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Student Incentives and Diversity in College Admissions

Ivan Pastine, University College Dublin and
Tuvana Pastine, National University of Ireland Maynooth

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Student Incentives and Diversity in College Admissions

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Ivan Pastine
University College Dublin
Tel: (353) 1 716 -8395
Fax: (353) 1 283-0068
Ivan.Pastine@ucd.ie

Tuvana Pastine
National University of Ireland Maynooth
Tel: (353) 1 708-6421
Fax: (353) 1 708-3934
Tuvana.Pastine@nuim.ie

Abstract

This paper examines student incentives when faced with a college admissions policy which pursues student body diversity. The effect of a diversify-conscious admissions policy critically depends on the design of the policy. If the admissions policy fails to incentivize students from a disadvantaged socioeconomic background it may lead to a deterioration in the intergroup score gap while failing to improve student body diversity in equilibrium.

Keywords: Affirmative Action, College Admissions, All-Pay Auction, Contest, Tournament

JEL: H0, J7

Corresponding Author:
Tuvana Pastine, Economics Department, National University of Ireland Maynooth, Maynooth, County Kildare, Ireland. Email: Tuvana.Pastine@nuim.ie

> “. . . the very existence of racial discrimination of the type practised by the Law School may impede the narrowing of the LSAT testing gap. [..] Whites scoring between 163 and 167 on the LSAT are routinely rejected by the Law School, and thus whites aspiring to admission at the Law School have every incentive to improve their score to levels above that range. [..] Blacks, on the other hand, are nearly guaranteed admission if they score above 155.[..] there is no incentive for the black applicant to continue to prepare for the LSAT once he is reasonably assured of achieving the requisite score.”
>
> Dissenting Opinion of Justice Thomas
> Grutter v. Bollinger (2003)

I. Introduction

In Grutter v. Bollinger (2003), the Supreme Court approved the use of race as a factor in the admissions decisions of the Law School of the University of Michigan in order to obtain educational benefits from a diverse student body. In practice this landmark ruling leaves the door open for a holistic background-sighted admissions policy allowing a less academically qualified candidate to be preferred in order to achieve diversity.

Opponents of diversify-conscious admissions policies often claim that students subject to preferential treatment have less incentive to put in effort to achieve higher academic quality. The students are therefore more likely to be polarized in quality based on their group affiliation. In the job market this may aggravate statistical discrimination. Justice Thomas argues that such Phelps (1972) style discrimination is likely to be exacerbated by affirmative action in the admissions process “either racial discrimination did play a role, in which case the person may be deemed ‘otherwise unqualified,’ or it did not, in which case asking the question itself unfairly marks those blacks who would succeed without discrimination.”[1]

[1] See the opinion of Justice Thomas in Grutter v. Bollinger (02-241) 539 U.S. 306 (2003).

However in an all-pay auction framework Fu (2006) finds that affirmative action creates positive cross-group interaction between applicants. It helps level the playing field inducing candidates from both disadvantaged and advantaged backgrounds to invest more heavily in educational attainment. Even if the college is purely academic-quality oriented, the optimal admissions policy gives preferential treatment to the applicant from a disadvantaged background.[2] Furthermore the college's objective of academic excellence is not in conflict with diversity. Diversity is simply a byproduct of the optimal admissions policy of a purely academic quality oriented college.

We also examine the effect of a preferential treatment rule on student incentives where students with diverse backgrounds compete for a scarce place. We depart from Fu (2006) in the form of the preferential treatment. Fu (2006) examines affirmative action policies with a multiplicative form: The "academic achievement score" of a candidate from an underprivileged background is multiplied by a constant greater than one. However preferential treatment policies often take, or may be perceived to take, an additive form. For example, the undergraduate admissions office of the College of Arts and Sciences at the University of Michigan added 20 points (out of 150) to the underprivileged candidate's score.[3] Espenshade, Chung and Walling (2004) estimates that in ten academically selective US universities on average the admissions policies implicitly add 230 extra

[2]Furstenberg (2005) has the same conclusion in a very different framework where racial identity contains relevant information about the applicant's true ability.

