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Announced climate policy and the order of resource use.

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Abstract

In this paper we study the optimal extraction of two fossil fuels when the economy faces an announced constraint on CO$_2$ emissions à la Kyoto. When high- and low-carbon resources are perfect substitutes, announcement of climate policy induces substitution towards the high-carbon input whenever this resource is abundant. Emissions can then increase at the instant of announcement when the future constraint is not too tight, and the period between announcement and implementation of climate policy is long enough. We present data that suggest that this effect might have occurred in the German electricity industry after announcement of the European Union Emissions Trading Scheme.

1 Introduction

Climate policy has significant real effects on economic behaviour. By changing the relative price of fossil fuels, e.g. through a carbon tax or through a cap and trade system, it induces firms in most sectors of the economy to substitute away from fuels with high carbon content. These behavioural adjustments, however, are restricted by the stocks of machines and equipment currently installed, as well as by available alternative technologies. For electric utilities the short-run response is largely determined by the installed stock of combustion devices. As a given coal-fired power station cannot easily be replaced by a gas-fired unit to reduce carbon dioxide emissions, the short-run response of a coal-fired plant is typically to switch towards cleaner coal types. Indeed, the preferred reaction for most power plants to the 1990 Clean Air Act’s policy to reduce sulfur dioxide emissions was switching from high- to low-sulfur coal (Carlson, Burtraw, Cropper, and Palmer, 2000). At the same time, power plants typically engage in long-term contracts with coal providers, which makes it harder to move from brown coal to cleaner hard coal. For all these reasons, announcement of climate policy can reduce the overall burden of the policy by giving agents time to prepare.

Announcement of climate policy still leaves agents free to emit in the period between announcement and implementation. This raises the question how carbon dioxide emissions respond to the announcement of future climate policy. In this paper, we study how emissions in this period are affected by the announcement, and provide conditions under which emissions...
of carbon dioxide may increase at the instant of announcement of future climate policy. We use a Hotelling (1931)-style model in which utility is derived from electricity consumption, which is produced using non-renewable resources. When the economy faces a future constraint on carbon dioxide emissions, resource owners face a future reduction in demand for their good. Since they want to sell their entire resource stock over time, they have an incentive to reduce the price of their resource to sell more of their resource in the short-run (scarcity rent effect). We show that whether this effect indeed leads to an increase in energy demand and emissions, depends on several factors. First of all, it depends on consumer preferences. As consumers prefer a smooth consumption path, and therefore a smooth path of resource use, an upward jump in demand and emissions is not possible in a 1-resource economy. With 2 resources that differ in their carbon content, this consumption smoothing argument plays a smaller role, as now firms in the power industry can substitute between high- and low-carbon resources at any point in time, without affecting the level of output and consumption. We show that an increase in emissions is then possible if (i) climate policy induces substitution towards the high-carbon input, which occurs when this resource is abundant, (ii) the constraint is not too tight, and (iii) the period between announcement and implementation is sufficiently long. Data on the use of hard and brown coal and on carbon dioxide emissions for the German power industry suggest that this scarcity rent effect might have been an underlying force for the substitution towards dirtier brown coal and for the increase in emissions after announcement of the European Union Emissions Trading System.

Few papers study the effects of announced climate policy, and none has studied the effect of announcement on emissions. Smulders and Van der Werf (2008) study climate policy in a 2-resource model when the resources are imperfect substitutes and show that announcement of a ceiling on the flow of emissions induces substitution towards the high- or the low-carbon input, depending on the marginal productivity of carbon for and the relative scarcity of the two fuels. However, they do not study the effect of announced policy on emissions of carbon dioxide. Chakravorty, Moreaux, and Tidball (2007) study a ceiling on the stock of pollution when both resources are perfect substitutes and do not explicitly study announcement effects either. Kennedy (2002) and Parry and Toman (2002) focus on domestic climate policies in the period between announcement and implementation of international climate policy, and argue that policies aimed at emission reductions in this period may be costly and inefficient. In our paper we do not look at additional emission reduction policies and only focus on the effects of announcement of the climate policy on emissions.

In section 2 we present the model...

*** To be completed ***

2 The model

A dirty or high-carbon nonrenewable resource, \( H \), and a lower-carbon one, \( L \), are perfect substitutes in the production of some output \( q \) (electricity, say), together with a renewable resource \( y \) whose use does not cause \( \text{CO}_2 \) emissions. The latter is available at constant marginal cost, but is initially not competitive, while the nonrenewables have zero extraction costs. The use of one unit of \( i \), with \( i \in \{ H, L \} \), entails the emission of \( \varepsilon_i \) units of carbon dioxide, with \( \varepsilon_H > \varepsilon_L \). Throughout the paper both extraction and stocks of all inputs are expressed in units of energy. At some point in time \( T \geq 0 \), the economy faces a ceiling on the amount of emissions, denoted by \( \bar{Z} \).

Consumers maximize consumer surplus over time, where the surplus is utility \( U \) coming from energy use \( q \), minus costs, where \( U(\cdot) \) is a \( C^2 \) function such that \( U' > 0 \) and \( U'' < 0 \).
Hence, the model reads:

$$\max_{\{R_H(t), R_L(t), \hat{y}(t)\}} \int_0^\infty \left[ U(q(t)) - c_b y(t) \right] e^{-\rho t} dt \quad (1.a)$$

s.t.  
$$q(t) = R_H(t) + R_L(t) + y(t); \quad (1.b)$$

$$S_H(t) = -R_H(t), \ R_H(t) \geq 0, \ S_H(0) = S_{H0}; \quad (1.c)$$

$$\dot{S}_L(t) = -R_L(t), \ R_L(t) \geq 0, \ S_L(0) = S_{L0}; \quad (1.d)$$

$$Z(t) \equiv \epsilon_H R_H(t) + \epsilon_L R_L(t) \leq \tilde{Z} \ \forall t \geq T; \quad (1.e)$$

$$y(t) \geq 0. \quad (1.f)$$

$R_i(t)$ denotes extraction of nonrenewable $i$ at time $(t)$, $c_b$ is the constant marginal cost of the backstop technology, and $\rho$ is the rate of time preference. Equations (1.c) and (1.d) show that the stock $S_i$ of each nonrenewable declines with extraction. The initial endowment of each resource, $S_{i0}, i \in \{H, L\}$, is given. Climate policy is described in (1.e): emissions $Z$ arise from resource use, but starting at time $T$, they cannot exceed $\tilde{Z}$. The problem above can be solved by dividing the time horizon in two phases, a first period when the constraint is not yet enforced, and a second period when instead, the constraint is enforced and (at least initially) binding.\(^1\) We refer to the first phase as the interim phase, the time interval preceding the enforcement of the announced regulation, i.e. $t \in [0, T)$; the remaining phase we call, rather more prosaically, the enforcement phase.

We start with solving the interim phase, where the maximization occurs over the horizon $[0, T)$. The Lagrangian for this part of the problem is:

$$\mathcal{L} = U(q(t)) - c_b y(t) - \sum_{i \in \{H, L\}} \lambda_i(t) R_i(t) + \sum_{i \in \{H, L\}} \gamma_i(t) R_i(t) + \gamma_b(t) y(t);$$

where $\lambda_i$ is the co-state variable for input $i$, and the $\gamma$’s are the Lagrange multipliers associated with the non-negativity constraints.

