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The influence of pre-existing vibrations on the dynamic response of medium span bridges

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Abstract: Critical static bridge loading scenarios are often expressed in terms of the number of vehicles that are present on the bridge at the time of occurrence of maximum lifetime load effect. For example, 1-truck, 2-truck, 3-truck or 4-truck events usually govern the critical static loading cases in short and medium span bridges. However, the dynamic increment of load effect associated with these maximum static events may be assessed inaccurately if it is calculated in isolation of the rest of the traffic flow. In other words, a heavy vehicle preceding a critical loading case causes the bridge initial conditions of displacement and acceleration to be non zero when the critical combination of traffic arrives on the bridge. Failure to consider these pre-existing vibrations will result in inaccurate estimation of dynamic amplification. This paper explores these dynamic effects and, using statistical analyses outlines the relative importance of pre-existing vibrations in the assessment of total traffic load effects.

Key words: bridge, dynamic, free vibration, traffic loading.

Résumé:

Mots clés:

1. Introduction

Accurate prediction of the behaviour of highway bridges under heavy loading is central to modern design techniques, and, more importantly, in the assessment of existing infrastructure. Load effects for typical medium-span highway bridges are known to be governed by free-flowing traffic (Bruls et al. 1996), when the dynamic increment due to bridge vibration is considered. Conversely, for longer bridges, the greatest load effect is generally the result of congested traffic. For static loading on bridges the defining variable is the gap or headway between vehicles (OBrien and Caprani 2005).

The worst static loading scenarios can be classified in terms of the number of vehicles that are present on the bridge at the time of occurrence of maximum load effect. Caprani et al. (2006) show that the statistical analysis of bridge traffic load effects requires the classification of load effects into statistically similar groupings based on the number of vehicles contributing to maximum load effect, for example 2-truck events, 3-truck events, etc.. The critical static loading events are commonly obtained using Monte Carlo simulation in conjunction with measured Weigh in Motion (WIM) data (Moses 2001; Nowak 1993; Nowak and Hong 1991; O'Connor and OBrien 2005). Once the worst static case is known, the total traffic load can be estimated through the application of a Dynamic Amplification Factor (DAF), to allow for the dynamic component contained in the bridge response that results from the vehicle-bridge interaction (Chatterjee 1991; Dawe 2003; Kirkegaard et al 1997).

Many researchers have assessed the influence of bridge and vehicle dependent dynamic parameters such as vehicle velocity, road profile, suspension, tyre stiffness on the total bridge response (Brady and OBrien 2005; Brady et al. 2006; Chompooming and Yener 1995; DIVINE 1997; Gonzalez et al. 2003; Green and Cebon 1994, 1995; Li 2006; Paultre et al. 1992; Rattigan et al. 2005). Nevertheless, the vibratory condition of the bridge prior to

loading is a consideration that, in the calculation of resultant dynamic amplification, has not received sufficient attention. Clearly, a bridge which is in free vibration due to a previously applied vehicle load cannot be expected to respond in the same way as a stationary bridge would. The response of a bridge under consideration to a critical truck event therefore can be expected to vary depending on the characteristics and proximity of the preceding vehicle.

The purpose of the study is to provide a preliminary investigation of the effects of pre-existing vibrations using numerical models. Simplified models based on constant forces are used to outline the problem first. Then, sprung vehicle models are used on a simple beam to investigate the effect of pre-existing vibration for some critical loading scenarios. The investigation is restricted to a 25 m-long simple supported beam model and a typical heavy 5-axle truck configuration. A comparison is made between the dynamic response when pre-existing vibrations are neglected and when they are considered, allowing the importance of pre-existing bridge vibrations to be shown.

