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<th><strong>Title</strong></th>
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EXAMINATION OF A NOVEL WAVELET APPROACH FOR BENDER ELEMENT TESTING

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Abstract
Accurate determination of shear wave arrival time using bender elements may be severely affected by a combination of near field effects and reflected waves. In most cases, the near-field effect masks the first arrival and it makes its detection difficult in the time domain. Nevertheless the arrival of a shear wave creates a detectable singular point. This paper tests a recent approach for the assessment of shear wave arrival time by analysing the output signal in the time-scale domain using a multi-scale wavelet transform. Indeed, one can follow the local maxima lines of the wavelet transform modulus across scales, to detect the location of all singularities leading to detection of the first arrival.

Keywords: bender elements, shear wave velocity, wavelet

1. Introduction
Geophysical techniques, and in particular seismic methods, have received considerable attention in civil engineering over recent years, their role steadily increasing to the point where they play an important part in site and material characterisation. This popularity likely arises from recent advances in both computational power and the geophysical techniques themselves. Furthermore, many geophysical methods are non-invasive which make them well suited and cost effective in profiling spatially and temporally. From a geotechnical engineering perspective the most popular geophysical techniques are seismic methods as they may directly measure a mechanical property, soil or rock stiffness. Seismic testing consists of monitoring seismic waves that occur either naturally or artificially from a source inside or on the surface of the earth. This usually involves strains of $10^{-3}$ % and less. The measurement of stiffness at this magnitude of strain is important for deformation prediction, as strains associated with most soil-structure interaction problems are generally less than 0.1% (Jardine et al. 1986). It has been shown by Stokoe et al. (2004) that stiffness-strain curves for a range of materials may contain considerable error if small strain stiffness values have not been incorporated. A significant overestimation of deformation may result, which could substantially increase the cost of a project. According to elastic theory the small strain shear modulus, $G_{\text{max}}$, may be calculated from the seismic parameter, shear wave velocity, using the following equation:

$$G_{\text{max}} = \rho . V_s^2$$  \hspace{1cm} (1)

where $G_{\text{max}} = $ shear modulus (Pa), $V_s = $ shear wave velocity (m/s) and $\rho = $ density (kg/m$^3$).
Several techniques are commonly used to measure the shear wave velocity both in the field (in-situ) and in the laboratory. Field testing methods may be classified as intrusive or non-intrusive depending on whether boreholes are required or if the seismic instrumentation is placed on the surface. Intrusive field methods include cross-hole, down-hole and seismic cone methods. In these surveys seismic sources and receivers are located either between boreholes or between the surface and a point in a borehole or cone. Non-intrusive field methods used to determine $V_s$ include seismic reflection and refraction and surface wave surveys. Laboratory methods commonly used to compute $V_s$ include the resonant column method and the bender element (BE) method. In the resonant column method $V_s$ is determined from the resonance frequency of a soil cylinder that is being vibrated. In bender element testing a shear wave is transmitted using a piezoelectric element and recorded with another piezoelectric element having travelled through a soil specimen. This paper tests a recent approach (Bonal et al. 2008) for the assessment of bender-element determined shear wave velocities using a multi-scale wavelet transform.

1.1 Bender Elements

The bender element method was first introduced by Shirley and Hampton (1978), and was further developed by Dyvik and Madshus (1985). A piezoelectric bender element is an electro mechanical transducer that may convert electrical energy to mechanical energy and vice versa. Bender elements consist of two piezoelectric sheets that are mounted together, separated by an electrode surface and bounded by two further electrode surfaces. The two piezoelectric sheets may be polarised in the same or opposite directions by wiring either in parallel or series, depending on whether an electrical signal is to be transmitted or received. In a series connected element (Fig. 1a) the wires are connected to the outer electrode surfaces and the two piezoelectric plates are polarised in opposite directions. In a parallel connected element (Fig. 1b) the wires are connected to both the outer electrode surfaces and by careful grinding away of a small portion of the element, the centre electrode. The polarisation of the two outer electrodes is the same, either positive or negative, while the centre electrode is the other pole.