The first-order necessary conditions for the solution are:

$$U'(q(t)) = \lambda_i(t) - \gamma_i(t), \ i \in \{H, L\}; \quad (2)$$

$$U'(q(t)) = c_b - \gamma_b(t); \quad (3)$$

$$\dot{\lambda}_i(t) = \rho \lambda_i(t), \ i \in \{H, L\}; \quad (4)$$

with complementary slackness conditions:

$$\gamma_i(t) \geq 0, \ R_i(t) \geq 0, \ \gamma_i(t) R_i(t) = 0, \ i \in \{H, L\}; \quad (5)$$

$$\gamma_b(t) \geq 0, \ y(t) \geq 0, \ \gamma_b(t) y(t) = 0. \quad (6)$$

Equation (4) shows the Hotelling rule for the nonrenewables. That is, the scarcity rent of the two resources grows at a constant rate $\rho$, hence, at each $t \in [0, T)$ we can write $\lambda_i(t) = \lambda_i(0) e^{\rho t}$. In the constrained phase, we need to include the emissions constraint to the problem and the corresponding Lagrangian reads:\(^2\)

$$\mathcal{L} = U(q(t)) - c_b y(t) - \sum_{i \in \{H, L\}} \tilde{\lambda}_i(t) \tilde{R}_i(t) + \tau(t) (Z - Z(t)) + \sum_{i \in \{H, L\}} \tilde{\gamma}_i(t) \tilde{R}_i(t) + \tilde{\gamma}_b(t) y(t).$$

---

\(^1\)Whether any given constraint is binding or not depends, as discussed below, on the initial endowments of the fossil fuels. We are interested in situations where regulation makes a difference and therefore we restrict ourselves to situation where the initial endowments are large enough to represent a problem in terms of climate change.

\(^2\)In what follows, we identify variables that pertain to the constrained phase with a tilde ($\sim$) above the corresponding symbol.
In parallel with the interim phase, here $\tilde{\lambda}_i$ is the co-state variable for input $i$, and the $\tilde{\gamma}$’s are the Lagrange multipliers associated with the non-negativity constraints. Additionally, $\tau$ is the multiplier associated to the emission constraint.

The first-order necessary conditions are, in this case:

$$U'(\tilde{q}(t)) = \tilde{\lambda}_i(t) + \epsilon_i \tau(t) - \tilde{\gamma}_i(t) \equiv \tilde{p}_i(t), \quad i \in \{H, L\};$$  \hspace{1cm} (7)

$$U'(\tilde{q}(t)) = c_b - \tilde{\gamma}_b(t);$$ \hspace{1cm} (8)

$$\tilde{\lambda}_i(t) = \rho \tilde{\lambda}_i(t), \quad i \in \{H, L\};$$ \hspace{1cm} (9)

with complementary slackness conditions:

$$\tau(t) \geq 0, \quad \tilde{Z} - Z(t) \geq 0, \quad \tau(t) \left[ \tilde{Z} - Z(t) \right] = 0, \quad \forall t \geq T;$$ \hspace{1cm} (10)

$$\tilde{\gamma}_i(t) \geq 0, \quad \tilde{R}_i(t) \geq 0, \quad \tilde{\gamma}_i(t) \tilde{R}_i(t) = 0, \quad i \in \{H, L\};$$ \hspace{1cm} (11)

$$\tilde{\gamma}_b(t) \geq 0, \quad y(t) \geq 0, \quad \tilde{\gamma}_b(t) y(t) = 0;$$ \hspace{1cm} (12)

and transversality conditions:

$$\lim_{t \to \infty} \tilde{\lambda}_i(t) S_i(t) e^{-\rho t} = 0, \quad i \in \{H, L\}. \hspace{1cm} (13)$$

Notice that also in this case we can integrate equation (9), and get $\tilde{\lambda}_i(t) = \tilde{\lambda}_i(0) e^{\rho t}$. Moreover, we assume that the cap is initially binding, so that $\tau(T) > 0$.

We have to impose certain restrictions on the solutions of the two separate parts of the problem to guarantee consistency and optimality. As refers to consistency, we must require that the final resource stocks of the interim phase exactly match the initial stocks of the constrained phase. Letting $\tilde{S}_i(T)$ represent the initial stocks ($i \in \{H, L\}$) of the constrained phase, we impose:

$$\lim_{t \to T^-} S_i(T) = \tilde{S}_i(T). \hspace{1cm} (R.1)$$

While, as far as optimality is concerned we have to impose a sort of no-arbitrage condition, in the sense that we must rule out jumps in utility going from one phase to the other. No path that violates such restriction can be optimal: given the strictly concave utility function, and perfect foresight there would be potential gains to be reaped from reallocating extraction. Thus energy consumption (and total extraction) has to be the same along both trajectories at time $T$. Using the first-order conditions (2) and (7), it is immediate to see that this second condition requires that the price path across periods be continuous:

$$\lim_{t \to T^-} \lambda(T) = \tilde{p}(T); \hspace{1cm} (R.2)$$

where $\lambda(t) \equiv \min[\lambda_H(t), \lambda_L(t)]$ and $\tilde{p}(t) \equiv \min[\tilde{p}_H(t), \tilde{p}_L(t)]$ are the relevant energy prices in the two phases.

Before characterizing the optimal solution to the problem posited above, it helps to discuss the benchmark case where no constraint is imposed.

### 3 The benchmark economy

According to (1.b), the two resources (and the back-stop) are perfectly substitutable as sources of utility. Hence, as long as their carbon content is irrelevant – as is the case in the absence of emissions controls – the two nonrenewables are de facto identical. As a consequence, we can define the total stock of available resources at time $t$ in an unconstrained economy as $S(t) \equiv S_H(t) + S_L(t)$, and the initial total stock as $S(0) \equiv S_0$. Being identical for all present purposes,
the two fossil fuels will have the same scarcity rent \( \lambda_H = \lambda_L = \mu \) and will be extracted as a single one.

From the transversality condition of the unconstrained economy, it follows that it will be optimal to completely exhaust the resource stocks, before their scarcity rent rises above the marginal cost of the backstop. Let’s denote the instant when this happens by \( T_b \). Given the dynamics of the resource price implied by the Hotelling rule in (9), it follows that the (endogenous) initial level of \( \mu, \mu(0) = \mu_0 \) say, needs to be determined such that \( \mu(T_b) = c_b \).

Using one of the first-order conditions (7), we can define the demand for energy at time \( t \) as \( d(\mu(t)) = U^{-1}(\mu(t)) = q(t) \). The initial scarcity rent then solves \( \int_0^{T_b} d(\mu_0 e^{\rho t}) dt = S(0) = S_0 \), where we made use of the fact that \( \mu(t) = \mu_0 e^{\rho t} \), and \( T_b \) is the (endogenous) moment at which the resource price equals the cost of the backstop, i.e. \( T_b = \frac{\rho \log \frac{\mu_0}{\mu(T_b)}}{p} \). Clearly, the larger the initial (total) resource stock, the lower \( \mu_0 \), the higher initial extraction – see (7) –, and the larger \( T_b \).

Along this type of unconstrained path the optimal extraction profile is determined purely by scarcity, in accordance with the analysis of Hotelling (1931). Thus we refer to this type of paths as to Hotelling paths.

### 4 The role of initial stocks

To study the optimal extraction paths in the presence of (announced or unannounced) climate policy, we first need to determine initial conditions under which an emissions constraint will be binding. Economic intuition tells us that the climate policy has a role to play only insofar as carbon-rich fuels are available in such abundance that their use has the potential to destabilize the global climate. The rest of this section formalizes our intuition, and defines what is meant by abundance in this context.