2. Formulation of the problem using a simplified P-load model

Each vehicle is first modelled as a single point load (P-load model) and the bridge is modelled as a simply supported beam. This simple P-load model is introduced to develop an understanding of the problem, and to idealise those bridge vibrations that may exist as a critical vehicle arrives on a chosen structure. As defined by, amongst others, Clough and Penzien (1993), the magnitude of displacement in free vibration of a beam, at its fundamental frequency, at a section location x and time t , $v(x,t)$, can be represented as

$$[1] \quad v(x,t) = [v(x,0) \cos \omega_b t + (\frac{\dot{v}(x,0) + v(x,0)\xi\omega_b}{\omega_b}) \sin \omega_b t] \exp(-\xi\omega_b t)$$

where $v(x,0)$ and $\dot{v}(x,0)$ are initial displacement and velocity at the time when free vibration starts ($t = 0$). ξ is the damping ratio and represents the level of bridge damping as a ratio of critical damping. ω_b represents the circular frequency of the damped system, and is defined as:

$$[2] \quad \omega_b = \omega_1 \sqrt{1 - \xi^2}$$

in which ω_1 is the first natural circular frequency of the beam. Typical values of ξ for concrete bridges are within the range $[0.003, 0.03]$ (Cantieni 1983), while ω_1 can be approximated as $(2\pi L/100)$, where L is bridge length in metres (DIVINE 1997; Heywood et al. 2001).

The response of a simply supported beam, with initial conditions ' $v(x,0) = 0$ ' and ' $\dot{v}(x,0) = 0$ ', when traversed by a single P-load is assessed first. Then, the corresponding case where the bridge has been excited by a previous P-load of magnitude P_1 will be investigated.

2.1. Response of a stationary beam to a single P-load

The solution for the total response of an Euler-Bernoulli beam of length L subjected to a P-load travelling at a velocity of c is provided by Frýba (1999). The beam deflection at point x and time t , $v(x,t)$ is given by:

$$[3] \quad v(x,t) = v_0 \sum_{j=1}^{\infty} \frac{\sin\left(\frac{j\pi x}{L}\right)}{j^2 \left[j^2 (j^2 - \alpha^2)^2 + 4\alpha^2 \beta^2 \right]}$$

$$\times \left\{ \begin{aligned} & j^2 (j^2 - \alpha^2) \sin(j\Omega t) \\ & - j\alpha [j^2 (j^2 - \alpha^2) - 2\beta^2] e^{-\omega_b t} \sin(\omega'_j t) \\ & - 2j\alpha\beta [\cos(j\Omega t) - e^{-\omega_b t} \cos(\omega'_j t)] \end{aligned} \right\}$$

where v_0 is deflection at midspan of a simply supported beam loaded at midspan by a static

force P ($v_0 = \frac{PL^3}{48EI}$ with E and I being modulus of elasticity and second moment of area,

respectively), j is mode shape number, α is a velocity parameter, β is a damping parameter, Ω

is the circular frequency of the moving load and ω'_j is the j^{th} circular damped frequency of

the beam. These parameters are defined in the following equations:

$$[4] \quad \Omega = \pi c / L$$

$$[5] \quad \alpha = \frac{\pi c / L}{\omega_1}$$

$$[6] \quad \beta = \frac{\zeta}{\sqrt{1 - \zeta^2}}$$

$$[7] \quad \omega'_j = \sqrt{\omega_j^2 - \omega_b^2}$$

in which ω_j is the circular natural frequency of the j^{th} mode of vibration of the beam.

Equation [3] can be differentiated twice with respect to x to relate it to the bending moment at beam section x and time t , $M(x,t)$, as follows:

$$[8] \quad M(x,t) = M_0 \sum_{j=1}^{\infty} \frac{\frac{8j^2}{\pi^2} \sin\left(\frac{j\pi x}{L}\right)}{j^2 [j^2 (j^2 - \alpha^2)^2 + 4\alpha^2 \beta^2]}$$

$$\times \left\{ \begin{aligned} & j^2(j^2 - \alpha^2) \sin(j\Omega t) - \frac{j\alpha[j^2(j^2 - \alpha^2) - 2\beta^2]}{\sqrt{j^4 - \beta^2}} e^{-\omega_b t} \sin(\omega'_t t) \\ & - 2j\alpha\beta[\cos(j\Omega t) - e^{-\omega_b t} \cos(\omega'_t t)] \end{aligned} \right\}$$

where M_0 is the midspan bending moment of a simply supported beam loaded at midspan by P ($M_0 = PL/4$).

