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PUBLIC POLICY TOWARDS R&D IN OLIGOPOLISTIC INDUSTRIES*

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ABSTRACT

This paper examines the free-market and socially optimal outcomes in a dynamic oligopoly model with R&D spillovers. First-best optimal subsidies to R&D are higher when firms play strategically against each other but lower when they cooperate on R&D (at least with high spillovers) and when they play strategically against the government. Second-best optimal subsidies to R&D are presumptively higher than first-best ones, but policies to encourage cooperation are likely to be redundant (since it is always privately profitable) and simulations suggest that the welfare cost of lax competition policy is high.

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1. Introduction

The importance of determining optimal policy towards R&D cannot be exaggerated, given the worldwide interest in fostering R&D and given the significant differences between European and U.S. policies towards interfirm cooperation on R&D.[1] Moreover, the problem is an inherently difficult one because of the complex nature of the R&D process. Since R&D is a component of fixed costs, industries where it is important tend to be concentrated. Hence R&D policy must go hand in hand with competition policy. At the same time, R&D is like any form of investment in that it precedes the production stage. Hence issues of time consistency and strategic commitment inevitably arise in considering the choice of R&D policy. Finally, R&D by one firm typically leads to spillovers which benefit other firms, so that R&D exhibits many of the characteristics of a public good, albeit one that is mostly privately produced. The degree to which such spillovers occur and can be internalised is another crucial influence on the desirable pattern of intervention.

All these aspects of R&D generate incentives for firms to behave strategically, but, as previous writers have shown, the effects of such incentives are ambiguous. Brander and Spencer (1983) showed that oligopolistic firms which invested strategically in R&D, with a view to improving their future competitive position *vis-à-vis* their rivals, would normally carry out *more* R&D than the cost-minimising level. In this respect, the strategic incentives to which R&D give rise are identical to those arising from investment in physical capital, as considered for example by Spence (1977), Dixit (1980) and Fudenberg and Tirole (1984). However, Brander and Spencer did not allow for any R&D spillovers between firms. Spence (1984) focused on this issue and noted that such spillovers dilute the strategic incentive for firms to engage in R&D (because each firm is adversely affected by the positive benefits which its own R&D confers on its rivals). Spence suggested that cooperation on R&D might internalise this negative externality, though he did not present a complete analysis. It was left to d'Aspremont and Jacquemin (1988) to formalise this argument, and to show that, with sufficiently large spillovers, cooperation on R&D (though with subsequent competition at the output stage) indeed

[1] See Jacquemin (1988) and Martin (1995).

leads to more output, R&D and welfare.

These papers and the subsequent literature they have inspired have thrown considerable light on aspects of R&D in oligopolistic markets.[2] Nevertheless, there remain a number of issues which require further exploration and which form the subject of this paper. The first of these is the need to disentangle the separate influences of strategic behaviour on the one hand and R&D cooperation on the other. Whereas Brander-Spencer and Spence compare strategic and non-strategic behaviour in the absence of cooperation, d'Aspremont-Jacquemin and subsequent writers take strategic behaviour for granted and concentrate on comparing the outcomes with and without R&D cooperation. Each of these approaches is incomplete and, as we shall see, potentially misleading. The first objective of this paper is therefore to present a comprehensive analysis of these issues within a generalisation of the d'Aspremont and Jacquemin model of two-stage duopoly to allow for non-linear demands and many firms. By focusing on the incentives to engage in R&D, by invoking stability conditions in a natural way and by making use of a new geometric technique, we are able to give a more comprehensive ranking of output, R&D and welfare in the different cases.

A ranking of welfare levels is essential if they are to be evaluated from a public policy perspective. However, it is not sufficient as a guide to whether intervention is desirable or not. To determine this, it is also necessary to examine which of the equilibria will be chosen in the absence of intervention. In this context, the second objective of this paper is to compare the levels of industry profits in the different equilibria. This allows in particular an investigation of the market incentives for firms to engage in R&D cooperation without any intervention.

Finally, the third objective of the paper is to consider explicitly the nature of optimal intervention in each equilibrium. While most previous papers have had a public policy

[2] The d'Aspremont-Jacquemin model in particular has generated considerable analysis. Its stability has been considered by Henriques (1990) and it has been extended to allow for differentiated products and price as well as quantity competition by de Bondt et al. (1992) and Kamien (1992). Suzumura (1992), like us, allows for many firms and general demands, as does Ziss (1994), who independently derives the same diagram. None of these papers calculate explicit expressions for the marginal return to R&D in different equilibria; they do not disentangle the separate influences of R&D cooperation and strategic behaviour; and they do not derive optimal policies nor consider industry profits.

focus, they have not attempted to characterise explicitly the optimal package. We do this both for the second-best case, when only R&D subsidies are available, and for the first-best case, when both output and R&D subsidies can be chosen optimally. We also address the problem of dynamic consistency which arises in the first-best case. If firms anticipate output subsidies which are increasing in output, and if the government cannot commit in advance to a subsidy rate, then they have a further incentive to invest in R&D. Anticipating this strategic behaviour, the government in turn has an incentive to offer a lower subsidy, just as strategic behaviour by firms enjoying learning by doing was shown in Leahy and Neary (1994) to justify lower rather than higher subsidies.

The plan of the paper is as follows. Section 2 introduces the model and Section 3 considers the free-market outcome, isolating the separate influences of strategic behaviour and R&D cooperation. Section 4 then shows how the first-best outcome can be attained by appropriate R&D and output subsidies, under different assumptions about the extent of commitment and cooperation. Section 5 turns to the case where the government can only use R&D subsidies and considers the optimal second-best subsidies in this case. Section 6 looks at explicit solutions of the model for particular functional forms and considers the robustness of the conclusions to the relaxation of a key assumption. Finally, Section 7 concludes with a summary of results and some suggestions for further research.

2. The Model

We consider an industry of n firms, each of which produces an amount q_i of a homogeneous product. Market demand is given by:

$$p = p(Q), \qquad Q = \sum_{i=1}^{n} q_i . \tag{1}$$

We assume that the inverse demand function is three times differentiable and define $b \equiv -p' > 0$ as its algebraic slope and $r \equiv Qp''/p' = Qb'/b$ as a measure of its concavity.

The model is a two-period one. Each firm chooses the level of R&D, x_i, it carries out in the first, pre-production, period, and the level of output, q_i, it produces in the second period. Marginal production costs are independent of output but are decreasing in R&D, both that of the firm itself and (through spillover effects) of its rivals':

$$c_i = c_i(x_i, X_{-i}), \tag{2}$$

where X_{-i} is the total R&D carried out by all $n-1$ firms other than firm i. We define:

$$\theta \equiv -\partial c_i/\partial x_i > 0, \tag{3}$$

as the direct cost-reducing effect of R&D per unit output; and

$$\beta \equiv \{\partial c_i/\partial X_{-i}\}/\{\partial c_i/\partial x_i\}, \qquad 0 \leq \beta \leq 1, \tag{4}$$

as the spillover coefficient, measuring (as a fraction of θ) the extent to which firm i benefits from R&D carried out by any other firm. The terms θ and β need not be constant, but we will focus on symmetric equilibria in which they are common across all firms.[3] Finally, we define the marginal social return to R&D per unit output as:

$$\xi \equiv \{1+(n-1)\beta\}\theta > 0. \tag{5}$$

It shows the effect on one firm's marginal cost of a unit increase in R&D by all firms; in symmetric equilibria this equals the effect on the marginal cost of industry output of a unit increase in R&D by a single firm.

Given demands and costs, each firm seeks to maximise profits, which equal revenue pq_i less costs of production c_iq_i and R&D costs $\Gamma_i(x_i)$, plus subsidies. We denote by σ and s the per-unit subsidies to R&D and output respectively. Hence profits are given by:

$$\pi^i = (p-c_i)q_i - \Gamma_i(x_i) + \sigma x_i + sq_i. \tag{6}$$

In specifying firm behaviour, there are two further issues to be considered, the *degree of cooperation* and the *order of moves*. Concerning the former, we are mainly interested in the implications of R&D cooperation. We therefore contrast the cases where firms either do or do not cooperate on their levels of R&D, assuming that they choose their output levels in a non-cooperative Cournot fashion. We also consider the cartel case, where both R&D and output levels are chosen cooperatively. As for move order, the decisions on R&D and output have a natural temporal sequence. However, firms may or may not be able to commit to their output levels at the same time as they choose their R&D; and the government may or may not be able to commit to both subsidies in advance of firms' decisions. We assume that the government always has the ability to commit *intra-temporally*: it can set the level of each period's subsidy (to R&D in period 1 and to output in period 2) before firms choose the corresponding variable. As in Leahy and Neary (1994), this leaves three alternative assumptions about the degree of *intertemporal* commitment, each implying a different game with a distinct order of moves:

[3] The consequences of asymmetric R&D spillovers are explored by de Bondt and Henriques (1994).

