A (Weak) Exogeneity Test of the German Leadership Hypothesis

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1. Introduction.

If a fixed exchange rate system has \( n \) participating currencies there can only be \( n-1 \) exchange rates and once monetary policy is determined in one currency it is automatically determined elsewhere in the system. Hence there can be only one independent monetary authority which may be a system-wide institution setting a common interest rate target for all currencies or the central bank of a single country. In either case the monetary authority may pursue an independent policy or accommodate preferences in other economies. In the context of the European Monetary System (EMS) the German Leadership Hypothesis (GLH) assumes that this single degree of freedom is taken by the Bundesbank and that other countries pursue monetary policies consistent with a DM peg. In its simplest form the hypothesis views the Bundesbank as setting policy independently of pressures elsewhere in the system so that credible maintenance of parity exchange rates against the DM must eliminate monetary independence in other participating economies. Alternatively it is possible that the Bundesbank may set policy to at least partly accommodate either inflation differentials with partner countries or movements in EMS exchange rates against non-participating currencies such as the U.S dollar, in which case we would expect to observe co-movements between German and other interest rates with possible feedback from the latter to the former.

To illustrate, suppose that a shock to, say, the Italian inflation rate leads to a loss of competitiveness against Germany. To the extent that the shock stimulates expectations of a Lira devaluation subsequent capital flows will contract (expand) the Italian (German) monetary base and increase the interest rate differential. The German authorities may accommodate the shock by permitting an increase in the German money supply and a
reduction in interest rates (symmetric adjustment), or act independently by sterilising inflows so that the adjustment pressure is taken by Italian interest rates alone (asymmetric adjustment). In either case a credible commitment to the DM parity should imply that the inflation differential will be eroded by some combination of German monetary expansion and/or Italian monetary contraction leading to a re-convergence of interest rate levels.

Recent studies have, however, produced conflicting evidence on the role played by German interest rates in the Exchange Rate Mechanism (ERM) and the symmetry of the adjustment process. Karfakis and Moschos (1990), using monthly data on short-term money market rates for Belgium, France, Germany, Ireland, Italy and the Netherlands over 1979:04 to 1988:11 report that: (a) all interest rate series are integrated of order one¹, (b) bivariate tests between the German rate and other ERM rates fail to detect evidence of cointegration suggesting that these series do not move together in the long-run, (c) bivariate vector autoregressions for first differenced series indicate uni-directional Granger causality running from German to other ERM interest rates. On the basis of this evidence Karfakis and Moschos conclude that movements in relative interest rates "may be attributed to the non-stationarity of either the expected exchange rate movements or the risk premia" and that Granger causation tests "highlight the dominant role of Germany in the EMS." (p. 393). That is, they find evidence of asymmetric adjustment. However it should be noted that in the absence of cointegration between integrated series Granger causation in first differences has no implication for long-run relationships between interest rate levels. All that Karfakis and Moschos establish is that changes in the German interest rate precede changes in other interest rates. Failure to detect cointegration, on the other hand, implies that long-run forecasts for

¹A time series is said to be integrated of order d if it is non-stationary in levels but is stationary after differencing d times. Hence a series is integrated of order one, denoted I(1), if it is stationary in first differences.
other EMS interest rates are independent of the current German rate.

Katsimbris and Miller (1993) using the same sample as Karfakis and Moschos confirm (a) and (b) but challenge (c) on the grounds that "bilateral cointegration tests and Granger causation tests may be subject to spurious findings due to the omission of an important third variable" (p. 772) - the U.S. interest rate. Their results suggest: (a) there is evidence of bilateral cointegration between ERM interest rates (including Germany) and the corresponding U.S. rate, (b) two-way Granger causality between German and other ERM rates when the vector autoregression is augmented with the U.S. rate and estimated as a trivariate error correction model. Consequently Katsimbris and Miller reject the hypothesis of asymmetric adjustment and conclude that "monetary policies within the EMS respond to each other, as well as to impulses from the rest of the world." (p. 779) While similar conclusions are reached by de Grauwe (1989) and Fratianni and von Hagan (1990), Biltoft and Boersch (1992), using daily data over 1987 to 1991, report unidirectional Granger causality between German rates and those in Belgium, Denmark, France and the Netherlands.

