(h\), implying that the foreign firm has an incentive to use experience as a strategic variable against its domestic rival.

I now consider the direct impact of the foreign firm’s first-period output on its second-period profits via its effect on the export subsidy that it is receiving from its own government. Using the foreign profit function (2.5) and the expression for
the foreign export subsidy in (4.7), yields:

\[(4.16) \quad \delta (dc^*/dy_i)(\partial \pi^*/\partial s^*)(\partial s^*/\partial c^*_2) = 3\delta \lambda (1 - 4h)y_2/(5 + 8h).\]

This shows that a marginal increase in foreign first-period sales will have the beneficial effect of inducing a larger government transfer to the foreign firm if and only if more than three quarters of the equity of that firm is owned by residents of the foreign country.

The total strategic effect of first-period output for the foreign firm is captured by:

\[(4.17) \quad -\delta ((\partial \pi^*_2/\partial x_2)[4\lambda (1 + h)b(5 + 8h)] + (dc^*/dy_i)(\partial \pi^*_2/\partial s^*)(\partial s^*/\partial c^*_2))\]

\[= \delta \lambda y_2(7 - 8h)/(5 + 8h).\]

This strategic effect will be negative for \(h > 7/8\). Now use (4.14) and (4.17) in (4.16) and then substitute in for \(y_2\) from (4.8(ii)). This gives the following:

\[(4.18) \quad \alpha_{21}x_1 + \alpha_{22}y_1 = \beta^*(a - c),\]

where:

\[(4.19) \quad \beta^* = b + 24\delta \lambda (5 + 8h)^2,\]

\[\alpha_{21} = b^2 + 48\delta \lambda^2(5 + 8h)^2,\]

\[\alpha_{22} = 2b^2 - 72\delta \lambda^2(5 + 8h)^2.\]

Combining (4.11) and (4.18) in matrix form gives:

\[(4.20) \quad \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \beta(a - c) \\ \beta^*(a - c) \end{bmatrix}.\]

The determinant \(D = \alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}\) is positive and increasing in \(h\). From (4.20) it is possible to obtain explicit expressions for home and foreign first-period outputs:

\[(4.21) \quad \begin{align*}
(i) \quad x^*_1 &= (\beta\alpha_{22} - \beta^*\alpha_{12})(a - c)/D, \\
(ii) \quad y^*_1 &= (\beta^*\alpha_{11} - \beta\alpha_{21})(a - c)/D.
\end{align*}\]

It can be shown that first-period home output exceeds first-period foreign output.
for \( h > 0 \):

\[
(4.22) \quad x_i^\circ - y_i^\circ = (\beta(\alpha_{22} + \alpha_{21}) - \beta^*(\alpha_{11} + \alpha_{12}))(a - c)/D > 0.
\]

The determinant can be shown to be decreasing in \( \delta \) and \( \lambda \), while the numerator in (4.22) is increasing in these. Therefore, the difference \( x_i^\circ - y_i^\circ \) is an increasing function of learning by doing and the discount factor (see the Appendix for a proof of this). The difference between home and foreign output in the second period obtained from (4.8), is:

\[
(4.23) \quad x_2^\circ - y_2^\circ = (4h(a - c) + \lambda[(10 + 8h)x_i - (10 + 4h)y_i])/(5 + 8h).
\]

This is positive for \( h > 0 \), and increasing in \( \delta \) and \( \lambda \) (see the Appendix). The subsidy differential, obtained from (4.7) is:

\[
(4.24) \quad s^\circ - s^* = (4h(a - c) + \lambda[5x_i - (5 - 4h)y_i])/(5 + 8h).
\]

It can also be shown that this differential is increasing in the discount factor and the speed of learning.

**Proposition 4:** When international rent diffusion occurs (\( h > 0 \)), (i) home sales in the first and the second period and the home export subsidy all exceed their foreign counterparts. (ii) These differentials are all increasing in the discount factor \( \delta \) and the speed of learning \( \lambda \). For a proof see Appendix.