1. *Full Commitment Equilibrium (FCE):* In this case, no agents play strategically and the game has two stages. The government first chooses both subsidies and firms then choose simultaneously their levels of R&D and output.
2. *Government-Only Commitment Equilibrium (GCE):* This game has three stages. As in FCE, the government first chooses both subsidies. Firms then choose their R&D levels at the second stage and their output levels at the third stage.
3. *Sequence Equilibrium (SE):* In this four-stage game, no intertemporal commitment is possible. The government chooses its R&D subsidy σ; each firm then chooses its R&D level; next the government chooses its output subsidy s; and finally each firm chooses its level of output.

We assume subgame perfection throughout, so at each stage each agent anticipates how its actions will influence the actions of all other agents at every future stage. Under this interpretation, the differences between the three games reflect differences in the constraints, institutional or other, on agents' ability to commit to future actions. Alternatively, following Spence (1984), the differences between the three games may be interpreted as arising from different degrees of strategic behaviour. In the next section, we examine the free-market outcome, where the government commits to zero subsidies in both periods (so "GCE" simply means that firms behave strategically).

3. The Market Outcome

3.1 Competition in Output

Provided firms choose their outputs non-cooperatively, their first-order condition for output is independent of the degree of R&D cooperation and the order of moves. For a typical firm, this first-order condition sets marginal revenue equal to marginal cost:

$$p(Q) - b(Q)q_i - c_i(x_i, X_{-i}) = 0, \qquad \forall i. \tag{7}$$

In symmetric equilibria the firm subscripts can be omitted. For later use, we note some properties of symmetric equilibria of the output game (see the Appendix for proofs):

Lemma 1: In a symmetric equilibrium, the firms' second-order conditions for profit-maximisation are met if and only if: $2n+r>0$.

Lemma 2: In a symmetric equilibrium, outputs are strategic substitutes (i.e., an increase in the output of firm i will reduce the marginal profitability of firm j, for any $i \neq j$) if and only if: $n+r>0$.

Lemma 3: The symmetric output game is stable if and only if: $n+1+r>0$.

The stability condition in Lemma 3 can be used to illustrate the symmetric equilibrium graphically. Totally differentiating (7), its slope is shown in the Appendix to equal:

$$\frac{dq}{dx} = \frac{\xi}{b(n+1+r)}. \qquad (8)$$

Provided the stability condition holds globally, the locus of (q,x) combinations satisfying (7) is upward-sloping, as illustrated by the HH schedule in both panels of Figure 1. Since this assumption is needed for some of our results, we state it explicitly:

Assumption 1: The stability condition given in Lemma 3, $n+1+r>0$, holds at every point along HH, the locus representing the output equilibrium condition, (7).

In addition, we assume that equilibrium output is positive even when no R&D takes place, so the schedule cuts the q axis above the origin as drawn.

3.2 Effects of Strategic Behaviour without R&D Cooperation

Consider next the firms' choice of R&D. The simplest case is that of no-cooperation FCE: firms neither cooperate on R&D nor behave strategically. The level of R&D is then chosen by setting the marginal private return to R&D equal to its marginal cost:

$$\frac{\partial \pi^i}{\partial x_i} = \mu_F^N q - \Gamma' = 0, \quad \forall i; \qquad \mu_F^N \equiv \theta. \qquad (9)$$

In this case, the marginal private return to R&D per unit output, which we write as μ_F^N (for "No-cooperation FCE"), is simply the reduction in the firm's own unit costs, θ.

By contrast, if firms behave strategically (so the equilibrium is GCE rather than FCE), they also take account of how their R&D affects the output choices of other firms:

$$\frac{d\pi^i}{dx_i} = \frac{\partial \pi^i}{\partial x_i} + \sum_{j\neq i} \frac{\partial \pi^i}{\partial q_j}\frac{dq_j}{dx_i} = 0, \quad \forall i. \qquad (10)$$

It is straightforward that firm i's profits are decreasing in the output of every other firm $(\partial \pi_i/\partial q_j = -bq_i)$. Hence the implications of strategic behaviour depend on how firm i's R&D affects the output of all other firms. This is shown in the Appendix to be given by:

$$\frac{dq_j}{dx_i} = -\frac{\alpha}{b}(\bar{\beta}-\beta)\theta, \qquad i\neq j, \qquad (11)$$

$$\text{where:} \quad \alpha \equiv \frac{2n+r}{n(n+1+r)} > 0 \quad \text{and} \quad \bar{\beta} \equiv \frac{n+r}{2n+r}. \qquad (12)$$

Since α is positive from the stability condition, the crucial issue is whether the R&D spillover parameter is greater or less than the threshold value $\bar{\beta}$. Summarising:

Lemma 4: In a symmetric equilibrium, an increase in the R&D of one firm raises the outputs of all others if and only if the spillover coefficient β exceeds $\bar{\beta}$. This threshold value is strictly less than one; it is positive if and only if outputs are strategic substitutes; and it is greater than one half if and only if demand is concave.

As the second part of the Lemma states, higher R&D by one firm always raises the outputs of all other firms when spillovers are at their maximum ($\beta=1$); and it lowers them in the absence of spillovers ($\beta=0$) provided outputs are strategic substitutes.

The first-order condition for R&D in a symmetric GCE may now be written as:

$$\frac{d\pi^i}{dx_i} = \mu_G^N q - \Gamma' = 0, \quad \forall i; \qquad \mu_G^N \equiv [1+(n-1)\alpha(\bar{\beta}-\beta)]\theta. \qquad (13)$$

Here, we denote by μ_G^N (for "No-cooperation GCE") the marginal private return to R&D per unit output when firms behave strategically. It is greater than θ if and only if R&D spillovers are sufficiently low; specifically, if and only if β is less than the threshold value $\bar{\beta}$. Could sufficiently high spillovers deter R&D completely? The answer is yes, as may be seen by rewriting μ_G^N as follows:

$$\mu_G^N = (n-1)\alpha\left[1-\beta+\frac{n(2+r)}{(n-1)(2n+r)}\right]\theta. \qquad (14)$$

Thus, sufficiently high spillovers (β close to one) and sufficiently convex demand (r very negative) imply that R&D has a negative return. In particular:

Lemma 5: If r is more negative than -2, then, for some level of spillovers, the marginal return to R&D is negative, and so no R&D will be carried out when firms behave strategically.

Lemmas 2 and 5 together imply that, if outputs are strategic complements, no R&D will be carried out when spillovers are sufficiently high. Moreover, with more than two firms, it is possible for no R&D to be carried out even if outputs are strategic substitutes (i.e., in the case $-2>r>-n$).

We now wish to compare the FCE and GCE equilibria. The comparison made above between μ_G^N and θ suggests that strategic behaviour leads to more R&D if and only if β is less than $\bar{\beta}$. But that result was only a local one, which cannot directly be extended to a comparison between two distinct equilibria, since μ_G^N and θ need not be constant. Fortunately, the comparison may be made rigorously with some additional assumptions:

Assumption 2: Equilibrium of each type is unique.
Assumption 3: The profit functions in either one of the two regimes to be compared exhibit the Seade (1980) stability condition with respect to R&D levels at all points along HH between the two equilibria.

The Seade stability condition requires that the first-order condition for profit maximisation by a single firm be decreasing in a uniform increase in R&D by *all* firms: $\partial^2\pi/\partial x_i\partial x<0$. We can now rank the levels of output and R&D in the two equilibria:

Proposition 1: Given Assumptions 1, 2 and 3, then, with no R&D cooperation, output and R&D are higher with strategic behaviour than without (i.e., in GCE than in FCE) provided the spillover parameter β is less than the threshold level $\bar{\beta}$, both evaluated at the equilibrium corresponding to the regime opposite to that specified in Assumption 3.

This proposition is proved in the Appendix and is illustrated in Figure 1, where Panel (a) represents the case of low spillovers ($\beta<\bar{\beta}$) and Panel (b) that of high spillovers ($\beta>\bar{\beta}$). The OF and OG schedules reflect the first-order conditions for R&D in FCE and GCE respectively, and their intersection points with HH (A_F^N and A_G^N) represent the FCE and GCE equilibria respectively. Stability requires that OF and OG cut HH from below, as shown. Proposition 1 generalises the findings of Brander and Spencer (1983) to allow for R&D spillovers: they showed that, with no spillovers, output and R&D are higher when firms behave strategically whereas our result shows that this is only true for $\beta<\bar{\beta}$.

3.3 Effects of Strategic Behaviour with R&D Cooperation

Suppose now that firms cooperate in their choice of R&D levels, though they continue to compete at the output stage. Following d'Aspremont and Jacquemin (1988), we assume that the cooperative level of R&D is chosen to maximise joint profits, but that cooperation does not affect the value of the spillover parameter β. (The implications of relaxing this assumption are considered in Section 6.3.) Once again, we must distinguish between FCE, where firms commit to both R&D and output levels, and GCE, where R&D levels are chosen in the anticipation of their strategic effects in the output game.