[3]The Supreme Court ruled out this rigid scoring practice in Gratz v. Bollinger (2003). Whether diversity conscious admissions policy is officially announced as a rigid scoring rule or simply perceived by students as an additional bonus makes no difference for the purposes of this paper.

SAT points (on a 1600 scale) to the scores of African American students.[4] Hence we analyze preferential treatment with an additive form.

We find that the exact form of the preferential treatment admissions policy matters. If an additive form is adopted, strong preferential treatment must be implemented in order to improve diversity. However with strong preferential treatment the expected academic achievement of the disadvantaged student falls, widening the intergroup score gap, sacrificing academic excellence and possibly exasperating statistical discrimination in the job market. If the preferential treatment is mild, it leaves the expected academic achievement of the disadvantaged student unaltered while the advantaged student puts in more effort. In equilibrium, this simply leads to a higher score gap between students from different backgrounds without improving student-body diversity. Our predictions contrast with Fu's and support the views of Justice Thomas. This is due to the difference in the incentive structure in the two preferential treatment rules.

This difference between the effects of additive and multiplicative affirmative action is in contrast to Chan and Eyster (2003) which finds that an additive affirmative action policy is optimal and equivalent to multiplicative policy if the applicant effort distribution is independent of the admissions policy. Fryer and Loury (2005) also finds that an additive rule is optimal to achieve academic excellence. However in contrast to our paper, in Fryer and Loury (2005) the college does not internalize students' response to affirmative action and students do not interact strategically. Anecdotal evidence suggests that at the top end of the performance distribution students often compete with classmates for admissions to selective universities. This may be due to the perception that elite colleges rarely accept more than one or two students from the same class. California and Florida guarantee a spot in the state university system for the top 4% and top 20% students of their

[4]While our discussion focuses on the case of the US, the existence of socioeconomic tiers is surely not unique to the US. For instance, in an effort to foster social mobility the National University of Ireland Maynooth adds 50 points out of 600 to the admissions score of students who can document that they are from a disadvantaged socioeconomic background. NUIM is one of the eight Higher Education Institutions in Ireland that participates in the Higher Education Access Route Program (HEAR). See http://access.nuim.ie/scope/hear.

highschool respectively. These sort of policies induce further head-to-head competition between students within the highschool.

We find that if students respond to the admissions policy and colleges internalize this response, the exact structure of preferential treatment matters. A multiplicative preferential treatment policy increases the marginal benefit of effort from a disadvantaged candidate. This induces greater academic effort on his part and in equilibrium this results in a greater probability that he will be admitted. On the other hand, an additive affirmative action policy does not alter the marginal benefit of effort for a disadvantaged candidate. The bonus points are added irrespective of the performance of the student. Hence it does not incentivize the student to put in additional effort.

Affirmative action policy was a ballot measure in two states in the 2008 U.S. elections. While Colorado voted for affirmative action (51%), Nebraska voted against it (58%). Public opinion is divided on the question of affirmative action with the opponents gaining momentum.[5] While some might oppose affirmative action based on their philosophical standing, others may do so due to the perceived practice of it.[6] Whether the policy is in place as an affirmative action policy or as a diversity-conscious admissions policy, it is important to pay attention to the incentive effects that it will induce. We find that the exact implementation of the policy can dramatically alter its effects. Our paper together with Fu's suggests that when contestants are able to react to the allocation policy, it should be tailored to increase the marginal benefit of effort for disadvantaged candidates.

We analyze three measures that are frequently cited as goals for admission policy. It is often considered desirable for admissions policy to improve student quality, to create a diverse student body and to reduce the academic polarization of student achievement based on group affiliation. However, we show that these three goals are not mutually inclusive consequences of admissions

[5] Affirmative action in all public colleges is banned either by court order, by referendum or by Governor's initiative in California (1996), Louisiana (1996), Mississippi (1996), Texas (1996), Washington (1998), Florida (2000) and Georgia (2001).

