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Bridge Roughness Index as an Indicator of Bridge Dynamic Amplification

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ABSTRACT
The concept of a road roughness index for bridge dynamics is developed. The International Roughness Index (IRI) is shown to be very poorly correlated with bridge dynamic amplification as it takes no account of the location of individual road surface irregularities. It is shown in this paper that a Bridge Roughness Index (BRI) is possible for a given bridge span which is a function only of the road surface profile and truck fleet statistical characteristics. The index is a simple linear combination of the changes in road surface profile; the coefficients are specific to the load effect and span of interest. The BRI is well correlated with bridge dynamic amplification for bending moment due to 2-axle truck crossing events. A similar process can be used to develop a BRI for trucks with other numbers of axles or combinations of trucks meeting on a bridge.

Keywords: DAF, DAE, BRI, dynamic, roughness, bridge, vehicle.

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1. Introduction

The Dynamic Amplification Factor (DAF) for a bridge is defined as the maximum total (dynamic plus static) load effect divided by the maximum static load effect. DAF is influenced by many factors. Speed is particularly important, as noted by many authors [1-6] and in particular the ratio of speed to bridge 1st natural frequency [7]. Road roughness is also a key factor [3-4, 8], particularly for short bridges.

In the highway industry, indices for the evaluation of pavement surface evenness have been developed since the 1960s. The most popular parameters are the International Roughness Index (IRI) [9-11] which was developed and recommended by the World Bank to evaluate pavement roughness, and the Power Spectral Density (PSD) [12]. [Li – can you fix up the ref numbering please]

Olsson (1985), Lin (1997), Henchi et al. (1997), Liu et al. (2002) and Majumder and Manohar (2003) are just some of the authors that use finite element analysis with six degrees of freedom per node to model bridges dynamically. Olsson (1985) compared the results of finite element analysis with an Euler-Bernoulli beam model [Li – did he compare them?]. Green and Cebon (1994) modelled two bridges in the U.K. with finite elements and compared the results to field measurements. [Li – I think this is true – is it?] Chompooming and Yener (1995) present a finite element model of a beam and slab bridge. This type of section is also modeled by Kou and Dewolf (1997) who use plate elements for the deck and beam elements for the girders. González (2001) couples displacements and velocities at the bridge/vehicle contact points at each time step in an iterative formulation. Yau and Yang (2004) allow for instability and inertial effects in a finite element model of a cable stayed bridge.
The effect of road surface irregularities on bridge vibration has been examined by DIVINE [1], Green et al. [4], Lay and Zhu [13], Kou and Dewolf [14], Lei and Noda [15] and Chatterjee et al. [16]. Vehicle and bridge models have been used to simulate the vehicle-bridge interaction system and to determine the effect of profile unevenness. However, these papers investigate the influence of different PSD levels. It has been found by Li et al. [17] that there are substantial differences in dynamic amplification between road profiles with the same PSD level and the same IRI value.

Michaltsos [18] and Pesterev et al. [19, 20] have shown that the position of an irregularity is important for bridge dynamic amplification. In a previous paper [17], the authors confirm this and show that if bridge deflections are negligible [20], the principle of superposition applies for the dynamic response to individual road surface irregularities. This makes possible the estimation of dynamic amplification by adding together the DAFs due to each irregularity that makes up the road surface profile. Very good agreement with critical DAF values is reported for a range of 'good' road surface profiles. A significant problem is that the resulting dynamic amplification estimate is specific to the properties (spring stiffness etc.) of that vehicle.

Yang et al. [12], Zhu & Law [21] and Brady & OBrien [22] examine cases of two following loads and compare the effects to single load crossings. Axle spacing is identified as being particularly important [22, 23] and it quickly becomes clear that an amplification factor derived from a 1-axle vehicle model is of limited value as an indicator of DAF for multiple-axle vehicles.
The goal of this paper is to develop the concept of a Bridge Roughness Index (BRI) which can be used as an estimator of DAF for a given vehicle class, and in particular in this paper, a BRI for 2-axle vehicles. The BRI will be a function of the road profile only; it will not be dependent on the speeds or properties of particular vehicles. A BRI potentially constitutes an extremely useful measure of road surface roughness that could be used by bridge maintenance managers as an indicator of the level of dynamic amplification that might be expected on a bridge.

