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Importance of the Tail in Truck Weight Modeling for Bridge Assessment

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Abstract

To predict characteristic extreme traffic load effects, simulations are sometimes performed of bridge loading events. To generalize the truck weight data, statistical distributions are fitted to histograms of weight measurements. This paper is based on extensive WIM measurements from two European sites and shows the sensitivity of the characteristic traffic load effects to the fitting process. A semi-parametric fitting procedure is proposed: direct use of the measured histogram where there are sufficient data for this to be reliable and parametric fitting to a statistical distribution in the tail region where there are less data. Calculated characteristic load effects are shown to be highly sensitive to the fit in the tail region of the histogram.

Keywords: Bridge, assessment, traffic, load effect, loading, weigh-in-motion, WIM, simulation, tail, semi-parametric.
1. **Introduction**

The accurate estimation of the characteristic site-specific lifetime maximum loading for existing highway bridges can result in significant cost savings in bridge maintenance and repair. The application of full design or assessment code loadings is, in many cases, unduly conservative (Bailey 1996). One method of estimating the characteristic loading is to fit statistical distributions to load effects (such as bending moments) calculated from measured traffic, and to use these distributions directly to predict the lifetime maximum loading (Nowak 1993; Nowak 1994). An alternative method, which is used in this study, is to run Monte Carlo simulation models for traffic which are representative of measured vehicle data for the site (Bruls et al. 1996, O'Connor and O'Brien 2005, Caprani et al. 2008). Measured traffic data include such parameters as Gross Vehicle Weight (GVW), number of axles, axle spacing, distribution of GVW between axles, and inter-vehicle spacing.

To perform Monte Carlo simulation, it is necessary to fit statistical distributions to histograms of measured data. The quality of these fits is important and has a significant influence on the accuracy of the results. GVW is used to illustrate the problem as it has a particularly strong influence on the load effects of interest. Data are analyzed from weigh-in-motion (WIM) systems at two European sites: 650 000 trucks weighed over a 20-week period in 2005 at Woerden in the Netherlands, and 750 000 trucks weighed over 19 months in 2005 and 2006 at Branisko in Slovakia. A notable feature is the significant number of extremely heavy vehicles, particularly in the Netherlands where 892 vehicles weighed over 70 t [154 kips], with a maximum recorded weight of 165 t [364 kips]. In Slovakia, there were 78 trucks over 70 t and a recorded maximum of 117 t [258 kips].
2. Modeling GVW

A critical bridge loading event may be caused by a single very heavy truck, or by a combination of trucks of different weights crossing the bridge at the same time. It is important, therefore, to model accurately the complete range of GVWs. Three different methods of modeling GVW are considered. All are based on histograms of the observed GVWs using a bin size of 1 t.

2.1. Parametric fitting

Perhaps the most widely used approach (O'Connor and O'Brien 2005) is to fit the “measured” histogram to a multimodal Normal (Gaussian) distribution, i.e., to a linear combination of a number of Normal distributions. This is similar to the approach used in reliability studies where heavy trucks (in this case the mode at 40 t in Fig. 1) are modeled with a Normal distribution (Kennedy et al. 1992). Maximum likelihood estimation is used here to estimate the parameters for a trimodal Normal distribution. As can be seen in Fig. 1, this gives a moderately good fit for most of the GVW range, but significantly underestimates the probabilities in the critical upper tail.

2.2. Non-parametric

Non-parametric fitting uses the measured histogram directly as the basis for simulating GVW. A uniformly distributed random variable is generated in the range $[0, \Sigma f_i]$ where $f_i$ is the measured frequency for interval (bin) $i$. The corresponding GVW is used in the simulation. This is a reasonable method for the range of commonly observed GVWs, but the
method presents problems in the upper regions of the histogram where observations are few and there are gaps with no measured data (Fig. 1). If a particular GVW is not in the set of measured data, it will not appear in the simulation and, most significantly, this method will never simulate a GVW heavier than the maximum measured value.

![Histogram of GVW](image)

*Fig. 1. GVW Histograms for Lane 3, Branisko, Slovakia with parametric and semi-parametric fits (close ups of tail region inset)*

### 2.3. Semi-parametric fitting

A third method, proposed here, is to use the measured histogram in the lower GVW range where there are sufficient data, and to model the upper tail with a parametric fit. This ensures much greater accuracy of the probabilities in the tail region (Fig. 1), allows for interpolation between sparse data points and provides a non-zero probability of GVWs above the highest observed value. The curve chosen here is the tail of a Normal distribution which is asymptotic towards zero probability and has been found by the authors to fit well to extreme truck weight data.
To apply the semi-parametric method, a threshold value for GVW must be selected. Below this threshold, the measured histogram is used, while above it, the parametric curve is used. The threshold must be not be too large – it is necessary to have sufficient data to the left of it for the bin counts to be “reliable”, i.e., there should be sufficient data in each GVW bin for it to be repeatable, and for the histogram to be smooth. It is also important that the threshold not be too small to ensure that there is a good fit to the histogram in the important tail region.