This paper proposes an alternative test of the German Leadership Hypothesis using the concept of weak exogeneity as defined in Engle, Hendry and Richard (1983) and the econometric methodology described in Johansen (1991). Following Engle et. al. a variable is said to be weakly exogenous for estimating a set of parameters if inference conditional on the variable involves no loss of information. For example, in OLS estimates of error correction models valid inference on the long-run parameters requires that conditioning

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2This paper does not challenge either the Karfakis and Moschos or Katsimbris and Miller bivariate results given their sample period. However in the Appendix it is shown that Katsimbris and Miller's error correction model is an inappropriate parameterization for valid Granger-causation tests.

3See also Johansen (1988, 1991) and Johansen and Juselius (1990, 1992).
variables, such as the German interest rate, be weakly exogenous with respect to the adjustment, or feedback, parameters. When this is not the case then the conditioning variables are error-correcting in that they respond to deviations from long-run equilibrium relationships which can be taken as evidence of feedback, or symmetric adjustment, between the variables of interest. These ideas are discussed in section 2 which also outlines the Johansen procedure for investigating cointegrating relationships. Results are given in section 3.

2. Econometric Methodology.

A conventional procedure for testing the GLH is to first establish cointegration between the variables of interest and then test for Granger-causation in the context of either a vector autoregression (VAR) framework or an error correction model (ECM). Typically a VAR model conditions the level of each series on lagged levels of other variables in the system while the ECM conditions changes in each series on an error correction term which models adjustments to previous divergences from equilibrium. However it is important to note that the VAR and ECM are alternative parameterizations of the same dynamic model so that inferences made on one must hold for the other. Further, if all variables in the system are error-correcting in that observed changes in their levels cannot be adequately explained without reference to error correction terms then we may not validly conduct inference from a ECM (or a VAR equation) for a single variable which is conditioned on other variables in the system. For example in a simple bivariate system there can be only one cointegrating vector and one error correction term. If this term is required in the ECM for each variable then we cannot make inferences on one without reference to the other. Alternatively, if the error correction term enters the conditional ECM for the first variable but not the second then valid inference may be made on the parameters of the former. When this is the case the
second variable is, in the sense of Engle et. al. (1983), weakly exogenous with respect to the long-run parameters.

In the context of the GLH the relevance of this discussion is as follows. If, for example, the German interest rate is not weakly exogenous then inferences such as Granger causation tests on single VAR or conditional ECM equations for equivalent rates in other countries become questionable. Further, a test for weak exogeneity may be regarded as a test for feedback in that rejection of an exogeneity hypothesis implies that the German rate responds to deviations in its long-run relationship with other interest rates. Hence it is important to estimate both equations and test the appropriate restrictions. More formally consider the VAR model:

\[ X_t = \mu + \sum_{i=1}^{p} A_i X_{t-i} + \epsilon_t \]  

(1)

where \( X_t \) is an \( nx1 \) vector of \( I(1) \) time series, \( \epsilon_t \) is a vector of \( I(0) \) error processes, \( A_i \) are coefficient matrices and \( \mu \) is a vector or constants. Johansen starts with an error correction representation of (1) given by:

\[ \Delta X_t = \mu + \sum_{i=1}^{p-1} \Gamma_i \Delta X_{t-i} + \Gamma X_{t-p} + \epsilon_t \]  