2. BRIDGE VIBRATION

2.1. Vehicle-bridge interaction model

A half-car model crossing a simply supported Bernoulli-Euler beam at a constant speed is used to simulate 2-axle vehicle events (Fig. 1). The motion controlling this system is defined by the ordinary differential equations [24]:

\[ -I \frac{d^2 \varphi(t)}{dt^2} + (-1)^i D_i (Z_i(t) + Z_{si}(t)) = 0 \]  
(1a)

\[ -m_i \frac{d^2 y_i(t)}{dt^2} - \sum_{i=1}^{3} [Z_i(t) + Z_{si}(t)] = 0 \]  
(1b)

\[ m_{s_i} g + m_i g - m_i \frac{d^2 y_i(t)}{dt^2} + Z_i(t) + Z_{si}(t) - R_i(t) = 0 \]  
\( i=1,2 \)  
(1c)

\[ EJ \frac{\partial^4 v(x,t)}{\partial x^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + 2 \mu \omega \frac{\partial v(x,t)}{\partial t} + \sum_{i=1}^{3} \epsilon_i \delta(x-x_i) R_i(t) = 0 \]  
(1d)

where \( I \) and \( \varphi(t) \) are the mass moment of inertia and rotational degree of freedom of the body mass respectively; \( D_i \) is the horizontal distance between the centroid of sprung mass and unsprung mass \( i \); \( m_1, m_2, m_3 \) are masses of the front axle, rear axle and vehicle
body respectively and \( y_1(t), y_2(t) \) and \( y_3(t) \) are the corresponding vertical displacement of their centers of gravity; \( g \) is acceleration of gravity; \( v(x,t) \) is the displacement of the bridge at location \( x \) and time \( t \); \( E, J, \mu \) and \( \omega_b \) are modulus of elasticity, inertia of the cross-section, mass per unit length and circular frequency of damping of the bridge respectively; \( \varepsilon_i = 1 \) if axle \( i \) is present on the bridge and \( \varepsilon_i = 0 \) if not; \( \delta(x-x_i) \) is the Dirac function. Therefore,

\[
Z_i(t) = K_i \left[ y_{i1}(t) - y_i(t) \right]
\]

(2a)

is the force in the spring between the \( i^{th} \) axle and the vehicle body, where \( K_i \) is the suspension spring stiffness.

\[
Z_{i\delta}(t) = C_i \left[ \frac{dy_{i1}(t)}{dt} - \frac{dy_i(t)}{dt} \right]
\]

(2b)

is the damping force between the \( i^{th} \) axle and the vehicle body, where \( C_i \) is a suspension linear damper.

\[
y_{i3}(t) = y_{i1}(t) - (-1)^i D \phi(t)
\]

(2c)

is the displacement of the contact point between the \( i^{th} \) axle and the vehicle body.

\[
m_{i31} = m_i \frac{D_2}{D_1 + D_2}
\]

(2d)

\[
m_{i32} = m_i \frac{D_1}{D_1 + D_2}
\]

are the masses of the sprung part of the vehicle by axle.

\[
R_i(t) = K_i \left[ y_i(t) - \varepsilon_i v(x_i, t) - r(x_i) \right] \geq 0
\]

(2e)

is the tire force imparted to the bridge by the \( i^{th} \) axle, where \( K_i \) is tire stiffness and \( r(x_i) \) is height of road profile at location of axle \( i \). No allowance has been made for separation.
of the tire from the road. Based on the work of Frýba [24], numerical results are found in the time domain through the Runge-Kutta-Nyström method [25].

