Monetary shocks with variable effort

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Abstract

In a model with rigid nominal wages, full information and competitive product markets, I show that when an effort augmented production function is incorporated into an analysis of supply and demand shocks, the outcomes are in line with traditional Keynesian analysis for a wide range of parameter values. Monetary shocks can increase output and employment.

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1. Introduction

Traditional Keynesian analysis assumes nominal wage rigidity in the face of nominal shocks. A problem with such analysis is that workers are assumed to be willing to supply more labor even though real wages have fallen. Lucas (1972) introduces imperfect information on the source of price changes to rationalize unanticipated monetary shocks having real effects. Models such as Akerlof and Yellen (1985) or Ball and Romer (1990) assume nominal rigidity of prices in models with imperfectly competitive labor and product markets, showing that small adjustment costs can rationalize nominal rigidities that can generate employment and output effects con-

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sistent with the Keynesian model. The model developed below has similar results but
with a perfectly competitive product market and no uncertainty.

Waller (1989) incorporates effort (as a function of the wage) into the production
function. In this model firms are price takers, firms and workers are fully informed
about all prices, and efficiency units of labor are the product of effort and employ-
ment as in Solow (1979). Waller shows that monetary shocks leave output un-
changed in the presence of nominal wage rigidity. This result arises because
workers’ labor supply is determined by a combination of wages and effort. While
a rise in the price level reduces the real wage, which induces the firm to increase
employment, the reduction in the real wage also induces a reduction in effort leaving
output unchanged. Waller also shows that, unlike models without effort, indexing
wages will not have a destabilizing effect on output in the face of supply shocks.

In this paper I generalize Waller’s (1989) model, replacing the Solow (1979)
Cobb–Douglas model of efficiency units with a constant elasticity of substitution
function. Accordingly, the output effect of monetary shocks will only be zero if
the elasticity of substitution is unity (the Cobb–Douglas case). For a wide range
of elasticities of substitution the results support the Traditional Keynesian story, that
is, a monetary shock with nominal wage rigidity causes higher output and employ-
ment in the short run and wage indexation increases the variability of output in re-
response to supply side shocks.

2. Background to the model

This section discusses some of the basic assumptions of the model and some
empirical regularities. Specifically I discuss the assumption that efficiency units of la-
bor should not be restricted to the product of effort and employment, the assumption
of nominal wage rigidity, and the observed regularity that productivity is pro-cyclical
in contrast with the predictions of the model.

Akerlof and Yellen (1986, pp. 14–16) question the appropriateness of the assump-
tion that efficiency units of labor are the product of employment and effort. 1,2 Ra-
mana and Rowthorn (1991) formalise Akerlof and Yellen’s argument in a general
model where effort and hours enter as separate inputs. It is argued, for example, that
with technologies where workers can do a large amount of damage by reducing effort
(assembly lines or airline pilots are examples given by Ramana and Rowthorn) it will
be more difficult to substitute between effort and employment. One can also think of
many possibly menial or low skill tasks where the ease of substitution between effort
and hours would possibly be greater than implied by the Cobb–Douglas assumption

1 Akerlof and Yellen (1986) argue that the elasticity of effort with respect to the wage will be less than
unity in contrast with Solow’s (1979) model. They argue that workers exerting less effort will damage the
firm via the effect on the return to other inputs in a model where labour is not the only input. Faria (2000)
develops an efficiency wage model where the Solow condition cannot hold in equilibrium.

2 Holmes and Hutton (2000) get a positive output effect by introducing Monopsony power into the
efficiency wage model.
(workers picking fruit for example). An additional feature of the Solow model is that effort and hours work have the same weight in producing efficiency units of labor. If effort has less weight than employment in the production of efficiency units of labor, then monetary shocks will increase output and employment even in the Cobb–Douglas case. Finally, with regard to the appropriateness of the Solow model, it is worth noting that introducing labor taxes into the wage effort model is enough to ensure that the Solow condition will not hold.