![Figure 1 - Connections of (a) Series and (b) Parallel Type Bender Elements](image)

The electrical impedance of a parallel element is half that of an equivalent series element. The series connected element is therefore twice as effective when converting mechanical energy to electrical energy and the parallel connected element is twice as effective at converting electrical energy to mechanical energy (Dyvic and Madshus, 1985). As a result the parallel connected element is twice as effective as a transmitter with the series connected element being twice as effective as a receiver.

For bender element testing of soil or rock specimens an electrical signal is sent to the transmitter element, which is in turn excited and generates a shear wave; this propagates parallel to the direction of the element down through the specimen. The receiver element is excited by this shear wave, thus creating an electrical impulse. In addition to testing samples
in their unconfined state (Donohue and Long 2008), bender elements have been incorporated into a range of geotechnical testing apparatus including the triaxial test (Dyvic and Madshus 1985; Viggiani and Atkinson 1995; Jovicic and Coop 1997), oedometer (Dyvic and Olsen (1989), Zeng and Ni, (1998)) and direct simple shear devices (Dyvic and Olsen, 1989).

A simple design schematic for the UCD bender element system is shown in Figure 2. In order to maximise mobility, each part of the system was chosen to be lightweight and portable. An arbitrary function generator (Tecstar FGA 2030) was used for signal generation and detection and observation of the resultant output was made using a portable digital storage oscilloscope (DSO), a Picoscope ADC 212/100. For even greater mobility some researchers (Mohsin et al. 2004) have used the laptops soundcard for generation of the shear wave, although generally only smaller amplitude (±2V) signals can be produced.

![Figure 2 – UCD Bender element system](image)

### 2. Analysis of Bender Element Signals

By accurately measuring the travel time of a shear wave from the transmitting element to the receiver element in a soil sample, the shear wave velocity ($V_s$) through the sample may be determined from:

$$V_s = \frac{L}{t}$$  \hspace{1cm} (2)

where $L$ is the distance between receivers and $t$ is the time it takes for the shear wave to propagate through the specimen. It is frequently taken that the first deflection of the received signal represents the first arrival of the shear wave, i.e. the time between the initial voltage change on the transmitted wave and the corresponding initial voltage change on the received trace. Ideally this would be simple and would resemble the illustration in Figure 3. However, due to attenuation, the influence of compressive waves (near-field effect) and wave reflections the received trace never appears as simply as that shown in Figure 3. In reality, the received trace is somewhat distorted (Figure 4) and in some cases does not bear any resemblance to the input trace. This has led to many researchers using a range of different methods to determine it. The methods adopted are commonly based in the time domain: (1) first arrival method; (2) peak to peak method; (3) using second arrivals, although a number of frequency domain approaches have also been used (1) cross correlation; (2) Phase-delay method. Due to the not insignificant differences in $V_s$ obtained from the different methods, however, the geotechnical
community has yet to propose a standard method for analysing bender element signals. Bonal et al. (2008) have recently proposed a novel wavelet based approach for analysing bender element signals. The purpose of this paper is to test this approach on real signals, in order to assess its usefulness.

![Input Wave](image1) ![Output Wave](image2)

**Figure 3** - Idealised first arrival waveform at receiver

![Figure 4](image3)

**Figure 4** - Typical output signal from a bender element test in a soil sample

### 3. Wavelet Analysis

A signal may be observed through two main domains: time domain and frequency domain (Fig. 5). The Fourier Transform and its inverse connect them and are the main mathematical tools for signal analysis. The Fourier Transform is perfectly adequate for stationary and periodic (or quasi-periodic) signals and provides a global description of frequency distribution, energy and overall regularity. However, it involves the complete loss of local time information such as the location of singularities. Keeping local time information makes non-stationary signal analysis possible. The Windowed (or Short Time) Fourier Transform is a time-frequency tool. However the resolution of this tool is limited by Heisenberg's principle: if the precision in the time domain increases, the precision in the frequency domain decreases.