**5 International Diffusion of Experience**

So far I have assumed that all the cost advantages of learning by doing accrue to the firm itself. That is there is no diffusion of learning that would directly affect the cost functions of other firms. In this section I consider a model similar to that
in section 2 except that now there can be some diffusion of cost improvements brought about by learning by doing. In the terminology of Ethier (1979,1982) there are both national scale economies, internal to the individual firms, and international returns to scale, external to firms. As before these returns to scale are of a dynamic type, today's output affects future costs. I will restrict attention to the symmetric equilibrium case in order to keep the analysis manageable. The cost functions take the following form:

\[(5.1) \quad (i) \quad c_2 = c - \lambda (x_1 + \varepsilon y_1),\]

\[(ii) \quad c^*_2 = c - \lambda (\varepsilon x_1 + y_1),\]

\[(iii) \quad c_1 = c^*_1 = c,\]

\[\varepsilon \in [0,1],\]

where \(\varepsilon\) is a measure of learning diffusion. When \(\varepsilon\) is equal to zero there is no diffusion, as in section 2, and when \(\varepsilon\) is equal to unity there is full diffusion of experience so that cost functions are affected equally by own and rival outputs.

The use of (5.1) in place of (2.2) and (2.3) in (2.5) yields the following second-period output levels:

\[(5.2) \quad (i) \quad x^*_2 = \{a - c + \lambda [(2 - \varepsilon)x_1 + (2\varepsilon - 1)y_1] + 2s - s^*\}/3b,\]

\[(ii) \quad y^*_2 = \{a - c + \lambda [(2\varepsilon - 1)x_1 + (2 - \varepsilon)y_1] - s + 2s^*\}/3b.\]

The equilibrium subsidies can be obtained by using (5.2) instead of (2.8) in (2.10) in order to give:

\[(5.3) \quad (i) \quad dW/ds = \{a - c + \lambda [(2 - \varepsilon)x_1 + (2\varepsilon - 1)y_1] - 4s - s^*\}/9b = 0,\]

\[(ii) \quad dW^*/ds^* = \{a - c + \lambda [(2\varepsilon - 1)x_1 + (2 - \varepsilon)y_1] - s - 4s^*\}/9b = 0.\]
These two equations in the two unknowns $s$ and $s^*$ can then be solved simultaneously to give:

\begin{align*}
(5.4) \quad (i) \quad s^o &= (a - c + \lambda((3 - 2\varepsilon)x_1 + (3\varepsilon - 2)y_1))/5, \\
(ii) \quad s^{*o} &= (a - c + \lambda((3\varepsilon - 2)x_1 + (3 - 2\varepsilon)y_1))/5.
\end{align*}

As before, an increase in the home firm's first-period output leads to an increase in the size of the home government's subsidy. This is because the higher level of home output reduces second-period costs of production so justifying a higher export subsidy to the home firm. However, an increase in home output can also lead to an increase in the foreign export subsidy if the diffusion of "learning by doing" is large enough. An interesting special case is full diffusion ($\varepsilon = 1$), when:

\begin{equation}
(5.5) \quad \frac{\partial s}{\partial x_1} = \frac{\partial s^*}{\partial x_1} = \frac{\partial s}{\partial y_1} = \frac{\partial s^*}{\partial y_1} = \frac{\lambda}{5} > 0.
\end{equation}

**Lemma 1:** When there is diffusion of experience, ($\varepsilon > 0$), an increase in the home firm's first-period output leads to an increase in the other government's subsidy iff $\varepsilon > 2/3$.

The diffusion weakens the strategic effects of first-period output on both subsidies as can be clearly seen from the following two comparative static derivatives:

\begin{align*}
(5.6) \quad \frac{\partial s^o}{\partial x_1} &= \lambda(3 - 2\varepsilon)/5, \\
\frac{\partial s^{*o}}{\partial x_1} &= -\lambda(2 - 3\varepsilon)/5,
\end{align*}

To obtain an expression for the strategic effect of first-period output on second-period outputs substitute (5.4) into (5.2) giving:

\begin{align*}
(5.7) \quad (i) \quad x_2'(x_1, y_1) &= 2(a - c + \lambda((3 - 2\varepsilon)x_1 + (3\varepsilon - 2)y_1))/5b, \\
(ii) \quad y_2'(x_1, y_1) &= 2(a - c + \lambda((3\varepsilon - 2)x_1 + (3 - 2\varepsilon)y_1))/5b.
\end{align*}
Now this yields the following intertemporal strategic effects:

\[(5.8) \quad \frac{\partial y_2(x_1, y_1)}{\partial x_1} = \frac{\partial x_2(x_1, y_1)}{\partial y_1} = \frac{2 \lambda (3 \epsilon - 2)}{5b}.
\]

**Lemma 2:** When there is diffusion of experience, \((\epsilon > 0)\), an increase in a firm's first-period output leads to an increase in the other firm's second-period output iff \(\epsilon > 2/3\).