Another feature of the model developed below is the assumption of nominal wage rigidity. In Romer (1996) Section 6.1.1 it is illustrated in a model with nominal price rigidity, imperfectly competitive firms, and a competitive labor market that the cost of adjusting prices needed to justify nominal rigidity would be unreasonably large unless the labour supply elasticity is also unreasonably large. Romer (1996) as well as Akerlof and Yellen (1985) show that small costs in adjusting prices can rationalize nominal price rigidity and generate relatively large changes in output and employment. In Akerlof and Yellen’s model product markets are imperfectly competitive while the labor market is characterised by Solow’s (1979) efficiency wage model. Walsh (2002) shows that the decrease in profits from failing to adjust nominal wages are of the first order and argues that small menu costs can justify nominal wage rigidity in the model developed below. The possibility that wage indexation can increase output variability also gives credence to the nominal wage rigidity assumption. A standard argument is that indexing wages to the inflation rate is a cheap mechanism for adjusting wages, making the existence of menu costs implausible (see Romer (1996, p. 280) for example). The results below show that such a strategy may in fact be costly because adjusting the nominal wage in line with price changes resulting from supply shocks would be inefficient.

At first glance the model is at variance with the empirical regularity that labour productivity is pro-cyclical. See Bernanke and Parkinson (1991) for a discussion and some evidence of different models that can explain pro-cyclical productivity. In Walsh (2002) I show that when one focuses on particular groups of workers where one surmises that alternative reasons for pro-cyclical productivity such as labor hoarding would not be important, the pro-cyclical productivity pattern following an unexpected monetary shock is less clear.

3. The model

In this Section 1 outline the model and conduct some comparative static analysis. It is shown for a wide range of elasticity’s of substitution that monetary shocks have positive output and employment effects and that indexing wages raises the output variability associated with supply side shocks.

Competitive firms have the following production function:

\[
y = \beta f \left\{ A e \left( \frac{W}{P} \right)^x + Bn^x \right\}^{\frac{1}{2}}, \tag{1}\]
where $y$ is output, $W$ the nominal wage, $P$ the price level, $e$ effort, $n$ employment and there is a technology parameter $\beta = 1 + \mu$ where $\mu \sim G(0, \sigma^2_\mu)$. $G(\cdot)$ is a distribution function that allows for either positive or negative supply side shocks. There is no uncertainty so all agents know the realization of the shock when we solve for equilibrium. Efficiency units of labor $x(e, n)$ are generated with a constant elasticity of substitution function:

$$x = \left[ Ae \left( \frac{W}{P} \right)^z + Bn^z \right]^\frac{1}{z}.$$  \hspace{1cm} (2)

The elasticity of substitution between effort and employment in producing efficiency units is

$$\sigma = \frac{1}{1 - x}. \hspace{1cm} (3)$$

The relative importance of effort and hours work in producing efficiency units of labor is determined by $A$ and $B$. The model is identical to Waller (1989), except that Waller assumed that $x = 0$ and $A = B$. That is, Waller used Solow’s model where $x = (en)$. As $x$ increases towards unity it gets easier to substitute between effort and workers, while larger negative values imply it is more difficult to substitute.

The elasticity of substitution between effort and employment in producing efficiency units is

$$\sigma = \frac{1}{1 - x}.$$  \hspace{1cm} (3)

The profit function divided by price is

$$\pi = \beta f \left\{ \left[ Ae \left( \frac{W}{P} \right)^z + Bn^z \right] \right\} - \frac{W}{P} n. \hspace{1cm} (4)$$

Profit maximization yields the following first order conditions for $W$ and $n$, respectively (where for notational convenience $[\cdot] = [Ae(\frac{W}{P})^z + Bn^z]$):

$$\beta f_w(x) = A \frac{\beta f[x] e^{x-1} e_w(\frac{W}{P}) [\cdot]^{\frac{1-x}{z}}}{P} - \frac{n}{P}, \hspace{1cm} (5)$$

$$\beta f_n(x) = B \beta f[x] n^{x-1} [\cdot]^{\frac{1-x}{z}} = \frac{W}{P}. \hspace{1cm} (6)$$

