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A multisector model of efficiency wages

Frank Walsh
Department of Economics
University College Dublin
Belfield Dublin 4
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Tel: +353-1-706 8697
Email: fwalsh@ollamh.ucd.ie
(1) Introduction

This essay fills a gap between the theoretical and empirical efficiency wage literature. Empirical papers such as Krueger and Summers (Romer 1988) or Katz and Summers (Brookings Papers 1989) often analyse wage differentials across sectors as evidence of efficiency wage payments, while the theoretical literature does not have a model with many sectors. Shapiro and Stiglitz (A.E.R. 1984) develop a one sector model where firms have an exogenously given monitoring technology and looked at the wage needed to extract a given level of effort from workers. Involuntary unemployment was necessary in equilibrium to make the threat of dismissal matter to workers and prevent shirking. While empirically observed wage differentials have not been explained by competitive models, they have not been directly linked with a theoretical model of efficiency wages.

Efficiency wage theory says the wage a firm will choose for a given amount of effort depends on the technology used by the firm. For example firms may use wage premiums as a substitute for costly monitoring. Workers value these high wage jobs and won't risk losing them by shirking. Additionally workers who do not expect to be with the firm for very long have a greater incentive to shirk, so firms may pay high wages to prevent this. Chapter 1 generalises the Shapiro and Stiglitz model to many sectors with different monitoring technologies and turnover rates. Firms in each sector choose the wage and effort to maximise profits.
Jobs in sectors where monitoring is difficult, turn out to be more valuable and we get a pattern of wage and effort differentials in equilibrium. Equilibrium wages in all sectors (including the competitive sector) are a function of wage contracts in all other sectors. Better jobs have low monitoring and a high effort requirement. Higher turnover sectors have higher wages, but they just compensate for the increased probability of job termination, there are no rents paid. Firms choose the optimal level of effort, rather than taking it as a given. Sectors with high monitoring will have high effort levels in equilibrium.

Kimbal's (1994) one sector results that there is no cyclical pattern to the wage differentials and that the pattern of differentials is independent of the pattern of employment growth across sectors are still true in the multisector model. If unemployment changes, wages in all sector change by the same amount. Many empirical studies, such as Krueger and Summers or Katz and Summers have found the pattern of wage differentials to be remarkably stable over time. Allowing on the job offers also makes the model theoretically consistent with full employment in the labour market.

The main theoretical critique of these models is the bonding argument. This says that by asking workers to post bonds, which they will receive on the termination of employment, or forfeit if caught shirking, firms can avoid paying wage premiums. Dickens et al. (1989) provide a simple model outlining the bonding critique and the counter arguments. The first two objections to the bonding critique are that workers may face liquidity constraints in posting bonds or that there may be a moral hazard problem that the firms will use the bonds as an instrument to blackmail workers once posted. If this were true we should still expect to observe bonding at some level across firms even if it were not complete. In addition we should see large firms with established reputations paying lower wages and engaging in bonding while these tend to be the high wage firms. The conclusion is that legal restrictions on penalties to shirkers and social norms on what workers find acceptable as a contract are the main obstacles to the bonding critique. Courts often will not enforce contracts that impose costs on the worker far in excess of the damage they caused. Both courts, individual workers and managers may find that ex post the provisions of the contract are cruel and unfair. Managers may find them difficult to impose without facing significant and costly disruption.

These arguments are made credible by the empirical facts. We do indeed observe widespread monitoring. 7% of workers in the U.S. nonagricultural private sector were inspectors or supervisors in 1983 (Dickens et al. 1989). We do not however, commonly observe either employees posting bonds or selling jobs.
(3) workers optimal effort function

In this section I derive the workers effort supply curve. Later it will be combined with effort demand from the firms problem to solve for equilibrium. Identical workers have the following utility function

\[ U = w - g(e) \quad e = \text{effort} \quad w = \text{Wage} \]  \hspace{1cm} (1)

\( g() \) is a convex function giving the disutility of effort. If a worker gets a job in sector \( i \) they get a wage stream \( w_i \) over time. In return they are expected to exert a specified effort level \( e_i \). If they are caught shirking they are fired immediately.

We think of a job in any sector as an asset in this model. The value of an asset at a point time "t" can be written as

\[ rA(t) - D(t) + \Delta(t) \]  \hspace{1cm} (2)

This says the value is the dividend stream plus the expected rate of capital gain divided by the discount rate \( r \).

