<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Can rational expectations sticky-price models explain inflation dynamics?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Authors(s)</strong></td>
<td>Whelan, Karl; Rudd, Jeremy</td>
</tr>
<tr>
<td><strong>Publication date</strong></td>
<td>2006-03</td>
</tr>
<tr>
<td><strong>Publication information</strong></td>
<td>American Economic Review, 96 (1): 303-320</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>American Economic Association</td>
</tr>
<tr>
<td><strong>Link to online version</strong></td>
<td><a href="http://dx.doi.org/10.1257/000282806776157560">http://dx.doi.org/10.1257/000282806776157560</a>; <a href="http://search.ebscohost.com/login.aspx?direct=true&amp;db=buh&amp;AN=19991983&amp;site=ehost-live">http://search.ebscohost.com/login.aspx?direct=true&amp;db=buh&amp;AN=19991983&amp;site=ehost-live</a></td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/199">http://hdl.handle.net/10197/199</a></td>
</tr>
<tr>
<td><strong>Publisher's version (DOI)</strong></td>
<td>10.1257/000282806776157560</td>
</tr>
</tbody>
</table>
Can Rational Expectations Sticky-Price Models Explain Inflation Dynamics?

By Jeremy Rudd and Karl Whelan*

In recent years, there has been a trend in macroeconomics toward analyzing business cycles and stabilization policy in the context of models that incorporate both nominal rigidities and optimizing agents with rational (i.e., model-consistent) expectations. One important way in which this “new-Keynesian” approach differs from earlier work in the Keynesian tradition involves the way in which expectations are assumed to affect price-setting behavior. In particular, rather than assuming adaptive inflation expectations on the part of wage- and price-setters, recent work draws on explicit models of price stickiness (such as that of Guillermo A. Calvo, 1983) in order to motivate a forward-looking inflation equation (a “new-Keynesian Phillips curve”) of the form

\[ \pi_t = \beta E_t \pi_{t+1} + \gamma y_t, \]  

where \( \beta \) is a parameter close to or equal to one, and \( y_t \) is a measure of the output gap.

An important implication of this model is that inflation should be independent of its own lagged values. As a result, this specification has often been criticized on the grounds that it cannot account for the important empirical role played by lagged dependent variables in inflation regressions. In response to this critique, several researchers have suggested an alternative to the pure forward-looking model that is intended to better capture observed inflation inertia. This “hybrid” specification modifies the new-Keynesian Phillips curve so that inflation depends on a weighted sum of its lag and its (rationally) expected future value,

\[ \pi_t = (1 - \theta) \pi_{t-1} + \theta E_t \pi_{t+1} + \gamma y_t, \]

with the weights constrained to sum to unity in order to preclude the existence of a long-run level tradeoff between inflation and real activity.²

Within the class of papers employing variants of this hybrid specification, the best-known studies have featured models in which \( \theta = 1/2 \). For example, the well-known model of Jeffrey C. Fuhrer and George R. Moore (1995) employs an assumption that workers bargain over relative real wages in order to obtain an equation with \( \theta = 1/2 \). More recently, Lawrence J. Christiano et al. (2005) have explicitly derived a specification of this form using a variant of the Calvo model in which those firms that are unable to reoptimize their price instead index it to last period’s inflation rate. In their framework, \( \theta \) equals \( \beta/(1+\beta) \) (where \( \beta \) is the factor used to discount firms’ profits); this directly implies that \( \theta \) will be less than \( 1/2 \).

In this paper, we assess whether hybrid models of this sort provide a good empirical characterization of the behavior of U.S. inflation. For the case in which \( \theta \leq 1/2 \), our tests are based on the observation that the hybrid specification implies an expression for the change in inflation of the form

\[ \Delta \pi_t = \lambda_1 \sum_{k=0}^{\infty} \lambda_k E_t y_{t+k} \]

² Examples of studies that use this pricing equation include Miguel Casares and Bennett T. McCallum (2000), Michael Ehrmann and Frank Smets (2003), and Glenn Rudebusch (2002).

* Rudd: Division of Research and Statistics, Board of Governors of the Federal Reserve System, 20th and C Streets NW, Mailstop 80, Washington, DC 20551-0001 (e-mail: jeremy.b.rudd@frb.gov); Whelan: Department of Economic Analysis, Research, and Publications, Central Bank and Financial Services Authority of Ireland, Dame Street, Dublin 2, Ireland (e-mail: karl.whelan@centralbank.ie). We thank Dale Henderson, Frank Smets, William Wascher, and two anonymous referees for helpful comments on earlier drafts. The views expressed are our own and do not necessarily reflect the views of the Board of Governors or staff of the Federal Reserve System, or of the Central Bank and Financial Services Authority of Ireland.

¹ See Richard Clarida et al. (1999) for a survey of much of this work, and Michael Woodford (2003) for a detailed treatment.

303
where $\lambda_2 \leq 1$. We focus on this prediction, rather than on the model’s ability to fit the level of inflation, in order to derive tests that are capable of distinguishing the hybrid model from reasonable alternatives. In practice, inflation can be predicted well from its own lagged value; hence, incorporating lagged inflation into the inflation equation should allow the hybrid model to fit the level of inflation relatively well. However, such a fit could also be obtained by any model that features an important role for lagged inflation—including models that rely on nonrational, backward-looking expectations. In contrast, the hybrid model’s predictions for the evolution of $\Delta \pi_t$ are quite clear-cut and allow us to precisely distinguish this model from a traditional backward-looking specification.