When the emissions constraint binds the behaviour of agents at some point in time, the optimal (unconstrained) solution described in section 3 cannot be achieved. Given the existence of a cap on carbon emissions, the level of fossil fuel extraction (and use) will have to be reduced for some period of time following the enforcement of the policy. This has important consequences for the whole optimal (constrained) path. We will show below that, when the cap is binding, the choice of extraction rates in the interim period is affected as well. The analysis of these announcement effects constitute the crux of our analysis, for this reason it is important to understand under what initial conditions the cap will be binding.

First, notice that for any given level of \( \bar{Z} \), whether the constraint actually affects the behaviour of economic agents, ultimately depends on whether the cap is binding at any point in time. If the optimal extraction path is such that the associated emissions are never above this maximum allowed level during the phase when the cap is enforced, the announced constraint will have no real effects on the economy. The whole profile of extraction, however, depends on the initial resource endowments, hence, it is this key dimension that we need to consider next.

Let the initial endowment vectors, describing the initial levels of the stock of the resources be vectors \( S_0 = \{S_H(0), S_L(0)\} \). For each fossil fuel \( i \in \{H; L\} \), let \( S_i^h \) be the maximum stock of fuel such that, if extracted independently of any other, would lead to a path not constrained by regulation. That is, let \( S_i^h \) be such that the associated initial scarcity rent \( \lambda_i^h(T) \) satisfies \( \epsilon_i d(\lambda_i^h(T)) = \bar{Z} \); i.e. the constraint is binding only at the instant \( t = T \), and never again. Since these correspond to the largest possible stocks of each resource that may be extracted along an unconstrained path when the constraint is enforced, we call these the maximal Hotelling stocks.

Let \( \bar{R}_i \) denote the maximum amount of \( i \) that can be extracted when the emission constraint is binding, i.e. \( \bar{R}_i = \bar{Z}/\epsilon_i \). The associated price of the resource at the cap is \( \bar{p}_i = U'(\bar{R}_i) \). From
the properties of the function $U$, it follows that $\bar{p}_H > \bar{p}_L$. Moreover, since the carbon coefficient $\epsilon_i$ determines the amount of resource that can be used at the ceiling, it follows that the cleaner the resource, the larger the maximal Hotelling stock, i.e. $S^h_L > S^h_H$.

Now, let $\delta_H(T)$ be the largest cumulative extraction of the dirty fuel that can be performed in the interim phase of an unconstrained path entailing the extraction of $H$ only. Since the price of the dirty input at the beginning of the constrained period cannot be below $\bar{p}_H$ – the price that characterize the maximal Hotelling path – and given the continuity condition (R.2), the size of $\delta_H(T)$ only depends on the duration of the interim period, and is given by $\delta_H(T) = \int_0^T d(\bar{p}_H e^{-\rho(T-t)})dt$.

In the same way, for $\delta_L(T)$ – the largest cumulative extraction of the clean fuel that can be performed in the interim phase of an unconstrained path entailing the extraction of $L$ only – we have $\delta_L(T) = \int_0^T d(\bar{p}_L e^{-\rho(T-t)})dt$.

We can represent initial endowment vectors $S_0$ in a graph having the stock of the low-carbon source on the horizontal axis, and the stock of the high-carbon one on the vertical one. In Figure 1, we have drawn the parallel iso-energy lines corresponding to the maximal Hotelling stocks $S^h_H$ and $S^h_L$, and we have indicated the size of the maximal cumulative extraction of $H$ and $L$ ($\delta_H$ and $\delta_L$, respectively) that is possible to undertake in the interim phase of an unconstrained path. If we then define the optimal controls associated to each initial endowment vectors as $R_1^i(t; S_0)$, the set of all initial endowment vectors leading to an unconstrained path in the presence of an announced constraint enforced at time $T$ is:

$$
\Pi \equiv \{ S_0 : \int_0^T R_1^i(t; S_0) dt = S_i(0) \cap \sum_i \epsilon_i R_1^i(T; S_0) \leq \bar{Z} \}.
$$

Figure 1: A taxonomy of initial stocks

The following result relates the definition above to the graphical representation in Figure 1:

**Lemma 1.** The set $\Pi$ is represented by the area $O A'B'D$ in Figure 1.

**Proof.** See Appendix A.
Moreover, we can formalize the fact that, for our purposes, we only need concentrate on initial endowments outside of $\Pi$ as follows

**Lemma 2.** Any path beginning from a vector of initial stocks $S_0 \notin \Pi$ will be constrained over some period of time.

*Proof.* It immediately follows from the definition in (14) and Lemma 1.

Thus only initial endowment outside the set $\Pi$ are “abundant”, and any path starting from such endowments will be constrained after the announcement of climate policy.

## 5 Optimal extraction in a constrained economy

According to Lemma 1, paths that start from endowment vectors outside $\Pi$ will necessarily be constrained over some interval of time. It follows that any optimal paths that solves problem (1.1)-(1.f) will be composed of three parts: An interim phase that precedes the period of actual enforcement of the cap; a second period, that we call the constrained phase, during which the cap is binding; a third phase where, as the resources get depleted and extraction (and emissions) declines, the cap eventually ceases to bind. This final phase ends with the exhaustion of the non-renewable resources, and the contemporaneous switch to the renewable back-stop technology. Since during this final period the extraction paths are purely dictated by scarcity, we refer to this as the *Hotelling phase*.

The easiest way to characterize the different phases, and to understand what kind of paths arise from different initial endowments outside of set $\Pi$, is to understand how the price of $q$ changes, relative to a pure Hotelling path, following the announcement of the cap.

Recall that an Hotelling path is characterized by a price (the pure scarcity rent) that, starting from some initial level $\mu(0)$, grows at a constant rate equal to the rate of time preference $\rho$ and reaches the choke level $c_b$ at precisely the instant when the resource stocks are exhausted. Starting from this benchmark, we can decompose the impact of the announced constraint in two effects: the *scarcity effect*, and the *announcement effect*.

On the one hand, the existence of the (binding) constraint limits the use of the resources over some time, and this has as an immediate consequence to reduce the scarcity of the fossil fuels. This implies that the underlying scarcity rent along the constrained path is everywhere lower than the Hotelling rent: $\tilde{\lambda}(t) < \mu(t) \ \forall t$; this we refer to as the scarcity effect of the constraint.

On the other hand, the fact that the constraint is announced rules out any discrete change in the path of utility. This fact is captured by the continuity condition (R.2), requiring that the co-state variable $\lambda(t)$ in the interim phase grows at rate $\rho$ and tends to the value of the price that will prevail along the constrained phase beginning at $T$. Thus $\lambda$ captures at the same time the scarcity rent $\tilde{\lambda}$, and the value of the announcement, $\eta(t) \equiv \lambda(t) - \tilde{\lambda}(t), \ \forall t \in [0,T)$. The fact that $\lambda(t) > \mu(t)$ captures the essence of the announcement effect.

Depending on the initial relative abundance of the two resources, the scarcity effect may affect the scarcity rents of the two fuels asymmetrically: this is the key insight to understand how initial stocks determine the optimal paths for the extraction of the resources. We make use of this insight to prove the following central result,

**Proposition 1.** The announcement of a binding emission constraint leads to the sole extraction of the high-carbon source ($H$) in the interim phase, whenever $H$ is abundant.

*Proof.* In Appendix B.1.
Here we sketch the optimal extraction paths that are formally proved in the Appendix, distinguishing situations when only one, or both fuels are abundant.