The first order conditions imply:

$$e_w \frac{W}{P} = \left( \frac{n}{e} \right)^z B \frac{x}{A}. \hspace{1cm} (7)$$

The demand and supply for money are given, respectively, as

$$M^D = kPy, \hspace{1cm} (8)$$

$$M^S = \delta M, \hspace{1cm} (9)$$

where $\delta = 1 + \varphi$ and $\varphi \sim H(0, \sigma^2_\varphi)$. The distribution function $H(\cdot)$ allows for random shocks to the money stock. As with the supply shock, the realization of the money supply is known to all agents when we solve for equilibrium. We do not make any assumptions on the shape of these distributions. In equilibrium one finds that:
$$k\Pi y = \delta \overline{M}.$$ \hfill (10)

139 Totally differentiating Eq. (1) one gets:

$$dy - \beta f_n dn - \beta f_W dW - \beta f_p dp - f d\beta = 0.$$ \hfill (11)

142 Using Eqs. (5) and (6) this can be rewritten as

$$dy - \frac{W}{P} dn - \frac{n}{P} dW + \frac{nW}{P^2} dp - f d\beta = 0.$$ \hfill (12)

145 Totally differentiating Eqs. (6) and (10) one gets the following equations:

$$\beta f_{nn} dn + \left(\beta f_{nW} - \frac{1}{P}\right) dW + \left(\beta f_{nP} + \frac{W}{P^2}\right) dP + f_n d\beta = 0,$$ \hfill (13)

$$\overline{M} d\delta = kP dy + ky dP.$$ \hfill (14)

146 The money stock is assumed to be constant apart from the random shock.

Eqs. (12)–(14) can be used to analyze the impact of various shocks on equilibrium. The first comparative static exercise is to analyze the output effect of a monetary shock when nominal wages are rigid. The details of the analysis are in Appendix A.1 where it is shown that:

$$\frac{dy}{d\delta} = \frac{\overline{M}}{P} \left(\frac{\alpha \frac{w}{P^2}}{\alpha \frac{w}{P^2} - f_{nn} \frac{y}{W}}\right).$$ \hfill (15)

167 It is only if \(\alpha = 0\) (the Cobb–Douglas case) that the output effect of a monetary shock is zero. If the elasticity of substitution between effort and employment (3) is greater than unity \((0 < \alpha < 1)\), then (15) is unambiguously positive. For the case where \(\alpha < 0\).

I show in the appendix that the sign of the output effect is more likely to be positive if the elasticity of substitution is small, if the elasticity of effort with respect to the real wage is large, the smaller are fixed costs and the less curvature there is in the production function. It is easy to construct examples where the firm’s problem is well behaved and the output effect can be positive or negative for different values of \(\alpha\).

Overall though, for a wide range of parameter values the output effect is positive.

The next comparative static exercise is to examine the employment effects of a monetary shock. In this case once again \(dW = 0\) and \(\beta = 1\). Solving Eqs. (11) and (12) and using Eq. (A.2) from the appendix one gets:

$$\frac{dn}{d\delta} = \frac{\overline{M}}{PKW} \left(\frac{\alpha \frac{w}{P^2} - f_{nn} \frac{y}{W}}{\alpha \frac{w}{P^2} - f_{nn} \frac{y}{W}}\right).$$ \hfill (16)

171 A sufficient condition for the employment effects to be positive is that the elasticity of substitution between employment and effort is greater than or equal to unity \((\alpha \geq 0)\).

For negative values of \(\alpha\) where it is more difficult to substitute between effort and hours the employment effect will be positive unless the following condition holds:

$$-f_{nn} \frac{y}{W} < -\alpha \frac{w}{P^2} < -f_{nn} \frac{y}{W}.$$ The condition for a positive output effect (15) is sufficient for a positive employment effect.
Having analyzed the impact of monetary shocks I continue by contrasting the impact of a supply side shock under nominal wage rigidity with the outcome when wages are indexed. Waller (1989) shows that the output effects are the same. A negative supply shock with rigid nominal wages increases the price level and lowers the real wage. Both effort and labor demand fall. On the other hand if wages are indexed, effort is constant and employment falls by more than in the sticky wages case. Thus Waller (1989) shows that a full indexation of wages does not increase the variation in output associated with supply shocks. This result depends on the Solow model as is shown below. Given that in reality it may be difficult to separate the parts of inflation that come from real and monetary factors, the results imply that indexation is not a simple way of maintaining the optimal real wage.

