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How does Unemployment Affect Direct and Indirect Tax Reform?

by

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How does Unemployment Affect Marginal Direct and Indirect Tax Reform?

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Abstract: This paper incorporates the stylised fact of labour market rationing into an analysis of marginal tax reform in Ireland. In the absence of weak separability between goods and leisure, labour-market rationing will have both substitution and income effects. This paper estimates "matched pairs" of demands for Ireland and investigates the sensitivity of marginal tax reform recommendations to the presence of rationing, both with and without weak separability between goods and leisure.

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How does Unemployment Affect Marginal Direct and Indirect Tax Reform?[1]

1. Introduction

Both the calculation of optimal tax rates and the identification of marginal welfare-improving tax reforms are sensitive to demand responses. Optimally calculated tax rates are functions of household demand responses, and the calculation of marginal revenue costs (MRCs), the central parameter in marginal tax reform analysis, requires knowledge of aggregate demand responses. The empirical sensitivity of such measures to assumptions regarding preferences and the functional form specified for the underlying demand system is thus an important issue and has been examined for optimal tax rates by Ebrahimi and Heady (1988) and Ray (1986) and for marginal tax reform analysis by Decoster and Schokkaert (1990) and Madden (1993a and 1993b).

In all of the above papers it was assumed that agents were completely unrationed in their demands for goods and leisure. This assumption is unlikely to be valid for certain goods. For example, many agents may be rationed in their demand for leisure (supply of labour), while in developing countries goods such as food and shelter may be subject to quotas and/or rations. The formal analysis of consumer demand under rationing goes back to Tobin and Houthhakker (1950-51). More recently Neary and Roberts

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(1980) show how "matched pairs" of demands may be obtained (i.e. both with and without rationing), using Rothbart's (1940-41) concept of "virtual prices", i.e. those prices which would have induced the rationed quantity to be demanded in the absence of rationing.

The incorporation of rationing into the area of optimal taxation may proceed in two ways. Firstly we could regard the ration itself as an additional policy variable at the disposal of the government (see for example Guesnerie and Roberts (1984)). Secondly, we could examine how the existence of rationing in some goods affects the first-order conditions of the familiar single and many-person optimal tax models. In this paper we follow the second approach but in the context of marginal tax reform, rather than optimal tax design.

From the viewpoint of optimal taxation and tax reform, rationing takes on even greater importance when the rationed good is leisure (labour). In this case the often used assumption of weak separability (WS) between goods and leisure becomes crucial, since in the presence of WS rationing will have income effects only, and will not affect marginal substitution between commodities. Thus the presence or absence of WS which has important effects for optimal taxation theory in the unrationed case (see for example the papers by Atkinson and Stiglitz (1976), Deaton (1979, 1981a) and Deaton and Stern (1986)) takes on even greater importance when rationing is present in the labour market.

Rationing is incorporated into tax reform by Van de gaer, Schokkaert and De Bruyne (1991, 1992) who examine marginal tax

reform recommendations under different macroeconomic regimes, including the cases of both Keynesian and Classical unemployment. They estimate conditional goods demand functions, conditioned on the total level of employment.[2] Through their use of conditional demand functions they assess the effect of the ration on the demand for different goods. A similar approach is also adopted by Wibaut (1989) who estimates a general equilibrium model for Belgium and examines tax reform under different assumptions regarding the macroeconomic regime. Once again he uses conditional demand functions, conditioned on the level of unemployment.

Neither of these papers, however, allow for the simultaneous determination of labour supply generated from the same set of preferences. The contribution of this paper is to take account of rationing within a more general framework via the estimation of "matched pairs" of rationed and unrationed demands generated from the same set of preferences in a jointly determined commodity demand-labour supply model.[3] Thus while we remain within a partial equilibrium setting, there is a richer analysis of demand responses and hence of marginal tax reform and its sensitivity to assumptions regarding rationing. This approach enables us to present empirical evidence on the sensitivity of directions of tax reform to the incorporation of rationing in the context of rationed demands which are generated from the same preferences as unrationed demands. We can also explicitly test

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[2] For another example of this approach see Browning and Meghir (1991). For an example which also incorporates marginal tax reform, see Madden (1993d).