**Only the clean fuel is abundant** We start from extraction paths originating from zone IV in Figure 1. In this area only the clean fuel is abundant. As mentioned above, any profile of extraction will be constrained over some time interval. Any path starting from an initial endowment in zone IV will then become unconstrained as the trajectory of the stocks enter zone II from the right. The third phase of the optimal extraction profile is a Hotelling phase, where the scarcity rent of both resources are the same. The price at the beginning of this phase is $\hat{p}_L$, and the extraction level $\hat{R}_L$.

Let us retrace the optimal path backwards from the time when the constraint ceases to be binding. In the preceding constrained phase, only the clean input is extracted, due to its lower pollution intensity. While at the cap it is indeed possible to obtain $\tilde{R}_L \equiv \tilde{Z}_L$ units of energy from the clean fuel, whereas only $\tilde{R}_H \equiv \tilde{Z}_{\epsilon}$ of energy from the dirtier source.

In the interim period, before the cap becomes binding, the shadow value of carbon is zero, and only the scarcity rents matter for the order of extraction. Relative extraction follows the same pattern as in the Hotelling period that follows the constrained phase. In this case, it follows that the composition of extraction in the interim phase is a matter of indifference, as the underlying scarcity rents are identical. The level of extraction is, instead, determined by the initial endowments.

Intuitively, since the clean input is abundant while the dirty fuel is not, the available amount of the clean resource suffices to see the economy through the constrained phase, irrespective of the extraction of the dirty one. Thus climate policy doesn’t introduce a wedge between the scarcity rents of the two fuels.

Before moving on to the next case, however, it is worth emphasizing that the dirty good approaches a situation of ‘abundance’ as the initial stocks approach the border between zone IV and VI. In such cases along the optimal extraction path it will be preferable to conserve more of the clean input for the enforcement phase. Thus, paths starting closer to this boundary tend to be characterize by a larger share on the dirty input in the initial part of the interim phase.

**Only the dirty fuel is abundant** Let us now focus on situations where only the dirty input is abundant. In this case the initial endowment vector $S_0$ is in zone V.

The initial abundance of the carbon-rich resource poses a challenge to the economy facing a constraint. The announcement of climate policy raises the value of the clean fuel during the enforcement phase. Using the clean fuel during the constrained phase, the economy is able to respect the cap and maximize energy supply. This makes the clean fuel scarcer, and hence more valuable. The scarcity rent of the dirty fuel will then fall more than the one for the cleaner one: $\tilde{\lambda}_H(t) < \tilde{\lambda}_L(t)$, $\forall t$. When this is the case, even taking into account the announcement effect $\eta(t)$, the higher-carbon fuel will prove cheaper in the interim phase and will be the sole resource used ahead of the enforcement.

As formally discussed in part III of Appendix B.1, the optimal path will then be characterized by the extraction of $H$ only in the interim phase, followed by joint extraction of both fuels at the cap, until the clean fuel runs out. A constrained phase where only the carbon-intensive fuel is used at the cap follows. Finally, the cap ceases to be binding and the final phase entails an Hotelling path with extraction of $H$ only.

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3 Hotelling paths starting in zone II are discussed in Appendix B.1, part I.

4 See part II of Appendix B.1 for the details.
Thus, the fact that climate policy makes the cleaner fuel relatively scarcer, and hence more valuable than the dirty one leads to a situation where the high-carbon fuel is the preferred source during the interim phase. As we will see below this has (potentially) serious consequences for the amount of carbon released to the atmosphere.

**Both fuels are abundant** Finally, let us investigate the situation that obtains when both inputs are abundant. This is represented by area $VI$ in the graph of Figure 1.

Also in this case the abundance of the dirty input increases the value of the clean one, during the enforcement phase. For the same reason as in the previous case, the scarcity rent of the dirty resource drops more than the one for the cleaner source. Thus, ahead of the enforcement, only the dirty fuel gets extracted to save the clean one for the enforcement phase.

Trajectories from initial stocks in zone $VI$ eventually enter zone $V$, after which extraction follows the path described for initial stocks in this zone. As a consequence, the pattern of extraction for both the Hotelling path after the constrained period, and for the period before the cap is enforced is the same as when initial stocks are in zone $V$. After the announcement only the high-carbon resource is extracted, while at the instant on which policy becomes binding resource use switches to the low-carbon input.\(^5\)

In sum, the announcement of the ceiling on the flow of carbon dioxide emissions induces a switch towards the use of the high-carbon input whenever this resource is abundant. Moreover, even when the dirty input is ‘almost’ abundant, as is the case in zone $IV$ close to the boundary with zone $VI$, the content of the dirty input in extraction tends to be high in the initial phases of the optimal path. We will now argue that this can lead to a temporary increase in total carbon dioxide emissions in the interim phase, which goes against the spirit of the policy.

### 6 Jumps in emissions

***Preliminary***

Now that we have derived the paths of extraction of the two resources in the presence of announced climate policy, we are able to present the paths of emissions during the interim phase. Before we move to the case of an economy with two resources, we first discuss what the path of emissions looks like in an economy in which there is only one resource.

When there is only one resource available, announcement of climate policy cannot induce substitution towards a higher- or lower carbon resource: the economy can only adjust the amount of the available resource extracted. As shown in the previous section, resource owners adjust their scarcity rents after announcement of future climate policy. After announcement they know that at some time in the future they will be restricted in their extraction, as this is the only way in which emissions can be reduced in the economy. Since they still want to exhaust the resource stock before the resource is replaced by the backstop, they will reduce the scarcity rent to such a level that the stock gets depleted at the instant on which the scarcity rent equals the marginal cost of the backstop. At the same time, however, consumers will want to avoid a downward jump in utility at any point in time: given the strictly concave utility function, a jump in utility implies that intertemporal utility can be increased by smoothing consumption and hence extraction over time. As the following proposition shows, in the 1 resource case, the consumption smoothing effect outweighs the scarcity rent effect:

**Proposition 2.** In an economy with only one nonrenewable resource, an announced emissions constraint cannot lead to an increase in emissions in the period between announcement and implementation.

\(^5\)The detailed discussion is presented in Appendix B.1, part IV.
This result seems encouraging for those in favour of announcement of climate policy. However, when two resources are available, substitution effects might play a role as well. As shown in the previous section, announcement of climate policy induces substitution towards the high-carbon input if this resource is abundant. The question then arises under which circumstances the scarcity rent effect (which induces substitution) might outweigh the consumption smoothing effect, such that announcement of climate policy induces an increase in carbon dioxide emissions.

In order to be able to answer this question, we need to put more structure on the utility function. In the remainder of this section, we assume that the utility function has constant relative risk aversion:

\[ U(q(t)) = \frac{q(t)^{1-\eta} - 1}{1 - \eta}, \quad (15) \]

where \( \eta \) is the coefficient of relative risk aversion. In an economy that is unconstrained at time \( t \), and in which there is only one resource \( R_i(t) \), we can use (2) and (4) to find (see Heal, 1993):

\[ R_i(t) = \frac{\rho}{\eta} \left( 1 - e^{\frac{\rho}{\eta}(t-T_u)} \right)^{-1} S_i(t), \quad (16) \]

where \( T_u \) denotes the instant at which the unconstrained economy will switch to the backstop. Secondly we need to assume what the path of extraction will be for each individual resource (and hence total emissions), in an economy with two perfectly substitutable resources. As argued in Section 3, the composition of extraction in this case is in principle a matter of indifference. The most natural assumption is for (16) to apply for each individual resource as well.