In this case $dW = 0$ as before and also $d\delta = 0$ and $\delta = 1$ as shown in Appendix A.2.

\[
\frac{dy}{db} = \frac{\beta f_{nn}f - f_n \frac{W}{P}}{\beta f_{nn} - \alpha \frac{W^2}{P}}.
\]

If $\alpha > 0$ then (17) is unambiguously positive. The possibility of a negative output effect arises because nominal wages are rigid in the face of falling output prices, leading to the possibility that real wages will be too high. Firms may find it profitable to produce less output in a more effort intensive fashion.

If wages are fully indexed then $d\delta = 0$ and $\delta = 1$. Because real wages are constant one finds that:

\[
dW = \frac{W}{P} dP.
\]

Appendix A.2 shows that:

\[
\frac{dy}{db} = \frac{\beta f_{nn}f - f_n \frac{W}{P}}{\beta f_{nn}}.
\]

It can be seen that it is only when $\alpha = 0$ (the Cobb–Douglas case) that the output effect of a supply shock is the same under nominal wage rigidity and wage indexation (Eqs. (17) and (19) are the same). If $\alpha > 0$ the supply shock will have a bigger effect on output with wage indexation as in more traditional models such as Gray (1976).

4. Conclusion

The addition of effort into the production function provides a means for monetary shocks to have real effects without appealing to asymmetric information on prices or wages and where firms are price takers in the output market.

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3 Solow (1979) showed that it is only in the Cobb–Douglas case that the real wage will be invariant to the firms output. The implication is that in any non-Cobb–Douglas case a supply shock will change the optimal real wage, so that wage indexation will not adjust to the "optimal" real wage.
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Appendix A

A.1. The output effects of a monetary shock

In this case I assume that there is no supply shock and there is a random money shock. This means $\beta = 1$ and $\sigma = 0$. Nominal wages are sticky so $dW = 0$. Solving from Eqs. (11)–(14) gives

$$
\frac{dy}{d\delta} = \frac{\overline{M}}{FK} \left[ \frac{f_{np} + \frac{W}{P^2} + \frac{n}{P} f_{nn}}{f_{np} + \frac{W}{P^2} + \frac{n}{P} f_{nn} - \frac{dW}{P}} \right].
$$

(A.1)

By substitution using (7) these equations imply:

$$
\beta f_{np} + \frac{n}{P} f_{nn} = (\alpha - 1) \frac{W}{P^2}.
$$

(A.2)

Using this in Eq. (A.1) results in Eq. (15) in the text.

In the case where $\alpha < 0$ one can see from (15) that there will be a positive output effect if

$$
\alpha \frac{W}{P^2} - f_{nn} < 0.
$$

(A.3)

Given Eqs. (1) and (2) one notes the following derivatives:

$$
f_n(x[n, w, p]) = f_\ell(x[n, w, p])x_n(w, n, p),
$$

(A.4)

$$
f_{nn} = f_{xx}x_n x_n + x_{nn}f_x,
$$

(A.5)

$$
f_{np} = f_{xp} x_n + x_{np} f_x,
$$

(A.6)

where

$$
x_n = Bn^{z-1} \left[ A \left( \frac{w}{p} \right)^z + Bn^z \right]^{\frac{1-z}{z}},
$$

(A.7)

$$
x_p = -\frac{Ae^{z-1}e_{xx}w}{p^2} \left[ A \left( \frac{w}{p} \right)^z + Bn^z \right]^{\frac{1-z}{z}},
$$