I turn now to the first stage of the game in which firms choose their first-period output levels, aware of how these will affect subsidies and second period output. This maximisation problem is described in (2.14). The first-order condition given in (2.15) must now be modified to take into account the possible diffusion of experience. If there are learning spillovers \((\epsilon > 0)\) a marginal increase in home first-period output has an additional effect on home second-period profit via the induced fall in \(c_2^*\). This effect is captured by:

\[(5.9) \quad \delta \frac{dc_2^*}{dx_1} \left[ \frac{\partial \pi_2}{\partial y_2} \left( \frac{\partial y_2}{\partial c_2^*} + \frac{\partial y_2}{\partial s} \frac{\partial s}{\partial c_2^*} + \frac{\partial y_2}{\partial c_2^*} \frac{\partial c_2^*}{\partial c_2^*} \right) + \frac{\partial \pi_2}{\partial s} \frac{\partial s}{\partial c_2^*} \right] = - \frac{8\epsilon}{5} \delta \lambda x_2^* < 0.
\]

An increase in \(x_1\) causes \(c_2^*\) to fall bringing about an increase in \(y_2\) and thus a fall in \(\pi_2\). This effect is captured by the term in \(\partial \pi_2/\partial y_2\) in (5.9). The fall in \(c_2^*\) worsens the relative cost competitiveness of the home firm and therefore reduces the optimal home export subsidy and home second-period profits. This is represented by the term in \(\partial \pi_2/\partial s\) in (5.9)\(^{11}\). The total effect of \(x_1\) on home profits is obtained by adding the term in \(dc_2^*/dx_1\), given in (5.9) to the other six terms in (2.15). When

\(^{11}\) If \(\epsilon > 7/8\), the total strategic effect will take on the opposite sign to its counterpart in the basic model of section 2. This is clear when (5.9) is combined with the strategic terms (iii) to (vi) in (2.15).
this is done it can be seen that even if learning spillovers occur, firms will still produce more in the first period than in a simple one-period Cournot duopoly. This much is clear from the modified first-order condition:

\[
(5.10) \quad a - b(2x_i + y_i) - c + 48\lambda x_2(3 - 2\varepsilon)/5 = 0.
\]

It is possible to eliminate the endogenous \(x_2^o\) from this using (5.7) to give:

\[
(5.11) \quad a - b(2x_i + y_i) - c + \frac{8\delta\lambda(3 - 2\varepsilon)(a - c)}{25b} = 0.
\]

From this it is possible to obtain the symmetric first-period output levels:

\[
(5.12) \quad x_1^o = y_1^o = \frac{(25b + 8\delta\lambda(3 - 2\varepsilon))(a - c)}{75b^2 - 8\delta\lambda^2(3 - 2\varepsilon)(1 + \varepsilon)}
\]

As before this is clearly increasing in \(\delta\) and \(\lambda\). The use of (5.12) in (5.7) now yields the following expression for second-period outputs:

\[
(5.13) \quad x_2^o = y_2^o = \frac{10[3b + \lambda(1 + \varepsilon)](a - c)}{75b^2 - 8\delta\lambda^2(3 - 2\varepsilon)(1 + \varepsilon)}
\]

In the special case of full diffusion, first and second-period outputs are:

\[
(5.14) \quad \begin{align*}
(i) \quad x_1^o &= y_1^o = \frac{(25b + 8\delta\lambda)(a - c)}{75b^2 - 16\delta\lambda^2} \\
(ii) \quad x_2^o &= y_2^o = \frac{10(3b + 2\lambda)(a - c)}{75b^2 - 16\delta\lambda^2}
\end{align*}
\]

A comparison of (5.14) with (2.19) and (2.20) yields the following result:

**Proposition 5:** In a symmetric equilibrium, for given \(\delta\) and \(\lambda\) (i) first-period output is higher when there is no diffusion (\(\varepsilon = 0\)) than when there is full diffusion (\(\varepsilon = 1\)), whereas (ii) second-period output is higher under full diffusion than under no diffusion.
Diffusion weakens the strategic effect of learning and so leads to a lower level of first-period output being produced. In the second-period more diffusion tends to reduce costs directly by spreading the benefits of experience. However, since it tends to reduce first-period output there will be less experience available. It is possible to show that a small increase in diffusion, starting form $\varepsilon = 0$, leads to an increase in second-period outputs.

6. Concluding Remarks

In this paper I have examined some of the ways in which the introduction of "learning by doing" and the intertemporal market linkage that this entails, affects the strategic interactions of governments and firms in an export subsidy game. In particular it is shown that firms use first-period outputs to play strategically, not just against other firms, as has already been shown in the Industrial Organisation literature, but now also against governments. Lower own firm marginal costs raises the government's optimal export subsidy by strengthening the rent-shifting impact of that subsidy. This means that firms have an incentive to produce a lot early, acquire experience, and reduce costs in order to be more worthy of subsidisation.

I have shown here that the incentive faced by a firm to produce a lot in the first period for strategic purposes is weakened if the third-country government introduces a tariff. It is also weakened if the firm cannot fully appropriate the benefits of experience.
In a situation in which a firm is partly foreign owned its government may find it optimal to use an export tax rather than a subsidy. Under these circumstances I have demonstrated that the tax is increasing in own first-period output. This will discourage a firm from producing a lot early.

Overall, it is clear that allowing for intertemporal market linkages even in the relatively simple setting of a two-period model adds considerably to the richness of strategic interaction between firms and governments.

**Appendix  Proof of Proposition 4**

I first show that $x_1^\circ - y_1^\circ$, $x_2^\circ - y_1^\circ$ and $s^\circ - s^{**}$ are all positive and then I show that they are all increasing in $\delta$ and $\lambda$. From (4.15) it is possible to obtain:

$$\alpha_{11} + \alpha_{12} = 3b^2 - \rho_1 \delta \lambda^2,$$

where $\rho_1 = 8[(3 + 4h)(1 + 2h)]/(5 + 8h)^2$, and from (4.24) it is similarly possible to obtain:

$$\alpha_{21} + \alpha_{22} = 3b^2 - \rho_2 \delta \lambda^2,$$

where $\rho_2 = 24/(5 + 8h)^2$. It is also possible to write

$$\beta = b + \rho_1 \delta \lambda,$$

$$\beta^* = b + \rho_2 \delta \lambda.$$  

Then use (A.1), (A.2) and (A.3) in (4.22) to get:

$$x_1^\circ - y_1^\circ = \frac{16h(5 + 4h)b \delta \lambda(3b + \lambda)(a - c)}{(5 + 8h)^2D} > 0$$

It is now obvious that $x_2^\circ - y_2^\circ$ and $s^\circ - s^{**}$ are both positive.

I will now show that all these differentials are increasing in $\delta$ and $\lambda$. Differentiation of the determinant $D(\phi)$ yields:
\[ \frac{dD(\phi)}{d\phi} = \alpha_{11}(\phi)\frac{d\alpha_{22}(\phi)}{d\phi} + \alpha_{22}(\phi)\frac{d\alpha_{11}(\phi)}{d\phi} \]

\[-\alpha_{12}(\phi)\frac{d\alpha_{21}(\phi)}{d\phi} - \alpha_{21}(\phi)\frac{d\alpha_{12}(\phi)}{d\phi} < 0, \]

where $\phi \equiv \delta \lambda^2$.

From (4.12) and (4.19) it is apparent that all the terms on the right-hand side of (A.5) are negative, proving that:

(A.6)  
(i) $\partial x_1/\partial \lambda - \partial y_1/\partial \lambda > 0,$  
(ii) $\partial x_1/\partial \delta - \partial y_1/\partial \delta > 0.$

It is then straightforward to show that the differentials $s^0 - s^\circ,$ and $x^0_2 - y^0_2$ are also increasing in $\delta$ and $\lambda.$
REFERENCES