(2)

where \( \Gamma_i = -(I + A_1 + \ldots + A_i) \), \( \Gamma = -(I - A_1 \ldots - A_p) \) and \( I \) is an unit matrix.\(^4\) Johansen shows that the number of independent cointegrating vectors is given by the rank of the matrix \( \Gamma \), denoted \( r \). The Johansen procedure estimates (2) subject to the hypothesis that \( \Gamma \) has reduced rank \( r < n \). Or there exists a representation \( \Gamma = \alpha B' \) where \( B \) is an \( nxr \) matrix of cointegrating vectors and \( \alpha \) is an adjustment or feedback matrix determining the response of

\(^4\)The ECM (2) is derived by adding \( X_{t-i} \) and \( A_{j-1}X_{t-j} \) to both sides of (1) where \( i = 1 \ldots k \) and \( j = 2 \ldots k \).
each element in $\Delta X_t$ to the error correction term. If all elements in $X_t$ are I(1) then the $r$ columns of $\beta$ are cointegrating vectors such that $\beta' X_{t-k}$ is I(0). But as the number of cointegrating vectors cannot exceed $(n - 1)$, $r$ must be less than $n$, the number of variables. For example, if $X_t = (x_{1t}, x_{2t})'$ then cointegration implies that the rank of $\Gamma$ equals one so that the null of no cointegration requires a test of $H(\tau = 0)$ against $H(\tau = 1)$. Note also that by the Granger representation theorem, if the variables in $X_t$ cointegrate the error correction form (2) must exist so that at least one of the $\alpha$’s will be non-zero.

Johansen derives the following likelihood ratio statistics for testing the hypothesis that there are at most $r$ cointegrating vectors. A trace test:

$$-2\ln Q[H(r) : H(n)] = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \quad (3)$$

and a maximal eigenvalue test:

$$-2\ln Q[H(r-1) : H(r)] = -T \ln(1 - \hat{\lambda}_r) \quad (4)$$

where $Q$ is the ratio of the restricted to unrestricted maximised likelihoods and $\hat{\lambda}_1 > \ldots > \hat{\lambda}_n$ are eigenvalues from residual product moment matrices of the underlying VAR(p) system. Critical values for (3) and (4) are tabulated by simulation in Osterwald-Lenum (1992).

To illustrate the relevance of this approach for tests of the GLH let $X_t$ be a 2x1 vector with $x_{1t}$ denoting an interest rate in an ERM currency and $x_{2t}$ the corresponding German interest rate. If $x_1$ and $x_2$ are cointegrated there can be only one cointegrating vector denoted $\beta_1 = (\beta_{10} \, \beta_{11} \, \beta_{12})$, where $\beta_{10}$ is an intercept term, and an adjustment vector $\alpha_1' = (\alpha_{11} \, \alpha_{12})$ such that $\Gamma = \alpha \beta'$. For $p = 2$ the ECM for each interest rate is:

$$\Delta x_{1t} = \sum_{i=1}^{2} b_{1i} \Delta x_{1t-i} + \alpha_{11}(\beta_{10} + \beta_{11}x_{1t-2} + \beta_{12}x_{2t-2}) + e_{1t} \quad (5)$$
\[ \Delta x_{2t} = \sum_{i=1}^{2} b_{2i} \Delta x_{it-1} + \alpha_{12}(\beta_{10} + \beta_{11} x_{1t-2} + \beta_{12} x_{2t-2}) + e_{2t} \] (6)

where \( b_{ij} \) are combinations of the elements of the coefficient matrices \( A_i \). A crucial feature of (5) and (6) is that the error correction term appears in both equations with the implication that the parameters of the former are not independent of those in latter. However, if \( \alpha_{12} = 0 \) then the \( x_{2t} \) is not error-correcting and we may validly conduct inference from the estimation of (5) alone. If this is the case \( x_{2t} \) is said to be weakly exogenous with respect to the long-run parameters of the conditional ECM for \( x_{1t} \). Further, as the \( \alpha \)'s may be interpreted as feedback parameters in the sense that they estimate the response of each \( x_{it} \) to divergences from equilibrium they have direct bearing on the concepts of symmetric and asymmetric adjustment between the variables. For example if \( \alpha_{12} = 0 \) then adjustment to an equilibrium error is asymmetric and taken by \( x_{1t} \) only. Alternatively if \( \alpha_{11} \) and \( \alpha_{12} \) are non-zero then both interest rates are error-correcting and respond symmetrically to a disequilibrium. Johansen (1991) shows that an appropriate likelihood ratio test statistic for \( s \) homogeneous restrictions on \( \Gamma = \alpha \beta' \) is given by:

\[ -\ln Q_s = -T \sum_{j=1}^{r} \ln \left( \frac{1 - \hat{\lambda}_j^*}{1 - \hat{\lambda}_j} \right) \] (7)

Where \( * \) denotes eigenvalues from the restricted system. The test statistic (7) is asymptotically distributed as chi-square with degrees of freedom equal to the number of restrictions.

Further, in the context of testing for cointegration between interest rates the Johansen procedure has an advantage over Dickey-Fuller methods in that it permits valid inference on the cointegrating vector. For example Karfakis and Moschos and Katsimbris and Miller each use the long-run uncovered interest rate parity (UIP) condition to test for cointegration. That is:

7
\[ x_t = \beta_1 + \beta_2 x_{i,t} + u_t \]  \hspace{1cm} (8)

where \( x_i \) and \( x_j \) are interest rates in currencies i and j and the \( \beta \)'s are potential cointegrating parameters normalized on \( x_{i,t} \). While strict UIP requires that \( u_t \) is stationary with a \((0, -1)\) cointegrating vector, Karfakis and Moschos suggest that differential tax rates and/or measurement errors may permit a cointegrating vector which diverges from these values. Hence they estimate (8) for EMS interest rates with the German rate as the base \((x_j)\) but fail to reject the null of no cointegration in all cases. These results are replicated by Katsimbris and Miller who also test the raw interest rate differentials for unit roots. With the possibly significant exception of the Netherlands, they find little evidence that these differentials are stationary. In other words they fail to detect cointegration between German and other EMS interest rates regardless of whether \( \beta_1 \) and \( \beta_2 \) are freely estimated or restricted to \((0, -1)\). However it is important to note that in bilateral cointegration tests, the cointegrating vector will, if it exists, be unique and found by simple least squares regression. Hence Dickey-Fuller tests on interest rate differentials, which effectively restrict \((\beta_1, \beta_2)\) to \((0, -1)\), cannot be considered as an alternative to similar tests on the residuals from (8). Also, under the null of no cointegration the parameters do not follow any standard distribution which implies that valid inferences cannot be drawn from conventional procedures such as F-tests. The correct procedure is to estimate the cointegrating vector by Johansen's method and test the appropriate homogeneity restrictions using (7).

3. Results.

The concepts of German leadership and symmetry in adjustment are testing using the following strategy:

(a) Test that all interest rate series are I(1).
(b) Given (a) test for bilateral cointegration between German and other ERM rates.

(c) Conditional on cointegration, test the hypotheses \( H_1: \alpha = A\delta \) and \( H_2: \beta_1 = B\theta \) where \( \beta_1 = (\beta_{10} \beta_{11} \beta_{12}) \) is a cointegrating vector for \((1 \times_1 \times_2)\). For example if \( x_2 \) is the German interest rate then a test for weak exogeneity implies that \( A' = (1 \ 0) \) and \( \delta \) is a scaler. Likewise a test for strict uncovered interest rate parity implies that \( B = (0 \ -1 \ 1) \).