The DAF(dynamic amplification factor) is defined here as the ratio of the maximum total moment to the maximum static moment over the period under consideration. Figure 1 shows the variation in the DAF with velocity for a 25m long bridge with 3% damping and a first natural frequency of 3.97 Hz ($E = 3.6 \times 10^{10}$ N/m², $P = 100$ kN, $I = 1.39$ m⁴ and beam mass per unit length $\mu = 20000$ kg/m). These parameters are representative of a typical medium span concrete bridge (Li 2006), with moderate to high damping, and first natural frequency in agreement with field measurements for a bridge of similar span (Heywood et al. 2001). From Figure 1 it can be seen that vehicle velocity significantly influences DAF, and may result in an increase, or a decrease in the maximum static load effect experienced by a bridge under dynamic vehicle loading. For critical speeds of about 30 m/s, maximum DAF values close to 1.1 can be reached. In practise, for two-lane bridges and bending moment, the Eurocode proposes a DAF value that goes from 1.3 for very short span bridges to 1.1 for bridges of 50 m and above (Dawe 2003). These recommended DAF values are relatively close to the ones derived for a simple P-load model. The main discrepancy lies in short-span bridges where a higher dynamic allowance is necessary to accommodate the stronger influence of the road profile.

2.2. Response of a beam with pre-existing free vibration to single P-load

To analyse the effect of pre-existing vibrations, the bridge is first traversed by a load P_1 and then by a load P_2 . The total bridge response due to the moving load P_2 will be

influenced by the magnitude of the pre-existing bridge vibrations (initial bridge displacements, velocities and accelerations when P_2 enters the bridge) induced by the load P_1 . A schematic of the problem is illustrated in Fig. 2. The two moving loads move at the same velocity c , and are separated by a distance D . For this preliminary study, the magnitude of the loads P_1 and P_2 are assumed to be identical. The minimum allowable distance D is defined as L , the bridge length, in order to ensure the bridge is in free vibration as the second load arrives. It is noted that in some 1-truck events the gap between the critical vehicle and its preceding vehicle may be less than L , however this is not considered here, as free vibration does not occur prior to critical loading. Thus the maximum static bending moment corresponds to that bending moment obtained when P_2 is located at beam midspan. The total response due to P_2 on the other hand, is dependent on the load velocity, the bridge dynamics and the level of excitation originated by P_1 . This excitation results from the damped free vibration response of the bridge, described in eq. [1].

This is a linear dynamic problem where the total response due to the moving load P_2 , is the superposition of the free-vibration response of the beam due to P_1 added to the moment response of the beam due to the load P_2 (Brady et al 2006; Chan & O'Connor 1990; Wu & Daj 1986). Thus the total moment at time t can be approximated by combining eqs. [1] and [8] and is given by eq. [9].

$$\begin{aligned}
 M(x, t) = M_0 \sum_{j=1}^{\infty} \frac{\frac{8j^2}{\pi^2} \sin\left(\frac{j\pi x}{L}\right)}{j^2[j^2(j^2 - \alpha^2)^2 + 4\alpha^2\beta^2]} \\
 \times \left\{ \begin{aligned} & j^2(j^2 - \alpha^2) \sin(j\Omega t) - \frac{j\alpha[j^2(j^2 - \alpha^2) - 2\beta^2]}{\sqrt{j^4 - \beta^2}} e^{-\omega_b t} \sin(\omega'_t t) \\ & - 2j\alpha\beta[\cos(j\Omega t) - e^{-\omega_b t} \cos(\omega'_t t)] \end{aligned} \right\} \\
 + \left\{ M(x, -t_0) \cos \omega_b(t + t_0) + \frac{\dot{M}(x, -t_0) + M(x, -t_0)\xi\omega_1}{\omega_b} \sin \omega_b(t + t_0) \right\} e^{-\xi\omega_1(t+t_0)}
 \end{aligned}
 \tag{9}$$

Equation 9 considers the superposition of the free-vibration response of the beam at its fundamental frequency added to moment response of the beam due to the load P_2 , where $M(x, -t_0)$ and $\dot{M}(x, -t_0)$ are the magnitude and rate of change of bending moment, respectively, t_0 seconds before the load P_1 begins to cross the bridge, and t_0 is the time difference between the first load leaving the bridge and the second load entering the bridge, which is equal to $(D-L)/c$.