With these results in hand, we find the following:

\textbf{Proposition 3.} Suppose a ceiling on carbon dioxide emissions \( \bar{Z} \) is announced at \( t = 0 \) and becomes effective at \( t = T \). Then emissions will jump up at the instant of announcement if

1. both resources are abundant and in addition
   \[ \bar{Z} e^{\frac{\rho}{\eta}(T-t)} < \frac{\rho}{\eta} \left( 1 - e^{\frac{\rho}{\eta}(T-T_u)} \right)^{-1} (\epsilon_H S_H(0) + \epsilon_L S_L(0)) < \frac{\rho}{\eta} \bar{Z} e^{\frac{\rho}{\eta}T}, \]
   or

2. only the high-carbon resource is abundant and in addition
   \[ \bar{Z} e^{\frac{\rho}{\eta}(T-t)} < \frac{\rho}{\eta} \left( 1 - e^{\frac{\rho}{\eta}(T-T_u)} \right)^{-1} (\epsilon_H S_H(0) + \epsilon_L S_L(0)) < \frac{\rho}{\eta} \left( e^{\frac{\rho}{\eta}T} - e^{-\frac{\rho}{\eta}t_H} \right)^{-1} ((\epsilon_H - \epsilon_L) S_L(0) - \bar{Z} (T - t_H)), \]
   where \( t_H \) is the (endogenous) instant at which \( S_L = 0 \).

\textbf{Proof.} See Appendix B.3. \qed

Part 1 of the proposition describes the case where both resources are abundant, i.e. when the vector of initial stocks is in zone VI of Figure 1. As the high-carbon resource is abundant, its scarcity rent will be lower than the scarcity rent of the low-carbon resource, and hence only the high-carbon resource will be used in the interim period. At the instant at which the constraint becomes binding, i.e. at \( t = T \), only the low-carbon resource will be used, while emissions have to comply with the ceiling, which directly determines the amount of energy used. The path of extraction for the interim period can then be solved backward starting from \( t = T \), keeping in mind that utility and hence energy use cannot jump down at any point in time.

\[6\] As a result, relative extraction \( R_H/R_L \) is constant over time and equal to relative stocks \( S_H/S_L \). Indeed, (16) would apply for each resource for any function \( q(t) \) that has a positive but finite constant elasticity of substitution between the high- and low-carbon input.
The left-hand side of the inequality in part 1 of the proposition gives the amount of emissions at \( t = 0 \) if emissions had declined with rate \( \rho / \eta \) (as they do in an unconstrained economy, see (B.9)) to the level of the constraint \( \bar{Z} \) at \( t = T \). The middle part of the inequality shows the amount of emissions at \( t = 0 \) if the economy were unconstrained at any point in time, hence the first inequality provides the maximum level of \( \bar{Z} \) for the constraint still to be binding at \( t = T \). The last part of the inequality denotes actual emissions at \( t = 0 \). As forward-looking consumers know that there will be a binding emissions constraint at \( t = T \) they want to avoid a jump in utility at this point in time. Hence they choose a level of energy use at \( t = 0 \) such that it exactly reaches the level of energy use at \( t = T \) (which is \( \bar{Z} / \varepsilon_L \)), while using the high-carbon input only as described in the previous section. Therefore the last inequality provides the conditions under which emissions will jump up at the instant of announcement of a ceiling on emissions: the period between announcement and implementation \( T \) has to be large enough and the level of the constraint \( \bar{Z} \) should not be set too low. Put differently: if the period between announcement and implementation is too long, and if the constraint is not too tight, consumers can still smooth their consumption in such a way that there is no jump in utility at the instant of implementation, while the switch to the high-carbon input at the instant of announcement causes emissions at this instant to increase.

As can be seen from the inequality in part 1 of the proposition, the exact conditions under which emissions can jump up depend on the marginal cost of the backstop as well. As \( dT_b^u / dc_b > 0 \), a decrease in the marginal cost of the backstop brings the instant of the switch to the clean renewable forward in time, causing the expression between the two inequality signs to increase. As a consequence, policy can be looser and the interim period can be made longer, provided that the first inequality (the condition that the constraint is binding at \( t = T \)) is satisfied.

Part 2 of the proposition gives the conditions under which the constraint is binding and emissions will jump up at the instant of announcement, when only the high-carbon input is abundant (zone \( V \) of Figure 1). As in the previous case, in order to prevent an increase in emissions after announcement of the policy, the interim period \( T \) should not be too long (note that an increase in \( T \) brings along an increase in \( t_H \)). The effect of the level of the constraint, on the other hand, is less clear.

***EXPAND THE DISCUSSION HERE ***

7 Emissions of the German power industry after announcement of the EU ETS

*** PRELIMINARY ***

In this section, we confront the theory of the previous section with data on coal use in the German electricity industry during announcement and implementation of climate policy in Europe.

The European Union Emissions Trading System (EU ETS) is a clear example of an announced climate policy. The first phase of the EU ETS covers the period 2005-2007, and includes some 12,000 installations covering approximately 40% of EU carbon dioxide emissions. It commenced operation on January 1, 2005, although for years in advance it was inevitable that the European Union would introduce climate policy: already in 1998 it was suggested by the
European Commission that the EU could start a cap and trade system for carbon dioxide (European Commission, 1998). To gain experience with emissions trading, it was suggested to start this system in 2005. We can therefore consider the EU ETS as being announced in 1998, while becoming effective in 2005.\footnote{Ex post it turned out that the first phase of the ETS suffered from over- allocation at the European level (i.e. total emissions were not effectively restricted). However, this became only clear in May 2006: before that, the price of Phase I allowances ranged from 10 to 30 euro. Phase II allowances (for the 2008-2012 period) are still (September 2007) valued around 20 euro.}

---

Figure 2 presents the use of bituminous coal (denoted hard coal) and sub-bituminous coal and lignite (denoted brown coal) in the German electric utility industry from 1990 to 2005. During the 1990s, the use of hard coal was quite stable, both in quantities and in shares (not shown). The use of brown coal was declining until 1998, in part due to the restructuring of the (former East-) German coal industry. After 1999 however, brown coal made a spectacular come-back (again both in quantities and in shares), it stabilised in the period 2002-2004, while in 2005 its use was lower than in the year before.

These data are fully consistent with our theory regarding the response of fuel use to announced climate policy. In the case of Germany, we can consider both hard- and brown-coal to be abundant in terms of Figure 1: Germany itself has vast amounts of brown coal, while neighboring Poland has large quantities of hard coal.\footnote{Source: BP Statistical Review of World Energy 2006, available at http://www.bp.com/statisticalreview} Since power plants typically have a life time of several decennia, even a 7 year adjustment period is not enough to fully restructure the power industry towards clean(er) fuels like gas and renewables. Hence, with given generators, fuel switching is an important means for electric utilities to cope with the ETS, as it was the preferred reaction when the US energy sector had to comply with the Clean Air Act (Carlson, Burtraw, Cropper, and Palmer, 2000). When it became clear that the European Union would introduce limits on the emissions of carbon dioxide in a few years time, utilities moved towards the use of the high-carbon input, in this case brown-coal, as predicted in section 5. The foreseen increase in the use of hard coal during the compliance period made the resource scarcer for decades.
to come, causing an increase in the scarcity rent for hard coal. This caused an increase in the (relative) price of hard coal, inducing firms to move from hard to brown coal when this price increase was larger than existing cost differences stemming from e.g. transport costs, before the policy was enforced. Due to long-term contracts of individual utilities with coal providers, the industry as a whole was only gradually able to substitute towards the high-carbon input.