[6] See Fryer and Loury (2005) for a survey of studies documenting the atmosphere of confusion about the aim of affirmative action policies and how they are implemented.

policies that give preferential treatment to one of the groups of students whether additive or multiplicative. However, if the discrepancy in the academic achievement of students from different socioeconomic backgrounds is at least partially generated due asymmetry in cost of effort and in net value from education, policies that target the source of the problem can yield improvement in all three policy goals at once.[7]

It will be useful to introduce some terminology that will be employed throughout the paper: A college with a "fully diversified student body" has the percentage of admitted students from a disadvantaged background and from an advantaged background equal to percentage of applicants from each group.[8] A "purely academic quality-oriented college" sets the admissions policy to maximize the expected academic achievement of the admitted student. The "score gap" is the difference between the expected academic achievement of advantaged and disadvantaged students.

Section 2 presents the model and the equilibrium. Section 3 discusses the equilibrium implications of the model. Section 4 discusses student-body diversity and academic excellence. Section 5 concludes.

II. The Model

There are two risk-neutral applicants for one seat at a college: an applicant from a disadvantaged background (D) and an applicant from an advantaged background (A). Each applicant has a valuation $V_i \in (0,\infty)\ \forall\ i \in \{A,D\}$ of a place in college. Applicants' academic efforts e_i result in a test score q_i via a non-stochastic symmetric linear technology $q_i = e_i\ \forall i \in \{D,A)$. The unit cost of effective effort is $c_i \forall i \in \{D,A)$. The value to cost ratio is asymmetric where $\frac{V_A}{c_A} > \frac{V_D}{c_D}$. The asymmetry may be because $c_D > c_A$, for instance students from a disadvantaged background lack access to effective tutorials for

[7]Austen-Smith and Fryer (2005) argues that the performance gap may also be due to peer pressure. Among minorities, aspiring to achieve may be labeled as "acting white" and may be a cause of rejection from the social peer group.

[8]For instance, Trinity College in Dublin admits a percentage of students from the Republic of Ireland and from Northern Ireland equal to the percentage of applicants from each group.

the SAT. The asymmetry may also arise because $V_A > V_D$. This may occur due to perceived inequality in opportunities for the candidates in the post-college job market. Often students coming from an underprivileged socioeconomic background have less promising job prospects for a variety of reasons. Examples range from overt discrimination to the more subtle, such as statistical discrimination, a lack of social networks or a lack of mainstream business community approved mannerisms.[9] Additionally applicants from disadvantaged backgrounds may find the monetary cost of attending a selective university more onerous. The payoff to the applicant is $V_i - c_i e_i$ if he gets accepted and $-c_i e_i$ if he is rejected.

We examine admission policies where the college adds a fixed number of points $\gamma \in \left(-\frac{V_D}{C_D}, \frac{V_A}{c_A}\right)$ to the score of the disadvantaged applicant.[10] The college admits the applicant D if $q_D + \gamma > q_A$, it admits the applicant A if $q_D + \gamma < q_A$, and randomly assigns the seat in case of equality. It is possible to present the results much more concisely if we define f as the applicant who is favored by the admissions policy and u as the applicant who is unfavored by the policy. So if $\gamma \geq 0$, the admissions policy favors students from a disadvantaged background $f=D$ and $u=A$. If the college favors students from a privileged background such as giving priority to children of alumnae then $\gamma < 0$ and $f=A$ and $u=D$. Since the cost of effort is sunk irrespective of the identity of the winner, the resulting competition for the seat in the college takes the same form as an all-pay auction.[11]

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[9]Based on statistics published by the Census Bureau, in 2003 black males with a bachelors degree had a median income of $41.916 which was 82% of the $51,138 median income of similarly educated white males. The American Association of University Woman presents a table with a state by state estimate of median earnings of men and women with a college degree or more in year round full-time work for the period 2004-2006 (see www.aauw.org). The national earnings gap is 24%.

[10]If $\gamma > V_A / c_A$ then even if the advantaged candidate exerts a level of effort with cost equal to his valuation and the disadvantaged candidate exerts no effort, the disadvantaged candidate will still win. Thus with such an extreme affirmative action policy neither candidate would exert any effort and the place would go to the disadvantaged applicant. Likewise if $\gamma < -V_D / c_D$ the equilibrium has neither candidate exerting any effort and the place going to the advantaged applicant.