A wavelet is a mathematical function used to divide a given function or continuous-time signal into different frequency components and study each component with a resolution that matches its scale. A wavelet transform is the representation of a function by wavelets.
The wavelets are scaled and translated copies (known as "daughter wavelets") of a finite-length or fast-decaying oscillating waveform (known as the "mother wavelet"). Wavelet transforms have advantages over traditional Fourier transforms for representing functions that have discontinuities and sharp peaks, and for accurately deconstructing and reconstructing finite, non-periodic and/or non-stationary signals.

The continuous wavelet transform (CWT) is a mathematical tool which was first introduced by Grossmann and Morlet (1984) and is still commonly used today. Let $\Psi(t)$ be a complex valued function, where $t$ is the time variable and is used as the main variable in our study. The function is said to be a wavelet if and only if its Fourier transform $\Psi(\omega)$ satisfies:

$$
\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^2}{\omega} d\omega = \int_{-\infty}^{0} \frac{|\Psi(\omega)|^2}{\omega} d\omega = C_{\Psi} < +\infty
$$

(3)

This condition for zero mean implies that:

$$
\int_{-\infty}^{\infty} \Psi(u) du = 0
$$

(4)

$\Psi$ is the mother wavelet that generates a large family by dilation. Let $\Psi_s(t) = (1/s)\Psi(t/s)$ be the dilation of $\Psi(t)$ by the scale factor $s$. This factor changes the local frequency by dilation or compression of the wavelet; this effect links the time-frequency domain with the time-scale domain. $\Psi(t)$ decays rapidly to zero with increasing $t$. The range of non-zero values is called the support, denoted by $K$. The CWT of a function $f(t)$ in Hilbert space $L^2(\mathbb{R})$, is defined by:

$$
Wf(s,t) = f * \Psi_s(t) = \int_{-\infty}^{\infty} f(u) \Psi_s(t-u) du
$$

(5)

This definition can be written using the main wavelet $\Psi$:

$$
Wf(s,t) = f * \frac{1}{s} \Psi\left(\frac{t}{s}\right) = \frac{1}{s} \int_{-\infty}^{\infty} f(u) \Psi\left(\frac{t-u}{s}\right) du
$$

(6)
The Wavelet Transform is motivated by the possibility of finding a singularity as it decomposes the signal into elementary building blocks that are well localized both in time and frequency (Mallat and Hwang 1992). The local detail is matched to the scale of the wavelet, so it can characterize coarse (low frequency) features on large scales and fine (high frequency) features on small scales. This mathematical tool is a time-scale and multi-resolution analysis and offers a way to avoid Heisenberg's principle. Indeed Heisenberg is verified at each scale but its effect is overcome by the presence of several scales.

By assuming the shear wave to be both plane and homogenous, the shear wave arrival in a bender element test is characterized by the arrival of a broad-band energy, or a sine if the input signal is a sine. The shear wave is, however, preceded by the near-field effect. The near-field effect creates a first singular point in the output signal ($t \approx 1000000$ ns) that can be observed in Figure 4. It induces an opposite phase wave preceding the S-wave. The main S-wave arrival creates a second singular point ($t \approx 1400000$ ns) however its location in the time domain is usually scrambled by the near-field effect and the noise. Nevertheless this singularity exists and is defined as a discontinuity in the first derivative at this time.