Consider first the case of FCE with cooperation. Write $\Pi=\Sigma\pi_j$ for industry profits:

$$\Pi(Q,x) \quad = \quad [p(Q) - c(x)].Q - n\Gamma(x). \tag{15}$$

Cooperation implies that the optimal level of R&D by each firm maximises Π:

$$\frac{\partial\Pi}{\partial x_i} \quad = \quad \mu_F^C q - \Gamma' \quad = \quad 0, \qquad \forall i; \qquad \mu_F^C \equiv \xi. \tag{16}$$

Private (μ_F^C) and social (ξ) returns to R&D coincide: cooperation fully internalises the externality arising from R&D spillovers. However, this ignores any strategic motive. By contrast, in GCE, each firm also takes account of the effect of its choice of R&D on industry profits. Since industry profits depend only on industry output, the resulting first-order condition is similar in form to, but simpler than, that in FCE (equation (10)):

$$\frac{d\Pi}{dx_i} \quad = \quad \frac{\partial\Pi}{\partial x_i} + \frac{\partial\Pi}{\partial Q}\frac{dQ}{dx_i} \quad = \quad 0, \qquad \forall i. \tag{17}$$

An increase in firm i's R&D must raise industry output (the exact expression is given in the Appendix, equation (37)); and a rise in industry output must lower industry profits by $-(n-1)bq$. Hence the second term in (17) is negative, and the full effect on profits is:

$$\frac{d\Pi}{dx_i} \quad = \quad \mu_G^C q - \Gamma' \quad = \quad 0, \qquad \forall i; \qquad \mu_G^C \quad \equiv \quad \frac{2+r}{n+1+r}\,\xi. \tag{18}$$

Recalling Lemma 5, highly convex demand ($r<-2$) is now sufficient as well as necessary for no R&D to be carried out. The coefficient of ξ is less than unity: with cooperation, the marginal return to R&D is lower in GCE than in FCE. Once again, we require regularity conditions similar to those for Proposition 1 to hold in order that this result hold globally:

Proposition 2: Given assumptions 1, 2 and 3, then, when firms cooperate in their choice of R&D, the levels of output and R&D are lower with strategic behaviour than without (i.e., in GCE than in FCE).

3.4 Effects of R&D Cooperation

We are now in a position to consider the effects of cooperation itself. The results of the last two sub-sections are summarised in Table 1, which gives the marginal private return to R&D in each of the four equilibria, denoted by μ_l^k, k=N,C; l=F,G. We have already compared the results in each column, and now wish to compare across rows.

The first row suggests why cooperation is superficially desirable: in FCE, it ensures that each firm takes account of the effects of its R&D on the costs of all other firms. Hence, with zero spillovers it gives rise to the same equilibrium and, with strictly positive spillovers, it leads to higher output and R&D (provided again that the regularity conditions hold). However, the GCE comparison in the second row shows that this conclusion is complicated by strategic behaviour. Comparing the two expressions at a given point, we see that, when firms behave strategically (i.e., in GCE), the marginal private return to R&D with cooperation, μ_G^C, is greater than that without, μ_G^N, if and only if β exceeds a new threshold value $\bar{\beta}'$:

$$\mu_G^C - \mu_G^N \quad = \quad \alpha'(\beta - \bar{\beta}')\theta\,, \tag{19}$$

$$\text{where:} \qquad \alpha' \;\equiv\; \frac{n-1}{n}\,\frac{4n+(n+1)r}{(n+1+r)} \qquad \text{and} \qquad \bar{\beta}' \;\equiv\; \frac{2n+r}{4n+(n+1)r}\,. \tag{20}$$

Assuming that $2+r$ is positive (so that there is some incentive to engage in R&D in both equilibria), α' is positive and $\bar{\beta}'$ lies strictly between zero and one. Moreover, $\bar{\beta}'$ lies on the opposite side of 0.5 from $\bar{\beta}$. Summarising:

Lemma 6: When demand is strictly concave ($r>0$): $0<\bar{\beta}'<0.5<\bar{\beta}<1$. When demand is strictly convex, though not sufficiently to make R&D unprofitable ($-2<r<0$), the ranking of threshold values is reversed: $0<\bar{\beta}<0.5<\bar{\beta}'<1$.

In the d'Aspremont-Jacquemin case of linear demand both threshold values equal 0.5.

Finally, having considered a local comparison of the two marginal returns to R&D, we may proceed as in previous sub-sections to extend this to a global comparison. Under the same regularity conditions as before, we obtain:

Proposition 3: Given Assumptions 1, 2 and 3, then, when firms do not behave strategically, cooperation leads to more output and R&D, provided spillovers are strictly positive; but when they do behave strategically, it leads to less output and R&D unless spillovers are sufficiently high that β is greater than $\bar{\beta}'$.

Propositions 2 and 3 allow the equilibria with R&D cooperation to be located in Figure 1, with both output and R&D levels ranked by the locations of the corresponding equilibria.[4] For low spillovers, $A_G^C < A_F^N \leq A_F^C < A_G^N$, with A_F^N and A_F^C coinciding when β is zero. By contrast, for high spillovers, $A_G^N < A_F^N < A_G^C < A_F^C$.

3.5 Industry Profits and Social Welfare

Having examined how different assumptions about strategic behaviour and R&D cooperation affect output and R&D, we turn to evaluate the different equilibria from both

[4] A comprehensive list of the location of all four equilibria for intermediate values of β requires a comparison of the values of μ along the diagonals of Table 1. Details of this are given in the Appendix, Section A4.

private and social perspectives. Consider first industry profits. In symmetric equilibria, we may totally differentiate (15) to determine how they vary with output and R&D:

$$d\Pi \quad = \quad (p-c-bQ)n\,dq + (\xi q-\Gamma')n\,dx. \tag{21}$$

Setting the coefficients of dq and dx equal to zero shows that a cartel, which seeks to maximise industry profits, will choose output such that marginal cost equals *industry* marginal revenue $p-bQ$; but for that level of output it will choose the efficient level of R&D. Moreover, we can draw iso-profit curves in (q,x) space which are horizontal where they cross the efficient R&D locus (along which ξq equals Γ'); and vertical where they cross the locus along which marginal cost equals $p-bQ$. Two such curves, centred on the cartel outcome A^M, are shown in Figure 2. This diagram is similar to Figure 1 in that the curve HH represents the oligopoly equilibrium locus (i.e., the locus of points which equate marginal cost to firm marginal revenue). The line MM, representing the cartel equilibrium locus (the locus of points which equate marginal cost to industry marginal revenue), must lie below HH. Finally, the line OR represents the efficient R&D locus, on which must lie both the cartel equilibrium A^M and the non-strategic cooperative equilibrium A_F^C. Changes in the spillover parameter will shift all the loci in Figure 2 but they do not affect the qualitative relationships between the loci shown in the diagram.

In order to rank the level of profits in the different equilibria, we first need to establish which of the oligopoly equilibria maximises industry profits. To do this, we use the oligopoly output first-order condition, (7), and its slope, (8), to simplify (21):

$$d\Pi \quad = \quad \left[\frac{2+r}{n+1+r}\,\xi q \;-\; \Gamma'\right] n\,dx\,. \tag{22}$$

Comparing this with equation (18), the coefficient of ndx is the first-order condition for R&D by a strategic cooperative. Hence, we may conclude:

Proposition 4: Of all the oligopoly equilibria in which outputs are chosen non-cooperatively, industry profits are maximised when R&D is chosen both cooperatively and strategically.

To rank the levels of profits in other equilibria, we must assume that industry profits are quasi-concave in x, and so iso-profit contours do not intersect, along HH. Given this, profits fall monotonically as we move away from A_G^C in either direction along the HH locus, and the rankings of equilibria given in the Appendix, Section A4, allow us to rank the levels of profits by inspection of Figures 1 and 2.

The result that R&D cooperation (at least when firms play strategically) always maximises industry profits raises crucial questions about the policy stance towards cooperation. However, before considering these, we must examine the levels of welfare. We measure it, following standard convention, as the sum of consumer surplus $(u(Q)-p.Q)$ and industry profits, net of subsidy payments:

$$W(Q,x) \quad = \quad u(Q) - c(x).Q - n\Gamma(x). \tag{23}$$

As with industry profits, we differentiate this to see how it varies with output and R&D:

$$dW \quad = \quad (p-c)n\,dq + (\xi q - \Gamma')n\,dx. \tag{24}$$

Naturally, first-order conditions for a welfare maximum are that price equal marginal cost and that the marginal social return to R&D equal its marginal cost. Away from the optimum, we can draw iso-welfare contours which are horizontal where they cross the efficient R&D locus (where ξq equals Γ'); and vertical where they cross the $p=c$ locus. The latter, denoted SS in Figure 2, must lie above the HH locus, since the socially optimal level of output must exceed the oligopoly level for a given level of R&D. Two such iso-welfare curves, centred on the social optimum A^O, are shown in Figure 2.