[11]Rather than modeling the competition between students, Epple, Romano and Sieg (2008) model competition between colleges using financial aid.

In our framework, the college adds a fixed number of points to the score of the student subject to preferential treatment, whereas in Fu (2006) the affirmative action policy is multiplicative. While the two models are basically identical except for the exact procedure of preferential treatment, the equilibrium implications will prove to be distinctly different.

We adapt the preferential treatment all-pay auction of Konrad (2002) to allow for asymmetric valuations of the prize and cost of effort. As in Konrad equilibrium does not exist in pure strategies. The best response to an effort of e' of the favored applicant is either to put $|\gamma|$ more effort than the rival or to drop out of the race altogether, so e' would not be optimal. In Konrad (2002) the bidder with the head-start advantage (the favored student in our framework) always has a positive expected value from the contest and the bidder without the head-start advantage has an expected value of zero. When students have different valuation to cost ratios this is not always the case depending on the intensity of the preferential treatment. Hence we need to study the equilibrium in two separate cases.

Lemma 1: *The equilibrium is only in mixed-strategies and it is characterized by unique probability density functions* $\mathrm{F}_f(e)$ *and* $\mathrm{F}_u(e)$ *for the favored applicant's and the unfavored applicant's academic effort, respectively. These distributions are continuous in* $\gamma \ \forall \ \gamma \in \left(-\frac{V_D}{c_D}, \frac{V_A}{c_A}\right)$.

> *(i) If the admissions policy favors the advantaged applicant,* $\gamma \in \left(-\frac{V_D}{c_D}, 0\right)$ *or if the policy "strongly favors" the disadvantaged applicant,* $\gamma \in \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_A}{c_A}\right)$*, the unique equilibrium distribution functions are given by:*

$$\mathrm{F}_f(e) = \begin{cases} \dfrac{c_u}{V_u}(e + |\gamma|) & for \quad e \in \left[0, \dfrac{V_u}{c_u} - |\gamma|\right] \\ 1 & for \quad e > \dfrac{V_u}{c_u} - |\gamma| \end{cases}$$

and

$$\mathrm{F}_u(e) = \begin{cases} \dfrac{V_f - (c_f / c_u)V_u + c_f |\gamma|}{V_f} & for \quad e \in [0, |\gamma|] \\ \dfrac{V_f - (c_f / c_u)V_u + c_f e}{V_f} & for \quad e \in \left(|\gamma|, \dfrac{V_u}{c_u}\right] \\ 1 & for \quad e > \dfrac{V_u}{c_u} \end{cases}$$

(ii) If the admissions policy "mildly favors" the disadvantaged candidate $\gamma \in \left[0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right]$, *the unique equilibrium distribution functions are given by:*

$$\mathrm{F}_f(e) = \begin{cases} \dfrac{V_u - (c_u / c_f)V_f + c_u e}{V_u} & \textit{for} \quad e \in \left[0, \dfrac{V_f}{c_f}\right] \\ 1 & \textit{for} \quad e > \dfrac{V_f}{c_f} \end{cases}$$

and

$$\mathrm{F}_u(e) = \begin{cases} 0 & \textit{for} \quad e \in [0, |\gamma|] \\ \dfrac{c_f}{V_f}(e - |\gamma|) & \textit{for} \quad e \in \left(|\gamma|, \dfrac{V_f}{c_f} + |\gamma|\right] \\ 1 & \textit{for} \quad e > \dfrac{V_f}{c_f} + |\gamma| \end{cases}$$

The unique equilibrium given in Lemma 1 can be derived using standard methods as in Hillman and Riley (1989). The proof is presented in the Appendix.