[3] For an early example of these type of models see Blundell and Walker (1982).

for separability and examine the sensitivity of tax reform recommendations to its imposition in the presence of rationing.

The layout of the paper is as follows: in section 2 we work through the model of tax reform in the presence of rationing. Section 3 addresses the issue of estimating matched pairs of rationed and unrationed demands, while section 4 briefly discusses the estimation results. In section 5 we present calculated measures of marginal tax reform for Ireland, while section 6 summarises and concludes.

2. The Model.

The model used here is an extension of that of Madden (1993b)[4] which incorporates the stylised fact that some agents may be totally rationed in the labour market and hence be forced to consume T hours of leisure, where T is the total time endowment. There are H households and we assume that a fraction σ (which we index over 1,..,r) are unemployed and consume T hours of leisure, while the remaining fraction $1-\sigma$ (which we index over r+1 to H) are employed and can freely choose leisure $x_0=T-l_0$, where l is labour supplied. Utility is defined over goods $x_1,..,x_n$ and leisure x_0. Thus we have

$$u^h = u(T, x_1^h, .., x_n^h), \quad h=1,..r \tag{1a}$$

$$u^h = u(x_0^h, x_1^h, .., x_n^h), \quad h=r+1,.., H \tag{1b}$$

[4] Marginal tax reform models of this type were introduced by Ahmad and Stern (1984) in the context of indirect taxes only.

"Full" expenditure per period is given by

$$M^h = w^h T + \sum_i q_i x_i^h, \quad h=1,\ldots,r \tag{2a}$$

$$M^h = w^h x_0^h + \sum_i q_i x_i^h, \quad h=r+1,\ldots,H \tag{2b}$$

where $w^h=w^*-t_0$ is the after tax wage i.e. the opportunity cost of leisure, with t_0 the specific amount of direct tax paid (we assume that income tax is levied on wage income only) and $q_i=p_i+t_i$ is the tax-inclusive price of good i, with p_i the producer price and t_i the specific indirect tax on good i. Total commodity expenditure per period is constrained to equal the sum of labour and non-labour incomes so that $\sum q_i x_i^h=w^h l_0^h+y^h$, where y^h is non-labour income for household h, and l_0^h is zero for rationed households[5]. Substitution into (2a) and (2b) gives the full income budget constraint, which holds for both rationed and non-rationed households

$$M^h = w^h T + y^h, \quad h=1,\ldots,H \tag{3}$$

The solution of the maximisation of (1a) and (1b) subject to (3) gives the indirect utility functions for rationed and unrationed households, defined over consumer prices, after-tax wages, non-labour incomes and the ration in the labour market, T, which affects only rationed households

[5] For rationed households we would expect welfare payments to constitute the major part of y^h.

$$v^h = \bar{v}^h(q,\ w^h,\ y^h,\ T),\quad h=1,\ldots,r \tag{4a}$$

$$v^h = v^h(q,\ w^h,\ y^h),\quad h=r+1,\ldots,H \tag{4b}$$

where the bar indicates the indirect utility function for rationed households.

We assume the government maximises a Bergson-Samuelson indirect social welfare function defined over the indirect utility functions of the individual households

$$V(q,w^h,y^h,T)=W(\bar{v}^1(q,w^1,y^1,T),.\bar{v}^r(.),v^{r+1}(q,w^{r+1},y^{r+1}),.,v^H(.)) \tag{5}$$

There is an aggregate demand vector given by

$$X(q,\ w^h,\ y^h,\ T)=\sum_{h=1}^{r}\bar{x}^h(q,\ w^h,\ y^h,\ T)+\sum_{h=r+1}^{H}x^h(q,\ w^h,\ y^h) \tag{6}$$

Aggregate labour supply is given by

$$L(q,\ w^h,\ y^h) = \sum_{h=r+1}^{H} l^h(q,\ w^h,\ y^h) \tag{7}$$

We assume that government revenue comes from both commodity and labour taxes. We also make the simplyifing assumption that direct taxation is levied at a single proportional tax rate.[6] Thus the revenue equation is

$$R=\sum_i t_i X_i(q_i,\ w^h,\ y^h,\ T)+\sum_{h=r+1}^{H} t_0 l^h(q,\ w^h,\ y^h) \tag{8}$$