2.2. Dynamic Amplification due to 2-axle vehicle

In a previous study [17], the authors have proven that the effect of individual road irregularities can be superposed for a quarter-car model traveling on a short-span bridge with a good road profile. Bending moment is found by adding the moment due to the vehicle traveling on a perfectly smooth surface and the moment due to each individual irregularity that exists on the surface. The approach is accurate for negligible bridge deflections and a good approximation when bridge deflections are small compared to the road irregularities. Based on this assumption of superposition, the total normalized midspan bending moment, $M(c,t)$, for a vehicle speed, $c$, and position of 1st axle $x_1$, can be calculated as

$M(c,x_1) = M_0(c,x_1) + \sum_{i=1}^{N} \left[ M_u(c,i,x_i) - M_0(c,x_i) \right] \times s(i)$

where $M_0$ is the normalized midspan bending moment caused by the vehicle on a smooth profile and $M_u$ is the normalized midspan moment due to a unit ramp at location $i$. The profile is discretized into $N$ ramps each 100 mm long and the measured fall in the $i^{th}$ ramp in mm is $s(i)$. All bending moments are normalized by dividing them by the maximum static bending moment. The calculation procedure is illustrated in Fig.2 for a road profile made of two ramps of heights $H_1$ and $-H_2$ located at -0.1 and 0 m respectively. The total moment $M$ due to a vehicle of given mechanical parameters and speed (Fig.2(a)) is the result of adding three moments: (1) $M_0$ corresponding to a smooth profile (Fig.2(b)), (2) $[s(1) \times (M_{u(x=-0.1)} - M_0)]$, or moment due to a unit ramp at $x =$
-0.1 (Fig. 2(c)) scaled by the height of the first ramp \( s(1) = H_1/0.001 \), and (3) \[ s(2) \times (M_{u(x=0)} - M_0) \], or moment due to a unit ramp at \( x = 0 \) (Fig. 2(d)) scaled by its height \( s(2) = -H_2/0.001 \). This approach involves the addition of bending moment values corresponding to different critical points near the centre of the bridge. Then, the dynamic amplification factor due to a vehicle traveling at speed \( c \) can be estimated from the maximum normalized total moment for any location \( x_1 \) while the vehicle is near the center of the bridge:

\[
DAE(c) = \max_{0.4L \leq x_1 \leq 0.8L} \left[ M(c, x_1) \right] \quad (4)
\]

It becomes clear that the nature of a given road profile relating bridge dynamics, can only be characterized as ‘good’ or ‘poor’ linked to a given vehicle type.

Unfortunately, while \( DAE \) provides an excellent estimate of dynamic amplification for a particular vehicle, it is only accurate for that vehicle. For example, on the road profiles described as Fig. 3, the 2-axle estimator is illustrated for three vehicles with axle spacings \( D (=5 \text{ m}) \), 0.7\( D \) and 1.3\( D \) in Fig. 4. Clearly the spacing used to generate the terms, \( M_0(c) \) and \( M_u(c, i) \) in Eq. (3), has a most significant influence on the result.

Further, there is no one spacing for a 2-axle vehicle that will provide good estimates of dynamic amplification for other vehicle spacings.

DAF and \( DAE \) are also strongly influenced by vehicle speed. The normalized bending moment due to the two-axle vehicle defined by parameters in Table 1 and traveling on a smooth road profile, \( M_0(c, x_1) \), is illustrated in Fig. 5(a) for \( x_1 = 0.5L \), i.e., front axle at
the center of the bridge. The corresponding normalized moments for unit ramp
excitations of the vehicle, \( M_u(c, i, 0.5L) - M_o(c, 0.5L) \), are given in Fig. 5(b). This
graph provides a valuable insight into the source of the total bending moment. It can be
seen that particular locations on the bridge are especially important. For example a ramp
at \(-0.4L\) is highly significant while a ramp at \(-0.2L\) makes no contribution. It can also be
seen that there are positive and negative effects – light and dark zones in the figure.
Hence the effects of some ramps will cancel out while others will be additive and there
will be particular combinations of ramps that will result in very high bending moment.