As with profits, it is helpful to determine where welfare is maximised along the HH locus. This is done by substituting from equations (7) and (8), to obtain:

$$\frac{1}{n}\frac{dW}{dx} \quad = \quad \bar{\mu} q - \Gamma' \quad = \quad 0, \qquad \text{where:} \quad \bar{\mu} \quad \equiv \quad \frac{n+2+r}{n+1+r}\,\xi. \tag{25}$$

Here, $\bar{\mu}$ is the social return to R&D per unit output at the second-best optimum. The coefficient of ξ is greater than one, implying that for given output the second-best optimum requires over-investment in R&D. Geometrically, this second-best optimum is the point of tangency of the HH (i.e., the $p-bx=c$) locus with the highest attainable iso-welfare locus. To establish where this point lies relative to the other equilibria, we first carry out a local comparison of $\bar{\mu}$ with the four values of μ_l^k given in Table 1. It is clearly greater than all of them except for the No-Cooperation GCE case, in which:

$$\bar{\mu} > \mu_G^N \qquad \text{IFF} \qquad \beta > \frac{n-2+(n-1)\frac{r}{n}}{(n-1)\left\{n+4+r+\frac{r}{n}\right\}}. \tag{26}$$

The denominator is positive and greater than the numerator, so the threshold value for β is less than one. Indeed it is likely to be low: for linear demands ($r=0$) it is zero for

$n=2$ and never rises above .08 (for $n=4,5$); and it may be negative if there are few firms (n is small) and demand is convex ($r<0$). As with earlier results, these local comparisons imply global rankings under appropriate regularity conditions:

Proposition 5: If outputs are chosen non-cooperatively, welfare is maximised when the level of R&D is such that its marginal return per unit output equals $\bar{\mu}$. Given Assumptions 1, 2 and 3, this second-best optimum lies to the right of and so has higher levels of output and R&D than any of the free-market outcomes, except for the No-Cooperation GCE equilibrium when β is low.

Finally, provided we assume that the welfare function is quasi-concave in x (so iso-welfare contours do not intersect) along HH, we may rank the welfare levels in different equilibria in terms of their distance from the second-best optimum $\bar{A}$. Inspection of the diagrams shows that the ranking of the equilibria with respect to welfare is almost exactly the reverse of the ranking with respect to profits when spillovers are low. By contrast, when spillovers are high, the two rankings are more similar. Keeping this in mind, we proceed to derive rules for optimal intervention in the next section.

4. Attaining the First-Best Optimum

When we come to consider optimal intervention, it is immediately obvious that R&D policy alone cannot attain the first-best optimum. With two targets to control (the levels of output and R&D of the representative firm), two instruments are required. If we assume that the second instrument is an output subsidy, its optimal value must equal the gap between price and marginal cost, i.e.:

Need output R+D subsidies

$$s^O \quad = \quad bq \quad > \quad 0. \tag{27}$$

This formula holds irrespective of the order of moves of firms and government.

Turning to R&D policy, assume first that the government can commit to its output subsidy before the firms choose their R&D. The optimal policy then follows immediately from the results of the last section. With a subsidy, the profit-maximising condition for R&D in each of the four equilibria becomes:

$$\mu_l^k q + \sigma_l^k \quad = \quad \Gamma', \qquad k=N,C\,;\; l=F,G. \tag{28}$$

where the marginal return to R&D per unit output can be read from Table 1. Since the first-best optimum requires that the marginal cost of R&D, Γ', should equal its marginal social return ξq, the optimal R&D subsidy must be:

$$\sigma_l^k \;=\; (\xi - \mu_l^k)q, \qquad k = N,C;\; l = F,G. \tag{29}$$

The exact values of this expression in each of the four equilibria are given in the first two rows of Table 2. All are necessarily non-negative, except for σ_G^N:

$$\sigma_G^N > 0 \qquad \text{IFF} \qquad \beta > \frac{n+r}{n(n+3)+(n+1)r}. \tag{30}$$

The threshold value of β is strictly positive provided outputs are strategic substitutes (from Lemma 2). This implies that, if firms behave strategically but non-cooperatively and outputs are strategic substitutes, R&D should be *taxed* if there are no spillovers. However, the threshold is relatively small: it is less than $\bar{\beta}$ and also less than $1/(n+1)$. So there is always some level of spillovers which justify subsidisation. Summarising:

Proposition 6: If the government can commit to the optimal output subsidy, then R&D should be subsidised, except in the case of strategic non-cooperation when spillovers are low and outputs are strategic substitutes.

We now wish to compare the values of the subsidies in the different equilibria. Unlike Propositions 1 to 5, such comparisons do not require any regularity conditions. As in Leahy and Neary (1994), all the subsidies are evaluated at the same point and so their values may be compared directly. And, since the optimal subsidies are directly related to the marginal returns to R&D by equation (29), comparison between them is straightforward. By applying the results of earlier sections, we may immediately state:

Proposition 7: When firms do not cooperate on their choice of R&D, strategic behaviour implies a higher optimal subsidy if and only if $\beta > \bar{\beta}$; when they do cooperate, strategic behaviour always implies a higher optimal subsidy; cooperation without strategic behaviour requires no subsidy; but cooperation with strategic behaviour requires a higher subsidy if and only if $\beta < \beta'$.

How are these results affected if the government cannot commit to an output subsidy, the case we call Sequence Equilibrium? Firms now anticipate that the output subsidy will be set by equation (27).[5] But the right-hand side of this depends on the level of R&D. Hence firms have a strategic incentive to alter their R&D in order to increase their output

[5] We confine attention to subgame perfect equilibria. Cases where the government first announces the GCE output subsidy and then reneges are considered in a related model by Leahy and Neary (1995).

subsidy. The government, in turn, anticipating this incentive, should take it into account in setting its R&D subsidy. Even though x is chosen before the output subsidy, the government can still achieve the first-best optimum since it has two instruments at its disposal and only two distinct targets, x and q (as in Leahy and Neary (1994)).

The resulting fully time-consistent optimal subsidies are derived in the Appendix and given in the third row of Table 3. Ψ', which equals $(1+r)\xi/n^2$, is the derivative of the optimal output subsidy s^o with respect to a single firm's R&D. This measures the extra payoff to investment in R&D, and so it affects the reduction in s^o needed to restrain firms from this strategic overinvestment. With cooperation, both the extra incentive and the necessary corrective by the government are greater, depending on $n\Psi'$ rather than Ψ'. Provided demand is sufficiently concave $(1+r>0)$, Ψ' is positive:[6] the output subsidy is a *strategic complement* for R&D. In that case, the optimal subsidy both with and without R&D cooperation is lower in SE than in the corresponding GCE case. Summarising:

Proposition 8: If the government cannot commit to an output subsidy, then the first-best optimum can still be achieved. Whether firms cooperate on R&D or not, the optimal R&D subsidies are unambiguously lower than in the case of government commitment if and only if $1+r>0$.

It is possible for the optimal subsidies to be negative in SE. Without cooperation on R&D, this is even more likely than in GCE, and must hold (provided $2+r>0$) for zero spillovers. With cooperation, the optimal subsidy is positive if and only if $n-2-r>0$; so, for example, with two firms, the time-consistent optimal subsidy in SE with cooperation is positive if and only if demand is convex.

5. Optimal Second-Best Subsidies to R&D

Suppose now that it is not possible to subsidise output. We must then calculate the second-best optimal subsidy to R&D.[7] Welfare is related to changes in output and R&D

[6] For a similar result in tariff theory, see Brander and Spencer (1984) and Jones (1987).

[7] Suzumura (1992) also considers the possibility that output subsidies are not feasible. However, he deals with it by using what he calls a "Second-Best Welfare Function," defined as $\bar{W}(x) = W[x, q(x)]$, where $q(x)$ is the solution to (7), the oligopoly output equilibrium condition setting marginal cost equal to marginal revenue. Geometrically, this amounts to looking at values of welfare along the HH locus in Figure 1. Suzumura's

in the same way as before. However, the only instrument now available to the government is the R&D subsidy σ. This alters the incentive for investment in R&D but cannot affect the first-order condition for output, equation (7). Hence the best outcome that policy can achieve is the second-best optimum, where the marginal return to R&D equals $\bar{\mu}q$. Substituting this value for Γ' into (28), the second-best optimal subsidy in each of the four equilibria in which firms operate along the HH locus is given by:

$$\bar{\sigma}_l^k = (\bar{\mu} - \mu_l^k)q, \qquad k=N,C;\ l=F,G. \tag{31}$$

Recalling that $\bar{\mu}$ is greater than ξ, this equation suggests a presumption that, per unit output, each second-best optimal subsidy is greater than the corresponding first-best optimal subsidy (which we recall equals $(\xi-\mu_l^k)q$). As always, since the two sets of subsidies are evaluated at different points, this presumption may be outweighed by variations in the values of the parameters.

The values of the optimal second-best subsidies are given in Table 3. All are strictly positive, except in the strategic non-cooperative case: from (26), the optimal policy in this case may be a tax, but only when spillovers are low and <u>either</u> there are more than two firms <u>or</u> demand is concave. Summarising:

Proposition 9: If governments cannot subsidise output, then second-best optimal subsidies are presumptively higher than their first-best counterparts. At the second-best optimum R&D should be subsidised, except under strategic non-cooperation when spillovers are low and <u>either</u> there are more than two firms <u>or</u> demand is concave.