The crucial parameter in marginal tax reform analysis is the MRC of changing the tax on good i so as to increase welfare by one unit. Intuitively, at the optimum the MRC for all revenue sources (i.e. goods, including leisure) will be equal. If they are not equal then revenue-neutral welfare improving tax reforms at the margin can be identified. The expression for MRC, which

[6] This means that a change in direct taxation will have no impact on non-labour income.

we label ρ_i is

$$\rho_i = -\frac{\partial R/\partial t_i}{\partial V/\partial t_i} \tag{9}$$

where we insert the minus sign to denote marginal cost. We now proceed to express ρ_i in terms that are readily calculable. From Roy's Identity we know that

$$\frac{\partial v^h}{\partial q_i} = -\alpha^h x_i^h, \quad h=1,..,H \tag{10}$$

where α^h is the private marginal utility of income of household h . We can also apply Roy's Identity to the price of leisure to derive

$$\frac{\partial \bar{v}^h}{\partial w^h} = 0, \quad h=1,..r \tag{11a}$$

$$\frac{\partial v^h}{\partial w^h} = \alpha^h l^h, \quad h=r+1,..,H. \tag{11b}$$

We can then say

$$\frac{\partial V^h}{\partial t_i} = -\sum_{h=1}^{H} \beta^h x_i^h \tag{12a}$$

$$\frac{\partial V^h}{\partial t_0} = -\sum_{h=r+1}^{H} \beta^h l^h \tag{12b}$$

where β^h is the social marginal utility of income of household h, i.e. its welfare weight.[7] Note from (11b) that an increase in direct taxation, t_0, will have no welfare implications for those

[7] We introduce welfare weights exogeneously using Atkinson's (1970) utility of income function:

$$U^h(I^h) = \frac{kI_{1-e}}{1-e}, e\geq 0, e\neq 1; = k\log(I), \ e=1$$

where I^h is expenditure per equivalent adult of the hth household. $\beta^h=U'(I^h)$ and after appropriate normalisaton we have $\beta^h=(I^1/I^h)^e$ and e can be regarded as an inequality aversion parameter. e=0 is the extreme utilitarian case while as e approaches infinity we have the extreme Rawlsian case.

households which are rationed in the labour market.

The revenue impact of changes in taxation are as follows:

$$\frac{\partial R}{\partial t_i} = X_i + \sum_j t_j \frac{\partial X_j}{\partial t_i} + \sum_{h=r+1}^{H} t_0 \frac{\partial l^h}{\partial t_i} \tag{13a}$$

$$\frac{\partial R}{\partial t_0} = \sum_j t_j \frac{\partial X_j}{\partial t_0} + \sum_{h=r+1}^{H} l^h + \sum_{h=r+1}^{H} t_0 \frac{\partial l^h}{\partial t_0} \tag{13b}$$

We note that a change in direct taxation t_0 may cause changes in labour supply at both the intensive and extensive margin i.e. it may change participation as well as hours supplied. For our purposes here, we do not need to distinguish between the two. All we need is the effect on revenue. As we shall see below, to obtain labour supply elasticities we will estimate a labour income function, which does not make this distinction.

Using equations (10) to (13) and some further manipulation to obtain expressions in elasticity terms, we now have expressions for ρ_i and ρ_0 which are readily calculable

$$\rho_i = \frac{q_i X_i + \sum_j \tau_j \epsilon_{ji} q_j X_j + \sum_{h=r+1}^{H} \tau_0 w^h l^h \epsilon_{li}}{\sum_h \beta^h q_i x_i^h} \tag{14}$$

$$\rho_0 = \frac{\sum_{h=r+1}^{H} l^h w^h - \sum_{h=r+1}^{H} \tau_0 w^h l^h \epsilon_{lw} - \sum_j \tau_j q_j X_j \epsilon_{jw}}{\sum_{h=r+1}^{H} \beta^h l^h w^h} \tag{15}$$

where τ_j refers to the tax as a proportion of the consumer price for good j, τ_0 is tax as a proportion of the net wage, ϵ_{ji} is the uncompensated cross-elasticity of demand between goods j and i, ϵ_{li} is the uncompensated elasticity of supply of labour with respect to commodity i, ϵ_{lw} is the uncompensated elasticity of labour supply with respect to the after-tax wage and ϵ_{jw} is the uncompensated elasticity of demand for good j with respect to the

after-tax wage.