We consider two different methods for assessing whether this formulation of the hybrid model provides a reasonable description of the data. The first employs the well-known methodology of John Y. Campbell and Robert J. Shiller (1987), which entails using a VAR to forecast future values of the driving process $y_t$. The second method involves estimating the equation using the generalized method of moments (GMM). Both methods turn out to yield useful insights—the first into the predicted time-series properties of $\Delta \pi_t$ that are implied by the model, and the second into the statistical significance of the model’s forward-looking component.

While variants of the hybrid specification in which $\theta \leq \frac{1}{2}$ have received a large amount of attention in recent work, there is no a priori reason to rule out the possibility that price setting is characterized by a preponderance of forward-looking behavior. We therefore also consider versions of the hybrid model with $\theta > \frac{1}{2}$, which imply the following closed-form solution:

$$\pi_t = \mu_1 \sum_{k=0}^{\infty} E_t y_{t+k} + \mu_2 \pi_{t-1}. \tag{4}$$

Here, the level of current inflation is related to lagged inflation (with $\mu_2 < 1$) and current and expected future values of the driving term, where these receive a unit weight in all periods. Again, the presence of lagged inflation ensures that this model will be able to fit $\pi_t$ relatively well; hence, the relevant question here concerns what contribution the forward-looking terms make to explaining inflation dynamics.

Taken as a whole, our results suggest that the hybrid model provides a poor description of empirical inflation dynamics. Specifically, we find that the empirical process for the change in inflation appears to bear very little resemblance to a discounted sum of current and expected future $y_t$ values. Moreover, we find that the coefficients on the discounted sum ($\lambda_1$ or $\mu_1$) are not significantly different from zero for any variant of the hybrid model that we consider, implying that inflation is unrelated to the expectation of future values of the driving term, and indicating that the type of rational forward-looking behavior hypothesized by the hybrid model is absent from the data. Importantly, these conclusions hold both when we use detrended output as $y_t$, and when we use labor’s share of income (real unit labor costs), as has been suggested by Jordi Galí and Mark Gertler (1999).

The contents of the paper are as follows. Section I briefly discusses the nature of the “persistence problem” that is faced by the new-Keynesian Phillips curve (and that motivates the use of hybrid inflation equations). Section II introduces the hybrid model and discusses its closed-form solutions. Section III assesses the fit of the hybrid model when $\theta \leq \frac{1}{2}$. Section IV presents GMM estimates of this model, and also examines whether its performance can be improved by incorporating a more complex “rule-of-thumb” for backward-looking agents. Finally, Section V considers the version of the model that obtains when $\theta > \frac{1}{2}$, and Section VI concludes.

I. The Persistence Problem

At first glance, it might appear as though the new-Keynesian inflation specification,

$$\pi_t = \beta E_t \pi_{t+1} + \gamma y_t, \tag{5}$$

would be difficult to distinguish from models in which inflation depends on its own lagged values. Inflation is highly autocorrelated, and so next period’s expected inflation rate is likely to be highly correlated with last period’s rate. When combined with the assumption of rational expectations, however, the new-Keynesian model makes a very pre-
cise prediction about the nature of inflation dynamics. This can be seen by applying repeated substitutions to equation (5) to obtain

\[
\pi_t = \gamma \sum_{k=0}^{\infty} \beta^k E_t y_{t+k}.
\]

(6)

The model predicts that inflation depends solely on current and expected future values of the output gap. Once we condition on this, no lagged variables—including lagged inflation—should have an impact on the current level of inflation.

There is, however, relatively wide agreement that this formulation does not sufficiently explain the role played by lagged inflation in reduced-form inflation regressions. Jeremy Rudd and Karl Whelan (2005a) provide an illustration of this problem by using the methodology of Campbell and Shiller (1987) to assess the fit of the new-Keynesian Phillips curve. Specifically, if we assume that \( y_t \) is the first variable in a multivariate VAR of the form

\[
z_t = A z_{t-1} + \varepsilon_t,
\]

(7)

then one can express the discounted sum of current and future values of \( y_t \) as

\[
\sum_{k=0}^{\infty} \beta^k E_t y_{t+k} = \varepsilon'_t (I - \beta A)^{-1} z_t
\]

(8)

where \( \varepsilon'_t \) denotes a vector with one in the first row and zeroes elsewhere. Rudd and Whelan demonstrate that the empirical fit of the new-Keynesian Phillips curve is poor across a wide variety of VAR specifications for \( z_t \). In addition, econometric specifications such as

\[
\pi_t = \gamma' \varepsilon'_t (I - \beta A)^{-1} z_t + A(L) \pi_{t-1}
\]

(9)

reveal that there is a statistically significant and economically large role for lagged inflation, despite one’s having proxied for the expected present value of the driving variable \( y_t \). This result is obtained whether one measures the output gap as detrended GDP or as labor’s share of income, as suggested by Galí and Gertler (1999). The result is also robust to the use of VAR specifications that include inflation itself, so that one can rule out the possibility that lags of inflation enter empirical Phillips curves only because they are proxying for expectations of future values of \( y_t \).

It is important to stress that it is this result—the failure of the pure forward-looking model to account for the empirical importance of lagged inflation—that defines the so-called persistence problem faced by the new-Keynesian Phillips curve. We make this observation because discussions of the empirical performance of sticky-price models have commonly focused on the high autocorrelation of inflation, with the implication being that it is this property of the data that these models should seek to match.\(^3\) However, despite their inability to account for the important role played by lagged inflation, empirical implementations of the new-Keynesian Phillips curve still predict that inflation should be highly autocorrelated: as long as \( y_t \) is highly autocorrelated (as is the case for detrended output and the labor income share), the predicted inflation series from the new-Keynesian Phillips curve will be highly autocorrelated.