Both Karfakis and Moschos and Katsimbris and Miller perform their cointegration and causality tests using data from 1979:4 to 1988:11. However it is reasonably clear that over these years the EMS operated as a crawling peg rather than a disciplined fixed exchange rate system. For example, there were eleven currency realignments between September 1979 and August 1986. The French franc, the Belgian franc and the Irish pound all depreciated by thirty to forty per cent against the DM while the Lira depreciated by approximately fifty per cent. The Dutch guilder, on the other hand, depreciated by less that five per cent leading some authors, such as Weber (1991), to suggest the possibility of a bi-polar EMS with the DM and guilder forming a hard currency zone and the others a soft currency zone. Given this apparent instability in EMS exchange rates it is hardly surprising that these studies fail to detect long-run cointegrating relationships in the data. Indeed as Katsimbris and Miller suggest, "The finding of non-stationary interest rate differentials with Germany as the base may be spurious, emanating from eleven currency realignments within our sample period. That is, currency realignments cause one-shot structural adjustments in interest rate differentials." (1993, p.775)

Given that the early years are characterised by possible instabilities the GLH becomes something of a "straw-man". That is, if there is little evidence to suggest that the EMS operated as a stable and disciplined exchange rate system there is little point in attempting to
Table 1. Stationarity Tests.

<table>
<thead>
<tr>
<th></th>
<th>Levels</th>
<th>First Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DF</td>
<td>ADF(4)</td>
</tr>
<tr>
<td>Belgium</td>
<td>-1.21</td>
<td>-1.41</td>
</tr>
<tr>
<td>France</td>
<td>-1.04</td>
<td>-1.42</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.20</td>
<td>-0.05</td>
</tr>
<tr>
<td>Ireland</td>
<td>-2.15</td>
<td>-2.25</td>
</tr>
<tr>
<td>Italy</td>
<td>-2.36</td>
<td>-2.51</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.52</td>
<td>-0.29</td>
</tr>
</tbody>
</table>

Note: Critical value for DF and ADF(4) = 2.89 (95%) from MacKinnon (1990).

Table 2. Johansen Bilateral Cointegrating Tests Against Germany.

<table>
<thead>
<tr>
<th>Country</th>
<th>Cointegration Test Statistics</th>
<th>Cointegrating Parameters</th>
<th>Adjustment Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace</td>
<td>Max.</td>
<td>$\beta_0$</td>
</tr>
<tr>
<td>Belgium</td>
<td>29.73</td>
<td>33.68</td>
<td>4.675</td>
</tr>
<tr>
<td>France</td>
<td>12.70</td>
<td>16.01</td>
<td>6.119</td>
</tr>
<tr>
<td>Ireland</td>
<td>18.35</td>
<td>23.43</td>
<td>8.722</td>
</tr>
<tr>
<td>Italy</td>
<td>31.42</td>
<td>35.31</td>
<td>10.467</td>
</tr>
<tr>
<td>Netherlands</td>
<td>18.00</td>
<td>21.39</td>
<td>1.849</td>
</tr>
</tbody>
</table>

Notes: Trace is Johansen's trace test with critical values 17.852(90%) and 19.969(95%). Max. is Johansen's maximal eigenvalue test with critical values 13.752(90%) and 15.762(95%) as tabulated by Osterwald-Lenum(1992). The cointegrating and adjustment parameters are normalised on $x_{it} = \beta_0 + \beta_1 x_{it}$.

identify a system leader. Consequently the tests reported in this paper use monthly data for the later EMS period 1986:1 to 1992:6 which is characterised by greater stability and convergence in both exchange rates and interest rates.\(^5\) Table 1 reports the results of Dickey-

\(^5\)The sample is terminated in June 1992 just prior to the "currency crisis" of late 1992/early 1993.
Table 3. Parameter Restriction Tests

<table>
<thead>
<tr>
<th>Country</th>
<th>Cointegration Parameters</th>
<th>Adjustment Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_1 = 0$</td>
<td>$\beta_1 = 1$</td>
</tr>
<tr>
<td>Belgium</td>
<td>19.307</td>
<td>19.252</td>
</tr>
<tr>
<td>France</td>
<td>8.504</td>
<td>8.415</td>
</tr>
<tr>
<td>Italy</td>
<td>23.727</td>
<td>17.812</td>
</tr>
</tbody>
</table>

Notes: Test statistics are distributed as chi-squared. Numbers in parentheses are marginal significance levels.