A section through Fig. 5(b) corresponding to a speed of 80 km/h is illustrated in Fig.
6(a). The product of this graph and the road profile of Fig. 3 is illustrated in Fig. 6(b).
The sum of all ordinates in the latter graph represents the total contribution of the road
profile to the normalized bending moment (see Eq. (3)). It can be seen that bending and
hence dynamic amplification is highly sensitive to the location of road surface
irregularities. The corresponding graph for a speed of 120 km/h, illustrated in Fig. 6(c),
shows the sensitivity to speed.

### 3. MONTE CARLO SIMULATION

It has been shown that dynamic amplification and the estimate of dynamic amplification
provided by \( DAE \) for a given vehicle, is strongly influenced by the inter-axle spacing
and speed of that vehicle. Similar variability in results can be shown for other vehicle
properties such as the ratio of the axle weights. A \( BRI \) does not need to be accurate but
must provide an indication of DAF for the range of vehicles and speeds that are likely to occur on the bridge.

To overcome the problem of multiple parameters, statistical data is used to determine the properties of ‘typical’ vehicles from a truck fleet. For this paper, truck fleet statistics were obtained from a Weigh-in-Motion station on the A1 road near Ressons in France. Data was collected over a period of 105 hours for 9498 trucks. Only 2-axle trucks of more than 15 tonne gross vehicle weight were considered. The histograms for gross vehicle weight, spacing, axle weight ratio and speed are illustrated in Fig. 7.

Monte Carlo simulation [26] is used to generate 2-axle truck properties representative of the measured data. This is achieved by a simple bootstrapping from the measured data, i.e., frequency is selected by generating a random number between zero and \( \sum_i f_i \), where \( f_i \) is the frequency of interval \( i \); the property from the corresponding interval in the histogram is then used in the simulation. While there may be some correlation between vehicle properties, all properties are assumed to be independent in this study. For each of 21 values of \( x_1 \) in the interval \( 0.4L \leq x_1 \leq 0.8L \), 500 sets of property values were generated and the mean and standard deviation for \( M_0(x_1) \) calculated. The results are illustrated in Fig. 8(a) and Fig. 8(c). Fig. 8(a) corresponds to some extent to a truck fleet equivalent of Fig. 5(a) – it represents the mean normalized moment on a smooth road profile for 500 trucks with speeds, axle spacings etc. that are typical of the fleet. However, while Fig. 5(a) provides the maximum normalized moment for a range of speeds and front axle at midspan, Fig. 8(a) provides the vehicle at all points, averaged over many speeds and other vehicle parameters. Hence it gives an indication of the
mean dynamic amplification that might be expected of 2-axle vehicles, if the road profile were smooth. For example, when the front axle is located at the center of the bridge \((x_1 = 0.5L)\), the mean normalized midspan moment, \(\bar{M}(0.5L)\), is 0.8501, which means that the mean midspan moment from the 500 trucks/speeds considered is 0.8501 of the static value.

Fig. 8(b) is in some sense the truck fleet equivalent of Fig. 5(b). It gives the mean moment due to a unit ramp only at a given point from a representative sample of 500 trucks/speeds for a range \(x_1\) of vehicle locations. Figs. 8(c) and (d) give the corresponding standard deviations of normalized bending moment: \(M_{0SD}(x_i)\) and 
\([M_s(x_i) - M_0(x_i)]^{SD}\) respectively.

Fig. 8 is used to evaluate the Bridge Roughness Index, \(BRI\), for a given vehicle class, defined as:

\[
BRI = \max_{0.4L \leq x_1 \leq 0.8L} \left[ \bar{M}(x_i) + M^{SD}(x_i) \right]
\]  
(5)

where,

\[
M_r(i, x_i) = M_s(i, x_i) - M_0(x_i)
\]  
(6a)

\[
\bar{M}(x_i) = \bar{M}_0(x_i) + \sum_{i=1}^{N} \left[ \bar{M}_r(i, x_i) \times s(i) \right]
\]  
(6b)

\[
M^{SD}(x_i) = \sqrt{\left[ M_{0SD}(x_i) \right]^2 + \sum_{i=1}^{N} \left[ M^{SD}_r(i, x_i) \right]^2 \times [s(i)]^2}
\]  
(6c)

The \(BRI\) represents a kind of mean estimate of dynamic amplification for vehicle properties and speeds representative of site measurements of key vehicle properties. Of course, considerable inaccuracy is introduced by taking the maximum of many averaged
components instead of averaging the maxima. However, this is necessary as it is only in this way that the $BRI$ can be evaluated directly from a knowledge of Fig. 8 and the road profile, $s(i)$.

