Learning by Doing, Precommitment and Infant-Industry Protection

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ABSTRACT

This paper examines the implications for strategic trade policy of different assumptions about precommitment. In a dynamic oligopoly game with learning by doing, the optimal first-period subsidy is lower if firms cannot precommit to future output than if they can; and is lower still if the government cannot precommit to future subsidies. In the linear case the optimal subsidy is increasing in the rate of learning with precommitment but decreasing in it if the government cannot precommit. The infant-industry argument is thus reversed in the absence of precommitment, which has important implications for economic policy in dynamic environments.

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1. Introduction

The central theme in strategic trade policy is that a government's ability to precommit to tariffs or subsidies may make intervention desirable in oligopolistic markets.¹ An important theme in industrial organisation is that a firm's ability to precommit to future actions gives it an incentive to behave strategically.² In this paper we make a start at integrating these insights. More specifically, we show that the assumptions made about the ability of different agents to precommit have major implications for the magnitude and even the sign of optimal intervention.³

The specific context in which we explore the issues of precommitment and strategic behaviour is that of learning by doing.⁴ Our model builds on that of Fudenberg and Tirole (1983) who, drawing on Spence (1981), compared equilibria in which oligopolistic firms can precommit to future outputs with equilibria in which they cannot.⁵ However, in discussing government policies in such a model, Fudenberg and Tirole assumed that the government can precommit to future policies. This ignores the fact

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¹ See, for example, Dixit (1984), Brander and Spencer (1985) and Eaton and Grossman (1986).

² By strategic behaviour we mean behaviour which is not optimal in a static context but which changes the future behaviour of other agents to the firm's advantage. See, for example, Spence (1977) and Dixit (1980) for strategic choice of capacity, Fudenberg and Tirole (1983) for experience as a strategic variable and Fudenberg and Tirole (1984) for an overview.

³ Precommitment in trade policy has been studied by Eaton and Grossman (1985), Staiger and Tabellini (1987), Lapan (1988) and Matsuyama (1990). The paper which is closest to ours is that of Maskin and Newbery (1990), who consider optimal tariffs for the importer of a natural resource with and without precommitment.

⁴ From the pioneering work of Arrow (1962) to recent work on endogenous growth theory (e.g., Grossman and Helpman (1991) and Young (1991)), learning by doing has long been recognised as a phenomenon of empirical and theoretical importance.

⁵ Spence claimed that the qualitative differences between these two equilibria (which he called "open-loop" and "closed-loop" respectively) were only minor. However, Bulow et al. (1985) showed that this result derived from the functional forms used by Spence.
(first noted by Leahy (1992)) that, if the government cannot precommit, then a firm which enjoys learning by doing has an incentive to play strategically against it. A forward-looking government in turn will anticipate this behaviour and take it into account in determining its optimal policy towards the firm in its learning phase.

From a trade policy perspective, learning by doing is often cited as a justification for intervention, especially to assist infant industries, although in a competitive environment this argument requires some additional distortion such as capital-market imperfections or externalities. However, the literature on strategic trade policy in imperfectly competitive markets has shown that shifting profits from foreign to home firms may justify intervention even in the absence of learning by doing. This raises the question of whether learning by doing strengthens the case for strategic trade policy. To investigate this issue, we incorporate dynamic elements into a model of strategic export subsidisation due to Brander and Spencer (1985) by allowing for the effects of current output on future costs.

The plan of the paper is as follows. Section 2 sketches the model and introduces the different assumptions about precommitment which are explored in the paper. Section 3 considers the benchmark case where firms and government precommit in the first period to outputs and subsidy levels in the second period. These assumptions are then relaxed in Sections 4 and 5, which consider in turn the effects of ruling out commitment by firms and by government. Section 6 explores the implications of constraining governments to a particular form of precommitment which provides only temporary subsidies. Section 7 examines the special case of linear demands and linear learning and considers the robustness of the results. Section 8 examines how the arguments are affected if firms are assumed to engage in price rather than in quantity competition. Finally, Section 9 concludes with a summary of the paper's results and a discussion of the implications for the infant-industry argument and for the general principles guiding government policy in dynamic environments.

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2. The Model

The model to be used in most of the paper builds on the Brander-Spencer (1985) export subsidy model, extending it to two periods which are linked by learning by doing.\(^7\) We distinguish two firms, a home firm producing output levels \(x_t\) and \(x_2\) in periods 1 and 2 respectively and a foreign firm producing output levels \(y_t\) and \(y_2\). The outputs are homogeneous and both firms sell into a third market where the inverse demand function is:\(^8\)

\[
p_t = p_t(x_t + y_t), \quad t=1, 2; \quad p_t' = -b_t < 0.
\]

(1)

For some results we specialise to time-invariant linear demands:

\[
p_t = a - b(x_t + y_t), \quad t=1, 2.
\]

(2)

in which case \(b\) is a constant.

Firms produce with constant marginal costs in each period. The foreign firm is assumed to have already reached its mature phase, so its marginal costs are fixed at \(c^*\).\(^9\) By contrast, the home firm has a given marginal cost, \(c_1\), in the first, learning, period whereas its marginal cost in the second period, \(c_2\), depends negatively on its output in period 1 and on a shift parameter, \(\epsilon\):

\[
c_2 = c_2(x_t, \epsilon), \quad \frac{\partial c_2}{\partial x_t} = -\lambda(x_t, \epsilon) < 0.
\]

(3)

where the signs indicate the signs of the corresponding partial derivatives. Here \(\lambda\)

---

\(^7\) Earlier studies of learning by doing and strategic trade policy include Krugman (1984), Baldwin and Krugman (1988) and Gatsios (1989). These papers assumed that governments can precommit to future policies. Leahy (1992) relaxed this assumption and explored the implications of firms playing strategically against governments. Neary (1994, Section 4) extended this model to the case where the government anticipates the firms' actions. The latter two papers followed Fudenberg and Tirole in considering strategic effects of learning only in linear examples.

\(^8\) Allowing products to be differentiated complicates the algebra without yielding any additional insights. Details are available on request.

\(^9\) Dasgupta and Stiglitz (1988) note that this asymmetric specification of learning seems to accord with most policy discussions of assistance to learning firms.
measures the rate of learning and is increasing in $\varepsilon$: $\lambda_\varepsilon > 0$. For some results we specialise to linear learning:

$$c_2 = c_1 - \lambda x_1,$$

(4)

in which case the rate of learning $\lambda$ is a constant and we set $\lambda_\varepsilon = 1$.