We use this equation to define the value of the job if you are a shirker or a non-shirker.

\[ rV_i^*(t) = w_i + (b_i + q_i)[V_i(t) - V_i^*(t)] + \Delta(t) \]  \hspace{1cm} (3)

\[ b_i \] is the arrival rate of job separations due to exogenous factors (layoffs, etc.). \( q_i \) is the arrival rate of supervisors (or monitoring intensity).

Since \( q_i \) is the same for any effort level below the specified level \( e_i \), if you shirk you will exert no effort. If you are caught however, you will be fired immediately. \( V_i \) is the value of unemployment which is defined below.

The right hand side of equation (3) is the dividend(wage) plus the probability of job loss times the change in asset value if you lose your job, plus the change in value of being an employed shirker over time. These last two terms are the expected rate of capital gain(loss) associated with this state. The equation for a non-shirker is of the same form. The disutility of effort is subtracted out of the wage since you exert effort, but the probability of job loss from being caught shirking \( q_i \) is zero. Once firms choose the desired effort level they must pay a wage such that workers have no incentive to shirk. The lowest wage that satisfies the No Shirk Condition is when \( V_i^* = V_i \).

We can use this condition to solve for the rent associated with a job in any particular sector.

\[ V_i^* - V_i = \frac{g(e_i)}{q_i} \]  \hspace{1cm} (5)
A job that specifies high effort, but where monitoring is difficult is the best. We note for example that a competitive sector where monitoring is perfect and \( q = w \), will have no rents. The exogenous probability of job loss will not effect the value of the job. You may get paid more in a high turnover job, but this just compensates for the higher probability of being fired.

The Value of Unemployment (Welfare plus other job offers) is

\[
rv_u - \bar{w} + \sum_{j=1}^{n} a_j[V_j(t) - V_u(t)] - \bar{V}_u(t) \tag{6}
\]

\( \bar{w} = \text{Welfare payments} \quad a_i = \text{probability of a job offer in sector } i \)

Initially we assume you must enter the unemployed pool to switch jobs. Now using equation (5) we can rewrite the above equation as

\[
rV_u(t) - \bar{w} \sum_{j=1}^{n} a_j \frac{g(e_j)}{q_j} \dot{V}_u(t) \tag{7}
\]

Once the N.S.C. is satisfied workers will not shirk in equilibrium, so we treat the expression for \( rV_u \) as the equilibrium value of a job. Next we can use equation (6) to solve \( V_u \) in terms of \( V_u \). Now, using equation (7) above for \( V_u \) we can solve for the relationship between wages, effort and the exogenous parameters such that the worker will not shirk. In other words we have the workers effort supply relationship.

\[
w_i(t) = g(e_i)[1 - \frac{b_i}{q_i}] \cdot \bar{w} \tag{8}
\]

\( X \) is the value of job offers the worker receives in unemployment, or

\[
X = \sum_{j=1}^{n} a_j \frac{g(e_j)}{q_j} \tag{9}
\]

We note that \( X \) is constant across sectors, a janitor who loses a job in a large manufacturing plant where monitoring is difficult, has no reason to do better searching for jobs than a janitor who worked in a small restaurant. Figure one graphs equation (8) illustrating a workers effort supply curve as he moves from sector one to sector two, where monitoring intensity is higher.
Job switching without unemployment

In the previous section, we gave the arrival rate of job separations, whether voluntary or involuntary, but for reasons unrelated to your performance. What I essentially do in this section is to net out voluntary job separations from the total term, where you only quit if you get a better offer. The distinction between the two types of job separation is important since in Shapiro and Stiglitz’s model high turnover is associated with high wages to satisfy the N.S.C., while in Stiglitz (1985) for example firms increase the wage to lower turnover. It will be clear in a later section that including on the job offers gives us an equilibrium that is consistent with full employment in the labor market. I allow workers to receive job offers while employed. Assuming the wage in all sectors satisfies the N.S.C. we can change equations (3) and (4) to allow for the probability of being offered a better job while employed in sector “i”. Getting offered jobs in worse sectors gives zero additional utility. The term below is added onto equations (3) and (4) reflecting the value of on the job offers (d is the arrival rate of job offers from sector j to employed workers).

\[ Y_i = \sum_{j=1}^{n} d_j \left( \frac{g(e_j)}{q_j} - \frac{g(e_i)}{q_i} \right) \]

(11)

Where \( m \ldots n \) are the better sectors.