(A.8)

$$
x_{nn} = (\alpha - 1)Bn^{z-2} \left[ A \left( \frac{w}{p} \right)^z + Bn^z \right]^{\frac{1-z}{z}}
+ (1 - \alpha)B^2n^{2(z-1)} \left[ A \left( \frac{w}{p} \right)^z + Bn^z \right]^{\frac{1-2z}{z}},
$$

(A.9)
243 and

\[ x_{np} = -(1 - \alpha)ABn^{x-1}e^{x-1}e_w \frac{W}{p^2} \left[ Ae\left(\frac{w}{p}\right)^x + Bn^x \right]^{\frac{1-2\alpha}{2}}. \]  

(A.10)

246 Using (A.5) condition (A.3) is written as

\[ \alpha \frac{W}{P^2} - (f_{xx}x_n^2 - x_{nn}f_x) \frac{y}{W} < 0. \]  

(A.11)

249 Using (A.7) and the first order condition on \( n \) note that

\[ x_{nn}f_x = \frac{W}{P^2} [\alpha - 1] \frac{py}{wn} \left( \frac{Ae^x}{Ae^x + Bn^x} \right). \]  

(A.12)

\[ f_{xx}x_n^2 = \frac{f_{xx}y}{f_x} \frac{W}{P^2}. \]  

(A.13)

254 Using Eqs. (A.12) and (A.13) in (A.11) one gets:

\[ \left[ \frac{(z - 1) - \varepsilon_{ew}}{1 + \varepsilon_{ew}} \right] \frac{1}{\sigma} + \left[ 1 - \varepsilon_{xx} / \varepsilon_x \right] < 0, \]  

(A.14)

where \( z = \frac{P}{w} \geq 1 \) is a parameter exogenous to the firm and determined by fixed entry costs. The elasticity of effort with respect to the wage is \( \varepsilon_{ew} = e_w \frac{w}{P} \), the elasticity of output with respect to efficiency units of labor is \( \varepsilon_x = f_x \frac{y}{x} \) and the elasticity of the marginal product of efficiency units of labor with respect to efficiency units is \( \varepsilon_{xx} = f_{xx} \frac{y}{x} \).

262 A.2. The output effect of a supply shock

263 From Eq. (14)

\[ -P \frac{dy}{dy} = y \frac{dP}{dy}. \]  

(A.15)

266 Using this in (13) one gets:

\[ \frac{dn}{f_{nn}} = \left( \frac{\beta f_{n'}}{n'} \frac{w}{P^2} \right) \frac{p}{y} \frac{dy}{f_n} - \frac{f_n}{\beta f_{nn}} d\beta. \]  

(A.16)

269 Using these equations in (11) gives:

\[ \frac{dy}{y} = \frac{\beta f_{n'}}{f_{nn}} \frac{W}{P} \frac{dy}{f_n} - \frac{W}{f_n} f_n \frac{dy}{\beta f_{nn}} + \frac{n w}{P y} \frac{dy}{f_n} + f \frac{d\beta}{d\beta}. \]  

(A.17)

272 This implies that:

\[ \frac{dy}{\beta f_{nn} - \frac{w}{y} \left( \beta f_{n'} + \frac{w}{P} + \frac{n}{P} \beta f_{nn} \right)} = \frac{dy}{\beta f_{nn} - \frac{w}{y} \left( \beta f_{n'} + \frac{w}{P} + \frac{n}{P} \beta f_{nn} \right)}. \]  

(A.18)

275 Using (A.2) in the denominator gives Eq. (17) in the text.
For indexed nominal wages using (18) in Eq. (11) one gets:

\[ dy = \frac{W}{P} \, dn + f \, d\beta. \] (A.19)

Also recognizing that \( \beta f_{nW} = -\beta f_{nP} \frac{P}{W} \) and using (A.17) in (13) one finds that:

\[ dn = -\frac{f_n}{\beta f_{nn}} \, d\beta. \] (A.20)

This gives the employment effect of a supply shock. This is unambiguously positive with indexed wages, while substituting (A.20) into (A.16) gives (19), the output effect, which is also positive.

References