3. Reliability of Bin Counts

Assuming that all observed data are drawn from the same distribution, this “parent” distribution (whether known or not) will give the expected value for the count in any bin for a given sample size. For each value observed there is a probability \( p \) that it will fall into a particular bin and \( (1 - p) \) that it will not – a Binomial trial. The total number of values observed is \( N \), and the expected (mean) number of values in the bin is \( Np \). When \( N \) is large and \( p \leq 0.05 \), which it typically is for 1 t GVW bins and certainly is in the upper tail, then the Poisson distribution gives a good approximation to the distribution of the number of values observed in the bin (Scott 1992). The probability of \( k \) observed values in the bin is:

\[
p(k) = e^{-k} \frac{(Np)^k}{k!}
\]  

(1)

The variance is \( \sigma^2 = Np \) and the coefficient of variation is:

\[
COV = \frac{\sigma}{Np} = \frac{1}{\sqrt{Np}}
\]  

(2)

This gives a measure of how reliable a particular bin count is. For GVWs, the parent distribution is unknown, and hence \( p \) is unknown. The maximum likelihood estimate for \( Np \)
is the mean observed number in the bin. As this is a single observation, the observed number
is used as the best estimate available.

To illustrate this, the theoretical COV based on the Poisson distribution is plotted in Fig. 2 for
a simple Normal distribution – $N_{50,10}$ with 20 000 values binned using a bin size of 1. As can
be seen, the COV is relatively low for the bulk of the distribution, but rises rapidly in the tail
regions.

Fig. 2. Coefficient of Variation for a Normal distribution - $N_{50,10}$

The threshold value for GVW should be below the point where the COV gets excessively
large, such as the 25% level indicated.

4. Estimating characteristic GVW and bridge load effects

GVW data from the Slovakian site are analyzed in Fig. 3. The weights are grouped randomly
into blocks of 750, and the block maxima are plotted on Gumbel (Type 1) probability paper.
If the parent distribution were a Normal distribution, this plot of extreme values would appear
as a straight line. As can be seen, it has two distinct linear sections, suggesting that there is a
mixture of two Gumbel distributions present, probably consisting of two different types of
truck (for example, it would be reasonable to speculate that the weights up to 65 t are 5- or 6-axle trucks while the data above this point consists of cranes and/or low-loaders). Fitting a Normal tail to the GVW histogram corresponds to fitting a straight line to the upper part of the block maximum data plotted on Gumbel probability paper. The point at which the extreme value curve begins to deviate significantly from this straight line gives a lower bound for the GVW threshold. The shape of this curve varies between the sites considered, but the upper portion has been found to be fairly linear which supports the choice of the Normal distribution for tail fitting.

An examination of similar curves for each individual lane at Branisko and at Woerden suggests that a COV value of 25% is a good basis for selecting a threshold for GVWs. This corresponds to a minimum bin count of 16 data below the threshold. This recommendation is, within reason, independent of bin size – for a greater bin interval width, the threshold moves to the right as the histogram becomes smoother and more repeatable in the tail region. For a bin size of 1 t, there are typically about 100 observed values above the threshold.
Extreme value theory (Castillo 1988, Coles 2001) is used to estimate characteristic bridge load effects. For the design of new bridges in accordance with Eurocode 1, this is the value with a 1000 year return period, i.e., 5% probability of being exceeded in a 50 year lifetime (EC1 2003, Bruls et al. 1996, Flint and Jacob 1996).

Sample simulation results are presented in Fig. 4 for the fast lane at Woerden where the maximum observed GVW is 75 t. The simulations are run for 2000 days (the equivalent of 8 years excluding weekends), and the daily maximum bending moments are calculated. To estimate the 1000-year characteristic moment, these curves need to be extrapolated to a value of 12.43 on the Y axis.

It can be seen that the parametric method gives a relatively low estimate for the characteristic bending moment. The non-parametric curve is becoming bounded in its upper region due to the fact that no GVWs greater than the observed maximum are simulated. The semi-parametric curve gives a significantly greater result for this example and is considered to provide a more realistic basis for extrapolation to the characteristic value.

![Bending moment graph](image)

*Fig. 4. Daily maximum mid-span bending moments, simply supported 35 m span, single truck load, fast lane, Woerden.*
5. Conclusions

The problems surrounding the simulation of traffic loading scenarios and extrapolation to find the characteristic value are reviewed using WIM data from two European sites. The problems with both parametric and non-parametric fitting to histograms of measured data are identified and a semi-parametric approach is recommended. The implications of each assumption are illustrated using a simulation in which characteristic 1000-year bending moments are estimated.

References