These findings suggest that it is the failure to capture the inertia in inflation, given fundamentals, that characterizes the pure forward-looking model’s persistence problem. Put differently, the persistence problem stems from the fact that lagged inflation enters reduced-form inflation equations with large coefficients \( e_1 \) after we have conditioned on driving variables (such as the output gap) that are themselves highly autocorrelated. This suggests that hybrid variants of the basic sticky-price model, which directly allow for a lagged inflation term, may perform better empirically. We now examine these models.

II. Closed-Form Solutions to the Hybrid Model

The approach we take to evaluate the empirical relevance of the hybrid inflation equation

\[
\pi_t = (1 - \theta) \pi_{t-1} + \theta E_t \pi_{t+1} + \gamma y_t
\]

(10)

closely follows the approach described in the previous section for assessing the pure forward-

\(^3\) Fuhrer and Moore (1995), John B. Taylor (1999), and Luca Guerrieri (2002) provide three examples of papers that discuss the new-Keynesian Phillips curve’s “persistence problem” in terms of its ability to match high autocorrelations for inflation.
looking model. Specifically, we focus directly on the hybrid model’s closed-form solutions, which express inflation in terms of its own lagged value and a composite forward-looking term of expected future output gaps. In this section, we first describe how to derive these expressions, and then contrast our method for evaluating the hybrid model with the procedures employed by Galí and Gertler (1999) and Fuhrer (1997) in previous work.

A. Derivation of the Model’s Closed-Form Solutions

Begin by rewriting the hybrid model (10) in terms of lead and lag operators:

\[
E_t \left[ F^2 - \frac{1}{\theta} F + \frac{1 - \theta}{\theta} \right] L \pi_t = -\gamma \theta y_t.
\]

(11)

It is straightforward to apply the quadratic formula to show that one root of this characteristic polynomial equals one, while the other equals \(\frac{1 - \theta}{\theta}\). Hence, the stochastic difference equation implied by the hybrid model can be written as

\[
E_t \left[ (F - 1) \left( F - \frac{1 - \theta}{\theta} \right) \right] L \pi_t = -\gamma \theta y_t.
\]

(12)

When \(\theta \leq \frac{1}{2}\), then \(\frac{1 - \theta}{\theta} \geq 1\) and the unique stable solution is found by multiplying through by the forward inverse \(\left[ F - \frac{1 - \theta}{\theta} \right]^{-1}\), which yields a solution of the form

\[
\Delta \pi_t = \frac{\gamma}{1 - \theta} \sum_{k=0}^{\infty} \left( \frac{\theta}{1 - \theta} \right)^k E_t y_{t+k}.
\]

(13)

Thus, hybrid models such as those of Fuhrer and Moore (1995) (which assumes \(\theta = 0.5\)) and Christiano et al. (2005) (which assumes \(\theta < 0.5\)) imply that the change in inflation should be proportional to a discounted sum of current and expected future values of the output gap.

Alternatively, when \(\theta > \frac{1}{2}\), the stable solution is found by multiplying through with the forward inverse \(\left( F - 1 \right)^{-1}\). This results in a solution of the form

\[
\pi_t = \left( \frac{1 - \theta}{\theta} \right) \pi_{t-1} + \frac{\gamma}{\theta} \sum_{k=0}^{\infty} E_t y_{t+k}.
\]

(14)

In this case, inflation depends on its own lag and on an undiscounted sum of current and expected future values of the output gap.

These derivations clearly show that the crucial feature of the hybrid model under rational expectations is the presence of a composite forward-looking sum of expected output gaps. It is this term that distinguishes these models from alternatives based on purely backward-looking inflation expectations. Hence, our approach in this paper involves assessing the role played by this forward-looking component. For example, by specifying a forecasting model for the output gap, we can construct an empirical proxy for the forward-looking term, which in turn will permit us to determine its contribution to the model’s fit. Alternatively, we can use GMM-based techniques to directly estimate the closed-form expressions (13) and (14), which will allow us to examine whether the composite sum is statistically significant.

B. Comparison with Other Procedures

It is useful to contrast briefly our method of assessing the hybrid model with the approach taken in previous studies. Here we discuss the differences between our approach and two alternative procedures followed by Galí and Gertler (1999) and Fuhrer (1997).

Comparison with Galí and Gertler (1999).—

These authors focus on estimates of \(\theta\) obtained from directly fitting equation (10) using GMM. Specifically, under this procedure \(E_t \pi_{t+1}\) is replaced with \(\pi_{t+1}\) and the model is estimated using instruments for \(\pi_{t+1}\). If the model is correct and expectations are rational, then any estimation error reflects the presence of an expectation error \((\pi_{t+1} - E_t \pi_{t+1})\) that should be unforecastable at time \(t\) or earlier. Thus, in theory, any variable dated \(t\) or earlier can serve as a valid instrument. Using this method, Galí and Gertler find that \(\theta\) is greater than one-half, and conclude that rational forward-looking behavior plays an important role in determining U.S. inflation.

It is possible to demonstrate, however, that a number of potential pitfalls can arise when GMM estimates of \(\theta\) from equations like (10) are used in order to assess the importance of forward-looking behavior in price setting. Al-
though this particular approach invokes the assumption of rational expectations to justify its choice of instruments, it does not actually impose the assumption of fully rational (that is, model-consistent) expectations on the estimation. GMM is equivalent to two-stage least squares in this case, and in practice there may be little correlation between the fitted value of $E_t \pi_{t+1}$ from the first-stage regression and its value under model-consistent expectations. Thus, the constructed proxy for expected inflation can receive a significant coefficient even if the model’s key prediction regarding the relationship between inflation and expectations of future output gaps is incorrect.\footnote{See Rudd and Whelan (2005b) for a detailed discussion of this problem.}

Our approach does not suffer from this drawback: by directly estimating the model’s closed-form solution, we ensure that model-consistent expectations are imposed. Moreover, focusing on the role played by expected future values of $y$, permits us to highlight precisely the specific contribution of rational forward-looking expectations to inflation dynamics.