Again using the N.S.C., we get the wage effort relationship

\[ w_i(t) = g(e_i) \left[ 1 - \frac{b_i * r}{q_i} \right] + \bar{w} + X - Y_i \]

(12)

The availability of on the job offers means that workers do not forego completely the possibility of getting good jobs when they accept a low paying job. This means that for any given effort level, the N.S.C. can be met at a lower wage in the bad sectors. If you already have a good job, on the job offers are not valuable and the N.S.C. remains unchanged. Thus the \( Y_i \) term increases as we move into worse jobs and allowing on the job offers actually lowers wages in the worse sectors.

In the above analysis we assumed offers flowed in costlessly, but we could impose search costs without changing the results. Say the probability of getting a job offer in sector “i” is a function of constant hourly search costs incurred by a worker in sector i, that is \( d_i(s_i) \), where \( s_i \) is total search costs of a worker in sector i incurred looking for a job in sector j. We would expect the marginal cost to be higher in the better sectors since the opportunity cost of an hours wages is bigger. Assume that marginal benefit of time searching is diminishing in each sector, but the marginal benefit of search in a better sector lies to the left of that.
in a worse sector (since there is less to gain in the better sector). Workers in the worst sectors would engage in the most search activity, but the benefit of on the job offers net of search costs would still be greatest in the worst sectors. Allowing search activity in this way offsets the effect of on the job offers on wages to some degree. We would just subtract $s_i$ out of $Y_i$ in equation (12). The $(Y_i - S_i)$ term is still biggest in the lowest sector and getting smaller as we progress into better sectors, reaching zero in the best sector.

(4) The firms profit maximisation problem

The point of this section is to recognise that in many cases firms can react to the monitoring difficulty not just by paying higher wages, but also by specifying lower effort levels for the job, which is an alternative way (and up to a point cheaper) of making workers value the job. This will not always be true, the example of an airline pilot used by Ramana and Rowthorn (1991) comes to mind. Clearly the firm must demand high effort levels in this case even at the cost of high wages and substituting into higher employment at lower effort is not an option. Competitive firms maximise profits:

$$\max_{e, P_i} P_i e(1) - wN$$

(13)

$P_i$ is the output price in sector $i$, $F()$ is the production function and $N$ is the number of workers. The first order conditions imply

$$w = e(1)$$

(14)

The elasticity of effort with respect to the wage will be one. In other words firms increase the wage to a point where the marginal cost of a one percent increase in efficiency units is the same, whether it comes from hiring one percent more workers, or increasing effort by one percent with higher wages. Ramana and Rowthorn (1991) show that if the relationship between effort and employment is given a more general form $F(N, e(1))$, which allows for different cases (such as the case of the airline pilot where the worker can damage the firm) we still get an equilibrium where the elasticity of the effort with respect to wages equals the ratio of the output labour elasticity over the output effort elasticity.

(5) Combining the workers and firms problems

I use equation (8) (The effort supply curve) as the $e(1)$ function the firm uses to choose the wage in equation (13) above. For simplicity I rewrite equation (8):

$$w = g(e)A_i - c$$

(15)

Note that $A_i$ is increasing with turnover in sector $i$ and decreasing with monitoring intensity.
Setting the elasticity of effort from a change in wages equal to one we get the equilibrium illustrated in figure two where a ray from the origin (effort/wage) equals the slope of the effort supply curve.

This condition implies the following relationship.

\[ \frac{C}{A_j} = \alpha g'(e) - g(e) \]  

(16)

If \( g(e) \) is convex (which is a necessary condition to get a solution to the firms problem), an increase in monitoring is associated with an increase in effort in equilibrium, and an increase in turnover is associated with a fall in effort.

As long as it is not easy to elicit additional effort: The elasticity of the disutility of effort with respect to effort is increasing, wages will fall in the high monitoring sectors.

(6) Effect of labour market conditions

We see in this section that since there are many different sectors we do not need unemployment as a threat to ensure non-shirking. Additionally the pattern of differentials is independent of wage differentials or whether different sectors have different employment growth rates.

In the Shapiro-Stiglitz framework unemployment was the "worker discipline device" and zero unemployment meant workers would certainly shirk since they could instantaneously find another job if fired.