We cannot in general say whether there is more or less R&D in the second-best optimum than in the first-best. (We return to this question for the linear case in Section 6.1.)

Finally, the rankings of the optimal subsidies which held in the first-best optimum continue to hold in the second-best optimum. Once again, this result is a general one and does not require any regularity conditions (though, of course, the parameters must now be evaluated at the second-best optimum itself). For completeness, we state this as follows:

Proposition 10: The rankings of subsidies given in Proposition 7 for the first-best optimum also hold at the second-best optimum.

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approach is equivalent to ours in a linear model. (See Section 6.1 below.) In more general contexts, our approach has the advantage of permitting an exact comparison of the subsidies in all four equilibria at the same second-best optimal point.

6. Intensions and Extensions

In this section, we explore the properties of the model in more detail for special functional forms and then discuss the consequences of relaxing a key assumption.

6.1 Linear Demands and Quadratic Costs of R&D

With linear demands, (1) reduces to: $p=a-bQ$, with a and b constant and r, the measure of concavity, equal to zero. On the cost side, we assume that the marginal cost of output is linear in R&D, so (2) becomes: $c_i = c_0-\theta(x_i+\beta X_{-i})$, with c_0, θ and β constant. In symmetric equilibria this simplifies to: $c=c_0-\xi x$, assumed nonnegative. As for the costs of R&D itself, we assume they are quadratic: $\Gamma(x)=\gamma x^2/2$, with γ constant.

Under these assumptions, the expressions derived for the general case simplify considerably: see the Appendix, Table A1. Comparisons between equilibria now depend on only three parameters: the number of firms n; the spillover parameter β; and a new parameter measuring the relative effectiveness of R&D, defined as $\eta\equiv\theta^2/b\gamma$. Table A1 helps resolve some ambiguous results. For example, it is shown in the Appendix that the level of R&D at the second-best optimum is always less than at the first-best optimum. It is also possible to obtain a feel for the quantitative implications of the model with the help of Figure 3. This illustrates the relationship between welfare in each equilibrium (relative to that in the social optimum) and the spillover parameter β, assuming $n=2$ and $\eta=0.4$.

The first conclusion suggested by Figure 3 is that the level of welfare in the second-best optimum closes only about half of the gap between the levels of welfare in the cartelized equilibrium and the first-best optimum. To see how robust this is, Figure 4 shows for different values of n how $(\bar{W}-W^m)/(W^o-W^m)$ varies as a function of a composite parameter $\tilde{\xi}^2$, which depends positively on *both* the effectiveness of R&D η and the spillover parameter β.[8] As Figure 4 shows, with large numbers of firms, welfare in the second-best optimum is close to the first-best level, and so R&D policy has the potential to close much of the gap between the cartel and the first-best levels. However,

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[8] Comparisons between all the equilibria, such as those in Figure 3, depend on three parameters, n, β and η. However, if we exclude equilibria in which R&D is chosen non-cooperatively, as in Figures 4 and 5, the comparisons depend only on two parameters: n and $\tilde{\xi}^2$, which equals $\xi^2/nb\gamma$, or, alternatively, $\{1+(n-1)\beta\}^2\eta/n$. The range for β in Figure 3 (0 to 1.0) corresponds to a range for $\tilde{\xi}^2$ in Figures 4 and 5 of 0.2 to 0.8. The explicit expressions underlying Figures 4 and 5 are given in the Appendix.

If output is not subsidised the level of R+D is lower even with a higher R+D subsidy for highly concentrated industries (small n), R&D policy alone (in the absence of competition policy) is at best a limited tool for improving market performance. The potential effectiveness of R&D policy is dramatically lower when R&D is highly effective and spillovers are high.

The second conclusion suggested by Figure 3 concerns the benefits of cooperation. In the absence of strategic behaviour, cooperation greatly enhances market performance: in FCE, the cooperative equilibrium always leads to a higher level of welfare than the non-cooperative one and the gap is significant when spillovers are high. However, when firms behave strategically, the superiority of cooperation is much reduced. In GCE, cooperation leads to higher welfare than no cooperation whenever β exceeds 0.5 (as we know from the theoretical results). But cooperation does not help that much, leading for high spillovers to a level of welfare only slightly higher than the cartel level. Figure 5 shows that this result is robust: except for cases with ineffective R&D, low spillovers and large numbers of firms, strategic cooperation leads to welfare levels which are not much better than the cartel level. Recalling from Proposition 4 that firms always have an incentive to cooperate, this suggests that the case for encouraging cooperation is much weaker than previous studies suggest: such encouragement is both limited in its potential for raising welfare and likely to be redundant in any case.

6.2 Stability of the Linear Model

It is necessary to check that any set of parameter values is consistent with stability. The details of this are lengthy and are relegated to the Appendix, Sections A1 and A6. In all cases, stability is more likely the lower is η, i.e., the higher the cost of and the lower the effectiveness of R&D. Heuristically, higher values of η impart an element of increasing returns to the model, increasing the incentive for each firm to deviate from the symmetric equilibrium. As for increases in β, they tend to enhance stability when firms do not cooperate but to make it less likely when they do. Instances of instability should be interpreted as a failure of the assumptions of the model, especially that of symmetry, requiring most plausibly some entry or exit of firms from the industry.

6.3 Cooperative Synergies

So far, we have assumed that the spillover parameter β is unaffected by the decision to cooperate. This is clearly unrealistic, and a number of authors have considered the implications of allowing β to rise when cooperation occurs, reflecting cooperative

synergies.[9] Kamien et al. (1992) go so far as to describe as an "R&D cartel" the type of cooperation which we have considered so far (in which β is unaffected), reserving the term "cooperation" for the case where spillovers are complete ($\beta=1$).

How are our results affected if R&D is subject to cooperative synergies? The first point to note is that all our conclusions concerning the effects of strategic behaviour for a *given* degree of cooperation are clearly unaffected. Thus it remains true that strategic behaviour tends to reduce output, R&D and welfare in most cases except the Brander-Spencer benchmark case of low spillovers and no cooperation. Our conclusions about the effects of cooperation itself must be amended, of course. The expressions given in Table 1 continue to hold, but comparisons across rows must allow for higher values of β and hence of ξ when firms cooperate on their R&D. This naturally tends to make cooperation more attractive from a welfare point of view. However, it also makes it more privately profitable, reinforcing the view that policy intervention to encourage cooperation is likely to be redundant whether or not it is desirable. Finally, the conclusions drawn from the simulations of the linear case in Figure 3 also continue to hold for high β while Figures 4 and 5 are unaffected by cooperative synergies.

7. Summary and Conclusions

This paper has considered a general model of an oligopolistic industry, in which firms first invest in R&D and then produce output. Our objective has been to present a unified treatment of a number of issues in order to establish the principles which should govern public intervention in industries where R&D is important. In particular, we have sought to disentangle the influences of strategic behaviour and R&D cooperation on the levels of output, R&D and welfare; to compare the private profitability and social performance of different equilibria; and to calculate explicitly the optimal subsidies to output and R&D under alternative assumptions.

In comparing the levels of output, R&D, profits and welfare in different equilibria, the paper offers two types of results. Firstly, Propositions 1 to 5 make global comparisons between these levels directly, subject to mildly restrictive regularity conditions.

[9] See, for example, Katz (1986) and Motta (1994). Katsoulacos and Ulph (1994) construct a model in which the spillover parameter is endogenised.

Secondly, Propositions 6 to 10 compare the optimal subsidies which are required to attain either the first-best optimum (where both output and R&D subsidies can be used) or the second-best optimum (where only R&D subsidies are available). The second set of results rely on local comparisons only and so need far fewer qualifications than the first.

As far as strategic behaviour is concerned, our results show that it tends to reduce output, R&D and welfare and to mandate higher subsidies in all cases except that considered by Brander and Spencer (1983). The exception is when firms choose their R&D levels non-cooperatively, R&D spillovers are low and demand is "not too" convex. In all other cases, strategic behaviour tends to lead to outcomes which are unambiguously less desirable from a social perspective.

Turning to R&D cooperation, its superficial attractiveness is highlighted by the fact that it unambiguously raises output, R&D and welfare (eliminating the need for any R&D subsidy) when firms do not behave strategically. However, with strategic behaviour, cooperation is less attractive: only when spillovers are high does it raise welfare and so require a lower subsidy. From Lemma 6, the threshold value of the spillover parameter lies on the opposite side of 0.5 from the threshold value which determines the effects of strategic behaviour (except in the linear case when both equal 0.5).

Our results do not overturn the finding of d'Aspremont and Jacquemin that cooperation is socially desirable when spillovers are high. However, they cast doubt on both its relevance and its usefulness. The result is less relevant because industry profits are *always* higher when firms choose their R&D strategically and cooperatively. Indeed, with higher spillovers, cooperation is more attractive from *both* private and social perspectives. So intervention to encourage cooperation is likely to be least needed when cooperation itself is socially desirable. As for the usefulness of encouraging cooperation, our simulations for the linear case suggest that the payoff from encouraging cooperation is likely to be low and that the welfare cost of lax competition policy is likely to be high.