Each firm seeks to maximise the present value of its profits in the two periods, with future profits discounted by a factor $\rho$. Thus, for the home firm: $\pi = \pi_1 + \rho \pi_2$, where profits in each period equal revenue $R$ less production costs plus subsidies:

$$\pi_t = R_t - (c_t - s_t)x_t = (p_t - c_t)x_t + s_t x_t, \quad t = 1, 2.$$

(5)

(Fixed costs are ignored for simplicity since the important issues relating to entry and exit of firms from the market are not considered in the paper.) Subsidies in turn are provided by the home government only, which seeks to maximise the present value of welfare:

$$W = (p_1 - c_1)x_1 + \rho (p_2 - c_2)x_2.$$

(6)

This can also be written as $W = W_1 + \rho W_2$, where (with no home consumption) welfare in each period equals profits net of subsidy payments, $W_t = \pi_t - s_t x_t = (p_t - c_t)x_t, \quad t = 1, 2$.

A key feature of our model is the order of moves by different agents. We follow Brander and Spencer throughout in two crucial respects. Firstly (except in Section 8), we assume that firms are Cournot competitors, making simultaneous, noncooperative decisions about their output levels in each period. Secondly, we assume that in each period the government can credibly precommit to a subsidy before outputs are chosen.\(^{10}\)

The government thus has the ability to precommit intratemporally in all cases. However, different assumptions about intertemporal precommitment are possible. Depending on the assumptions made we can distinguish three different types of equilibrium:

**Full Precommitment Equilibrium (FPCE):** In this two-stage game, all agents take decisions for both periods at the beginning of period 1. Thus, in the first stage of the game, the government chooses the subsidies $s_1$ and $s_2$, and in the second stage the two firms

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\(^{10}\) The implications of reversing the intratemporal order of moves by government and firms in a static export subsidy game have been considered by Carmichael (1987), Gruenspecht (1988) and Neary (1991 and 1994, Section 5).
government chooses the subsidies $s_1$ and $s_2$, and in the second stage the two firms choose both their present and future outputs noncooperatively.

**Government Precommitment Equilibrium (GPCE):** In this three-stage game the government can precommit in period 1 to its second-period action but the firms cannot. Thus the first stage is the same as in FPCE; in the second stage the firms choose their period-1 outputs; and in the third stage they choose their period-2 outputs. (The two cases of FPCE and GPCE correspond to the equilibria compared by Fudenberg and Tirole (1983).)

**Sequential Equilibrium (SE):** The final case is a four-stage game, in which no agents can precommit to future actions. In each period, the government first sets the subsidy and the firms then choose their outputs.

Throughout the paper, we assume that all the equilibria considered are subgame perfect, in the sense of Selten (1975). All agents thus take account of the influence of their current actions on the future actions of other agents and it is this forward-looking behaviour which opens up the possibility of agents playing strategically. Different move orders imply different strategic incentives for the home firm and the home government. The key issue we address is how government policy takes account of the different strategic possibilities open to the home firm.

### 3. Optimal Subsidies with Full Precommitment

We consider first the benchmark case of FPCE. For the foreign firm, with no intertemporal links in its decisions, the first-order conditions are straightforward. Using asterisks to denote variables relating to the foreign firm, marginal revenue ($R_y = \partial R^* / \partial y_i$) less marginal cost is zero in each period:

$$R_y^* - c^* = 0, \quad t=1,2. \tag{7}$$

Each of these two first-order conditions may be solved for the foreign firm's reaction function in each period, which fully determines its output in each period as a function of the home firm's output: $y_t = \psi_t(x_t), \quad t=1,2$. Provided foreign output is a strategic substitute (in the sense of Bulow et al. (1985)) for home output in each period, these
reaction functions are downward-sloping: $\psi_1^\prime < 0$.\(^{11}\)

Since the home firm benefits from learning by doing, its optimisation problem is more complex. It seeks to maximise the present value of profits, given by:

$$
\pi = \pi_1(x_1, y_1, s_1) + \rho \pi_2(x_2, y_2, c_2, s_2),
$$  

(8)

where the signs indicate the signs of the corresponding partial derivatives. Optimal choice of second-period output sets the partial derivative of $\pi_2$ with respect to $x_2$ equal to zero, yielding a standard first-order condition:

$$
R_2^2 - c_2 + s_2 = 0.
$$  

(9)

However, the firm's choice of output in the first period must take account of the effects on profits in both periods. In FPCE, it takes the foreign firm's outputs and the subsidy levels in both periods as given. There remains the pure (non-strategic) learning by doing effect, whereby the home firm's choice of $x_1$ affects its future costs $c_2$. Differentiating $\pi$ with respect to $x_1$ and using equation (3) and the fact that $\partial \pi_2 / \partial c_2$ equals $-x_2$ gives:

$$
\frac{d\pi}{dx_1} = \frac{\partial \pi_1}{\partial x_1} + \rho \frac{\partial \pi_2}{\partial c_2} \frac{\partial c_2}{\partial x_1} = R_2^1 - c_1 + s_1 + \rho \lambda x_2 = 0.
$$  

(10)

Note that marginal revenue is set below marginal cost in the first period ($R_2^1 - c_1 + s_1$ is negative). The home firm has an incentive to produce beyond the point of short-run profit maximisation in order to learn faster and bring down its costs in period 2.

What is the optimal policy for the government, anticipating this behaviour? Since the home firm fully internalises the gains from learning and, in FPCE, has no incentive to behave strategically, there is no basis for intervention other than that which arises in the static Brander-Spencer model. The only role for the government is to use its ability to precommit within each period to offer a subsidy to the home firm which shifts profits towards it and away from the foreign firm. Totally differentiating the welfare function

---

\(^{11}\) This follows from totally differentiating (7): $\psi_1^\prime = -R_{x_2}^{S_f}/R_{x_1}^{S_f}$. The denominator is negative from the foreign firm's second-order condition, so $\psi_1^\prime$ is negative provided the numerator $R_{x_2}^{S_f}$ is negative. This is equivalent to requiring that the marginal profitability of foreign output is decreasing in home output.
(6) and substituting from the home firm’s first-order conditions gives: \[ dW = -(s_1 + b_1 x_1 \psi'_1) dx_1 + \rho (s_2 + b_2 x_2 \psi'_2) dx_2. \] (11)

From the firms’ first-order conditions, outputs \( x_1 \) and \( x_2 \) are fully determined by the two instruments \( s_1 \) and \( s_2 \). Hence, optimal policy choice requires only that the coefficients of both \( dx_1 \) and \( dx_2 \) be set equal to zero. This gives:

\[ s_t F = -b_x \psi'_t, \quad t=1,2. \] (12)

This is identical to the formula for the optimal subsidy in the static Brander-Spencer game. Hence, in FPCE, the optimal subsidy is positive in each period if and only if foreign output is a strategic substitute for home output.