Figure 3 shows the response of a beam of length 25 m, with pre-induced vibrations, to a P-load of 100 kN travelling at 30 m/s. The initial bridge vibrations occur due to an identical P-load travelling 30 m ahead of the following load. In this case, a DAF of 1.16 is obtained for the case where the existing bridge vibrations are considered. For the case of no pre-existing vibrations the DAF is 1.085 (Fig. 1). In this case the load travelling 30 m ahead of the following load has the effect of approximately doubling the dynamic increment.

Figure 4 presents a sample of responses to a range of cases, and illustrates the possibility for both increase and reduction of DAF. The influence of pre-existing vibrations on DAF is clearly seen from Figs. 4a and 4b. The reason for the peaks in bending moment is that the peaks of the bending moment due to the moving force and that due to the waveform of the bridge vibration superpose in some particular cases of vehicle speeds and gaps. I.e., in Fig. 4a there is no matching of peaks, and the DAF is 0.903 and 0.960 if neglecting or allowing pre-existing vibrations respectively. However, the combination of vehicle speed and gap of Fig. 4b leads to a matching of peaks and a DAF value of 1.080 without pre-existing vibrations. When pre-existing vibrations are considered, this DAF rises up to 1.115. Significantly, in Fig. 4c, the DAF due to two P-loads, crossing at a velocity of 30 m/s, and separated by a gap of 29.4 m, is 1.004, whereas for the same loads, travelling at the same velocities, but separated by a gap of 33 m, a DAF of 1.158 is recorded, as shown in Fig. 4d. In other words, a small change in gap can result in a significant change in DAF.

2.3. Maximum and minimum DAF in a vibrating beam subjected to a P-load model

It is apparent that, for each combination of P-loads travelling at the same velocity, there exists an optimum gap between loads that results in minimum DAF, and a critical gap that results in maximum DAF. This effect is assessed by considering all possible gaps between loads for each velocity within the range (10-50 m/s). For each velocity, the distance between loads, D , is varied in increments of 0.05 m between a minimum value given by the bridge length and a maximum value given by the product of the vehicle speed and the time, t_f , by which free vibrations have decayed considerably (by approximately 95%) and are deemed to have become negligible. The value of t_f is dependent on the damping characteristics of the beam (for 3% damping, t_f is approximately 5 seconds). If $t_0 > t_f$, the problem reverts to that of section 2.1. It is noted that the maximum and minimum influence on response due to pre-existing bridge vibrations generally occurs when the second load reaches the bridge within the first cycle of free vibration of the beam after the first axle has left.

Figure 5 illustrates the maximum and minimum DAF values when pre-existing bridge vibrations are considered (DAF(2P)), and compares them to the case of a single P-load (DAF(1P)). Similar patterns are apparent for the three DAF curves, with similar values for the critical velocities. For example velocities of between 12 and 14 m/s, 16 and 19 m/s, and 23 and 34 m/s result in highest levels of DAF. However it can be seen that the maximum DAF occurring for 2P loading is significantly higher than that occurring for a single P-load. This proves the theory that ignoring the possible pre-existing bridge vibrations may lead to an incorrect estimation of bridge DAF. On the one hand, initial bridge conditions can result in significant increases in the overall DAF. For the lower range of velocities (< 25 m/s), maximum DAF has been found to occur for some low value of gap (25-30 m), while for higher velocities larger gaps may become critical. On the other hand, it is possible to find a

gap at all velocities (e.g., below 40 m/s) that will enable the pre-existing vibrations to reduce the dynamic increment of loading. If further proven this finding may have implications for the development of intelligent bridge management systems, allowing for an increase in bridge allowable load limits.

The importance of the pre-existing vibrations is also a function of the level of damping of the bridge under study. Figure 6 shows the influence of different levels of bridge damping on the maximum and minimum DAFs obtained from varying the gap. As expected, maximum or minimum DAF are quite sensitive to damping and the curves of maximum DAF are closer to the single P-load case for higher levels of damping.

3. Influence of pre-existing free vibration on articulated sprung vehicle models

This section extends the investigation to more realistic heavy vehicle models. Numerical models are used to simulate the crossing of two articulated 5-axle vehicles, one following the other, to assess the relative importance of the preceding truck on the response due to the following truck. A schematic of the problem is shown in Fig. 7. The vehicle model consists of a tractor supported by two suspension systems and a semi-trailer supported by the tractor and a tridem suspension. The bridge is the same 25 m simply supported beam model with 3% damping defined in section 2.1.