Comparison with Fuhrer (1997).—The methodology employed in this paper shares a similarity with our own approach, in that Fuhrer’s estimation procedure also imposes model-consistent expectations on the inflation equation (which ensures that it will take the form of either equation (13) or equation (14)). There is a fundamental difference, however, between our method for assessing the relevance of forward-looking behavior in price setting and the method used by Fuhrer.

Fuhrer’s estimation procedure yields the value of the $\theta$ parameter that best fits the data. Based on the low values of $\theta$ that he obtains, he concludes that forward-looking behavior plays essentially no role in observed inflation dynamics. It should be emphasized, however, that the estimate of $\theta$ produced by this method does not necessarily allow one to discriminate between forward- and backward-looking models of inflation. Indeed, this procedure can yield significant positive estimates of $\theta$ even when the true model for inflation features only backward-looking behavior (in which case the term involving future output gaps is immaterial). To see this, suppose that the best-fitting specification is the one given by equation (14). In this case, the estimated value of $\theta$ will be completely determined by the estimated coefficient on lagged inflation. Given the empirical importance of lagged inflation, $\theta$ will typically be estimated to be highly statistically significant (with a point estimate that will be greater than one-half so long as the coefficient on lagged inflation is less than one)—and this can be true even if the coefficient on the sum of future output gaps is itself statistically insignificant.

For this reason, we focus directly on the importance of the composite forward-looking term (i.e., the sum of current and expected future output gaps). Furthermore, because some of our results are in fact consistent with significant positive estimates of $\theta$, our rejection of forward-looking behavior in price setting is based on a reading of the empirical evidence that is different from what is in Fuhrer’s paper.\footnote{It is worth noting two other significant differences between the two papers. First, Fuhrer’s paper uses detrended output as a proxy for the output gap; we use both detrended output and the labor share measure recommended by Gali and Gertler (1999). Second, to apply Fuhrer’s maximum-likelihood methodology, one must explicitly specify a driving process for the output gap proxy; in contrast, this is not required for the GMM procedures that we consider in Sections IV and V.}

### III. Fit of the Hybrid Model with $\theta \leq \frac{1}{2}$

We now apply the Campbell-Shiller methodology to assess the fit of equation (13), which gives the closed-form solution of the hybrid model with $\theta \leq \frac{1}{2}$. As described above, we can assume that $y_t$ is the first variable in the vector $z_t$, and calculate the discounted sum as

\[
\sum_{k=0}^{\infty} \left( \frac{\theta}{1 - \theta} \right)^k E_t y_{t+k} = e_t \left( I - \frac{\theta}{1 - \theta} A \right)^{-1} z_t
\]

where $z_t$ is modelled as a VAR expressed in the companion form $z_t = A z_{t-1} + \varepsilon_t$. The “discount factor” associated with the infinite sum, $\theta/(1 - \theta)$, is unknown, so the approach that we take here involves using a grid search (over the interval zero to one) to obtain the value of the discount factor that yields the highest correla-
tion between the resulting discounted sum and the first difference of inflation.

We consider two versions of the model. The first equates $y_t$ with a traditional output gap measure, defined here as the deviation of log real nonfarm GDP from a quadratic trend. The second follows Galí and Gertler (1999) in using (the log of) labor’s share of income, again defined for the nonfarm business sector. The motivation for this latter measure stems from the observation that the sticky-price models underpinning the new-Keynesian Phillips curve imply that the correct driving variable for inflation is actually real marginal cost. Because the theoretical restrictions required in order for real marginal cost to move with the traditional output gap are restrictive, Galí and Gertler (and others) have instead proposed using average unit labor costs—nominal compensation divided by real output—as a proxy for nominal marginal cost. The resulting measure of real marginal cost is labor’s share of income (nominal compensation divided by nominal output).

A. Output Gap Model

To forecast future values of the output gap, we use a standard two-lag, three-variable VAR which includes the output gap, the federal funds rate, and inflation, which we measure as the log-difference of the price deflator for the nonfarm business sector. The sample period extends from 1960:Q1 to 2002:Q1. This simple VAR forecasts the output gap well and has been used in a number of papers, including John H. Cochrane (1994), Fuhrer and Moore (1995), and Julio J. Rotemberg and Michael Woodford (1997).

The results from this exercise provide little support for the hybrid model. The model explains only about 3 1/2 percent of the variance in the first-difference in inflation, and the grid search reveals that zero is the best-fitting non-negative value of the discount factor, implying an equation that reduces to $\Delta \pi_t = \gamma y_t$. In this model, then, expectations of future output gaps do nothing to improve the equation’s fit. The model’s poor fit is illustrated graphically in Figure 1. The top panel of the figure plots the time series for the first-difference of inflation, along with the time series for the model’s fitted values. Because the change in inflation is such a volatile series, it is somewhat difficult to assess accurately the model’s fit from this chart; hence, the lower panel of the figure presents a simple scatter diagram. As can be seen from the almost random distribution of the data points, the ability of this model to predict even the sign of the change in inflation is quite poor.6

B. Labor Share Model

To test this version of the model, we augment our existing three-variable VAR with the log of the labor share. The results for this version of the hybrid model are not much more encouraging. In this case, the grid search reveals that the best-fitting hybrid model implies a value for $\theta/(1 - \theta)$ of 0.97 (and thus $\theta = 0.49$), so the discounted sum does not vanish. As is illustrated in Figure 2, however, this model does an even worse job than the output gap model in fitting the first difference of inflation (its $R^2$ is only 0.01). In addition, a simple regression of $\Delta \pi_t$ on the discounted sum of labor income shares yields a $t$-statistic of only 1.40. Because the explanatory variable in this case is a generated regressor, and because we are arbitrarily treating the discount factor as known, this statistic cannot be interpreted as being drawn from a standard distribution (an issue that we will address in Section IV). But, together with the model’s low $R^2$, these results serve to question whether there is statistical evidence for any link between the first-difference of inflation and current and future values of the labor income share.