We can say more about the nature of the optimum by substituting the subsidy expressions (12) into the first-order conditions (9) and (10). This gives:

\[ H_1: \quad R_x^1 - c_1 - b_x \psi'_1 + \rho \lambda x_2 = 0. \] (13)

\[ H_2: \quad R_x^2 - c_2 - b_x \psi'_2 = 0. \] (14)

These equations may be totally differentiated to show that outputs in both periods and total welfare are increasing in the rate of learning. (Details are given in the Appendix.) Finally, how do the subsidies themselves vary with the rate of learning? To answer this question, note that, from (12), the subsidy in each period is a function of that period’s output only (recalling that foreign output is directly related to home output by the foreign firm’s reaction function). Differentiating totally, we may express the elasticity of the subsidy with respect to output in each period as:

\[ \sigma_t = \frac{x_t}{s_t F} \frac{ds_t F}{dx_t} = 1 + \mu_t + (1 + \psi'_t) r_t, \quad t=1,2. \] (15)

Stability implies that \( 1 + \psi'_t \) must be positive (and, using a result from an earlier footnote, it must be less than unity if outputs are strategic substitutes); \( r_t \) is defined as \( (x_t + y_t)b'_t/b_t \).

\[ \text{We also make use of the facts that, in each period: marginal revenue } R_x^t \text{ equals } p_t - b x_t; \text{ and the change in price is given by } dp_t = -b_t(dx_t + dy_t) = -b_t(1 + \psi'_t) dx_t. \]
a measure of the concavity of the demand function; and \( \mu_i \) is defined as \( x_i \psi_i''/\psi_i' \), a measure of the concavity of the foreign firm's reaction function. When demands are linear the right-hand side of (15) equals unity, and with general demands a sufficient condition for it to be positive is that both \( r_i \) and \( \mu_i \) are non-negative. Summarising:

Lemma 1: Higher home output in period \( t \) is associated with a higher value of the optimal subsidy in the same period, provided neither the demand curve nor the foreign firm's reaction function in the same period is "too" convex.

Combining these results:

Proposition 1: When both firms and government can precommit to future actions, outputs in each period and total welfare are increasing in the rate of learning; and, provided the optimal subsidies are positive and the conditions of Lemma 1 apply, the optimal subsidies are also increasing in the rate of learning.\(^{13}\)

In the general case it does not appear to be possible to rank the output levels in the two periods. However, this can be done under some additional assumptions:

Proposition 2: If we assume (i) time-invariant demands; (ii) no discounting; and (iii) linear learning; then outputs in the first and second periods are equal in FPCE.

The proof follows immediately by noting that the home firm's first-order conditions in the two periods, equations (13) and (14), are identical under the stated assumptions. A corollary is that the subsidies are also identical under the same assumptions.

4. Optimal Subsidies when Only the Government can Precommit

How are the conclusions of the previous section affected when the government can still precommit to a future subsidy but firms take their output decisions sequentially? Equations (7) and (9), giving respectively the first-order conditions for the foreign firm in both periods and for the home firm in the second period, are unaffected. However, as Fudenberg and Tirole (1983) emphasised, the home firm now has an additional strategic incentive to increase its output in the first period. Not only will the resulting fall in second-period costs raise its future profits directly, but it will also improve its future

\(^{13}\) It is straightforward to show in a similar manner that, under identical conditions, outputs and subsidies in each period as well as total welfare are increasing in the discount factor, \( \rho \).
strategic position in competition with the foreign firm. The home firm therefore chooses \( x_1 \) to maximise profits (8), recognising its influence on the foreign firm's future output \( y_2 \) (with \( \partial x_2/\partial y_2 = -b_2 x_2 \)). This yields a new first-period first-order condition:

\[
\frac{d\pi}{dx_1} = \frac{\partial \pi}{\partial x_1} + \rho \left( \frac{\partial \pi_2}{\partial x_2} \frac{dy_2}{dx_2} \frac{\partial x_2}{dx_1} \right) \frac{dc_2}{dx_1} = R_x^{\prime} - c_1 + s_1 + \rho \lambda x_2 \left( 1 + b_2 \psi_2 \frac{\partial x_2}{dc_2} \right) = 0. \tag{16}
\]

Since \( \frac{\partial x_2}{\partial c_2} \) is negative,\(^\text{14}\) this shows that (given strategic substitutability, so \( \psi_2 \)' is negative) the home firm has an even greater incentive to produce beyond the point of short-run profit maximisation than in FPCE.

Of course, the actual levels of output chosen by the firm depend on the subsidies which, as in FPCE, are chosen by the government in the first stage of the game. As in the last section, the optimal subsidies are found by differentiating the welfare function (6). A series of substitutions similar to those which led to equation (12) in FPCE now yields:

\[
s_1^G = -b_1 x_1 \psi_1 + \rho \lambda s_2^G \frac{\partial x_2}{\partial c_2} \quad \text{and} \quad s_2^G = -b_2 x_2 \psi_2. \tag{17}
\]

The full expression for the optimal second-period subsidy and the first term in the expression for the optimal first-period subsidy are identical to the corresponding expressions in the full precommitment equilibrium. However, the latter has an additional second term which (given strategic substitutability) is negative: the government provides a lower subsidy in the first period in order to restrain the home firm from "over-producing" strategically in an attempt to drive down the foreign firm's output next period.

We might expect that the first-period subsidies in the two cases of FPCE and GPCE are evaluated at different points and hence cannot be compared exactly. However, this is not the case: even with general demands the levels of output in each period are identical in the two equilibria. To see this, substitute for \( s_1^G \) and \( s_2^G \) into the home firm's first-order conditions in the government precommitment equilibrium, (9) and (16). This yields a

\[^{14}\text{This partial derivative allows for the endogenous adjustment of } y_2 \text{ along the foreign firm's reaction function for a given value of } s_2. \text{ Standard calculations give: } \frac{\partial x_2}{\partial c_2} = R_x^{s_2}/\Delta_2. \text{ The numerator is negative from the foreign firm's second-order condition while the denominator is the determinant of the second-period coefficient matrix and is positive from stability.}\]
pair of equations identical to those obtained when the corresponding substitutions are made in the full precommitment equilibrium, (13) and (14). Since the foreign firm’s first-order conditions (given by equation (7)) are also the same in both equilibria, the levels of output are identical in both equilibria. And, since (from (6)) welfare depends only on output levels, it follows that total welfare must also be the same in the two equilibria. Finally, it is shown in the Appendix that $s^f_2$ is increasing in the rate of learning at a slower rate than $s^f_1$. Summarising these results:

Proposition 3: Comparing the full and government-only precommitment equilibria: outputs in each period, total welfare and the optimal second-period subsidy are identical; and the optimal first-period subsidy is lower and increases less rapidly in the rate of learning in the government-only than in the full precommitment equilibrium.

Intuitively, the values of output are the same in the two equilibria because in each case the government has two instruments ($s_1$ and $s_2$) and only two targets ($x_1$ and $x_2$, since $y_1$ and $y_2$ are fully determined by the foreign firm’s reaction functions). This allows the government to bring about the same real equilibrium (and attain the same level of welfare) in each case, the only difference being that the first-period subsidy required to achieve this is lower when the home firm behaves strategically against the foreign firm.

5. Optimal Subsidies in Sequential Equilibrium

Sections 3 and 4 have examined optimal intervention in the cases of precommitment and sequential decision-making by firms. In this section we break new ground by considering the case where neither the government nor the firms can precommit inter-temporally. For the first time, this implies that government policy is partly endogenous.