As can be seen from Figure 3, carbon dioxide emissions from the German electricity industry followed exactly the same pattern as the use of brown coal: after it became unavoidable that the EU would try to reduce carbon dioxide emissions, these emissions initially increased. As noted in the previous section, emissions can only rise after announcement of climate policy when the constraint is not too tight and the period between announcement and implementation is not too short. In 2006 it became clear that individual countries were quite generous with the allocation of permits for the first phase of the ETS (with the exception of Germany and the UK, see Climate Action Network Europe, 2006), and hence the constraint was not too tight. It should be noted, however, that it might have been binding after all: although usually permit allocations for the first phase are compared with 2005 emissions, they should be compared with emissions of earlier years as well. Emissions from the German electricity industry started to decline after 2003, which suggests that the EU ETS reduced carbon dioxide emissions after all.

The immediate impact of the announcement of European climate policy, however, seems to have been a move towards high-carbon inputs, and an increase in carbon dioxide emissions. This of course goes directly against the spirit of the policy, as the aim of climate policy is to stabilize atmospheric concentrations (Article 2 of the UNFCCC), which implies that emissions in 2003 are about as bad as emissions in 2005.

8 Conclusions

*** PRELIMINARY ***
The announcement of some restrictive policy some time before it becomes active can reduce the policy’s burden on the economy by giving firms time to prepare and adjust. We have shown, however, that in the case of climate policy and with perfectly substitutable nonrenewable inputs, announcement may be harmful for the environment when the high-carbon resource is abundant. As cleaner inputs become scarcer in the longer term due to more intensive use in the intermediate run (when carbon dioxide emissions are restricted), their scarcity rent increases. This changes the relative price of high- and low-carbon inputs: as the high-carbon input becomes relatively cheaper, firms might switch to the dirty fuel in the period between announcement and implementation of the policy. When the restriction on carbon dioxide emissions is not too tight, and the adjustment period is not too short, this might induce an increase in emissions immediately after announcement of the policy. That is, announcement of a policy to stabilize concentrations of carbon dioxide in the atmosphere might have beneficial economic effects by reducing the burden of the policy, but might have environmental results that go directly against the policy’s ultimate target.

We have used data for the German electricity sector to argue that the EU Emissions Trading System might have led to this adverse effect. After it became clear in the late 1990s that EU-wide climate policy was inevitable, electric utilities started to use more high-carbon coal, and to increase their emissions of carbon dioxide. Hence, the initial effect of the Kyoto Protocol might have been an increase in carbon dioxide emissions.

Our results suggest that there is a trade-off with respect to climate policy: reducing costs of compliance through pre-announcement of the policy, versus the environmental risk that this will increase carbon dioxide emissions. Recognizing the existence of this trade-off has consequences for countries that do not yet face binding carbon dioxide emission targets: the risk of inducing an increase in emissions, should be taken into account and the interim period be kept as short as possible.

References


A Appendix: proof of Lemma 1

Proof. Consider first the Hotelling phase. The emission constraint is not binding, hence $\tau(t) = 0$. We have the following two polar cases:

i. Along the path only the dirty fuel is extracted. From the FOC’s (7)-(9) we know that the energy price, $p_H$, increases at rate $\rho$, while extraction declines with it. Thus, the peak of extraction occurs at the beginning of the phase, at time $T$. For the path to be unconstrained initial extraction cannot exceed $\bar{R}_H = \bar{Z}/\varepsilon_H$, thus the price cannot be less than $\bar{p}_H$. From section 4 we know that the corresponding stock is $S_H^h$. This corresponds to point $A$ in figure 1.

ii. Along the path only the clean fuel is extracted. In this case the maximum extraction level cannot exceed $\bar{R}_L = \bar{Z}/\varepsilon_L$, corresponding to a price of $\bar{p}_L$. The corresponding maximal stock is $S_L^h$. This corresponds to point $C$ in figure 1.

$S_H^h$ and $S_L^h$ have the same carbon content, and so do initial endowments laying on the line joining point $A$ and $C$ in Figure 1. Thus, any initial endowment laying (on and) below this line unequivocally leads to an unconstrained extraction path. This asserts that any path starting inside zone $I$ is unconstrained.

Consider point $A$ again. Adding an arbitrarily small amount $\mu_L > 0$ of the clean fuel increases the total amount of resources available, $S_T > S_H^h$, and the total carbon content. To exhaust this larger stock before the back-stop becomes competitive, the initial price needs to fall, and extraction to rise. Given that the clean fuel has a lower carbon content, it is possible to extract more while emitting strictly less CO$_2$; as the initial price falls below $\bar{p}_H$, we get $\sum_i R_i(T) > \bar{R}_H$, while $\sum_i \varepsilon_i R_i < \bar{Z}$. This “dilution” of the carbon content of emissions is possible until $R_L(T) < \bar{R}_L$, that would happen along the iso-energy line $S_L^h - C$, when the limit price of $\bar{p}_L$ has been reached.

This shows that for initial endowments strictly to the east of the $A - C$ line (and to the the west of the iso-energy line $S_L^h - C$), i.e. endowments with strictly more of the clean input, the emission constraint is slack.

Thus, starting at point $A$, there is scope to extract larger stocks of both the dirty and the clean fuel, along a path that is just constrained, i.e. an Hotelling path with emissions continuously at the ceiling over a period of time.

The locus of such initial endowments is: $U = \{S : \sum_i R_i^*(t) > \bar{R}_H \cap \sum_i \varepsilon_i R_i^*(t) = \bar{Z} \forall t \in [T, t_H)\}$, where $t_H$ represents the moment when the clean fuel runs out and extraction proceeds as an Hotelling path from point $A$ (see Figure 1).

The set $U$ is represented graphically by the curve joining $A$ and $B$ in Figure 1. The concave shape is due to the fact that for any increase in the dirty input, a proportionally larger amount of the clean one is necessary to dilute emissions. This proves that any path starting inside zone $II$ is unconstrained.

We now consider the interim phase. In this case no constraint on emissions is enforced, and the requirement for the optimal path is to satisfy the FOC’s (2)-(4), and the restrictions (R.1) and (R.2).

The set of endowment vectors consistent with pure Hotelling paths gets larger.

We start once more from our polar cases. Consider point $A$. The price there is $\bar{p}_H$, hence the maximal additional extraction of the dirty input alone there is $\delta_H(T) \equiv \int_0^T d(\bar{p}_H e^{-\rho(T-t)}) dt$. Thus, the largest stock of the dirty fuel that can be extracted along a pure Hotelling path, depends on the duration of the interim period $T$, and is given by $S_{H0} = S_H^h + \delta_H(T)$. This stock corresponds to point $A'$ in Figure 1.
Consider now point \( C \). There, and along the iso-energy line starting there, the price of energy is \( \bar{p}_L \). Hence, the increase in resource extraction over the time interval leading to the enforcement period, is given by \( \delta_L(T) \equiv \int_0^T d(\bar{p}_L e^{-\rho(T-t)})dt \). Hence, the linear portion of the boundary of zone \( II \) shifts out by \( \delta_L(T) \), as can be seen from Figure 1.\(^9\)

To conclude let’s consider the \( A - B \) curve. Along the curve the price increases from \( \bar{p}_L \) at point \( B \), to \( \bar{p}_H \) at \( A \). Hence, the outwards (upwards) shift that occurs is largest – and equal to \( \delta_L(T) \) – at \( B \), and smallest at \( A \), where it equals \( \delta_H(T) \).