The expressions above for ρ_i and ρ_0 are similar to those in Madden (1993b), with the difference being that terms involving labour supply are summed over a subset of households rather than total households, and demand responses are estimated from matched pairs of demands. Thus rationing is incorporated in both the numerator and denominator of ρ_i and ρ_0. It is incorporated in the numerator since it is the *actual* commodity demands and labour incomes which are used in the calculation of the welfare effect of changing taxes. The actual commodity demands and labour incomes give consistently aggregated commodity demands and labour incomes for employed and unemployed households. For example, labour income for the lowest expenditure households tends to be very small in proportion to non-labour income (in particular social welfare payments) since, in general, the poorest households are also those facing rationing in the labour market. Thus the utility loss for these households following an increase in direct taxation will be quite small and this is accurately captured by the use of actual labour incomes.

The inclusion of rationing affects the denominator via demand and revenue responses. As we shall see, our demand responses will be a weighted average of rationed and unrationed responses. The use of aggregate time series data and the consistent aggregation of rationed and unrationed responses does limit the functional forms available for estimation, however. We now specify household behaviour and derive expressions for matched pairs of commodity demands and labour supplies.

3. Household Behaviour.

To estimate demand and revenue responses when some agents are rationed we need to estimate a "matched pair" of rationed and unrationed demands i.e. rationed and unrationed demands which are generated from the same preferences. Since we are restricted to aggregate time-series data we also have to satisfy conditions of exact linear aggregation.[8]

Under fairly general assumptions we can represent preferences in (1a) and (1b) by the consumer cost or expenditure function. The consumer cost functions for the unrationed and rationed cases (where the consumer faces a ration z, in this case equal to T, the total time endowment, on leisure hours and we drop household superscripts for notational convenience) are given by

$$c(q,\ w,\ u) = \min\ wx_0 + q'x \quad s.t.\ u(x_0,\ x) \geq u \tag{16a}$$

$$\bar{c}(q,\ w,\ z,\ u) = \min\ wz + q'\bar{x} \quad s.t.\ u(z\ ,\bar{x}) \geq u \tag{16b}$$

Neary and Roberts (1980) show how the restricted and unrestricted cost functions are related via the "virtual" wage i.e. that wage at which the ration would be freely chosen. In particular they show that

$$\bar{c}(q,\ w,\ z,\ u) = c(q,\ \bar{w},\ u) + (w-\bar{w})\,z \tag{17}$$

Deaton and Muellbauer (1981) and Murphy and Thom (1987) have shown the restrictions which must be imposed to enable exact linear aggregation over rationed and unrationed demands. They

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[8] For an example of estimation of "matched pairs" using household rather than aggregate data, and which uses more flexible functional forms but solves numerically, see Kooreman and Kapteyn (1986).

show that any one of the following three assumptions must be made: (1) all rationed agents face the same ration z; (2) all agents face the same wage w; or (3) weak separability between goods and leisure. Each of these assumptions appear somewhat restrictive, but we will choose to adopt assumption (1), a common ration level, as the least unappealing.

To permit exact linear aggregation over unrationed agents Muellbauer (1981) shows that the "macro" cost function must be of the Gorman polar form

$$c(q,\ w,\ u) = a(q) + wd(q) + w^{k}b(q)^{1-k}u \tag{18}$$

where a(q) and b(q) are homogenous of degree one in q and d(q) is homogenous of degree zero. If leisure and total commodity expenditure are both normal then $0<k<1$. Goldman and Uzawa (1964) show that goods and leisure are weakly separable if $c_{i0}=sc_{iu}$ where s is a constant for all i. For (18) this holds when $d_i(q)=0$ for all i. Additivity holds when $a_{ij}=0$ or $b_i(q)b_j(q)=0$ for all i and j. The cost function (18) yields the following uncompensated leisure and commodity demands.