Consider first the second period. This is now a standard static Brander-Spencer game in which the government first sets $s_2$ to maximise $W_2$ and the firms then choose outputs. With outputs set at their profit-maximising levels, second-period welfare is a function of the subsidy and (through the cost function $c_2$) the level of output in period 1:

$$W_2 = (p_2 - c_2(x_1))x_2 = W_2(s_2, x_1).$$  \hspace{1cm} (18)

Differentiating this with respect to $s_2$ and substituting as in previous sections leads to the
government’s first-order condition:\(^{15}\)

\[
\frac{\partial W_2}{\partial s_2} = -(s_2 + b_2x_2\psi'_2) \frac{\partial x_2}{\partial s_2} = 0. \tag{19}
\]

This yields the expression for the optimal second-period subsidy:

\[
s_2^* = -b_2x_2\psi'_2 = \Psi(x_1). \tag{20}
\]

This is identical to the expressions for the optimal second-period subsidies in FPCE and GPCE, while the notation \(\Psi(x_1)\) emphasises that we may view the government’s choice of \(s_2\) in the third stage of the game as moving along a reaction function whose location depends on the firm’s first-period choice of output.

Before considering the first period of this game, we need two preliminary results which allow us to characterise the slope of the reaction function \(\Psi(x_1)\). The first derives from the government’s second-order condition. Differentiating (19) with respect to \(s_2\) and evaluating at \(s_2 = s_2^*\):

\[
\frac{\partial^2 W_2}{\partial s_2^2} = - \left\{1 - \frac{\partial x_2^*}{\partial x_2} \frac{\partial x_2}{\partial s_2}\right\} \frac{\partial x_2}{\partial s_2} < 0. \tag{21}
\]

The expression in brackets must be positive for an interior maximum. This has a useful corollary, since direct calculation yields:

\[
\frac{\partial x_2}{\partial c_2} = \left\{1 + \frac{\partial x_2}{\partial s_2} \frac{\partial s^*}{\partial x_2}\right\}^{-1} \frac{\partial x_2}{\partial c_2}. \tag{22}
\]

From the second-order condition (21) (recalling that \(\partial x_2/\partial c_2 = -\partial x_2/\partial s_2\)), it follows that the right-hand side of (23) is negative. Summarising:\(^{16}\)

\[\]

\(^{15}\) The partial derivative \(\partial x_2/\partial s_2\) in this equation is positive and allows for the endogenous adjustment of \(y_2\) along the foreign firm’s reaction function, for a given value of \(c_2\). It is equal but opposite in sign to \(\partial x_2/\partial c_2\), defined in a previous footnote.

\(^{16}\) Lemmas 1 and 2 throw light on an issue considered by de Meza (1986) and Neary (1994), the relationship between a home firm’s cost competitiveness and the value of the static Brander-Spencer optimal subsidy. Equation (16) gives a necessary and sufficient condition for providing higher subsidies to more cost-competitive firms.
Lemma 2: A rise in period-2 costs, allowing for the endogenous adjustment of the period-2 subsidy, must lower period-2 output.

The second preliminary result needed is the effect on (19), and hence on the optimum subsidy \( s_2^* \), of an increase in period-1 output \( x_1 \). Differentiating (19) with respect to \( x_1 \) and evaluating at \( s_2 = s_2^* \):

\[
\frac{\partial^2 W_2}{\partial s_2 \partial x_1} = \left( \frac{\partial s_2^*}{\partial x_2} \frac{\partial x_2}{\partial x_1} \frac{\partial c_2}{\partial x_1} \right) \frac{\partial x_2}{\partial s_2}.
\]  

(23)

This expression is positive, implying that \( s_2^* \) is a strategic complement for \( x_1 \), if and only if \( ds_2^*/dx_2 \) is positive. Combining (21), (22) and (23):

\[
\Psi' = -\frac{\partial^2 W_2}{\partial s_2 \partial x_1} \left( \frac{\partial^2 W_2}{\partial s_2^2} \right) = -\lambda \frac{ds_2^*}{dx_2} \frac{dx_2}{dc_2}.
\]  

(24)

Recalling Lemma 1, we may conclude:

Lemma 3: The optimal subsidy in period 2 is a strategic complement for output in period 1, provided neither the demand curve nor the foreign firm's reaction function in period 2 is "too" convex.

Finally, (22) implies that, provided \( s_2^* \) is a strategic complement for \( x_1 \), the total effect of a cost change on output is algebraically greater than the partial effect: \( dx_2/dc_2 < dx_2/dx_1 < 0 \).

Armed with these results, which characterise the dependence of the second-period optimal subsidy on first-period output, we can turn now to consider the behaviour of agents in the first period. Unlike the equilibria considered in previous sections, the home firm can now play strategically against the government as well as against the foreign firm. Its first-order condition is now:

\[
\frac{d\pi}{dx_1} = \frac{\partial \pi_1}{\partial x_1} + \rho \frac{d\pi_2}{dx_1} = 0,
\]  

(25)

where the total effect of current output on future profits includes (from (8)) the direct effect of cost changes as well as their indirect effects working through both future foreign output and the future subsidy.
\[
\frac{d\pi_2}{dx_1} = \frac{\left[ \frac{\partial \pi_2}{\partial x_2} \frac{dx_2}{dc_2} + \frac{\partial \pi_2}{\partial c_2} \frac{ds_2}{dc_2} \right] dc_2}{\left[ \frac{\partial y_2}{\partial x_2} \frac{dx_2}{dc_2} + \frac{\partial c_2}{\partial c_2} \frac{ds_2}{dc_2} \right] dx_1}.
\]

Collecting terms and substituting in (25) gives the first-order condition:

\[
R_2' - c_1 + s_1 + \rho \lambda x_2 \left( 1 + b_2 \psi_2' \frac{dx_2}{dc_2} \right) + \rho x_2 \Psi' = 0.
\]  

(27)

Comparing this with the corresponding formula in the case of GPCE, (16), we see that the home firm has an additional incentive to increase output in the first period, provided the future subsidy is a strategic complement for first-period output.

The final step is to consider the government's optimisation problem in the first stage of the game. The welfare function is now:

\[
W = (p_1 - c_1)x_1 + \rho W_2(s_2, x_1).
\]  

(28)

Differentiating this with respect to \( s_1 \) (using the fact that, by the envelope theorem, \( dW_2/dx_1 = \partial W_2/\partial x_1 = \lambda x_2 \)) and substituting from the firm's first-order condition (27) gives the expression for the first-period optimal subsidy in SE:

\[
s_1^* = -b_1 x_1 \psi_1' + \rho \lambda s_2 \frac{dx_2}{dc_2} - \rho x_2 \Psi'.
\]  

(29)

Comparing this with the optimal first-period subsidy in GPCE (equation (17)), the second effect is more negative and the new third effect is also negative provided the second-period subsidy is a strategic complement for first-period output. The government thus has an additional incentive to tax the firm in the first period to restrain it from "overproducing" strategically in order to increase the subsidy it receives in the second period.