Bulow and Summers have a two sector model with a fixed wage competitive sector that could clear the market. The model still relies on equilibrium unemployment though, since workers must enter the unemployment pool to switch jobs. Unemployment is rationalized by assuming heterogeneous workers, that is it is assumed that some workers would sooner accept unemployment and the chance of a good job than the competitive job which foregoes such opportunities. However there is no clear empirical support for this rationale.

To look at the effect of labour market conditions, first we define the growth of employment in any sector i:
The first term is the arrival rate of job offers in sector \(i\) to each worker in the unemployed pool, times the size of the unemployed pool. This just goes to zero in the full employment case. \(b_i\) is job losses to workers in sector \(i\) and \(f_i\) is job offers to workers in sector \(i\) from better sectors (which would be accepted). The final term is the inflow of employed workers from other (worse) sectors.

The full employment equilibrium is where the first term is zero and the summation of these growth rates across sectors is zero for a given labour force \(N\). It might be reasonable to expect some frictional unemployment for example, but the point here is that the model is perfectly consistent with full employment, there is no reason to expect unemployment in this model that we would not find in a standard competitive model. \(f_i\) is the arrival rate of job offers to each worker in sector \(i\) which they would quit for and \(b_i\) the involuntary job separation rate. The final term represents job offers to employed workers in all sectors not as good as sector \(i\) (these represent inflows to this sector).

We can divide by employment in this sector to get the growth rate of employment:

\[
L_i \cdot a_i(t)[N(t) \cdot L_i(t)] \cdot (b_i + f_i) L_i(t) \cdot \sum_{j=1}^n d_j L_j
\]

\[
\lambda_i(t) \cdot \frac{L_i(t)}{L_i(t)} = \frac{a_i(t) u(t)}{\theta_i} - (b_i + f_i) \cdot \sum_{j=1}^n d_j \frac{L_j}{L_i}
\]

\(\theta_i\) is industry \(i\)'s employment as a share of the labour force.

We can use the above equation to solve for the equilibrium relationship between acquisition rates, unemployment, employment growth and the exogenous probability of job loss.

\[
a_i(t) = \frac{\theta_i}{u(t)} \left( \lambda_i(t) \cdot b_i + f_i \cdot \sum_{j=1}^n d_j \frac{L_j}{L_i} \right)
\]

For any given disutility of effort function \(g(e)\), this can be combined with the workers objective function, equation (8), to give the effort wage relationship that satisfies the N.S.C and takes account of labour market conditions:

\[
\omega_i(t) = g(\theta_i) A_i \cdot \frac{1}{u(t)} \sum_{j=1}^n \theta_j [b_j + f_j \cdot \sum_{k=1}^n d_k \frac{L_k}{L_i}] g(e_i) \cdot \frac{1}{q_i}
\]

Note from equation (20) that wages in any sector will change by the same amount with a change in unemployment. In other words, the pattern of wage differentials will not vary cyclically or as industries grow at different rates. The stability of the pattern of wage differentials over time is one of the striking features of the empirical literature.
(7) Simulated wage differentials

Having developed the equilibrium model above we can simulate how large we should expect wage differentials to be for reasonable parameter values. To do this we note that imposing the Solow condition on equation (8) gives:

\[ w = gA + c - g'Ae \]  

This lets us define percentage wage differentials as follows:

\[ w_j - w_i = \frac{gA_j}{w_i} \]  

(22)

\[ \frac{E_i}{E_j} \]

(23)

\[ E = 1 + \frac{c}{w - c} \]

The table below looks at the wage differential in equation (22) assuming industry "j" is the competitive sector. This means monitoring is perfect and \( A_j = 1 \). We assume the arrival rates of supervisors and exogenous job separations follow a Poisson process and \( P_j \) and \( P_i \) are the probabilities of one of these arrivals respectively, in any given year. Assume the disutility of effort in the non-competitive sector is 10% higher than the competitive sector (Note this implies an effort differential of less than 10% since \( E > 1 \)). To get values for \( E_i \) in equilibrium we can make assumptions about what fraction of the wage is given by opportunities foregone in unemployment (including welfare payments). Table one assumes benefits of unemployment represent 20% of the wage and table two assumes 50%.