$$x_0 = d(q) + \frac{k}{w}[M - a(q) - wd(q)] \tag{19}$$

$$x_i = a_i(q) + wd_i(q) + (1-k)\frac{b_i(q)}{b(q)}[M - a(q) - wd(q)] \tag{20}$$

(20) permits exact linear aggregation over agents with different time endowments, wage rates and non-labour incomes. The labour

supply function $l=T-x_0$ is non-linear in the wage rate. However, the labour income supply function is linear in the wage

$$wl = w[T-d(q)]-k[M-a(q)-wd(q)] \tag{21}$$

If the unrestricted cost function is given by (18) then it can be shown that the associated virtual wage and uncompensated rationed demands are

$$\bar{w} = b(q)\,(ku)^{1/(1-k)}\,(z-d(q))^{-1/(1-k)} \tag{22}$$

$$\bar{x}_i = a_i(q) + \left[\frac{b_i(q)}{b(q)} + \frac{k}{1-k}\,\frac{d_i(q)}{z-d(q)}\right][w(T-z)+y-a(q)] \tag{23}$$

(23) can only be linearly aggregated if z is a constant for all rationed agents or if leisure and goods are weakly separable i.e. if $d_i(q)=0$, for all i.

We now have to choose functional forms for a(q), b(q) and d(q). Following Blundell and Walker (1982) and Deaton (1981b) we choose

$$a(q) = \sum_{i=1}^{n} q_i\gamma_i \tag{24a}$$

$$b(q) = \eta_0\prod_{i=1}^{n} q_i^{\eta_i}, \quad \sum_{i=1}^{n}\eta_i=1 \tag{24b}$$

$$d(q) = \delta_0\prod_{i=1}^{n} q_i^{\delta_i}, \quad \sum_{i=1}^{n}\delta_i=0 \tag{24c}$$

This gives us the following unrationed commodity expenditure and labour income supply functions

$$q_i x_i = q_i\gamma_i + \delta_0\delta_i w\prod_{j=1}^{n} q_j^{\delta_j} + (1-k)\eta_i\left[wT+y-\sum_{j=1}^{n} q_j\gamma_j - w\delta_0\prod_{j=1}^{n} q_j^{\delta_j}\right] \tag{25}$$

$$wl = wT - w\delta_0 \prod_{j=1}^{n} q_j^{\delta_j} - k[wT + y - \sum_{j=1}^{n} q_j\gamma_j - w\delta_0 \prod_{j=1}^{n} q_j^{\delta_j}] \tag{26}$$

Leisure hours are rationed at z, and following our discussion above we set z equal to the common value of T. Thus all rationed agents are unemployed, with z=T and all employed agents are unrationed. The associated virtual wage for these functional forms and this value of the ration is

$$\bar{w} = \eta_0\delta_0 \prod_{j=1}^{n} q_j^{\eta_j} (ku)^{1/(1-k)} (T - \delta_0 \prod_{j=1}^{n} q_j^{\delta_j})^{-1/(1-k)} \tag{28}$$

and the associated rationed uncompensated demands are

$$q_i\bar{x}_i = q_i\gamma_i + [\eta_i + \frac{k}{1-k} \frac{\delta_0\delta_i \prod_{j=1}^{n} q_j^{\delta_j}}{(T - \delta_0 \prod_{j=1}^{n} q_j^{\delta_j})}] [y - \sum_{j=1}^{n} q_j\gamma_j] \tag{29}$$

Note the importance of separability for the above expression. If separability holds then $\delta_i=0$ and the ration will affect commodity demands via the supernumerary full income term only i.e. full income equals non-labour income less minimum expenditures on goods.[3] In the absence of separability, the ration can also exert substitution effects via the term in δ_i, k and T.

Thus (26) and (29) give us a matched pair of unrationed and rationed demands generated from the same preferences. We can then aggregate the rationed and unrationed demands

$$q_iX_i = Nq_ix_i + Uq_i\bar{x}_i \tag{30}$$

The total labour force equals L=N+U, with the unemployment rate

[3] Note that since the γ_i are expressed as linear functions of prices, q_i, the imposition of $\delta_i=0$ implies additive rather than weak separability. A possible generalisation of this would be to make the a(q) function a second order flexible functional form, in which case $\delta_i=0$ would imply weak separability.

σ=U/L. Observed aggregate demands are thus a weighted average of rationed and unrationed demands where σ and 1-σ are the weights.