These findings underscore a point made in the previous section; namely, that a positive estimate of $\theta$ should not on its own be construed as evidence that the forward-looking component of these models adds anything to the models’ overall fit (even when the estimate of $\theta$ is obtained from a procedure that imposes the model-consistent solution).

While we do not have the space to report these results here, we note in passing that our finding that both the output gap and labor share models fit poorly is robust to various changes in specification, including the use of alternative

---

6 The fact that the model cannot predict the magnitude of these inflation changes can also be seen from the scatterplot: while the $x$-axis, which plots actual changes in inflation, has a range of 15 percentage points, the fitted values on the $y$-axis have a range of less than 2 percentage points.
inflation and output gap measures and estimation over pre- and post-1983 subsamples.

C. Comparison with Reduced-Form Regressions

Of course, because the first-difference of inflation is such a volatile variable, we would not necessarily expect such parsimonious specifications as these to fit very well. That said, a useful benchmark that illustrates just how poor these models are can be obtained from a simple regression of $\Delta \pi_t$ on a constant and its own lag. This regression has an adjusted $R^2$ of 0.14; its fit is illustrated graphically in Figure 3. While it is
difficult to predict the exact magnitudes of quarterly changes in inflation, this model does much better than either of the hybrid models in matching the direction and size of these changes.

The simple regression achieves this improvement in fit by capturing an important feature of inflation dynamics that is absent from the hybrid model. The coefficient on the lagged change in inflation in this regression is $-0.38$, which reflects the fact that the first-difference of inflation is negatively autocorrelated. In contrast, the discounted sum of the output gap (which here is merely the output gap itself) and of the labor income share are both highly positively auto-
correlated, with first-order autocorrelation coefficients that exceed 0.9. Hence, the discounted sums fundamentally fail to describe a key feature of the $\Delta \pi_t$ process.

Table 1 reports some additional reduced-form regressions for $\Delta \pi_t$. Adding a second lag (column 2) raises the regression’s $R^2$ a touch, to 0.15. More interestingly, the inclusion of the output gap also improves the fit of this regression: for the two-lag case, the $R^2$ is 0.22 and the output gap’s $t$-statistic equals 4.06. In contrast, the addition of the labor income share (column 4) yields essentially no improvement in this regression’s fit. These patterns demonstrate that...
The ability of a standard reduced-form Phillips curve regression—which relates the level of inflation to its own lags (restricting the sum to one) and a measure of slack such as the output gap—to replicate important aspects of the empirical behavior of inflation is not at all shared by these hybrid sticky-price models.\footnote{See Douglas Staiger et al. (1997) and Robert J. Gordon (1998) for two typical implementations of a reduced-form Phillips curve.}

Finally, column 5 of Table 1 reports the effects of adding two lags of commodity price inflation to the basic reduced-form specification, where commodity prices are defined as the producer price index for crude materials. The purpose of adding this variable is to assess to what degree the observed negative autocorrelation in \( \Delta \pi \), reflects volatility in commodity prices. It seems unlikely that the kinds of frictions envisaged by sticky-price models hold for these types of prices, which are often determined in auction markets. And, as might be expected for a competitively determined price, changes in commodity prices are quite random. As a result, one would expect the change in commodity price inflation to be negatively autocorrelated, and this pattern does indeed hold in the data. Table 1 shows, however, that while including commodity prices improves the fit of the reduced-form regression, with the \( \overline{R^2} \) rising to 0.32 (see also Figure 4), it does little to alter the pattern of negative coefficients on the lagged changes in inflation.

### D. Results Using Annual Data

An additional factor that could contribute to the negative autocorrelation that we observe in \( \Delta \pi \), is the presence of serially uncorrelated measurement error (or some other type of transitory high-frequency shock) in inflation. Noise of this sort would have an effect similar to that described above for commodity prices, and could act to obscure any relationship between the first-difference of inflation and the discounted sum of the driving variable.

To test this possibility, we use annual data to reestimate the output gap and labor share variants of the hybrid model. When we do so, we find that none of our principal conclusions is altered; in particular, we still find that the expected discounted sum of the labor income share explains very little of the variance in \( \Delta \pi \), while the best-fitting value of the discount factor in the version of the hybrid model that uses detrended GDP remains zero (thus implying that forward-looking behavior is completely absent from the model).

The reason for the hybrid model’s inability to fit annual data is closely related to the source of the model’s failure in quarterly
Recall that, in quarterly data, $\Delta \pi_t$ was negatively autocorrelated while the estimated discounted sum of the driving term was highly positively autocorrelated. Using annual data smooths away much of the high-frequency variation in $\Delta \pi_t$, and leaves the first-difference of inflation essentially uncorrelated with its own lags. However, the discounted sums of both the output gap and labor’s share remain strongly positively autocorrelated at an annual frequency. Hence, our finding that the hybrid model provides a poor characterization of the $\Delta \pi_t$ process does not depend on the use of quarterly data.
E. Summary

The results of this section can be summarized as follows.