4. APPLICATION AND ASSESSMENT OF $BRI$

The application of the $BRI$ is illustrated using the road profile of Fig. 3. The contribution of the different vehicles on a smooth profile can be seen to be varying from about 0.85 to 0.99 in the range of $0.5L$ to $0.75L$ as described in Fig. 8(a). The influence of the road profile roughness is illustrated in Fig. 9(a), which gives a positive contribution of up to 0.02 at the critical position of $0.68L$, and the negative contribution at the position of $0.6L$. Fig. 9(b) provides a breakdown of the contributions of the ramps that make up the profile to the mean normalized moment, namely, $\bar{M}, (i, x) \times s(i)$. It is interesting to note that for this particular road profile, irregularities in the approach and at the start of the bridge are not significant (this would of course be different if there were large irregularities due to damage at a joint). The most important contributions of the road profile to bending moment occur in the range of $0.3L$ to $0.6L$ and depend on the location of the critical moment. At the time when $x_1 = 0.68L$, the critical point for this example, the contributions are illustrated in Fig. 9(c). Taking into account the standard deviation of the normalized bending moment, the dynamic amplification in Equ. 5 is described as Fig. 9(d), where the maximum value of 1.04 at the critical position of $0.7L$ represents $BRI$. 
The accuracy of the *BRI* as a measure of profile roughness is assessed through comparison with DAF for 500,000 vehicle/speed/profile combinations. For this purpose, one thousand random road profiles are simulated with the Class ‘B’ [27, 28]. Together, the profiles represent a wide range of conditions with different positions of the ramps. For each road profile, 500 sets of vehicle properties are generated by Monte Carlo simulation. In all cases, the exact dynamic amplification, DAF, is calculated for comparison. The DAF is related to IRI in Fig. 10(a). Each point in this figure corresponds to one road profile, the mean of DAF plus the standard deviation is plotted against the IRI. It can be seen that there is no discernable correlation between IRI and DAF. (However, it is of note that better correlation is evident if a wider range of road roughnesses is considered). The coefficient of correlation for this set of Class ‘B’ profiles is 0.185. The ratio of DAF to *BRI* is illustrated in Fig. 10(b). While there is considerable scatter, there is a clear correlation and the correlation coefficient is 0.762.

The effect of road roughness is strongly influenced by the span of bridge considered. Figs. 11(a) and (b) give the mean normalized moment on a smooth profile and due to a unit ramp for a 15 m bridge respectively (equivalent of Figs. 8(a) and (b)). While Fig. 11(a) is similar to Fig. 8(a), considerable differences are evident in Figs. 8(b) and 11(b) in the magnitudes of the effects of unit ramps. The corresponding standard deviation of normalized moment caused by the 2-axle truck fleet when ignoring and taking into count the road roughness are shown in Figs. 11(c) and (d) respectively. Nevertheless, the principle of superposition still applies and the *BRI* has a good correlation with DAF as can be seen in Fig. 12.
5. DISCUSSION AND CONCLUSIONS

The concept of a bridge roughness index, BRI, is developed in this paper. It is shown that IRI is poorly correlated with dynamic amplification for roads of average roughness. While there is considerable scatter in the relationship between the proposed BRI and dynamic amplification, this index is well correlated and considerably more so than IRI.

The index is independent of individual vehicle properties such as speed and axle spacing but is a function of the vehicle fleet properties, represented by histograms (Fig. 8). For a given bridge, it is calculated from the profile information only using factors derived from the fleet histograms, represented here by Fig. 7. Once the fleet-specific factors are known, the calculation of the BRI is quite simple (Eq. (5)).