Fuller and Augmented Dickey-Fuller tests on the levels and first differences of short-term money market in six countries - Belgium, France, Germany, Ireland, Italy and the Netherlands. In all cases these tests fail to reject the null of non-stationarity in levels, but the hypothesis is clearly rejected for first differences suggesting that all the series are I(1). That is, non-stationarity in levels but stationary in first differences.

Given that all series are I(1) Table 2 reports Johansen bilateral cointegration tests against Germany. As the lag structure must be the same for each equation in the VAR. Hence $p$ was selected using a log likelihood ratio test. This gave $p = 3$ for Belgium/Germany and $p = 2$ for the other pairings. With the exception of France both the Johansen trace and maximal eigenvalue tests accept cointegration, at the 95% significance level, with the German rate. In the case of France the hypothesis of no cointegrating vector is accepted on the trace.

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6These are the same six countries considered by Karfakis and Moschos and Katsimbris and Miller. All series are taken from the OECD Main Economic Indicators Data Base supplied with the RATS time series package. The series are: Belgium (TRATE), France (PIBOR), Germany (FIBOR), Ireland (TBILL), Italy (IBOR), Netherlands (AIBOR).
test but rejected on the maximal eigenvalue test. Hence for all five interest rates there is some evidence of cointegration with the German rate. The middle columns of Table 2 give the estimated cointegrating parameters normalised on $x_{it}$ while the final columns give the adjustment parameters. In each case the estimated adjustment parameter for the German rate ($\alpha_2$) is smaller than the corresponding parameter for the partner country indicating that most of the adjustment is taken by the latter.

Table 3 reports restriction tests on the cointegrating and adjustment parameters. In all cases the data rejects strict UIP implying that although the series are cointegrated the raw interest differentials are non-stationary. The last two columns of Table 3 report test statistics for the hypothesis that the adjustment parameters are zero. For Belgium, France and Italy these tests reject the hypothesis that $\alpha_1 = 0$ but accept that $\alpha_2 = 0$. Hence in these cases the German rate may be assumed weakly exogenous implying that all of the adjustment is taken by the corresponding rate and that the process is asymmetric. For Ireland and the Netherlands, on the other hand, both parameters are significantly different from zero suggesting that the adjustment process is symmetric and characterised by feedback. While this result may be expected for the Netherlands, which has consistently followed a strict DM peg, it is somewhat surprising for Ireland as there is no apparent reason as to why the Bundesbank should respond to a disequilibrium with the Irish interest rate as opposed to, say, the French interest rate.

4. Summary and Conclusions.

Tests for the influence of German interest rates in the EMS are conventionally based on the concept of Granger-causation in a single equation model. In some cases VAR methodology is employed but in the absence of cross-equation restrictions this is equivalent
to hypothesis testing on a series of single and independent equations for each variable in the system. In this type of test it is be important to first check that the conditioning variables in each equation (typically the German interest rate) are weakly exogenous with respect to the long-run parameters before inference is made on the estimates. If weak exogeneity cannot be supported then inference cannot be drawn from one equation without reference to the others and appropriate cross-equation restrictions may be required. Further, in the case of bivariate interest rate models the concept of weak exogeneity can be shown to be directly linked to the symmetry of the adjustment process. Specifically if, in a given equation, a conditioning variable can be assumed weakly exogenous then the adjustment process may be classified as asymmetric. In this sense tests of zero restrictions on the adjustment parameters of a bivariate error correction model can be viewed as a test for feedback between German and other ERM interest rates.

Using Johansen techniques the empirical analysis concluded that the German rate is weakly exogenous with respect to models for Belgian, French and Italian rates implying asymmetric adjustment in these cases. The hypothesis was rejected for Ireland and the Netherlands suggesting symmetric adjustment or feedback. While there is no obvious explanation for the Irish result symmetric adjustment with the Dutch interest rate is not too surprising given close integration with the German economy and credibility of the DM peg. While no firm conclusion can be reached the evidence suggests asymmetric adjustment with the weaker EMS currencies and symmetry with stronger currencies.
Literature Cited.