Appendix

A1. Proofs of Lemmas 1-3: The derivatives of the marginal profitability of firm i with respect to its own output and to the output of any other firm j are as follows:

$$\lambda \equiv \pi^i_{ii} = 2p' + q_i p'' = -b(2+\alpha_i r) = -b(2n+r)/n, \qquad (32)$$

$$\rho \equiv \pi^i_{ij} = p' + q_i p'' = -b(1+\alpha_i r) = -b(n+r)/n. \qquad (33)$$

Here, α_i is the market share of firm i, equal to $1/n$ in symmetric equilibrium. Lemmas 1 and 2 follow immediately. As for Lemma 3, we give a proof for a more general case, which is also useful in Section A7 below. From Dixit (1986, equation (36-ii)), necessary conditions for stability in a symmetric n-firm oligopoly are that:

$$\text{(i)}\ \ \lambda < 0 \qquad \text{and} \qquad \text{(ii)}\ \ (-1)^n(\lambda-\rho)^{n-1}[\lambda+(n-1)\rho] > 0. \qquad (34)$$

Moreover, from Seade (1980, Theorem 1), $\lambda+(n-1)\rho>0$ is *sufficient* for *in*stability and so $\lambda+(n-1)\rho<0$ is *necessary* for stability. Combining this with (34) (i) gives a general necessary condition: $\lambda<Min\{0,-(n-1)\rho\}$. And combining it with (34) (ii) when n is even gives: $\lambda<Min\{\rho,-(n-1)\rho\}$. As for sufficient conditions, Hahn (1962) implies: $\lambda<\rho<0$; and Seade (1980, Appendix) implies: $|\lambda|>(n-1)|\rho|$. Combining these with (34) (ii) gives a general sufficient condition: $\lambda<Min\ \{\rho,-(n-1)\rho\}$. Hence:

Lemma A1: In a symmetric n-firm oligopoly model: (i) $\lambda<Min\{0,-(n-1)\rho\}$ is always necessary for stability; (ii) $\lambda<Min\{\rho,-(n-1)\rho\}$ is always sufficient; and (iii) when n is even, $\lambda<Min\{\rho,-(n-1)\rho\}$ is both necessary and sufficient.

Corollaries of Lemma A1 are that the Seade sufficient condition (a) is necessary when $n=2$; but (b) does not nest the Hahn condition when $n>2$. Applying Lemma A1 to the output game: $\lambda-\rho=-b<0$ and $\lambda+(n-1)\rho=-b(n+1+r)$; which gives Lemma 3.

A2. Proof of Lemma 4: We begin by totally differentiating the first-order condition for output of a typical firm i, equation (7) in the text with the subsidy term added:

$$-bdq_i - b(1+\alpha_i r)dQ + \theta dx_i + \beta\theta dX_{-i} + ds = 0, \qquad \forall i. \qquad (35)$$

Consider now two shocks to a symmetric equilibrium. The first is a uniform increase in R&D by each firm, so $dQ=ndq_i$, $dX_{-i}=(n-1)dx_i$ and subscripts can be suppressed. Solving (35) in this case gives (8), the slope of the HH schedule. The second shock, the subject of Lemma 4, is an increase in R&D by firm i alone, so $dX_{-i}=0$. To eliminate dQ in this case, totally differentiate the first-order condition for output for firm j:

$$-b\,dq_j - b(1+\alpha_j r)dQ + \beta\theta dx_i + ds \quad = \quad 0, \qquad \forall j \neq i. \tag{36}$$

Now, sum equation (36) over all $n-1$ firms indexed j, add to equation (35) and solve for the effects of an increase in firm i's R&D and in the subsidy on industry output:

$$b(n+1+r)dQ \quad = \quad \xi dx_i + n\,ds. \tag{37}$$

Substituting into (36), setting $ds=0$ and collecting terms gives the first part of Lemma 4. The second part is established by comparing the threshold value $\bar{\beta}$ with the values one, zero and 0.5 in turn, and making use of Lemmas 1 to 3.

A3. Proofs of Propositions 1, 2, 3 and 5: We give a general proof, which applies to all of these propositions. In each case, we wish to compare two equilibria: (x^1,q^1) which satisfies $\mu^1(x^1,q^1)q^1=\Gamma'(x^1)$, the first-order condition for maximisation of $\pi^1(x_1, \ldots x_n)$ with respect to x_i; and (x^2,q^2) which satisfies $\mu^2(x^2,q^2)q^2=\Gamma'(x^2)$, the first-order condition for maximisation of $\pi^2(x_1, \ldots x_n)$ with respect to x_i. Both equilibria lie along HH, so from Assumption 1 q and x are monotonically related: $q=q(x)$. This allows us to eliminate q, writing $m(x)\equiv\mu[x,q(x)]$ and $g(x)\equiv\Gamma'(x)$.

The problem may now be stated compactly. Given: two first-order conditions, (a) $m^1(x^1)q(x^1)=g(x^1)$; and (b) $m^2(x^2)q(x^2)=g(x^2)$; a local ranking of μ's at the first equilibrium, (c) $m^1(x^1)>m^2(x^1)$; and Assumption 3 (the Seade condition) holding for the profit function of the second equilibrium, (d) $\partial^2\pi^2/\partial x_i\partial x < 0$; we wish to prove that $x^1>x^2$. The proof is immediate. From (b), (d) and Assumption 2: $m^2(x)q(x)<g(x)$ for all x if and only if $x>x^2$. But from (a) and (c): $m^2(x^1)q(x^1)<g(x^1)$. Hence $x^1>x^2$. Q.E.D.

The assumptions made do not rule out the possibility of the local ranking of the μ's being reversed at the second equilibrium: (e) $m^1(x^2)<m^2(x^2)$. However, in that case, Assumption 3 cannot hold for the profit function of the first equilibrium; i.e., $\partial^2\pi^1/\partial x_i\partial x < 0$ cannot hold. For, if it did, we would have from (a) and Assumption 2: $m^1(x)q(x) < g(x)$ for all x if and only if $x>x^1$. But, since $x^1>x^2$, this implies that $m^1(x^2)q(x^2) > g(x^2)$. From (b), this in turn implies that $m^1(x^2)>m^2(x^2)$, which contradicts (e).

A4. Threshold Values of β and the Ranking of Equilibria: The ranking of equilibria for values of β close to zero and one are given in the text following Proposition 3. To get a full ranking for intermediate values, we first derive two more threshold values of β:

$$\mu_G^C > \mu_F^N \quad \text{IFF} \quad \beta < \tilde{\beta} \equiv \tfrac{1}{2+r}, \tag{38}$$

$$\mu_F^C > \mu_G^N \quad \text{IFF} \quad \beta < \tilde{\beta}' \equiv \tfrac{n+r}{n(n+3)+(n+1)r}. \tag{39}$$

When $r=0$, $\tilde{\beta}=0.5$ and $\tilde{\beta}'=1/(n+3)<0.5$. The full set of local comparisons is now:

Case (a) $-2<r<0$: $0<\tilde{\beta}'<\bar{\beta}<0.5<\bar{\beta}'<\tilde{\beta}<1$, implying:

- For $\tilde{\beta}'<\beta<\bar{\beta}$: $\mu_G^C<\mu_F^N<\mu_G^N<\mu_F^C$.
- For $\bar{\beta}<\beta<\bar{\beta}'$: $\mu_G^C<\mu_G^N<\mu_F^N<\mu_F^C$.
- For $\bar{\beta}'<\beta<\tilde{\beta}$: $\mu_G^N<\mu_G^C<\mu_F^N<\mu_F^C$.

Case (b) $0<r<n$: $0<\tilde{\beta}'<\tilde{\beta}<\bar{\beta}'<0.5<\bar{\beta}<1$, implying:

- For $\tilde{\beta}'<\beta<\tilde{\beta}$: $\mu_G^C<\mu_F^N<\mu_G^N<\mu_F^C$.
- For $\tilde{\beta}<\beta<\bar{\beta}'$: $\mu_F^N<\mu_G^C<\mu_G^N<\mu_F^C$.
- For $\bar{\beta}'<\beta<\bar{\beta}$: $\mu_F^N<\mu_G^N<\mu_G^C<\mu_F^C$.

Case (c) $0<n<r$: $0<\tilde{\beta}<\tilde{\beta}'<\bar{\beta}'<0.5<\bar{\beta}<1$, implying:

- For $\tilde{\beta}<\beta<\tilde{\beta}'$: $\mu_F^N<\mu_G^C<\mu_F^C<\mu_G^N$.
- For $\tilde{\beta}'<\beta<\bar{\beta}'$: $\mu_F^N<\mu_G^C<\mu_G^N<\mu_F^C$.
- For $\tilde{\beta}<\beta<\bar{\beta}$: $\mu_F^N<\mu_G^N<\mu_G^C<\mu_F^C$.