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<th>( P_1 )</th>
<th>( P_2 = 0.1 )</th>
<th>( P_2 = 0.2 )</th>
<th>( P_2 = 0.3 )</th>
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<tr>
<td>0.7</td>
<td>14%</td>
<td>16%</td>
<td>24%</td>
</tr>
<tr>
<td>0.8</td>
<td>8%</td>
<td>9%</td>
<td>11%</td>
</tr>
<tr>
<td>0.9</td>
<td>0%</td>
<td>1%</td>
<td>3%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( P_2 = 0.1 )</th>
<th>( P_2 = 0.2 )</th>
<th>( P_2 = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>9%</td>
<td>10%</td>
</tr>
<tr>
<td>0.8</td>
<td>5%</td>
<td>6%</td>
</tr>
<tr>
<td>0.9</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

In most cases the model predicts wage differentials of less than 10% for any sector compared to the competitive sector. That is except for jobs which combine low levels of monitoring, a high probability of job separation and where the firm cannot easily substitute away from workers effort. While it may be possible to think of cases where this combination occurs it is unlikely to be true except in a small number of cases. It should be noted that the above wage differentials are deviations from the competitive sector (the lowest wage sector). In the empirical literature wage differentials are calculated as deviations from the mean whereas here they are the deviation from the lowest wage sector, this implies that the wage differentials here are over-stated compared to differentials we would compare to in the empirical literature.
(8) Minimum wages

Higher welfare payments will increase wages and effort in all sectors by the same amount as can be seen in the equilibrium wage expressions in table one and two. Welfare payments will also lower employment in the usual way. A minimum wage forces some low wage firms to set wages above the level in equation (15) and the elasticity of effort with respect to wages will be less than one. These firms will still extract the maximum effort possible given the wage they pay, so we still use equation (13) to get the relationship between effort and wages. Since the rent associated with jobs in affected sectors equation (5), will be higher given that effort is higher the value of $X$ (equation (9)) will increase driving up wages and effort in all other sectors. We note that any lowering of employment and increase in unemployment resulting will mitigate the upward pressure on wages in other sectors. This result is consistent with some empirical findings. Grossman (1983) has some evidence of minimum wage payments increasing wages across skill groups. Katz and Krueger (1992) analysing the Texas fast food industry find that in response to the 1991 minimum wage increase, firms that were initially above the minimum wage increased their starting wage on average.

(9) Unemployment-skill correlations

Akerlof and Yellen (1990) document the negative correlation between unemployment rates and skill levels. They make the point that models such as Shapiro and Stiglitz predict the opposite correlation if as they expect, monitoring of skilled workers is more difficult. This is because the unemployment threat is the worker discipline device. Since losing a job in the good sector is the threat in the model presented here, this is no longer true. We do not get higher equilibrium unemployment to maintain discipline for high skill groups, in fact we can have full employment and still satisfy the N.S.C. as was shown earlier. We expect lower quit rates in high wage sectors other things being equal. Pencavel (1970) has observed a negative correlation between quit rates and wage differentials. So if high wage sectors have a tendency to hire more high skill groups we can rationalise the lower unemployment rates for high skill groups in this model.

(10) Labour subsidies

The interconnection of wages across sectors has implications for the application of dual labour market models to the industrial policy debate. Papers such as Bulow and Summers and Katz and Summers argue for subsidies in high wage sectors to generate increased labour market rent for workers. This analysis assumes that wages in the competitive and unsubsidised sectors will remain unchanged, which is not correct. Wages in other sectors may rise, lowering employment and welfare. Another issue is the impact of per unit labour subsidies (advocated by Katz and Summers) on the individual firms problem. Working through the firms problem with such subsidies, we see that firms use the subsidy (on employment) to hire more employees at a lower wage and effort level.
Subsidising workers gives the firm the incentive to substitute from effort into additional workers and wage premiums fall. The implication is that output subsidies to the particular sectors may be preferable to labour subsidies.

(11) Conclusion

Allowing for a multiplicity of firms and allowing firms to choose effort endogenously has important implications for the predictions of an efficiency wage model of the labour market. We can get a pattern of high wage low effort jobs, although jobs where firms must demand high effort from workers will give bigger rents to workers. The model is consistent with full employment, showing that the efficiency wage model predicts a pattern of good and bad jobs but is not a rationale for involuntary unemployment.

Given that firms have other instruments at their disposal other than wages eg. they can adjust how hard the job is (the expected level of effort), the model predicts that we would not expect to see widespread wage differentials across industry of the size observed in the empirical literature.

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