4. Estimation and Results.

Estimation was carried out using the non-linear regression option on the SHAZAM package. The data consisted of annual observations of ten categories of consumption[10], numbers at work and after-tax wages[11] for the period 1958-88. Actual household commodity demands and labour incomes will be taken from the Irish Household Budget Survey (HBS), from which we obtain our household data ranked by expenditure per equivalent adult.[12] Two versions of the model were estimated, one unrestricted and one with δ_i=0 i.e. additive separability imposed. To overcome the singularity in the system the labour supply equation was deleted. Tables 1 and 2 show the estimated parameters for the two models with standard errors in brackets.

Overall, the results are quite plausible and as Table 3 shows the estimated elasticities seem reasonable. In terms of both relative and absolute magnitude the own-price elasticities are quite close to those estimated by Madden (1993a) using more flexible functional forms, but assuming goods-leisure separability. The most disturbing estimate is probably the

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[10] The ten goods were food, alcohol, tobacco, clothing and footwear, fuel and power, petrol, transport and equipment, durables, other goods, and services.

[11] To ensure proper linear aggregation we estimated a labour income function. As our measure of "hours" we used numbers at work, and so our labour supply/income function is more properly regarded as a labour force participation/income function.

[12] Equivalence scales are obtained from Conniffe and Keogh (1987).

negative value for k. This implies that leisure is an inferior good. The model was re-estimated a number of times with different starting values for k but the result appears to be robust. We also note in the unrestricted model the relatively low estimate for T and the *negative* estimates for γ_i for a number of goods.[13]

We also see that additive separability is decisively rejected. The value of the likelihood ratio test statistic is 144 compared to a critical value at 99% of 21.7.

5. Estimated Marginal Revenue Costs (MRC).

In tables 4 and 5 we present the estimated MRCs for the different goods for different values of inequality aversion. In tables 6, 7, and 8 we present rank correlation coefficients between ρ_i calculated from the different models, which allows us to see how different assumptions regarding inequality aversion and separability affect the rankings of goods.

Table 4 shows transport and equipment as having the highest MRC in the unrestricted model. Thus we would recommend that its tax be increased. This may appear surprising, in light of its relatively high own-price elasticity (see Table 3). However, we must remember that MRCs are based on cross-price as well as own-price elasticities and it is the relatively high substitutability between transport and equipment and services which gives rise to the high MRC for the former good.[14]

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[13] T refers to numbers at work and is in units of thousands.

[14] Details of cross-price elasticities are available from the author on request.

Looking across table 4 we can see that the ranking of ρ_i across goods is not very sensitive to the degree of inequality aversion assumed, except in the case of labour and this is borne out by the relatively high rank correlation coefficients in table 6. Thus at low levels of inequality aversion we would recommend no change or a marginal reduction in direct taxation, while at a high level of inequality aversion we would recommend that it be raised. We may note that this result is in contrast to much of the debate concerning tax reform in such countries as the US, the UK and Ireland (see for example the discussion in Atkinson (1990)) where the stress has been on reducing tax rates for upper bands.[*] The explanation lies in our measure for $l^h w^h$. We use direct income, not including state transfers, which in Ireland are untaxed. Thus relatively poor households, for whom state transfers constitute a relatively higher proportion of direct income, would suffer relatively little utility loss from an increase in direct taxation. The relatively greater loss is borne by relatively better off households. However, when the social welfare function incorporates a high degree of inequality aversion, these households' utility receives a very low weight, thus giving us a high MRC for direct taxtion. We should remember however, that in this relatively simple model we have assumed that all households face the same marginal rate of direct taxation. In reality households will face differing marginal tax rates and we may wish to recommend direct tax increases for some

[*] Of course our model assumes one band only and so it is not necessarily inconsistent with lowering top rates of tax. Also it could be argued that the advocates of cuts in top rates of income tax had a low degree of inequality aversion, i.e. a low value of e.

households accompanied by reductions for other households. It does, however, indicate the recommendations that might follow from a tax system which had only one rate of direct taxation and where the government was highly averse to inequality.