Any pair of these local comparisons may now be extended to a global comparison of R&D and output levels given Assumptions 1-3. Finally, as discussed in the text, these rankings along HH may be translated into rankings of profits and welfare using Propositions 4 and 5, provided the appropriate functions are quasi-concave in x.

A5. Optimal Subsidies in Sequence Equilibrium: The first step is to rework Section A2 allowing each firm to anticipate the effects of its choice of R&D on the output subsidy. Since each firm contemplates deviating from a symmetric equilibrium, we write the subsidy as a function of industry output only: $s=b(Q)Q/n$. Totally differentiating:

$$n\,ds \quad = \quad b(1+r)dQ. \tag{40}$$

Combining with (36) and (37) we solve for the effects of an increase in R&D by firm i:

$$\frac{ds}{dx_i} \quad = \quad \Psi' \quad = \quad \frac{1+r}{n^2}\xi, \tag{41}$$

$$\frac{dQ}{dx_i} \quad = \quad \frac{\xi+n\Psi'}{b(n+1+r)} \quad = \quad \frac{\xi}{bn}, \tag{42}$$

$$\frac{dq_j}{dx_i} \quad = \quad -\frac{\alpha}{b}(\bar{\beta}-\beta)\theta + \frac{1}{b(n+1+r)}\Psi' \quad = \quad -\frac{(n-1)-(2n-1)\beta}{bn^2}\theta. \tag{43}$$

These results can now be used to recalculate equations (13) and (18) to obtain the returns to R&D, and hence the optimal R&D subsidies, in SE, with and without cooperation.

A6. Linear Demands and Quadratic Costs of R&D: Table A1 gives the values of output, R&D and welfare under the assumptions of Section 6.1. The results are most conveniently expressed in terms of two new parameters $\tilde{\xi}$ and $\tilde{\mu}$, which equal ξ and μ respectively, deflated by $\sqrt{(nb\gamma)}$. For each oligopoly equilibrium the appropriate value for μ should be read from Table 1, with r set equal to zero.

From Table A1, R&D at the first-best optimum is greater than at any of the oligopoly equilibria if and only if $\mu < (n+1)\xi/n$. This holds in all cases (including the second-best optimum) except for μ_G^N when $n>2$ and β is low.

Figure 4 is based on the following expression:

$$\frac{\bar{W}-W^M}{W^O-W^M} = 1 - \frac{(2-\tilde{\xi}^2)^2}{(n+1)^2-n(n+2)\tilde{\xi}^2}. \tag{44}$$

This equals $1-4/(n+1)^2$ at $\tilde{\xi}^2=0$, rises very slightly until it reaches its maximum value at $\tilde{\xi}^2=1/n(n+2)$, and then falls monotonically to reach zero when $\tilde{\xi}^2$ itself attains its maximum permissible value of unity. The expression underlying Figure 5 is:

$$\frac{W_G^C-W^M}{W^O-W^M} = 1 - (2-\tilde{\xi}^2)^2\,\frac{(n+1)^2+n(n-1)(n^2+n+2)\tilde{\xi}^2}{\{(n+1)^2-2n\tilde{\xi}^2\}^2}. \tag{45}$$

Like (44), this equals $1-4/(n+1)^2$ at $\tilde{\xi}^2=0$. It then falls steadily (actually falling below zero for high $\tilde{\xi}^2$ when n is greater than 3) to reach zero at $\tilde{\xi}^2=1$.

A7. Stability: To apply Lemma A1 to the linear R&D game under GCE, differentiate the first-order conditions (13) and (18), making use of (11) and the corresponding expression for dq_i/dx_i:

$$\lambda = -\gamma + \mu\frac{dq_i}{dx_i} = -\gamma + \mu\frac{n-(n-1)\beta}{b(n+1)}\theta, \tag{46}$$

$$\rho = \mu\frac{dq_j}{dx_i} = -\mu\frac{1-2\beta}{b(n+1)}\theta. \tag{47}$$

From (47), $\beta>0.5$ is equivalent to $\rho>0$, implying that R&D levels are strategic

complements and so R&D reaction functions are upward-sloping. Substituting the appropriate values of μ for the non-cooperative and cooperative cases, we can calculate threshold values for η consistent with the stability conditions. The results are given in Table A2 and are illustrated for the non-cooperative case when $n=3$ in Figure A1. All of the latter are increasing in β, while those for the cooperative case are decreasing in β, except for the sufficient condition in the range $\beta\in\{(n-2)/2(n-1),0.5\}$. Finally, the result of Henriques (1990) emerges as a special case of ours: assuming $n=2$ and $\eta=1$ with no cooperation gives a threshold value for β of $(3-\sqrt{7})/2\approx 0.177$.

	No Cooperation on R&D	Cooperation on R&D
FCE	θ	ξ
GCE	$[1+(n-1)\alpha(\bar{\beta}-\beta)]\theta$	$\frac{2+r}{n+1+r}\,\xi$

$$\xi \;\equiv\; [1+(n-1)\beta]\theta \qquad \alpha \;\equiv\; \frac{2n+r}{n(n+1+r)} \;>\; 0 \qquad \bar{\beta} \;\equiv\; \frac{n+r}{2n+r}$$

Table 1: Marginal Private Return to R&D per unit Output in Different Equilibria

	No Cooperation on R&D	Cooperation on R&D
FCE	$(\xi-\theta)q \geq 0$	0
GCE	$\sigma_F^N - (n-1)\alpha(\bar{\beta}-\beta)\theta q \gtrless 0$	$\frac{n-1}{n+1+r}\,\xi q > 0$
SE	$\sigma_G^N - \frac{2+r}{n+1+r}\,\Psi' q \gtrless 0$	$\sigma_G^C - \frac{2+r}{n+1+r}\,n\Psi' q = \frac{n-2-r}{n}\,\xi q \gtrless 0$

$$\Psi' \;\equiv\; (1+r)\xi/n^2$$

Table 2: First-Best Optimal Subsidies to R&D in Different Equilibria

	No Cooperation on R&D	Cooperation on R&D
FCE	$\left[\frac{n+2+r}{n+1+r}\,\xi - \theta\right] q > 0$	$\frac{1}{n+1+r}\,\xi q > 0$
GCE	$\bar{\sigma}_F^N - (n-1)\alpha(\bar{\beta}-\beta)\theta q \gtrless 0$	$\frac{n}{n+1+r}\,\xi q > 0$

Table 3: Second-Best Optimal Subsidies to R&D in Different Equilibria

	Output (q)	R&D (x)	Welfare (W)
First-Best Optimum	$\frac{1}{1-\tilde{\xi}^2}\,\frac{A}{nb}$	$\frac{1}{1-\tilde{\xi}^2}\,\frac{\xi A}{nb\gamma}$	$\frac{1}{1-\tilde{\xi}^2}\,\frac{A^2}{2b}$
Cartel	$\frac{1}{2-\tilde{\xi}^2}\,\frac{A}{nb}$	$\frac{1}{2-\tilde{\xi}^2}\,\frac{\xi A}{nb\gamma}$	$\frac{3-\tilde{\xi}^2}{(2-\tilde{\xi}^2)^2}\,\frac{A^2}{2b}$
Oligopoly	$\frac{n}{n+1-n\tilde{\mu}\tilde{\xi}}\,\frac{A}{nb}$	$\frac{n}{n+1-n\tilde{\mu}\tilde{\xi}}\,\frac{\mu A}{nb\gamma}$	$\frac{n(n+2-n\tilde{\mu}^2)}{(n+1-n\tilde{\mu}\tilde{\xi})^2}\,\frac{A^2}{2b}$

Table A1: Levels of Output, R&D and Welfare with Linear Demands and Quadratic Costs of R&D

Notes:

$A \equiv a-c_0$; $\tilde{\xi} \equiv \xi/\sqrt{(nb\gamma)} = [1+(n-1)\beta]\sqrt{(\eta/n)}$; $\tilde{\mu} \equiv \mu/\sqrt{(nb\gamma)} = (\mu/\theta)\sqrt{(\eta/n)}$; $\eta \equiv \theta^2/b\gamma$.

For each oligopoly equilibrium, the appropriate value of μ, the marginal private return to R&D per unit output, should be read from Table 1, with r set equal to zero.

For the second-best optimum, the appropriate values of output, R&D and welfare are given by the corresponding formula for the oligopoly equilibria, with μ set equal to $\bar{\mu}$; i.e., from (25), to $(n+2)\xi/(n+1)$.