As in the last section, the final step is to compare the outputs in the different equilibria. Once again, substituting the optimal subsidies (20) and (29) into the home firm's first-order conditions (9) and (27) gives exactly the same equations as in earlier cases (i.e., (13) and (14)). The fact that the government has two instruments, even though one of these is subject to strategic manipulation by the home firm, continues to

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17 The derivations make use of (22) and of the derivatives of the second-period profit function: \( \partial \pi_2/\partial y_2 = -b_2 x_2; \partial \pi_2/\partial c_2 = -x_2; \) and \( \partial \pi_2/\partial s_2 = x_2. \)
give it the ability to attain the same real equilibrium. The only difference is that a much lower subsidy (possibly even a tax) is required in the first period to induce the home firm to produce the optimal level of output. Summarising:

Provision 4: Comparing the government precommitment and sequential equilibria: outputs in each period, total welfare and the optimal second-period subsidy are identical; and the optimal first-period subsidy is lower in the sequential than in the government precommitment equilibrium, if and only if the second-period subsidy is a strategic complement for the home firm’s first-period output.

It follows from Propositions 3 and 4 that outputs and welfare are identical in all three equilibria. To compare the optimal subsidies in FPCE and SE, rewrite (29) as:

\[ s_1^s = -b_1x_1\psi_1 + \rho \lambda s_2^s(1+\sigma_2) \frac{dx_2}{dc_2}. \]  

(30)

where \( \sigma_2 \) was defined in (15). Comparing this with (12), it is clear that the condition for the optimal subsidy in SE to be less than that in FPCE is even more stringent than the condition in Proposition 4 for the optimal subsidy in SE to be less than that in GPCE.

Finally, the Appendix shows that \( s_1^s \) is likely to increase less rapidly in the rate of learning than \( s_1^f \). Summarising:

Provision 5: Comparing the full precommitment and sequential equilibria: outputs in each period, total welfare and the optimal second-period subsidy are identical; the optimal first-period subsidy is lower in the sequential than in the full precommitment equilibrium, if and only if \( \sigma_2 \), the elasticity of the second-period subsidy with respect to the home firm’s second-period output, is greater than \(-1\); and it is likely to increase less rapidly in the rate of learning.

6. Temporary Subsidies

So far, we have identified government precommitment with the case where the government sets the welfare-maximising level of the period-2 subsidy in the first stage of the game. However, an inherent feature of precommitment is that it can take an infinite number of forms. For example, Fudenberg and Tirole (1983) consider the case where the government precommits to a balanced-budget policy package, taxing output in the first period and subsidising it in the second. An alternative form of sub-optimal precommitment, more in keeping with the spirit of the infant-industry argument, is for the government to precommit to a zero subsidy in the second period. In this section we
consider the implications of this form of precommitment.

As before, we must still consider two different cases, depending on whether the firms can precommit to future output or not. Assume first they can, so we are in the case of FPCE. The relationship between changes in welfare and changes in outputs is given, as before, by equation (11). We set $s_2$ to zero in this equation and note that the government can influence future output only by controlling current output which affects future marginal cost: $dx_2 = (\partial x_2/\partial c_2)dc_2 = -(\partial x_2/\partial c_2)\lambda dx_1$. This gives the expression for the optimal temporary first-period subsidy in FPCE:

$$s_1^{FT} = -b_1x_1\psi'_1 + \rho \lambda b_2x_2\psi'_2 \frac{\partial x_2}{\partial c_2}.$$  \hspace{1cm} (31)

Comparing this with $s_f^P$ in equation (12), the new second term is positive, suggesting that the optimal first-period subsidy is greater when the second-period subsidy is constrained to be zero than in the full optimum. Intuitively, this is a standard type of second-best result: the inability to offer a positive subsidy to second-period output justifies a higher subsidy to its complement, first-period output.

A similar result holds for the optimal temporary subsidy in GPCE, when firms cannot precommit. Equation (17) immediately implies that this equals:

$$s_1^{GR} = -b_1x_1\psi'_1.$$  \hspace{1cm} (32)

Once again, the inability to subsidise in period 2 encourages a higher subsidy in period 1 relative to the fully optimal case, $s_f^G$ in (17). Moreover, because the firm chooses its current output before $y_2$ is chosen, it internalises the strategic effect on $y_2$. The only role left to the government is thus to choose the level of the first-period subsidy to exert a strategic effect on the foreign firm’s first-period output. That is why equation (32) is identical to the standard static Brander-Spencer formula.\footnote{This result is obtained by Gatsios (1989).}

Of course, to compare the different subsidies exactly requires knowledge of the output levels. It turns out that outputs in each period are identical under the two types of temporary subsidy. To see this, substitute from the optimal temporary subsidy in FPCE, (31), into the home firm’s first-period first-order condition in FPCE, (10), and set $s_2$
equal to zero in its second-period first-order condition (9):

$$H_1: \quad R^1_x - c_1 - b_1x_1\psi_1'/\psi_1 + \rho \lambda x_2 \left\{1 - b_2\psi_2' \frac{\partial c_2}{\partial c_2}\right\} = 0. \quad (33)$$

$$H_2: \quad R^2_x - c_2 = 0. \quad (34)$$

Exactly the same equations follow when similar substitutions are made in the case of GPCE. It follows that outputs in the two equilibria are identical. Once again, the fact that the government has the same number of instruments (one rather than two) allows it to attain the same real equilibrium, irrespective of whether firms can precommit to future output levels or not. Summarising:

Proposition 6: If the government precommits to a zero period-2 subsidy, then, comparing the full and government-only precommitment equilibria: outputs in each period, total welfare and the optimal second-period subsidy are identical; and (provided second-period outputs are strategic substitutes) the optimal first-period subsidy is lower in the government-only than in the full precommitment equilibrium.

However, comparison of (33) and (34) with the corresponding conditions in the full optimum, (13) and (14), does not allow us to draw unambiguous conclusions about the relative output levels in the two cases. Hence, although each of the formulae for the optimal temporary subsidies (31) and (32) contains an additional positive term relative to the corresponding formulae in the fully-optimal FPCE and GPCE, we cannot conclude in general that temporary subsidies must be higher than their permanent counterparts.

7. Optimal Subsidies with Linear Demands and Linear Learning

From a technical perspective, perhaps the most surprising feature of earlier sections is that we have been able to compare different equilibria without placing restrictions on the demand and learning functions. Nonetheless, it is still of interest to consider the special case of linear demands and linear learning in order to illustrate more forcefully some of the general results of previous sections. Consideration of the special case also allows us to examine explicitly the sensitivity of the different optimal subsidies to changes in the rate of learning and to explore the implications of relaxing other assumptions. The detailed results for each of the cases considered below are given in Table 1 and their
implications can be more vividly seen from Figures 1 to 5. In each figure, the optimal period-1 subsidies under different assumptions are given as a function of a parameter $\gamma$, defined as the rate of learning normalised by the inverse demand slope (a measure of the market size): $\gamma = \lambda/b$. In all cases, the optimal subsidies are illustrated for a range of values of $\gamma$ over which the government’s second-order conditions are satisfied; the exact range differs with the assumptions made but is generally between zero and at least 0.85. A violation of such conditions should be interpreted as a failure of the assumptions of the model, in particular the assumption that there is no entry or exit of firms.