Turning now to table 5, which gives the MRCs for the model with separability imposed, we note that the spread between MRCs for different goods is much narrower, thus indicating less scope for marginal tax reforms. We should also remember that the MRCs are transformations of elasticities, which are themselves transformations of estimated parameters which have standard errors. Thus we need to take care in constructing a marginal tax reform package. We also note that the rankings of MRCs are more sensitive to the degree of inequality aversion assumed than is the case with the unrestricted model (witness the relatively low rank correlation coefficients in table 7). Once again we note that the relative MRC for labour tends to rise quite steeply with the degree of inequality aversion assumed.

Next we examine the sensitivity of the rankings of goods by MRC to assumptions regarding separability. Recall our discussion in the introduction where we noted the sensitivity of optimally calculated tax rates to assumptions regarding separability and conjectured that such results may be even more pronounced in the presence of rationing. Deaton (1987) conjectured that tax reform recommendations might be less sensitive to assumptions regarding separability than optimally calculated tax rates, and the results of Madden (1993c) appeared to confirm this. Those results were obtained for a model that did not incorporate rationing, and a rank correlation coefficient of 0.842 was obtained between the

model with additive separability imposed and that with separability not imposed. The rank correlation coefficient reported in table 8 of 0.636, while still reasonably high, shows that, when account is taken of rationing, the sensitivity of rankings by MRC to separability appears to increase, thus modifying Deaton's conjecture. We would regard this finding as being in accordance with intuition.

Finally in tables 9, 10 and 11 we present rank correlations for *goods* only i.e. those for *indirect* tax reform recommendations only. The results are quite similar, except that they show that the rankings by goods only tend to be *less* sensitive to the degree of inequality aversion assumed. Once again we note that some sensitivity to the imposition of separability (a rank correlation of 0.600) but there is still a reasonable degree of consistency in the tax reform recommendations arising from the two models.

6. Conclusion.

This paper has incorporated labour market rationing into a model of marginal direct and indirect tax reform and examined the sensitivity of tax reform recommendations to assumptions regarding separability in the presence of rationing. We obtain "matched pairs" of rationed and unrationed demands and use these demand responses in calculating marginal revenue costs (MRCs) of taxation for both direct and indirect taxes for Ireland. We find that calculated MRCs do exhibit some sensitivity to assumptions regarding separability and that the degree of sensitivity to assumptions regarding inequality aversion are also affected by

the imposition of separability. We feel that these findings are nevertheless consistent with the conjecture of Deaton regarding tax reform in that the relevant rank correlations are still quite high.

Table 1

Estimates from Unrestricted Model, Standard Errors in Brackets

Good	γ_i	δ_i	β_i
Food	2043.9 (130.2)	0.94 (0.47)	0.06 (0.02)
Alcohol	430.5 (76.5)	-1.02 (0.45)	0.11 (0.01)
Tobacco	429.5 (19.0)	-0.28 (0.14)	0.01 (0.002)
C & F	-148.8 (119.0)	-1.60 (0.53)	0.135 (0.01)
F & P	338.0 (34.1)	-1.25 (0.38)	0.04 (0.005)
Petrol	367.8 (37.3)	0.29 (0.15)	0.01 (0.005)
T & E	-476.9 (169.7)	1.12 (0.50)	0.16 (0.01)
Durables	-161.8 (57.8)	-0.23 (0.33)	0.08 (0.007)
Other Goods	-319.8 (112.2)	-3.16 (1.06)	0.18 (0.07)
Services	-303.9 (193.6)	5.19	0.22

LLF=-1188.6

δ =4.13 (2.28)

k=-0.89 (0.12)

T=827.4 (111.5).

Table 2

Estimates from Model with Separability Imposed

Standard Errors in Brackets.

Good	γ_i	β_i
Food	2059.2 (112.0)	0.11 (0.02)
Alcohol	715.3 (66.6)	0.12 (0.01)
Tobacco	455.4 (18.9)	0.01 (0.004)
C & F	383.0 (91.0)	0.10 (0.01)
F & P	450.2 (28.8)	0.04 (0.01)
Petrol	318.5 (36.2)	0.03 (0.01)
T & E	97.3 (135.1)	0.17 (0.02)
Durables	42.7 (68.9)	0.09 (0.01)
Other Goods	335.1 (98.5)	0.15 (0.01)
Services	719.7 (318.4)	0.18

LLF=-1260.0

T-δ =1296.3 (237.0)

k=-0.189 (0.41).