Given that the boundary of zone \( III \) represents the maximal additional extraction that can be performed in the interim phase leading to unconstrained Hotelling paths, this shows that the sum of zones \( I \), \( II \) and \( III \) exhausts the possibilities for initial stocks leading to optimal unconstrained paths in our model, and graphically represent set \( \Pi \).

\[ \square \]

### B Appendix: proof of Propositions 1, 2, and 3

#### B.1 Proof of Proposition 1

**I. Hotelling paths.** Along any unconstrained path \( \tau(t) = 0, \forall t \geq 0 \). So the interim and the enforcement phase are one and the same. From the first-order conditions (2) and (7), it follows that the scarcity rent for the two resources are equal over the whole time horizon. Given the initial stocks \( S_0 = \{S_{L0}, S_{H0}\} \), let the total initial stock be \( S_0 = S_{H0} + S_{L0} \). Since the two resources are perfect substitutes, let \( \lambda_H(0) = \lambda_L(0) = \mu(0) \), where \( \mu(t) \) is the scarcity rent along an Hotelling path. From the Hotelling rule, and the transversality conditions (13), it follows that the resource will have to exhausted along any optimal path. Hence, the price path is \( \mu(t) = \mu(0)e^{\rho t} \), where \( \mu(0) \) and \( T_b \) solve \( S_0 = \int_0^{T_b} d(\mu(0)e^{\rho t})dt \), and \( T_b = \frac{\log c_b - \log \mu(0)}{\rho} \).

Then paths for extraction and price is given as:

\[
R(t) = \begin{cases} 
    d(\mu(0)e^{\rho t}), & \forall t \in [0, T_b) \\
    0, & \forall t \in [T_b, \infty)
\end{cases}
\]

(B.1)

\[
p(t) = \begin{cases} 
    \mu(0)e^{\rho t}, & \forall t \in [0, T_b) \\
    c_b, & \forall t \in [T_b, \infty)
\end{cases}
\]

(B.2)

**II. Only \( L \) is abundant.** Let’s us divide this case in two subcases, depending on whether the initial endowment vector is below or above the horizontal line through point \( B \).

i. When initial endowments are below the line, the final phase of the path will necessarily be a pure Hotelling path starting along the iso-energy line \( B-C \). Consider the alternative, i.e. a path entering zone \( II \) along the horizontal axis. Such a path requires that the scarcity rent for \( L \) be below the scarcity rent for \( H \). If this were indeed a case, however, the interim phase (and the constrained phase, as shown below) would be characterized by extraction of \( L \) only. Hence, it would be impossible to reach the horizontal axis in the first place. A contradiction.

Along such a path, the scarcity rents of the two fuels are equal as shown above. The price at the beginning of this phase is \( \bar{p}_L \), and the extraction level \( \bar{R}_L \). In the preceding

\[ \text{\footnotesize\textsuperscript{9}It is immediate to see that } \delta_L(T) \equiv \int_0^T d(\bar{p}_L e^{-\rho(T-t)})dt > \delta_H(T) \equiv \int_0^T d(\bar{p}_H e^{-\rho(T-t)})dt, \text{ as } \bar{p}_H > \bar{p}_L. \]
constrained phase, only the clean input is extracted as $R_L \equiv \frac{Z}{\varepsilon_L} > R_H \equiv \frac{Z}{\varepsilon_H}$. In the interim period, only scarcity rents matter for the order of extraction. Given that the rents are equal, the composition of extraction in the interim phase is a matter of indifference. The level of extraction is, determined by initial endowments.

ii. When initial endowments are above the line, two cases are possible, either the paths cross the horizontal line through $B$ and evolves as in case i. above, or the path crosses into zone $V$. For the second case to obtain it must be the case that either both fuels are used in the interim phase, or only the clean. That is a situation where only $H$ is extracted is ruled out. However, as we will show below paths starting in zone $V$ are characterized by a lower scarcity rent for $H$ than for $L$. In this situation the interim phase must be such that $H$ alone gets extracted. Hence, we have a contradiction. Paths starting above the line cross the line ahead of the enforcement phase, and proceed as the paths discussed in the previous case.

The extraction and price dynamics are as follows:

\[
R(t) = \begin{cases} 
   d(\lambda(0)e^{\rho t}), & \forall t \in [0, T) \\
   R_L(t) = \frac{Z}{\varepsilon_L}, \text{ and } R_H(t) = 0, & \forall t \in [T, T + \Delta_L) \\
   d(\lambda(0)e^{\rho t}), & \forall t \in [T + \Delta_L, T_h) \\
   0, & \forall t \in [T_h, \infty) 
\end{cases} 
\]

\[
p(t) = \begin{cases} 
   \lambda(0)e^{\rho t} + \beta(t), & \forall t \in [0, T) \\
   \bar{p}_L, & \forall t \in [T, T + \Delta_L) \\
   \lambda(0)e^{\rho t}, & \forall t \in [T + \Delta_L, T_h) \\
   c_r, & \forall t \in [T_h, \infty) 
\end{cases} 
\]

where the duration of the constrained phase depends on the total initial endowments only. Let $S_0 = S_{0L} + S_{0H}$ be the total initial endowment, then independently of the choice of extraction during the interim phase, the duration of the phase at the cap is given by $\Delta_L = \frac{s_1 - s_{1}(0)}{R_L}$, the time required to reduce the initial stock to the maximal Hotelling stock.

III. Only H is abundant. When paths start in zone $V$, it is easiest to consider separately the enforcement phase and the interim phase. Let us start from the enforcement phase. At the time the cap becomes binding, it would be suboptimal to use only the clean input until exhaustion and then switch to the carbon-richer one, as this would entail a suboptimal jump in utility at the time of the switch. It is then optimal to use jointly the two fuels until exhaustion if the clean one and then switch, smoothly to extraction of $H$ only. This implies that $\lambda_H(t) + \varepsilon_H \tau(t) = \lambda_L(t) + \varepsilon_L \tau(t)$, and since $\varepsilon_H > \varepsilon_L$, it follows that $\lambda_H(t) < \lambda_L(t)$, that is the scarcity rent of the dirty fuel is below the one for the cleaner one.

Along this first part of the constrained phase the price will increase until it gets to $\bar{p}_H$ at time of the switch. This part of the path will be parallel to the $A-B$ curve, as emissions are exactly at the cap. When the clean input is exhausted, the path proceed as a constrained path with $H$ only being extracted. Finally, the trajectory enters zone $I$ along the vertical axis and, the remaining phase is an Hotelling path until exhaustion of $H$.

Given that the scarcity rent for $H$ is lower than the one for $L$, it is optimal to use the cheaper fuel only during the interim phase. Thus in the time leading to enforcement, only $H$ is extracted.
Formally, the description of the path is as follows:

\[
R(t) = \begin{cases} 
R_L(t) = 0, \text{ and } R_H(t) = d(\lambda_H(0)e^{pt}), & \forall t \in [0, T) \\
R_L(t) = \bar{x}_L, \text{ and } R_H(t) = 0, & \forall t \in [T, t_L) \\
d(\bar{\lambda}_i(0)e^{pt} + \varepsilon_i\tau(t)), i \in \{L, H\}, & \forall t \in [T + \Delta_{HL}, T + \Delta_{HL} + \Delta_H) \\
R_L(t) = 0, \text{ and } R_H(t) = \bar{x}_H, & \forall t \in [T + \Delta_{HL} + \Delta_H, t_b) \\
R_L(t) = 0, \text{ and } R_H(t) = d(\bar{\lambda}_H(0)e^{pt}), & \forall t \in [t_b, \infty) \\
0, & \forall t \in [0, T)
\end{cases}
\] (B.5)

\[
p(t) = \begin{cases} 
\lambda_H(0)e^{pt} + \beta(t), & \forall t \in [0, T) \\
\bar{\lambda}_i(0)e^{pt} + \varepsilon_i\tau(t), i \in \{L, H\}, & \forall t \in [T + \Delta_{HL}) \\
\bar{p}_H, & \forall t \in [T + \Delta_{HL}, T + \Delta_{HL} + \Delta_H) \\
\bar{\lambda}_H(0)e^{pt}, & \forall t \in [T + \Delta_{HL} + \Delta_H, t_b) \\
c_r, & \forall t \in [T_b, \infty)
\end{cases}
\] (B.6)

Where $\Delta_{HL}$ is the duration of the joint extraction phase, and $\Delta_H$ the duration of the $H$-only constrained path.