A. The Base Case

Figure 1 compares the optimal subsidies under five different assumptions about precommitment for a base case of Cournot duopoly with no discounting (so $\rho$ equals one). Units of measurement have been chosen such that the static Brander-Spencer subsidy (which continues to apply when the rate of learning is zero) equals unity. (For reference, in the static Brander-Spencer model, output equals $\phi/2b$ at the optimum and the optimal subsidy equals $bx/2$ or $\phi/4$. Here, $\phi$ is defined as $a-2c+c^*$, a measure of the relative cost competitiveness of the home firm, which must be positive for an interior solution.)

The outstanding feature of Figure 1 is that all four of the subsidies which assume any kind of government precommitment are increasing in the rate of learning. By contrast, the optimal subsidy in sequential equilibrium is decreasing in the rate of learning and becomes negative at $\gamma$ equal to 0.5, well within the admissible range. Clearly, it makes a much greater difference to assume that government decisions are sequential than to vary the assumptions made about the type of government commitment (whether temporary or fully optimal and whether in the context of sequential behaviour or precommitment by firms). The difference between the fully-optimal FPCE and GPCE subsidies is exactly one third of the difference between the FPCE and SE subsidies. Summarising:

Proposition 7: With linear demands and linear learning, the optimal period-1 subsidy is increasing in the rate of learning if any intertemporal precommitment is possible. However, it is decreasing in the rate of learning in sequential equilibrium.

The two temporary subsidies in Figure 1 are also of interest since we were unable to

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19 The calculations underlying Table 1 are lengthy but straightforward. Details are sketched in an appendix, available on request from the authors.
rank them with the corresponding fully-optimal subsidies in the general case. In fact, each of them lies above the corresponding fully-optimal subsidy over the range of $\gamma$ illustrated, although they are overtaken at still higher values of $\gamma$ which are nonetheless consistent with the second-order condition.\textsuperscript{20}

In the remainder of the paper we concentrate on the fully-optimal subsidies. It may be checked by inspecting Table 1 that (except in row 5, discussed in Section 7.E below) the average of the FPCE and SE subsidies equals the static Brander-Spencer subsidy. Hence we lose nothing by concentrating on a comparison of the GPCE and SE subsidies.

B. Sensitivity to the Discount Factor

Figure 2 shows how the GPCE and SE subsidies are affected as the discount factor falls below unity. Clearly, both converge towards the value of unity (independent of $\gamma$) which they take when $\rho$ is zero (the case where agents do not care about the future). Discounting the future thus weakens but (provided $\rho$ is positive) does not eliminate the tendency for the optimal subsidy to increase with $\gamma$ when government precommitment is possible and to decrease with it otherwise. This justifies our concentrating on the case of zero discounting in the remainder of the paper.

C. Sensitivity to the Number of Foreign Firms

Dixit (1984) has shown that, as the number of foreign firms increases for a given number of home firms, so does the optimal subsidy: with one home firm, $m$ foreign firms and no learning, the static optimal subsidy (using the same normalisation, $\phi/4=I$, as before) is $2m/(m+1)$.\textsuperscript{21} This can be seen along the vertical axis in Figure 3, where the optimal subsidy rises from 1 towards 2 as $m$ rises. The figure also shows that increasing the number of foreign firms makes another difference: both types of subsidy become more sensitive to the rate of learning. This makes the second-order condition more restrictive: the more foreign firms and the higher the rate of learning, the more likely is it

\textsuperscript{20} The (normalised) temporary subsidy in FPCE equals $(3+2\gamma)^2/(9-8\gamma^2)$, which is overtaken by the fully-optimal FPCE subsidy at a value for $\gamma$ of 0.866. The (normalised) temporary subsidy in GPCE equals $(9+8\gamma)/(9-8\gamma^2)$, which is overtaken by the fully-optimal GPCE subsidy at a value for $\gamma$ of 0.968.

\textsuperscript{21} The normalisation is now less innocent, since the cost competitiveness term $\phi$ depends on the number of foreign firms: $\phi=a-(m+1)c+mc^*$. Strictly speaking, Figure 3 is thus drawn on the assumption that $c=c^*$. 
that a domestic subsidy will squeeze some foreign firms out of the market. Otherwise, the qualitative relationship between the GPCE and SE optimal subsidies is robust to relaxing the assumption of a single foreign firm.

D. Sensitivity to the Degree of Product Differentiation

We have assumed so far that the home and foreign firms produce identical goods. To investigate whether this affects the conclusions, we have examined the case where demands are parameterised as follows:

\[ p_t = a - b(x_t + ey_t), \quad q_t = a - b(e x_t + y_t), \quad t=1,2. \]  \hspace{1cm} (35)

Here \( e \) can be interpreted as an inverse measure of the degree of product differentiation. The resulting optimal subsidies in GPCE and SE are given in the fourth row of Table 1 and are illustrated in Figure 4.\(^{22}\) Once again, the results are robust to this relaxation of our assumptions, with the responsiveness of both types of subsidy to the rate of learning falling as products become more differentiated.

E. Sensitivity to the Social Cost of Funds

Section 5 showed that in sequential equilibrium the government was able to attain the same levels of output and welfare as with precommitment but with a lower period-1 subsidy. This immediately suggests that, if the social cost of funds exceeds unity, then welfare may be higher in SE than in equilibria with precommitment. Of course, this is not inevitable, since a change in the value of the social cost of funds changes all the optimal subsidies. To investigate this issue, we follow Gruenspecht (1988) and Neary (1994) and use a parameter \( \delta \) to measure the social cost of funds, so that the welfare function in each period becomes:

\[ W_t = \pi_t - \delta x_t = (p_t - c_t - (\delta - 1)s_t)x_t. \]  \hspace{1cm} (36)

The resulting optimal subsidies are given in the fifth row of Table 1 and their values in GPCE and SE are illustrated in Figure 5. In both cases, a higher social cost of funds increases the incentive for the government to tax rather than to subsidise the firm, since

\(^{22}\) As in previous diagrams, the subsidies are normalised such that their value in the absence of learning, \( e^2\phi'/4(2-e^2) \), equals unity. This ignores the dependence of the cost competitiveness term \( \phi' \) (which is defined in Table 1) on \( e \).
the opportunity cost of subsidies is higher. In the absence of learning, the (normalised) optimal subsidy equals \((4 - 3\delta)/(3\delta - 2)\), which falls from 1.0 towards -1.0 as \(\delta\) rises. For low values of \(\delta\), the optimal subsidies behave in the same way as in earlier cases: that in GPCE increases in \(\gamma\) while that in SE decreases in it. But as \(\delta\) rises, both these relationships are reversed. In particular, the second-period optimal subsidy in SE is proportional to \((4 - 3\delta)\). Higher values of \(\delta\) thus lead the home firm to anticipate a lower subsidy, or even (for \(\delta\) greater than 4/3) a tax, next period, so reversing the negative relationship between \(\delta\) and \(\gamma\) which applies when \(\delta\) is equal or close to unity.