Stability Condition	Description	Threshold Values of η	
		Without Cooperation	With Cooperation
$\lambda < -(n-1)\rho$	Necessary; nec and suff for $\beta > 0.5$	$\frac{(n+1)^2}{2[1+(n-1)\beta][n-(n-1)\beta]}$	$\frac{(n+1)^2}{2[1+(n-1)\beta]^2}$
$\lambda < \rho$	Sufficient; nec and suff for $\beta < 0.5$ and n even	$\frac{n+1}{2(1-\beta)[n-(n-1)\beta]}$	$\frac{n+1}{2(1-\beta)[1+(n-1)\beta]}$
$\lambda < 0$	Necessary; only relevant for $\beta < 0.5$ and n odd	$\frac{(n+1)^2}{2[n-(n-1)\beta]^2}$	$\frac{(n+1)^2}{2[1+(n-1)\beta][n-(n-1)\beta]}$

Table A2: Threshold Values of η Consistent with Stability in GCE with Linear Demands and Quadratic Costs of R&D

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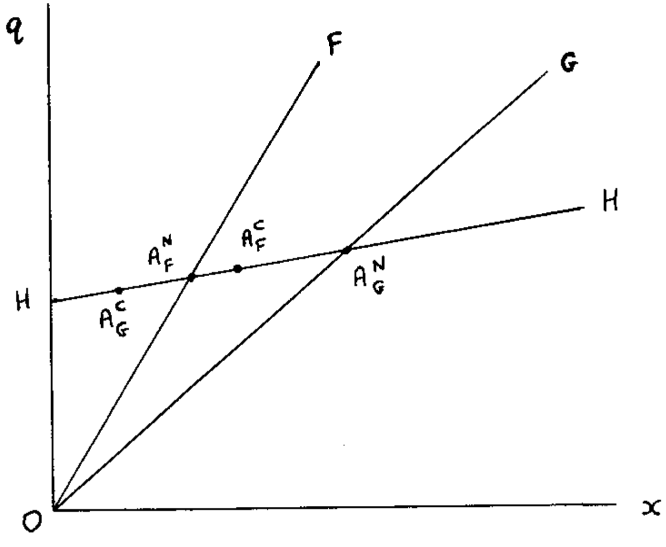

Figure 1(a): Oligopoly Equilibria with Low Spillovers
(A_F^C coincides with A_F^N when $\beta = 0$)

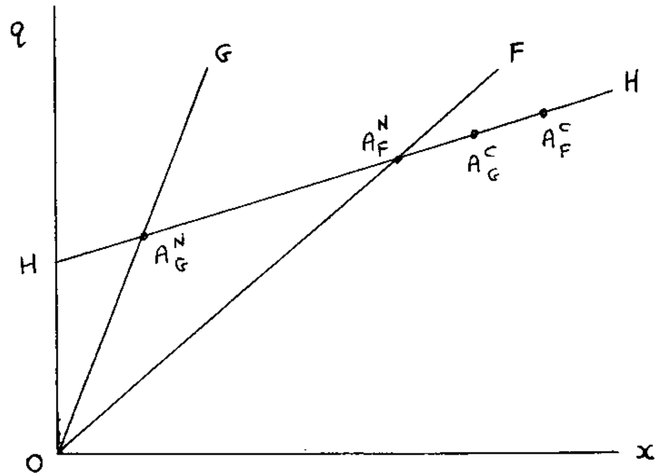

Figure 1(b): Oligopoly Equilibria with High Spillovers

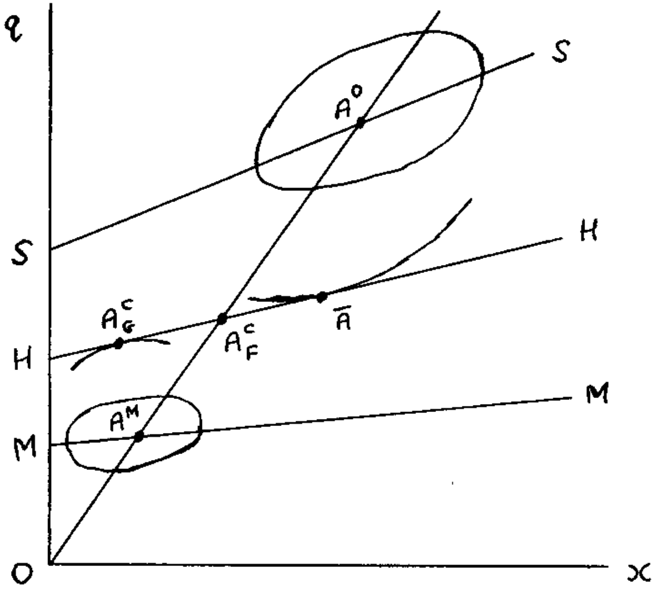

Figure 2: Iso-Profit and Iso-Welfare Loci

Figure 3: Levels of Welfare in Different Equilibria
(Linear demand; quadratic costs of R&D; eta=0.4; n=2)

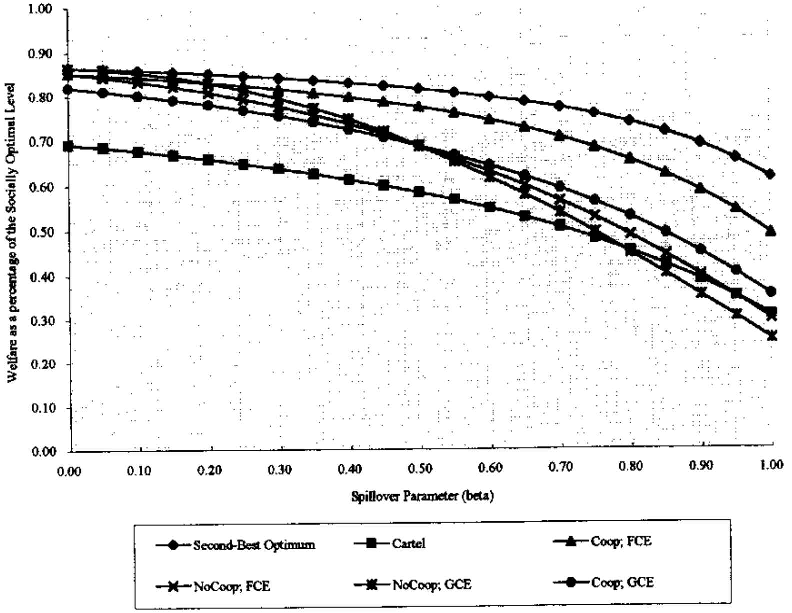

Figure 4: Maximum Potential Welfare Gain from R&D Policy Alone

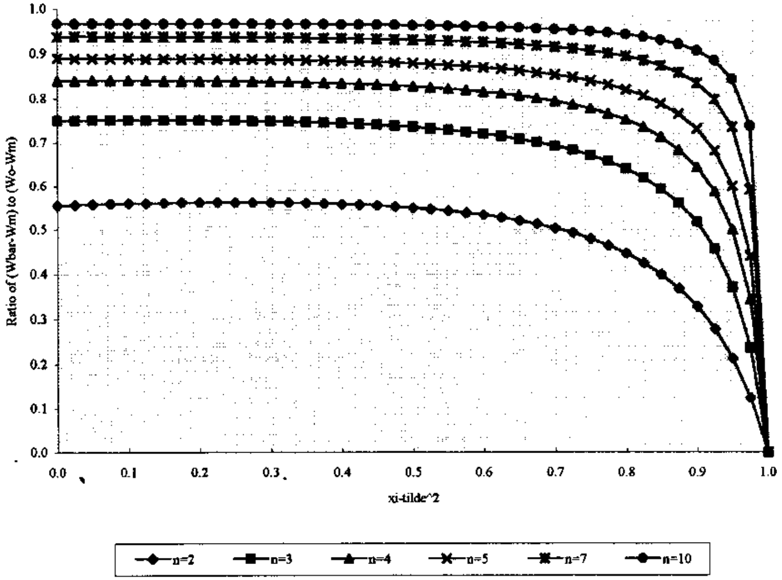

Figure 5: Welfare Gain from Strategic Cooperation on R&D

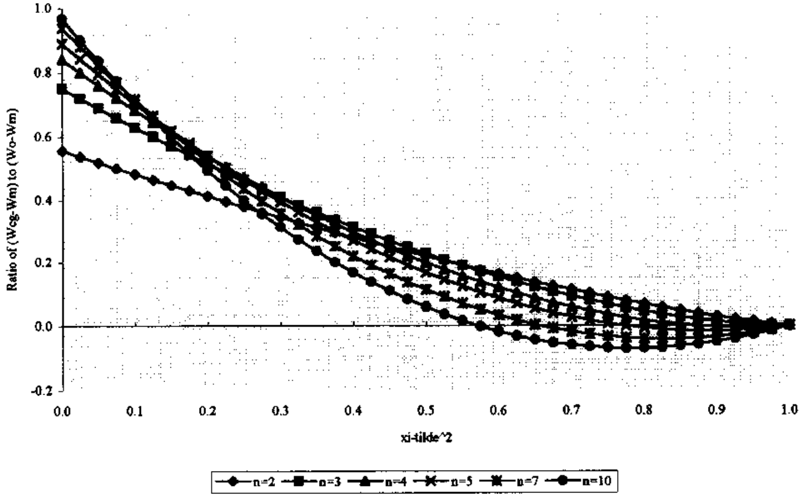

Figure A1: Maximum Values of eta Consistent with Stability
in Linear N-GCE Game: n=3