3.1. Vehicle-bridge interaction model

The vehicle-bridge interaction model utilised follows the fundamental approach presented by Hwang and Nowak (1991), in which vehicle suspension forces and bridge displacements are first assumed (based on previous time-step) and then updated iteratively to satisfy equations of both bridge and truck motion. The motion of the beam due to two 5-axle trucks can be represented by the following equation (Li 2006):

$$\begin{aligned}
EI \frac{\partial^4 v(x,t)}{\partial x^4} + \mu \frac{\partial^2 v(x,t)}{\partial t^2} + \mu \omega_b \frac{\partial v(x,t)}{\partial t} &= \sum_{i=1}^5 \mathcal{E}_i^1 \partial(x-x_i^1) [R_i^1(t) + P_i^1] \\
&+ \sum_{i=1}^5 \mathcal{E}_i^2 \partial(x-x_i^2) [R_i^2(t) + P_i^2]
\end{aligned}
\tag{10}$$

The left side of eq. [10] represents the motion of the beam, while the right side of the equation represents the moving load and interaction forces. In this, P_k^i is the static weight of the i^{th} axle of the k^{th} vehicle, x_i^k defines the position on the bridge of the i^{th} axle of the k^{th} vehicle, and $R_i^k(t)$ represents the dynamic tyre force imparted to the bridge by i^{th} axle of the k^{th} vehicle. In addition, $\mathcal{E}_i^k$ is the dirac function corresponding to the i^{th} axle of the k^{th} vehicle, that can be 1 when the axle is on the bridge or 0 otherwise. This equation can be solved using the Runge-Kutta-Nyström method, as shown by Li (2006). The total dimensionless bending moment is given by:

$$M(\chi, \tau) = \sum_{k=1}^2 \sum_{i=1}^5 M_{R_i^k}(\chi, \tau) + M_\mu(\chi, \tau)
\tag{11}$$

where χ is the normalised beam location given by x/L , while τ is normalised time given by t/L .

Therefore the instantaneous vehicle forces applied to the beam, $M_{R_i^k}(\chi, \tau)$ is:

$$M_{R_i^k}(\chi, \tau) = \begin{cases} 4\mathcal{E}_i^k R_i^k \chi_i^k (1-\chi) & \text{for } \chi_i^k \leq \chi \\ 4\mathcal{E}_i^k R_i^k \chi (1-\chi_i^k) & \text{for } \chi_i^k > \chi \end{cases} \quad i=1,2,3,4,5 \quad k=1,2
\tag{12}$$

and the response due to the inertial forces of the beam, $M_\mu(\chi, \tau)$ is:

$$M_\mu(\chi, \tau) = - \sum_{j=1}^{\infty} \frac{8l^2 \mu}{j^2 \pi^6 EI} \ddot{q}_j(\tau) \sin(j\pi\chi)
\tag{13}$$

where $\ddot{q}_j(\tau)$ is the acceleration of the generalised modal coordinate of the beam.

A measured road profile is input in the model to better represent a realistic vehicle – bridge interaction model. This road profile was taken from a section of main road at a bridge over the river Sava in Slovenia, and has an IRI of 2.69 m/km, assessed using ProVAL software (Federal Highway Administration 2007). Table 1 gives the parameters of the 5-axle articulated vehicle used in the simulations. Simulations incorporate an approach length of 100 m and terminate once the following truck has fully departed the bridge. The time increment used for simulations varies based on vehicle velocity, but does not exceed 0.01 seconds.

3.2. Analysis of the influence of a pre-existing bridge vibrations on DAF due to an articulated 5-axle truck

Both the critical vehicle (2nd truck) and the preceding vehicle (1st truck) cross the bridge at the same velocity c . This assumption is reasonable since heavy vehicles following each other at a relatively close gap will typically have a similar velocity within the traffic flow. Varying vehicle velocity introduces a large number of further questions regarding the importance of a preceding vehicle's velocity, the significance of the variable gap between vehicles, etc., and is therefore not considered here. The second truck has been defined with greater mass, since it is the total bridge response to this truck-crossing that is under investigation, and the dynamic amplification associated with the crossing of a 'critical' vehicle that is being sought. The gap is defined as the time or distance between the rear axle of the preceding vehicle and the front axle of the following vehicle, and is denoted D in Fig. 7. For gaps over t_f seconds, the influence of the vehicle causing the pre-existing vibrations on the total response is considered to be negligible, as free-vibrations will have damped out considerably.