Finally, Figure 6 investigates the conjecture that, because the period-1 subsidy is lower in SE than in GPCE, the level of welfare may be higher when the government cannot precommit and \(\delta\) is greater than one. (Recall that welfare is the same in both equilibria when \(\delta\) equals one.) The conjecture is refuted for high values of \(\delta\) but for values between 1.0 and 1.2, it is confirmed. Because of the social cost of raising revenue, there is therefore no guarantee that society loses because of the government’s inability to precommit to future policies.

8. Optimal Policy in Bertrand Competition

The final case we consider is where firms compete on price rather than quantity.\(^{23}\) Eatón and Grossman (1986) have shown that this reverses the Brander-Spencer result in a one-period game: provided prices are strategic complements (the "normal" case in Bertrand competition), the optimal policy is an export tax rather than a subsidy. To explore how this argument is affected when the home firm enjoys learning by doing, we have calculated the optimal subsidies when firms compete on price and demands are given by the linear, differentiated products specification, (36).\(^{24}\) The expressions for the

\(^{23}\) We confine attention to the cases where firms compete on either prices or quantity in both periods. An obvious extension is to the case considered by Kreps and Scheinkman (1983), where firms choose capacity outputs in the first period and prices in the second. However, this seems less appropriate in a learning-by-doing context, where periods should be thought of as years rather than weeks or months in length.

\(^{24}\) The qualitative results given here also hold with non-linear demands, subject to additional qualifications analogous to those made earlier in the Cournot case. Following Eaton and Grossman, we assume throughout that products are sufficiently differentiated that an equilibrium in pure strategies exists.
optimal subsidies are given in the final row of Table 1 and those for GPCE and SE are illustrated (for different values of the product differentiation parameter $e$) in Figure 7.\textsuperscript{25}

The results in Figure 7 are very different from those in the Cournot quantity-competition case of previous sections. Now, the optimal subsidy is negative and decreasing in $\gamma$ in FPCE but in SE it is increasing in $\gamma$ and, for high but plausible values of $\gamma$, it actually becomes positive. As for the optimal subsidy in GPCE, it lies between those in FPCE and SE and is increasing in $\gamma$, as in the Cournot case. These results, at first sight paradoxical, can be explained in terms of the same mechanisms explored in earlier sections. In FPCE, there is no basis for government intervention other than that in the analysis of Eaton and Grossman: the government taxes the home firm in order to restrain it from competing too vigorously and driving down its output price. This argument is not affected by higher rates of learning but the level of output is higher so the tax (which is positively related to output) rises too. In GPCE, the home firm anticipates the future actions of its foreign rival and tends to reduce its first-period output accordingly; this provides the government with a motive for subsidisation which is sufficient to reverse the Eaton-Grossman motive for taxation. Finally, in SE, the firm anticipates that it will be taxed in the second period. In order to reduce the tax it will face, it has an incentive to reduce output still further in the first, learning, period. Anticipating this tendency to "underproduce," the government counteracts it by offering a subsidy relative to the Eaton-Grossman benchmark (normalised at minus one in Figure 7). And, for sufficiently high values of $\gamma$, this motive for subsidisation can outweigh the Eaton-Grossman motive for taxing. Once again, sequential decision-making by government tends to reverse the standard conclusions reached on the assumption that the government can precommit.

9. Summary and Conclusions

This paper has examined the implications for strategic trade policy of different assumptions about precommitment. We considered the choice of export subsidies in a dynamic oligopoly game with learning by doing. In general, and with surprisingly mild qualifications, we found that in Cournot competition the optimal first-period subsidy is

\textsuperscript{25} The normalisation is now that $e^2 \phi"/4(2-e^2)$ equal unity. As in Figure 4, this ignores the dependence of $\phi$" (defined in Table 1) on $e$. 

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lower if firms cannot precommit to future output than if they can; and it is lower still if the government cannot precommit to future subsidies. In the linear case the optimal subsidy is increasing in the rate of learning with precommitment but *decreasing* in it if the government cannot precommit. Strategic considerations thus strengthen the infant-industry argument if the government can precommit to future subsidies but reverse it in the absence of precommitment.

So far, we have remained neutral on which combination of assumptions about precommitment is most plausible in the context of learning by doing. It should be clear, however, that we view the case which we have called "sequential equilibrium" as the most reasonable approximation of reality, since it is consistent with two stylised facts: on the one hand, governments are able to set subsidies which will remain in force for a period of weeks or months during which decisions on outputs will be made; on the other hand, they are unable to precommit to the subsidies which will prevail a few years ahead, when the firm has reached its mature phase. At the level of abstraction typically adopted in this literature, this seems a reasonable description of the legislative and administrative environment in which current policy decisions must be taken. Naturally, readers who do not share this prejudice will interpret our results differently but they cannot disagree with our general conclusion that the choice of assumptions about precommitment is crucial for both the sign and the magnitude of the optimal policy.

It is instructive to compare our approach to the macroeconomics literature, where precommitment and time consistency have been much discussed since the work of Kydland and Prescott (1977). Macroeconomic theorists have devoted much attention to devising measures which would enable governments to precommit to future macroeconomic policies, such as constitutional amendments, conservative central banks or a social security system. The starting point for such analyses has been the fact that welfare is higher with government precommitment than without. By contrast, in our framework, welfare is generally independent of the assumptions made about precommitment and, in the case of Section 7.E with a social cost of funds greater than unity, it may be higher when precommitment is not possible.