- The popular class of hybrid models for which $\theta \leq \frac{1}{2}$ can generate predicted series for the level of inflation that are both highly correlated with actual inflation (for either driving variable, this correlation equals 0.85 in quarterly data) and highly autocorrelated.
- There appears, however, to be very little evidence that the models’ success in matching the level of inflation requires any of the rational forward-looking behavior posited by the hybrid models. In particular, the prediction of these models that distinguishes them from backward-looking alternatives—that the change in inflation should move with a discounted sum of output gaps or labor income shares—is strongly rejected.
- Moreover, these specifications completely fail to capture important features of the data that can be summarized by simple reduced-form Phillips curves that feature the GDP gap and several lags of inflation.

These results still leave some important questions unanswered. The first involves the certainty with which we can rule out the presence of forward-looking behavior in the hybrid inflation specifications: we have not yet been able to formally assess the statistical significance of the discounted sum. The second issue relates to whether a patched-up version of the class of hybrid models with $\theta \leq \frac{1}{2}$—based, for example, on an alternative rule-of-thumb for backward-looking agents—can do better in matching the data, perhaps thereby revealing an important role for forward-looking behavior. Finally, there is the question of how models based on the assumption of $\theta > \frac{1}{2}$ perform. These topics are addressed next.

IV. GMM Estimation

The usefulness of the Campbell-Shiller approach comes from its ability to provide an explicit prediction for the values of $\Delta \pi_t$ that are implied by the hybrid model. However, one drawback of this method is that it cannot be used to derive statistical inferences about the model’s parameters—in particular, we cannot determine whether the discounted sums of output gaps or labor shares make a statistically significant contribution to observed inflation dynamics.

GMM provides an alternative methodology that does not suffer from this problem. While GMM does not yield an explicit predicted series for $\Delta \pi_t$ (and thus does not allow an assessment of the model’s fit), it has the advantage of not requiring us to specify an explicit process for the driving term $y_t$. And GMM allows us to estimate $\lambda_1$ and $\lambda_2$ consistently (with their standard errors) in the closed-form representation

$$\Delta \pi_t = \lambda_1 \sum_{k=0}^{\infty} \lambda_2^k E_t y_{t+k}. \tag{16}$$

Note that we have deliberately written our equation for GMM estimation in this form as opposed to in the form of equation (13). This is because we are interested in assessing directly whether $\lambda_1$ is statistically significant, rather than in testing hypotheses about the coefficients $\gamma$ and $\theta$. While one set of estimates clearly implies values for the other (and we report both), the question we are asking is whether the composite forward-looking term has a statistically significant effect on inflation—which in turn is a direct question about the statistical significance of $\lambda_1$.

A. The Basic Hybrid Model

GMM estimation of equation (16) requires us to specify a set of instruments $z_t$ that are known by agents at time $t$. Under rational expectations, the orthogonality condition

$$E \left[ \left( \Delta \pi_t - \lambda_1 \sum_{k=0}^{\infty} \lambda_2^k y_{t+k} \right) z_t \right] = 0 \tag{17}$$

should hold in the data. One practical issue that must be dealt with is the presence of an infinite sum in (17); we address this problem by following the approach of Rudd and Whelan.
Table 2—GMM Estimates of Hybrid Inflation Equation

<table>
<thead>
<tr>
<th>Driving variable ($y_t$)</th>
<th>Reduced-form parameters</th>
<th>Structural parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_1$</td>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>Detrended output</td>
<td>0.039</td>
<td>0.614</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.372)</td>
</tr>
<tr>
<td>Labor income share</td>
<td>0.017</td>
<td>0.769</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.498)</td>
</tr>
</tbody>
</table>

Notes: Table gives estimated parameter values from the basic hybrid model $\Delta \pi_t = \lambda_1 \sum_{i=0}^{K} \lambda_2^i \Delta y_{t+i}$, with structural parameters implicitly defined as $\lambda_1 = \gamma/(1 - \theta)$ and $\lambda_2 = \theta/(1 - \theta)$. Standard errors in parentheses; ** or * denotes significant at 1- or 5-percent level, respectively.

(2005b) and rewriting the orthogonality conditions as

$$E\left[\left(\Delta \pi_t - \lambda_1 \sum_{k=0}^{K} \lambda_2^k \Delta y_{t+k} - \lambda_2^{K+1} \Delta \pi_{t+K+1}\right) z_t\right] = 0.$$  

The estimates of $\lambda_1$ and $\lambda_2$ that we obtain using this procedure are reported in Table 2. For the models that use labor’s share as a proxy for $y_t$, the instrument set $z_t$ consists of two lags each of the change in inflation, the output gap, the labor share, and wage inflation (measured as the log-difference in nonfarm compensation per hour). When detrended output is used as the driving term, we replace log-differenced hourly compensation—which makes no contribution to first-stage fit—with the federal funds rate, which is a highly significant predictor in the first-stage regressions. We set $K$ equal to 12.

The results confirm an empirical finding that was suggested by our VAR-based exercises: for both the output-gap and labor-share versions of the models, the estimated values of $\lambda_1$ are not statistically different from zero. Hence, not only do the discounted sums of future labor shares and output gaps explain very little of the variation in $\Delta \pi_t$, they actually appear to have no statistically discernable influence on this variable whatsoever.

This finding was robust to the value of $K$ used, as well as to different definitions of inflation and detrended output, and estimation over pre- and post-1983 subsamples. In addition, this result was robust to the specific instrument set used: $\lambda_1$ was estimated to be statistically insignificant across a wide range of instrument sets that included various lags of additional instruments such as commodity price inflation, yield spreads, and short-term interest rates.