The vehicle length, axle spacings and the distribution of gross vehicle weight between axles are kept constant for the study and are based on mean values for a 'typical' European

route as described by Grave (2001). Other typical vehicle dynamic parameters are fixed.

Table 1 contains the values employed in the simulations.

Figure 8 shows DAF as a function of vehicle velocity and gap. The DAF in Fig. 8(a) corresponds to the total response obtained by single 5-axle vehicle of 60 000 kg with no allowance for pre-existing bridge vibrations. The gap has no relevance for this case. The DAF in Fig. 8(b) corresponds to the total response obtained by a 5-axle vehicle of 60 000 kg preceded by a 5-axle vehicle of 30 000 kg travelling at the same speed. As the gap between vehicles increases the influence of the preceding vehicle diminishes, since the free-vibration of the bridge is damped out over time. The variation in the peaks and valleys of DAF with gap in Fig. 8(b), are a result of the beam free vibration due to the preceding vehicle, which oscillates between positive and negative displacement in the form of a damped sinusoidal function. As shown in section 2.3, there exists the potential for DAF to be both positively and negatively influenced by pre-existing free vibrations.

Figure 9 compares the DAF obtained for the crossing of a single 5-axle truck of 60 000 kg (DAF(1 Truck)) with the maximum and minimum DAF resulting when the pre existing vehicle is considered (DAF(2 Truck)). The gaps causing the maximum and minimum DAF can be extracted from Fig. 8. For example, for this particular traffic event and bridge, the maximum possible DAF for a 60 tonne vehicle, travelling at 22 m/s is approximately 1.08 without consideration of pre-existing vibrations, but can reach approximately 1.1 when pre-existing bridge vibrations are considered. While this represents a 25% increase in the dynamic component, the absolute difference is quite small.

Although the results indicated here are dependent on a large number of bridge specific properties such as span length, bridge frequency, road profile, etc., it is reasonable to assume that similar trends are likely to be found for other bridges subject to excitation by traffic

loading. The assessment of the level of significance of each of these individual properties is however beyond the scope of this paper.

4. Statistical evaluation of the importance of a preceding vehicle

The gap between vehicles and the vehicle velocity have been chosen as variables for the simulation described below. For site-specific free flowing traffic, statistical information on the truck population is available in the form of WIM data obtained from an existing WIM site, on the A12 south, in the Netherlands. The data set consists of vehicle data recorded over 19 weeks (12 consecutive weeks from January 2004, and 7 consecutive weeks from November 2004), and contains over 200,000 articulated 5-axle vehicles (2-axle tractor + 3-axle semi-trailer type). Through analysis of this data, it will be possible to attach probability of occurrence to the velocity and gap variables of each vehicle-crossing event, and hence the overall importance that the presence of a preceding vehicle may have on the dynamic amplification resulting from a single vehicle event.

4.1. Determination of the distribution of design variables

The velocity histogram, from WIM data, was fitted to a weighted tri-modal normal distribution, as illustrated in Fig. 10a. This type of fit incorporates both major peaks of the distribution (23 m/s and 24.5 m/s), and allows for the significant frequencies of occurrence of velocities between 10 m/s and 20 m/s. For critical bridge load evaluation, it is the gap between heavy trucks that is of importance, the statistical distribution of which is dependent on the flow of heavy vehicles. However, OBrien and Caprani (2005) found, using data from a range of European WIM sites, that for headways of less than 1.5 seconds, the correlation between hourly flow and headway was weak. Hence, for headways of less than 1.5 seconds, it is reasonable to assume a distribution of headway that is independent of flow. OBrien and

Caprani (2005) noted that for headways between 1.5 and 4 seconds, there was a correlation between headway and flow, and that this correlation was dependent on average hourly flow (AHF). An AHF of 100 trucks/hour is chosen here, consistent with the mean AHF from the sampled WIM data. The total cumulative distribution function (CDF) for headway is represented in Fig. 10b, and is based on the equations given by OBrien and Caprani (2005) for similar AHF ranges. For headways of greater than 4 seconds a uniform distribution is chosen.