IV. Both fuels are abundant. Paths start in zone $VI$, will eventually enter zone $V$ and evolve as described in case III. above. The trajectory will enter into zone $VI$ while already constrained, after a phase with extraction of $L$ only at the cap. Prior to this, the interim phase, for the same reasons as in the previous case, will be characterized by the extraction of the high-carbon fuel only:

\[
R(t) = \begin{cases} 
R_L(t) = 0, \text{ and } R_H(t) = d(\lambda_H(0)e^{pt}), & \forall t \in [0, T) \\
R_L(t) = \bar{x}_L, \text{ and } R_H(t) = 0, & \forall t \in [T, t_b) \\
d(\bar{\lambda}_i(0)e^{pt} + \varepsilon_i\tau(t)), i \in \{L, H\}, & \forall t \in [T + \Delta_{HL}, T + \Delta_{HL} + \Delta_H) \\
R_L(t) = 0, \text{ and } R_H(t) = \bar{x}_H, & \forall t \in [T + \Delta_{HL} + \Delta_H, t_b) \\
R_L(t) = 0, \text{ and } R_H(t) = \bar{\lambda}_H(0)e^{pt}, & \forall t \in [T + \Delta_{HL} + \Delta_H, t_b) \\
0, & \forall t \in [T_b, \infty)
\end{cases}
\] (B.7)

\[
p(t) = \begin{cases} 
\lambda_L(0)e^{pt}, & \forall t \in [0, T) \\
\bar{p}_L, & \forall t \in [T, t_b) \\
\bar{\lambda}_i(0)e^{pt} + \varepsilon_i\tau(t), i \in \{L, H\}, & \forall t \in [T + \Delta_{HL}, T + \Delta_{HL} + \Delta_H) \\
\bar{p}_H, & \forall t \in [T + \Delta_{HL} + \Delta_H, t_b) \\
\bar{\lambda}_H(0)e^{pt}, & \forall t \in [T + \Delta_{HL} + \Delta_H, t_b) \\
c_r, & \forall t \in [T_b, \infty)
\end{cases}
\] (B.8)
Figure B.1: Price paths for endowments in zone IV.

Figure B.2: Price paths for endowments in zone V.

Figure B.3: Price paths for endowments in zone VI.
B.2 Proof of Proposition 2

If the economy were unconstrained, extraction would follow some path $R^u(t), \forall 0 \leq t \leq T$. With an announced constraint, the path of extraction $R^d(t)$ satisfies $R^d(T) < R^u(T)$, as otherwise the constraint would not be binding, and hence $p^d(T) > p^u(T)$. Continuity of the Hamiltonian rules out a jump in $R^d(t)$. As $\dot{p}^d(t) = \dot{p}^u(t) = \rho \forall 0 < t \leq T$, we must have $R^d(T) < R^u(T) \forall 0 \leq t \leq T$ and hence $Z^d(T) < Z^u(T) \forall 0 \leq t \leq T$.

B.3 Proof of Proposition 3

We prove each part individually.

1. From (16), for both resources, and (1.e), we find emissions for an economy that is unconstrained at any point in time:

$$Z^u(t) = \frac{\rho}{\eta} \left(1 - e^{\frac{\rho}{\eta}(t-T^u)}\right)^{-1} (\varepsilon_H S_H(0) + \varepsilon_L S_L(0)),$$

where $T^u$ denotes the instant at which this economy switches to the backstop. From Section 5 we know that

$$R(T) = R_L = \frac{\bar{Z}}{\varepsilon_L}$$

and that, when both resources are abundant, in the interim period only the high-carbon resource is used. Continuity of the Hamiltonian requires continuity of utility. As $\dot{\rho}(t) = \frac{dU(t)}{dt} = \rho$, it follows from (B.10), (16) in growth rates, and from the fact that in the interim period only the high-carbon resource is used, that $R(t) = R_H(t) = \frac{\bar{Z}}{\varepsilon_L} e^{\frac{\rho}{\eta}(T-t)} \forall t \in [0, T)$. This gives

$$Z(t) = \frac{\varepsilon_H}{\varepsilon_L} \bar{Z} e^{\frac{\rho}{\eta}(T-t)} \forall t \in [0, T)$$

For emissions to jump up at $t = 0$, unconstrained emissions at this point in time must be smaller than actual emissions: $Z^u(0) < Z(0)$. Combining these results gives part 1 of the proposition.

2. Denote the instant at which the low-carbon resource is no longer used due to exhaustion of the stock (see Appendix B.1) by $t_H$: the instant from which onward only the high-carbon resource is used. Hence $R_L(t_H) = 0$. From (7) and the fact that both resources are extracted during the interval $(T, t_H)$ (see Appendix B.1), it follows that $\dot{\rho}(t) = \rho$, which, together with (15) and (7) in growth rates, implies

$$\dot{q}(t) = -\frac{\rho}{\eta}, \forall t \in (T, t_H).$$

Combining this with (1.b) and (1.e) in growth rates, we find the following differential equation for the low-carbon resource:

$$R_L(t) = -\frac{\bar{Z}}{\varepsilon_H - \varepsilon_L} \frac{\rho}{\eta} - \frac{\rho}{\eta} R_L(t) \forall t \in (T, t_H).$$

Combining this result with (1.d) and $S_L(t_H) = 0$, and using $S_L(T) = S_L0$ gives

$$R_L(t) = -\frac{\bar{Z}}{\varepsilon_H - \varepsilon_L} + \frac{\rho}{\eta} e^{\frac{\rho}{\eta} T} - e^{\frac{\rho}{\eta} t_H} \left(S_L0 - \frac{\bar{Z}}{\varepsilon_H - \varepsilon_L} (T - t_H)\right) e^{-\frac{\rho}{\eta} t} \forall t \in (T, t_H).$$

(B.14)
To find an implicit solution for $t_H$, use $R_L(t_H) = 0$ to find

$$
\frac{1}{e^{-\frac{T}{\eta}(T-t_H)}-1} \left( S_{L0} - \frac{Z}{\epsilon_H - \epsilon_L} (T - t_H) \right) - \frac{Z}{\epsilon_H - \epsilon_L} = 0. \quad (B.15)
$$

This gives $d t_H / d T = 1$, from which follows that $d \left( e^{-\frac{T}{\eta}} - e^{-\frac{T}{\eta} t_H} \right) / d T < 0$, while the sign of $d t_H / d \tilde{Z}$ is indeterminate.

To find emissions at $t = 0$, we first need to find the amount of energy used at $t = T$. Using (B.14), (1.e) and (1.b) we get:

$$
q(T) = \frac{1}{\epsilon_H \eta} \frac{1}{1 - e^{-\frac{T}{\eta} (t_H - T)}} \left( (\epsilon_H - \epsilon_L) S_{L0} - \tilde{Z} (T - t_H) \right). \quad (B.16)
$$

From (B.12) and the fact that during the interim period only the high-carbon resource is used, we find the second inequality of part 2 of the proposition. The first inequality follows from the proof of part 1 of this proposition.