Table 2 also reports the estimates of $\gamma$ and $\theta$ obtained from applying GMM estimation to equation (13). Both the output gap and labor share versions of the model imply estimates of $\theta$ that are significantly greater than zero. It should be stressed, however, that the estimate of $\theta$ obtained from this procedure is only a function of the estimated forward root $\lambda_2$ (because here $\theta = \lambda_2/(1 + \lambda_2)$). The fact that we obtain a significant value of $\theta$ suggests that a discounted sum with a nonzero discount factor may yield the best-fitting model. But even this best-fitting discounted sum may make no significant contribution to explaining the change in inflation—and, indeed, in both of the cases considered here we are unable to reject the hypothesis that the coefficients on the discounted sums are zero. (This result closely parallels the VAR-based results for the labor share model, in which the grid search selected a nonzero value of $\theta$ even though the discounted sum made no contribution to the model’s fit.)

B. More General Hybrid Models

Our earlier results suggest one potential route for improving the performance of this model. Table 1 shows that an implicit assumption underlying the simple hybrid specification—namely, that incorporating a single lag of inflation would allow the model to match the empirical nature of inflation inertia—was incorrect. In particular, the negative autocorrelation of $\Delta \pi_t$ implies that the underlying model for the level of inflation should include more than one lag of the dependent variable. One way to address this is to assume that the underlying structural equation contains an additional inflation lag, thereby taking the form:

$$\pi_t = \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + (1 - \theta_1 - \theta_2)E_t \pi_{t+1} + \gamma y_t.$$
Such a specification could be motivated, for example, by assuming a fraction of nonrational price-setters who use the last two observations of inflation to formulate their expectations, or—within the Christiano et al. (2005) framework—a more complex indexation rule for those firms that do not set an optimal price this period.

Equation (19) has the following closed-form solution:

\[
\Delta \pi_t = \lambda_1 \sum_{k=0}^{\infty} \lambda_2^k E_t y_{t+k} + \lambda_3 \Delta \pi_{t-1}
\]

where the parameters \(\lambda_1\), \(\lambda_2\), and \(\lambda_3\) represent highly nonlinear functions of the underlying parameters \(\theta_1\), \(\theta_2\), and \(\gamma\). In Table 3, we report GMM estimates of \(\lambda_1\), \(\lambda_2\), and \(\lambda_3\) that are obtained using the same procedure and the same instrument sets that were used in estimating equation (18). Again, the key question is whether we obtain statistically significant and economically sensible values for \(\lambda_1\) and \(\lambda_2\) (i.e., whether allowing for extra lags of inflation improves the case for the existence of a forward-looking rational expectations term).

As expected, Table 3 indicates that the coefficient on \(\Delta \pi_{t-1}\) is negative and highly statistically significant. But this exercise still fails to produce any convincing evidence of forward-looking behavior. For the output-gap version of the model, the coefficient on the discounted sum, \(\lambda_1\), is statistically significant, but the estimated forward root, \(\lambda_2\), is negative, which is not reasonable in this context. For the labor share version, the estimated forward root is positive, but the coefficient on the discounted sum receives a \(t\)-statistic of only 0.65. On the whole, then, these results do little to endorse the presence of forward-looking rational expectations, and thus the case for a more complex hybrid model featuring extra lags of inflation.

V. The Hybrid Specification with \(\theta > \frac{1}{2}\)

The versions of the hybrid model that we have considered up to this point involve values of \(\theta\) that are less than or equal to one-half. We now examine the version of the model for which \(\theta > \frac{1}{2}\). Specifically, we examine the role played by the forward-looking term in the following closed-form solution for the level of inflation:

\[
\pi_t = \mu_1 \sum_{k=0}^{\infty} E_t y_{t+k} + \mu_2 \pi_{t-1}.
\]

As before, and for the same reasons, we focus on estimating the equation in this form, rather than in the form given by equation (14).

A. VAR-Based Method

Figure 5 summarizes the results obtained from applying the Campbell-Shiller methodology to assess the contribution of the forward-looking term in equation (21). We again run the VAR specifications described in Section III and measure the forward-looking term as

\[
\sum_{k=0}^{\infty} E_t y_{t+k} = e'_1 (I - A)^{-1} z_t.
\]

In this case, we do not need to estimate the best-fitting forward root because this model imposes the assumption that the forward root is one. Instead, we run a regression of inflation on its own lag and our measure of the discounted sum to arrive at our estimate of \(\mu_2\); i.e., we estimate \(\mu_1\) and \(\mu_2\) from

\[
\pi_t = \mu_1 e'_1 (I - A)^{-1} z_t + \mu_2 \pi_{t-1}.
\]

Our results suggest an extremely limited role for the forward-looking terms in determining the behavior of inflation. For the output gap model, adding the discounted sum improves the fit somewhat, but the estimated coefficient \(\mu_1\) has

<table>
<thead>
<tr>
<th>Driving variable ((y_i))</th>
<th>(\lambda_1)</th>
<th>(\lambda_2)</th>
<th>(\lambda_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detrended output</td>
<td>0.146**</td>
<td>-0.990**</td>
<td>-0.364*</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.050)</td>
<td>(0.141)</td>
</tr>
<tr>
<td>Labor income share</td>
<td>0.024</td>
<td>0.764</td>
<td>-0.392**</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.465)</td>
<td>(0.053)</td>
</tr>
</tbody>
</table>

Notes: Table gives estimated values for the parameters from the augmented hybrid model \(\Delta \pi_t = \lambda_1 \sum_{i=0}^{\infty} \lambda_2^i E_t y_{t+i} + \lambda_3 \Delta \pi_{t-1}\). Standard errors in parentheses; ** or * denotes significant at 1- or 5-percent level, respectively.
an incorrect (negative) sign. For the labor share case, the model has an $R^2$ of 0.71, which is exactly the same as what is obtained from a regression of inflation on its own lag only.\footnote{Experimentation with various specifications for the labor share VAR showed that some yield series for $e_t'(1 - A)^{-1}z_t$ that can improve the model’s fit somewhat. In each case, however, these VARs required exclusion restrictions that were strongly rejected by the data—in particular, exclusion of the output gap, which is invariably highly statistically significant in the labor share equation of the VAR.}

It is worth emphasizing here that focusing on the implied estimates of $\theta$ will again yield a
Noting from equation (14) that with looking behavior in price setting (i.e., models derived from the alternative basic hybrid model \( \pi_t = \mu_1 \sum_{i=0}^{\infty} E_t \pi_{t+i} + \mu_2 \pi_{t-1} \), with structural parameters implicitly defined as \( \mu_1 = \gamma \theta \) and \( \mu_2 = (1 - \theta) / \theta \). Standard errors in parentheses; ** or * denotes significant at 1- or 5-percent level, respectively.