4.2. Determination of the distribution of DAF

Since the correlation between velocity and gap is deemed to be negligible, the probability of occurrence of a truck of velocity c following a preceding vehicle at a gap of D , P_{cD} , is obtained by multiplying the probability of a truck of velocity c , P_c , by the probability of a gap D between two trucks, P_D . Therefore, P_{cD} is defined as being the frequency surface where

$$[14] \quad P_{cD} = P_c P_D$$

The DAF associated to two trucks following each other, DAF_{cD} , is a function of both initial design variables; in addition, the probability of occurrence of a DAF of value DAF_0 is the sum of all probabilities of all combinations of c and D that result in DAF_0 as shown by eq. [15]:

$$[15] \quad P(DAF_0) = \int_0^\infty \int_0^\infty [P_{cD} \mid DAF_{cD} = DAF_0] dc dD$$

For the data presented in Fig. 8, the probability of occurrence of each combination of velocity and gap, and thus the corresponding probability of occurrence of each value of DAF contained in the design space, were calculated and are presented in Fig. 11. This figure shows the variation in the CDF of DAF when: (a) no pre-existing vibrations are considered; and (b) when the presence of a preceding vehicle is considered. The distributions are significantly different near the upper tails of the distribution (high DAF). While a single vehicle (60 tonne) analysis yields a maximum DAF of 1.08, the inclusion of a preceding vehicle (60 tonne following 30 tonne) can result in DAFs up to 1.125 with probability 0.1% (1 in 1000). This represents a 56% increase in DAF allowance for this particular traffic event. This example illustrates that greater than anticipated maximum values can happen due to the presence of pre-existing vehicles. In extreme value statistics it is reasonable to assume that such extreme values will occur at least once in the remaining life of the bridge.

4.3. Influence of pre-existing vibrations on DAF for different road conditions

Simulations are carried out to evaluate differences in dynamic response when considering single vehicle and vehicle following events for different road conditions. It is assumed that a 60 tonne vehicle represents the half-daily maximum and that each year consists of 250 days of normal traffic flow. The 1 in 75 year DAF values (a probability of occurrence of $\frac{1}{2 \times 250 \times 75} = 2.67 \times 10^{-5}$, which leads to a $CDF = 1 - 2.67 \times 10^{-5} = 0.99997$) are shown in Figure 12, with and without pre-existing vibrations, for 20 road profiles classified according to IRI (Park et al. 2005). Results vary considerably depending on the roughness and location of the road irregularities, with the highest relative increase in the dynamic component of DAF being a change from 1.105 to 1.255.

Figure 13 compares the percentage increment in the dynamic component of DAF when allowing for pre-existing vibrations, for 75-year and 1000-year return periods. For a

number of profiles, the influence of a preceding vehicle on DAF is very similar for both return periods.

DAFs specified in codes are usually the result of extensive field testing, where the maximum responses are sometimes measured under free flowing traffic and therefore account for pre-existing vibrations. However, the component of the dynamic increment attributable to pre-existing vibrations is unknown. For the assessment of traffic loads on a real bridge, characteristic static load effects due to critical events can be obtained using WIM data and traffic simulations (Nowak 1993; Nowak and Hong 1991). Then, the required dynamic allowance can be estimated through accurate dynamic modelling of those critical loading cases. Although the investigations in this paper have been limited by a number of assumptions, they have revealed the significance of considering pre-existing vibrations (and hence, vehicle speed and gaps) as part of this modelling.

5. Conclusions

This paper has highlighted the importance of bridge initial conditions prior to critical traffic loading. A simple model was used to identify the key parameters influencing the possible over-estimation or under-estimation of DAF in bridge loading. For a particular bridge, it has been seen that a preceding load may have a positive or negative influence on the total response of the following load depending on bridge damping, inter-load gap and velocity. There exist critical combinations of velocity and gap between loads that result in significant increases in DAF. Conversely there exist combinations which can mitigate the level of dynamics, and reduce the total load effect. Damping reduces the effect of pre-existing free vibration significantly and highly damped bridges are less affected by pre-existing loading.