The BRI factors developed here are applicable to 2-axle vehicles over 15 tonnes and to mid-span bending moment in 25 m simply supported bridges. It is shown that the corresponding factors for a 15 m bridge are quite different. Clearly the same procedure can be used to develop similar factors for other fleets, spans and load effects. For example, it would be straightforward to develop factors for 5-axle trucks representative of Western European highway traffic. This could be repeated for a number of spans and load effects. However, this would still only represent an indicator of dynamic amplification for a single truck crossing event and would not be accurate for two or more trucks meeting or passing on the bridge. The great variety of combinations of truck speeds, properties and other parameters is such that there appears to be no simple roughness measure of a road surface that is universally applicable and has a strong correlation with dynamic amplification. Nevertheless, the BRI is valuable in that it gives insight into the contribution that road roughness makes to dynamics. The importance of
irregularity location and the sensitivity of dynamic amplification to irregularities at particular points both on and in the approach to the bridge are identified.
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Fig. 1 Schematic of the two-axle vehicle and bridge interaction system
Fig. 2 Concept of superposition of unit ramps: (a) Total moment. (b) Moment due to a smooth profile. (c) Moment due to a unit ramp at the first location. (d) Moment due to a unit ramp at the second location.
Fig. 3 Road Profile adjacent to and on the bridge (0 corresponds to the start of the bridge)
Fig. 4 Influence of axle spacing on dynamic amplification
Fig. 5 (a) Normalised moment for smooth surface and a range of speeds (front axle at mid-span). (b) Normalised moment for a range of unit ramp locations and speeds (front axle at mid-span)
Fig. 6 (a) Contribution to dynamic amplification due to a range of unit ramp locations for vehicle travelling at 80 km/h. (b) Contribution to dynamic amplification due to each road irregularity in 3(a) for vehicle at 80 km/h. (c) Contribution to dynamic amplification due to each road irregularity in 3(a) for vehicle at 90 km/h.
Fig. 7 Histograms of gross vehicle weights, axle spacings, axle weight distribution and speeds for a two-axle truck fleet.
Fig. 8 Normalized mid-span moments in a 25 m bridge due to a truck fleet: (a) Mean normalized moment for smooth surface and a range of vehicle locations. (b) Mean normalized moment for a range of vehicle and unit ramp locations. (c) Standard deviation of normalized moment for smooth surface and a range of vehicle locations. (d) Standard deviation of normalised moment for a range of vehicle and unit ramp locations.
Fig. 9 (a) Mean contribution to dynamic amplification due to road profile in Fig. 3 for a range of vehicle locations. (b) Mean contribution to dynamic amplification due to each road irregularity for a range of vehicle locations. (c) Mean contribution to dynamic amplification due to each road irregularity when front axle is at 0.68L. (d) Total mean plus the standard deviation normalised moment due to the road profile.
Fig. 10 (a) DAF versus IRI for a 25 m bridge. (b) DAF versus BRI for a 25 m bridge
Fig. 11 Normalized mid-span moments in a 15 m bridge due to a truck fleet: (a) Mean normalized moment for smooth surface and a range of vehicle locations. (b) Mean normalized moment for a range of vehicle and unit ramp locations. (c) Standard deviation of normalized moment for smooth surface and a range of vehicle locations. (d) Standard deviation of normalised moment for a range of vehicle and unit ramp locations.
Fig. 12 (a) DAF versus IRI for a 15 m bridge. (b) DAF versus BRI for a 15 m bridge
\[ y_1(t) = m_1 \]
Fig. 2
Fig. 3
Fig. 4
Fig. 5

(a) 

(b) 

Fig. 5
Fig. 6
Fig. 7
Fig. 8
Dynamic Amplification

Position of ramp / Length of bridge

(a)

(b)
Fig. 9

(c) Dynamic Amplification vs. Position of ramp / Length of bridge

(d) Dynamic Amplification vs. $x_f$
Fig. 10

(a) 

(b)

Fig. 10
Figure 11

(c) Position of ramp / Length of bridge

(d) Fig. 11
Fig. 12

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### Table 1. Vehicle and Beam Properties.

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