<table>
<thead>
<tr>
<th>Driving variable (( y_t ))</th>
<th>Reduced-form parameters</th>
<th>Structural parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detrended output</td>
<td>-0.007 0.622** -0.004 0.617**</td>
<td>(0.005) (0.069) (0.003) (0.026)</td>
</tr>
<tr>
<td>Labor income</td>
<td>0.017 0.485** 0.011 0.674**</td>
<td>(0.014) (0.072) (0.010) (0.033)</td>
</tr>
<tr>
<td>Share</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Table gives estimated values for the parameters from the alternative basic hybrid model \( \pi_t = \mu_1 \sum_{i=0}^{\infty} E_t \pi_{t+i} + \mu_2 \pi_{t-1} \), with structural parameters implicitly defined as \( \mu_1 = \gamma \theta \) and \( \mu_2 = (1 - \theta) / \theta \). Standard errors in parentheses; ** or * denotes significant at 1- or 5-percent level, respectively.

VI. Conclusions

The observation that lagged inflation plays an important role in empirical inflation regressions poses a major challenge to the rational-expectations sticky-price models that underpin the new-Keynesian Phillips curve. Indeed, it has now become relatively well accepted that purely forward-looking models of inflation cannot account for the degree of inflation inertia that we actually observe in the data, and that this failure significantly reduces these models’ usefulness in assessing practical policy questions. In response, researchers have increasingly adopted

9 The bottom panel of Figure 5 is analogous to Figure 2 of Galí and Gertler (1999), which compares actual inflation with what they term “fundamental inflation.”

10 The implied values of \( \theta \) shown in Table 4 are statistically significant. Note again, however, that this tells us nothing about the role played by forward-looking behavior: given the empirical importance of lagged inflation—and, hence, of \( \mu_2 \)—in our estimated equation, we would invariably expect to obtain a statistically significant value of \( \theta \).
hybrid pricing specifications, in which lagged inflation is allowed to have an explicit role in pricing behavior. This class of model is widely seen as striking a reasonable compromise between the desire to fit a key empirical characteristic of the inflation process (its inertia), and the desire to preserve an important role for forward-looking, rational expectations in price setting.

The goal of this paper has been to determine whether this reformulation of the basic sticky-price model yields a pricing specification that is capable of capturing empirical inflation behavior. We have shown that the hybrid specification generates precise predictions about the inflation process that are easily tested—and firmly rejected. In fact, we find no evidence in postwar U.S. data that inflation dynamics reflect the type of rational forward-looking behavior that the model hypothesizes. Hence, while the addition of a lagged inflation term permits the hybrid model to better capture certain features of the inflation process, ultimately this fix is cosmetic in that the feature of the model that truly distinguishes it from alternative models of inflation—such as a traditional Phillips curve based on backward-looking expectations—appears to be empirically irrelevant.

One conclusion that can be drawn from these results is that the hybrid model’s approach to patching up the new-Keynesian Phillips curve—which involves a direct attempt to deal with its persistence problem—may merely be addressing a symptom of what is in fact a much more deeply rooted problem with this type of model. Specifically, our findings suggest that pricing models of this sort suffer from a more serious (and less easily addressed) weakness—namely, their reliance on a strict form of rational expectations. The new-Keynesian inflation equation makes three assumptions about price-setting behavior: first, that prices are sticky; second, that agents optimize their behavior given that their prices are fixed; and third, that agents’ expectations are formulated in a rational—i.e., model-consistent—manner. Empirical studies suggest that a significant degree of price stickiness is present in the U.S. economy, and thus that firms almost surely attempt to make some prediction about future inflation when determining their current price. What appears to be less reasonable, however, is the assumption that these predictions are formulated in the manner implied by the new-Keynesian model under rational expectations.

Put differently, it may well be that \( E_t \pi_{t+1} \) has an important influence on current inflation. But if this is so, the evidence indicates that this expectation is not determined in the manner that the current generation of rational expectations sticky-price models would predict. This conclusion does not rule out a role for some sort of rational optimizing behavior in explaining inflation dynamics; indeed, there may be an optimization-based rationale for why the reduced-form Phillips curve models discussed in this paper fit so well. For example, in the absence of any agreement among economists on what the correct models for inflation (or the rest of the economy) actually are, and given most individuals’ limited ability to understand or model these uncertainties, a procedure in which agents base their expectations for future inflation on extrapolations of the recent past may itself constitute a form of optimizing behavior.

We conclude, then, that further research in this area is probably best aimed toward developing models that deviate from the standard rational expectations framework in favor of alternative descriptions of how agents process information and develop forecasts. Work in this vein by Christopher A. Sims (1998, 2003) and N. Gregory Mankiw and Ricardo Reis (2002) may prove to be a promising start in this direction.

REFERENCES

Calvo, Guillermo A. “Staggered Prices in a Utility-Maximizing Framework.” Journal of