Biltoft, Karsten and C. Boersch, "Interest rate Causality and Asymmetry in the EMS", *Open Economies Review* 3 (1992), 297-306


Appendix: Comment on Katsimbris and Miller (1993)

Katsimbris and Miller base their Granger causality tests on an error correction model (ECM) specified as:

\[
\Delta x_{it} = \mu + \sum_{j=1}^{3} \delta_{ji} \Delta x_{j,t-1} + \sum_{i=1}^{2} \alpha_{i} (x_{it-1} - x_{3t-1}) + e_t
\] (A1)

Where, for example, \(x_1, x_2\) and \(x_3\) are the French, German and U.S. interest rates. Based on prior Dickey-Fuller and cointegration tests each series, in levels, is assumed to be integrated of order one but the EMS interest rates \(x_1\) and \(x_2\) are found to be cointegrated with the U.S. rate. Hence all terms in (A1) are assumed stationary. Katsimbris and Miller then conduct Granger-causation tests that \(x_j (j = 2,3)\) does not Granger-cause \(x_1\) using the null hypothesis that the parameters \(\delta_{ji}\) are jointly zero.

However the ECM (A1) is simply a re-parameterization of the generalized autoregressive model (or vice-versa):

\[
x_{it} = \mu + \sum_{j=1}^{3} \beta_{ji} x_{j,t-1} + e_t
\] (A2)

Using (A2) the test that \(x_j\) does not Granger-cause \(x_1\) is conventionally based on the hypothesis that the parameters \(\beta_{ji}\) are jointly zero. Using \(x_{t+i-1} = x_{t-1} - \Delta x_{t-1}\) (A2) can be re-parameterized to the equivalent error correction form:

\[
\Delta x_{it} = \mu + \sum_{j=1}^{k} \sum_{i=1}^{n_j} \delta_{ji} \Delta x_{j,t-1} + \sum_{i=1}^{k} \delta_{ji} x_{i,t-1} + e_t
\] (A3)

Where:

\[
\delta_i = \sum_{i=1}^{n_i} \beta_{ji} ; \quad \delta_{ji} = \sum_{i=1}^{n_j} \beta_{ji} ; \quad \delta_{ji} = \sum_{p=1}^{n_p} \beta_{ji}
\] (A4)

\(i = 1...n_j-1 ; \quad j = 1...k\)
Note that (A2) and (A3) are exactly the same model. They simply package the information in different ways. Also (A3) is equivalent to (A1), the model used by Katsimbris and Miller with the restrictions $\alpha_1 = (\delta_1 - \delta_3)$ and $\alpha_2 = (\delta_2 - \delta_3)$. However the parameters $\delta_{ji}$ cannot be used to test for Granger-causation as they exclude the parameter at lag one or $\beta_{j1}$. The general form of $\delta_{ji}$ is given by:

$$\delta_{ji} = -(\beta_{j2} + \ldots + \beta_{jm_j}) : \delta_{j2} = -(\beta_{j3} + \ldots + \beta_{jm_j}) \text{ etc.} \quad (A5)$$

For example with $n_j = 3$ for all variables:

$$\delta_{j1} = -(\beta_{j2} + \beta_{j3}) : \delta_{j2} = -\beta_{j3} \quad j = 1 \ldots 3 \quad (A6)$$

As these terms exclude $\beta_{j1}$ the coefficients on the $x_{jt-1}$ in (A2) the hypothesis that $\delta_{ji}$ are jointly zero for $i$ and $j = 1 \ldots 3$ (3 variables, 3 lags) is not equivalent to the hypothesis that $\beta_{ji}$ are jointly zero (A2). However it is the latter which is required for Granger-causality tests.