Bonal et al. (2008) have presented a novel method of analyzing bender element signals using wavelets based on the Lipschitz Exponent (Mallat and Hwang, 1992). The Lipschitz exponent is a well known tool used to estimate function differentiability. To find function discontinuity or singularity, the exponent is applied together with the wavelet transform. Figure 6, shows the decomposition of a typical bender element signal at five scales and clearly illustrates the difficulties, due to noise, involved in selecting a first arrival. In order to accurately determine the first arrival of a shear wave using wavelets, Bonal et al. (2008) followed several maxima lines of the wavelet transform modulus across scales, sorted out from a range of local Lipschitz exponents. They observed the first arrival to be among these singular points with a local Lipschitz exponent of almost 1. They then used this approach to write a shear wave arrival detector algorithm. The program decomposes a signal at several scales with an appropriate wavelet. It determines all maxima lines and assesses their local Lipschitz exponents, in order to pick the remarkable points. The user sets the range of exponents of the analysis to lead the algorithm to analyze a particular area of the signal. The program returns a list of particular points and a figure showing the input and output signals at these points. The wavelet singularity detection does not give an automatic result, but nevertheless can lead us to points of interest within the signal. The points which are recurrent across the input periods are easily found. This approach is tested in this paper on signals from bender element tests on Bogganfin clay, a normally consolidated lacustrine clay from the midlands of Ireland described by Donohue and Long (2008).

Wavelet analysis was performed using the algorithm of Bonal et al. (2008) on bender element signals over a range of frequencies on a specimen of Bogganfin clay. Two of these frequencies are presented here. The input frequency which generated the first signal (Fig. 7a) was 2 kHz, the sampling frequency is 500 kHz (ratio $f_r/2f_i$ is 125). The range of local Lipschitz exponents is $[0.8, 1]$, in order to detect the first arrival times of both the near-field wave and the direct shear wave. The program returns a list of singular points and displays them on the graph (vertical lines in Fig. 7). The second signal is extracted from the same test. The input frequency is 12.5 kHz, the sampling frequency is 500 kHz, (ratio $f_r/2f_i$ is 20). The range of local Lipschitz exponents is $[0.6, 1]$. As with the previous test the program returns a list of singular points and displays them on the graph (Figure 7b).

Several points are recurrent and a visual analysis of different waveforms from a range of frequencies makes the choice possible. In this case the singular point at $t = 800 \mu s$ (corresponding to a $V_s = 119$ m/s & $G_{max} = 27$ MPa) occurs consistently over a range of frequencies unlike most of the other singularities detected which change from frequency to
frequency. Despite the considerable amount of noise in Figure 7b, it is encouraging that the wavelet analysis has detected a singularity at the same point. From a visual inspection of the data, most authors (e.g. Kawaguchi et. al., 2001) have traditionally taken the circular point in Figure 7a (the intercept of the horizontal line and the received waveform after the near field) as being equal to the first arrival of the shear wave. Using this traditional visual assessment method, a velocity $V_s$ of 109 m/s and a resulting $G_{\text{max}}$ of 22 MPa would have been determined. The wavelet approach has shown that this represents an 8% underestimation in the shear wave velocity and an 18% underestimation of stiffness.

![Figure 6 - Decomposition of a signal at different scales](image)

**Figure 6** – Decomposition of a signal at different scales

![Figure 7 – Testing the wavelet approach (Bonal et al. 2008) on a bender element signal from a test on Bogganfin clay with input frequencies of (a) 2kHz and (b) 12.5kHz.](image)

**Figure 7** – Testing the wavelet approach (Bonal et al. 2008) on a bender element signal from a test on Bogganfin clay with input frequencies of (a) 2kHz and (b) 12.5kHz.
4. Conclusions
Accurate determination of shear wave arrival time using bender elements may be severely affected by a combination of near field effects and reflected waves. Due to significant differences in measurements of arrival times determined using a range of analysis methods, the geotechnical community has yet to propose a standard method of signal analysis. This paper has reviewed and tested a novel wavelet based approach for arrival time determination. To detect function discontinuity or singularity, a Lipschitz Exponent was applied together with the wavelet transform. When tested on signals obtained from a bender element test in Bogganfin soft clay the wavelet approach yielded encouraging results. Across a range of frequencies the shear wave first arrival was characterised by a consistent singularity. Interestingly this point is located earlier in the received signal than the traditional visual assessment approach. One limitation of the wavelet approach appears to be that it is unsuitable for automation.

References