Our model is far from being the last word on strategic trade policy with learning by doing. By ignoring exit and entry of firms, and assuming that learning is fully internalised by the firm with no spillovers to rival firms at home or abroad, we have
neglected many of the key issues in real-world policy discussions. The assumptions that learning ceases after one period, that the foreign firm does not learn and that the foreign government does not subsidise are other features of our model which suggest that its specific policy conclusions should be interpreted with caution. All these extensions deserve to be explored and should be tractable in extended versions of the model we have considered.\footnote{Leahy (1992) considers foreign learning and foreign subsidies in a linear model.}

A different set of issues raised by this paper concerns the relevance of our findings to other issues in microeconomics. Similar considerations apply to any area in which governments have an incentive to intervene in future periods and in which the extent of such intervention can be influenced by current private sector actions. Applications to topics such as capacity choice, research and development, advertising and natural resources immediately suggest themselves. At a deeper level, our results suggest the need for a fundamental rethink of economic policy in dynamic environments. The fact that governments cannot precommit to future policies may mandate optimal intervention which is opposite in sign to the intervention which would be justified on the basis of a static model or (equivalently) a model in which government precommitment is possible.\footnote{For similar suggestions, see Hammond (1993).}

Appendix

Proof of Proposition 1: To prove that outputs are increasing in the learning parameter \( \epsilon \), totally differentiate equations (13) and (14) to obtain:

\[
\begin{bmatrix}
\xi_1 - b_1 \psi_1' \sigma_1 + \rho x_2 \lambda_x \\
\lambda
\end{bmatrix}
\begin{bmatrix}
\frac{dx_1}{dx} \\
\frac{dx_2}{dx}
\end{bmatrix}
= \begin{bmatrix}
\rho x_2 \lambda_e \\
c_2(\epsilon)
\end{bmatrix} \, d\epsilon. \tag{37}
\]

(Here \( \xi_1 \) is defined as \( R_L' + R_c' \psi_1' \), which is negative, and \( c_2(\epsilon) \) is the derivative of \( c_2 \) with respect to \( \epsilon \).) Since the government is in effect choosing \( x_1 \) and \( x_2 \) to maximise welfare, the second-order conditions are that the diagonal terms in the left-hand-side coefficient matrix be negative and that the determinant of the matrix be positive. Calculating the changes in \( x_1 \) and \( x_2 \), making use of these properties and the fact that, from (3), \( \lambda_e > 0 \) and
\( c_t < 0 \) gives the desired result. Finally, differentiating the welfare function (6) and applying the envelope theorem:

\[
\frac{dW}{d\epsilon} = \frac{\partial W}{\partial \epsilon} = -\rho x_2 c_{2e} > 0. \tag{38}
\]

Hence welfare is also increasing in the rate of learning.

**Proof of Proposition 3:** The only part of the proposition not proved in the text is the effect of an increase in \( \epsilon \) on the optimal first-period subsidy. From (17):

\[
\frac{ds^G_1}{d\epsilon} - \frac{ds^F_1}{d\epsilon} = \rho \lambda_1 \left\{ s^F_2 \frac{d}{d\epsilon} \left( \frac{dx_2}{dc_2} \right) + s^G_2 \frac{dx_2}{dc_2} \right\} + \rho s^F_2 \frac{dx_2}{dc_2} \left( \lambda_\epsilon + \lambda_2 \frac{dx_1}{d\epsilon} \right). \tag{39}
\]

The first term is indeterminate in sign but may be ignored for small values of \( \lambda \) (and it vanishes for linear demands); the second term is negative; and the third and fourth terms combined are negative provided \( d\lambda/d\epsilon \) (the total effect on the rate of learning of an increase in \( \epsilon \)) is positive, which we assume.

**Proof of Proposition 5:** Again, all the results are proved in the text except for the effect of an increase in \( \epsilon \), on the optimal first-period subsidy. From (29):

\[
\frac{ds^G_1}{d\epsilon} - \frac{ds^F_1}{d\epsilon} = \rho \lambda_1 \left\{ s^F_2 \frac{d}{d\epsilon} \left( \frac{dx_2}{dc_2} \right) + s^G_2 \frac{dx_2}{dc_2} \right\} + \rho s^F_2 \frac{dx_2}{dc_2} \left( \lambda_\epsilon + \lambda_2 \frac{dx_1}{d\epsilon} \right) \tag{40}
\]

\[-\frac{\rho x_2 \Psi'/\mu}{x_1} \frac{dx_1}{d\epsilon} - \frac{\rho \Psi'/dx_2}{d\epsilon} \]

The first four terms are identical to those in (39) except that they involve the total derivative \( dx_2/dc_2 \) rather than the partial derivative \( dx_2/dc_2 \). Hence these terms are more negative than the corresponding terms in (39), at least for small \( \lambda \). The final two terms are also negative, given two conditions. The first is Lemma 3, which ensures that \( \Psi' \) is positive. The second is that \( \mu \), defined as \( x_1 \Psi''/\Psi' \), is not too negative; i.e., that the government's reaction function is not "too concave."
References


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<td>( \frac{4-3\delta + 2(\delta-1)\gamma}{3\delta^2 - 2(\delta-1)\gamma} \frac{\phi}{4} )</td>
<td>( \frac{(4-3\delta)(3-2\gamma)}{3(3\delta^2 - 2(\delta-1)\gamma)} \frac{\phi}{4} )</td>
<td>( \frac{(4-3\delta)(1-\theta^2)\gamma - 2(6-5\delta)\delta^2 \gamma^2 + 8(\delta-1)\theta^4 \gamma^3}{3\delta^2 - 2(\delta-6)\theta^2 \gamma^2} \frac{\phi}{4} )</td>
</tr>
<tr>
<td>Bertrand</td>
<td>( \frac{1}{1-\beta \gamma / 4(1-\epsilon^2)} \frac{e^2 \phi''}{4\beta} )</td>
<td>( \frac{1-\beta \gamma / ((4-\epsilon^2)(1-\epsilon^2))}{1-\beta \gamma / 4(1-\epsilon^2)} \frac{e^2 \phi''}{4\beta} )</td>
<td>( \frac{1-\beta \gamma / 2(1-\epsilon^2)}{1-\beta \gamma / 4(1-\epsilon^2)} \frac{e^2 \phi''}{4\beta} )</td>
</tr>
</tbody>
</table>

### Table 1: Values of the Optimal Period-1 Subsidy under Different Assumptions

- **\( \gamma \):** Rate of learning, normalised by the demand slope (\( \gamma = \lambda / \beta \))
- **\( \rho \):** Discount factor (\( \rho \leq 1 \))
- **\( m \):** Number of foreign firms
- **\( e \):** Inverse measure of product differentiation (\( 0 \leq e \leq 1 \))
- **\( \delta \):** Social cost of funds (\( \delta \geq 1 \))

\( \phi = \frac{a-(m+1)c+mc^*}{\delta/3\delta-2} \)

\( \phi' = \frac{(2-e)\alpha-2c+e^*}{\delta/3\delta-2} \)

\( \phi'' = \frac{(\beta-e)\alpha-\beta e+c+e^*}{\delta/3\delta-2} \)

\( \beta = 2-e^2 \)

\( \theta = \frac{\delta}{3\delta-2} \)
Figure 3: Sensitivity of Optimal Subsidies to Number of Foreign Firms

(Depicted are period subsidy with one foreign firm normalized to equal one)

Figure 4: Sensitivity of Optimal Subsidies to Degree of Product Differentiation

(For each value of \( s \), the optimal per-period subsidy is normalized to mean one)
Figure 5: Sensitivity of Optimal Subsidies to the Social Cost of Funds

Figure 6: Sensitivity of Welfare to the Social Cost of Funds
Figure 7: Optimal Subsidies in Bertrand Competition

(For each value of $\alpha$, the optimal one-period subsidy is normalized at minus one)