Using WIM data, the significance of a vehicle preceding a critical truck loading event was analysed statistically. A 5-axle articulated sprung vehicle model with 8 independent degrees of freedom was employed in the simulations. Contour plots of DAF versus velocity and gap reveal the presence of peaks (combinations of velocity and gap causing maximum DAFs) and valleys. These contour plots were integrated with distributions of gap and velocity from WIM data to derive two cumulative distribution functions of DAF for a number of road profiles: one where the presence of pre-existing vibrations is ignored and one where it is included. A comparison between these distributions shows that, at significant levels of probability, an underestimation of DAF may occur when pre-existing vibrations are not considered.

Although further validation of the findings presented here is necessary it is concluded that calculation of dynamic response of bridges should address the issue of trucks preceding the critical loading events. The potential for dynamic amplification to be both increased and reduced depending on bridge initial condition may have significant implications in future bridge traffic load assessment. By implementation of intelligent bridge management systems a bridge's load carrying capacity may be significantly increased, by ensuring an optimal spacing between consecutive heavy vehicles.

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List of symbols

c	velocity of the moving load
CDF	cumulative distribution function
D	distance between two loads or gap
DAF	dynamic amplification factor
E	modulus of elasticity of the beam
i	axle number
I	second moment of area of the beam
j	mode number
k	vehicle number
L	bridge length
$M(x,t)$	bending moment at beam section x and time t
$M(\chi, \tau)$	dimensionless bending moment at normalised position χ and time τ
$M_{R_i^k}(\chi, \tau)$	dimensionless bending moment due to the dynamic tyre forces
$M_\mu(\chi, \tau)$	dimensionless bending moment due to the bridge vibration
M_0	midspan bending moment of a simply supported beam loaded at midspan by P
P	constant force
P_c	probability of occurrence of a truck of velocity c
P_{cD}	probability of occurrence of a truck of velocity c , following a preceding vehicle at a gap of D

P_D	probability of occurrence of gap D
P_k^i	static weight of the i^{th} axle of the k^{th} vehicle,
$P(DAF_0)$	probability of occurrence of a DAF of value DAF_0
$\ddot{q}_j(\tau)$	second derivative of the generalised modal coordinate of the beam at normalised time τ
$R_i^k(t)$	dynamic tyre force imparted to the bridge by i^{th} axle of the k^{th} vehicle
t	time
t_0	time difference between a first vehicle leaving the bridge and a second vehicle entering the bridge
$v(x, t)$	instantaneous deflection of the beam at section x and time t
$\dot{v}(x, 0)$	instantaneous velocity of the beam at section x and time t
v_0	static deflection at midspan of a simply supported beam loaded at midspan by P
x	position
x_i^k	the position on the bridge of the i^{th} axle of the k^{th} vehicle
Ω	circular frequency of the moving load
α	velocity parameter
β	damping parameter
χ	normalised position of a section on the bridge
χ_i^k	normalised position of the i^{th} axle of the k^{th} vehicle
$\mathcal{E}_i^k$	dirac function corresponding to the i^{th} axle of the k^{th} vehicle. The value of this function is 1 when the axle is on the bridge (otherwise zero)

ξ damping ratio

τ normalised time

μ mass per unit length of the beam

ω_b vibration circular frequency of a damped beam

ω_j circular natural frequency of the j^{th} mode of vibration of the beam

ω_j' circular damped frequency of the j^{th} mode of vibration of the beam

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Table 1. Configuration of 5-axle vehicle model

(a)

(b)

(c)

(d)

(a)

(b)

(a)

(b)

(a)

(b)

Axle No.	% of total axle and body mass	Axle spacing (m)	Suspension stiffness ($\times 10^3$ N/m)	Tire stiffness ($\times 10^3$ N/m)	Suspension damping (Ns/m)	Axle mass (kg)
1	12.7	—	1800	1000	5000	700
2	27.7	3.1	300	2000	5000	1100
3	19.86	5.1	1800	2000	5000	750
4	19.87	1.1	1800	2000	5000	750
5	19.87	1.1	1800	2000	5000	750