Table 3

Estimated Own Price Elasticities

Good	Unrestricted	Separability Imposed
Food	-0.253	-0.288
Alcohol	-0.665	-0.490
Tobacco	-0.052	-0.001
C & F	-0.973	-0.616
F & P	-0.415	-0.376
Petrol	-0.111	-0.256
T & E	-1.380	-0.903
Durables	-1.318	-0.911
Other Goods	-0.731	-0.742
Services	-0.532	-0.689
Labour	0.689	0.683

Table 4: MRCs for Unrestricted Model

e=0		e=1		e=5	
1. T & E	1.07	1. T & E	1.66	1. Services	5.49
2. Services	1.04	2. Services	1.65	2. T & E	5.27
3. Durables	0.95	3. Durables	1.45	3. Labour	5.15
4. C & F	0.88	4. C & F	1.36	4. Durables	4.23
5. Other G	0.80	5. Other G	1.18	5. C & F	4.16
6. F & P	0.68	6. Labour	1.08	6. Other G	3.04
7. Labour	0.64	7. F & P	0.94	7. F & P	1.95
8. Food	0.45	8. Food	0.64	8. Alcohol	1.79
9. Tobacco	0.44	9. Alcohol	0.63	9. Food	1.45
10. Alcohol	0.41	10. Tobacco	0.60	10. Tobacco	1.25
11. Petrol	0.06	11. Petrol	0.10	11. Petrol	0.30

Table 5: MRCs - Separability Imposed

e=0		e=1		e=5	
1. Services	0.84	1. Services	1.33	1. Labour	5.35
2. F & P	0.82	2. C & F	1.24	2. Services	4.44
3. C & F	0.80	3. Durables	1.18	3. C & F	3.81
4. Durables	0.77	4. F & P	1.13	4. T & E	3.55
5. Food	0.75	5. Labour	1.12	5. Durables	3.45
6. Other G	0.73	6. T & E	1.12	6. Petrol	3.07
7. T & E	0.72	7. Other G	1.07	7. Other G	2.77
8. Petrol	0.66	8. Food	1.05	8. Food	2.39
9. Labour	0.66	9. Petrol	1.01	9. F & P	2.34
10. Tobacco	0.61	10. Tobacco	0.83	10. Tobacco	1.74
11. Alcohol	0.38	11. Alcohol	0.58	11. Alcohol	1.66

Table 6: Rank Correlations - Unrestricted Model

Model U0	1.000		
Model U1	0.982	1.000	
Model U5	0.873	0.927	1.000
	Model U0	Model U1	Model U5

Table 7: Rank Correlations - Separability Imposed

Model S0	1.000		
Model S1	0.845	1.000	
Model S5	0.373	0.727	1.000
	Model S0	Model S1	Model S5

Table 8: Rank Correlation between Unrestricted and Separability Imposed Model, e=0.

Model U0	1.000	
Model S0	0.636	1.000
	Model U0	Model S0

Model U0: Unrestricted Model, e=0.
Model U1: Unrestricted Model, e=1.
Model U5: Unrestricted Model, e=5.
Model S0: Separability Imposed, e=0.
Model S1: Separability Imposed, e=1.
Model S5: Separability Imposed, e=5.

Table 9: Rank Correlations - Unrestricted Model (Goods Only)

Model U0	1.000		
Model U1	0.988	1.000	
Model U5	0.951	0.976	1.000
	Model U0	Model U1	Model U5

Table 10: Rank Correlations - Separability Imposed (Goods Only)

Model S0	1.000		
Model S1	0.915	1.000	
Model S5	0.600	0.818	1.000
	Model S0	Model S1	Model S5

Table 11: Rank Correlation between Unrestricted and Separability Imposed Model (Goods Only), e=0

Model U0	1.000	
Model S0	0.600	1.000
	Model U0	Model S0

Model U0: Unrestricted Model, e=0.
Model U1: Unrestricted Model, e=1.
Model U5: Unrestricted Model, e=5.
Model S0: Separability Imposed, e=0.
Model S1: Separability Imposed, e=1.
Model S5: Separability Imposed, e=5.

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