III. Equilibrium Implications

With a group-blind admissions policy (γ=0), in equilibrium the student from an underprivileged background is less aggressive than the student from a privileged background due to the higher cost of effort and/or lower expected return from the degree. With a positive probability the disadvantaged student simply chooses not to put in any effort due to his dim prospects (the probability mass at zero in Lemma 1). Hence a student from a privileged background has a chance of facing such a weak competitor. This implies that in equilibrium, for the same level of effort the student from an underprivileged background has a lower chance of being admitted than the student from a privileged background.

Proposition 1: *With a group-blind admissions policy, γ=0, with the same effort an applicant from a disadvantaged background has a lower chance of being admitted than an applicant from an advantaged background, for any level of effort below the suprimum of the equilibrium support,* $e \in \left(0, \frac{V_D}{c_D}\right)$.

The proof of Proposition 1 is in the Appendix. It is straightforward to derive the expected scores and the admissions probabilities of the applicants implied by the equilibrium distribution functions when there is preferential treatment.

Lemma 2: *If the admissions policy implements additive preferential treatment where the favored student is admitted when* $q_f > q_u - |\gamma|$ *then:*

a. *If the admissions policy favors the student from an advantaged background (the student with the high valuation to cost ratio),* $\gamma \in \left(-\frac{V_D}{c_D}, 0\right)$, *then the stronger the preference*

(i) the lower is the expected academic achievement of students from both backgrounds

(ii) the less diverse is the student body

b. *If the admissions policy mildly favors the candidate from a disadvantaged background (the student with the low valuation to cost ratio),* $\gamma \in \left[0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right]$, *then*

(i) it has no effect on the expected academic achievement of the disadvantaged student

(ii) it cannot achieve full diversity of the student body and a stronger preference in this range has no effect on the diversity of the student body

(iii) it widens the test score gap

c. *If the admissions policy strongly favors the candidate from a disadvantaged background (the student with the low valuation to cost ratio),* $\gamma \in \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_A}{c_A}\right)$

(i) it can achieve fully diversity of the student body

(ii) but it results in decreased expected academic achievement of the disadvantaged student compared to a group-blind policy

The proof of Lemma 2 is in the Appendix. Figure 1 and 2 graph the expected academic achievements of the students from advantaged and disadvantaged backgrounds as a function of the preferential treatment parameter. Figure 3 gives the probability of acceptance of the unfavored applicant.

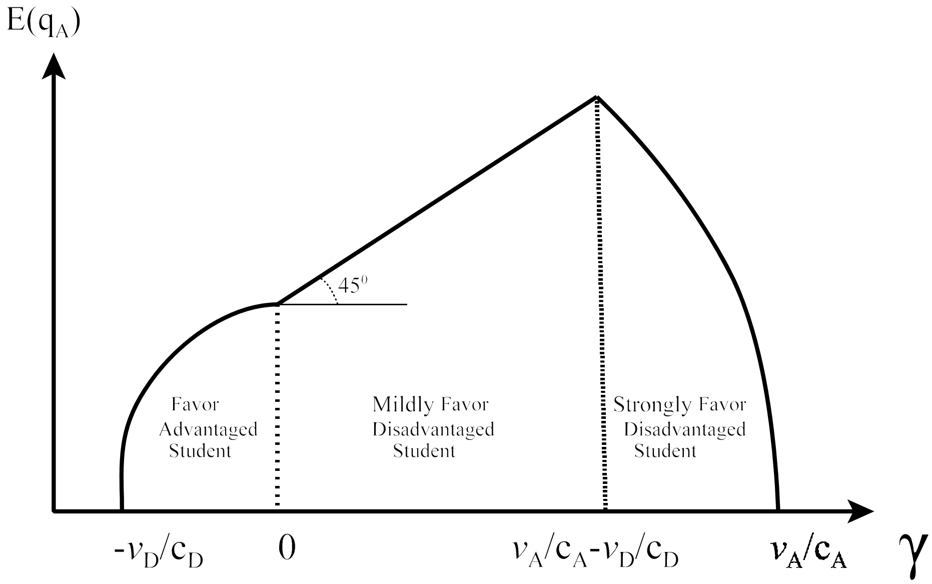

Figure 1:
Expected Achievement of the Student from Advantaged Background

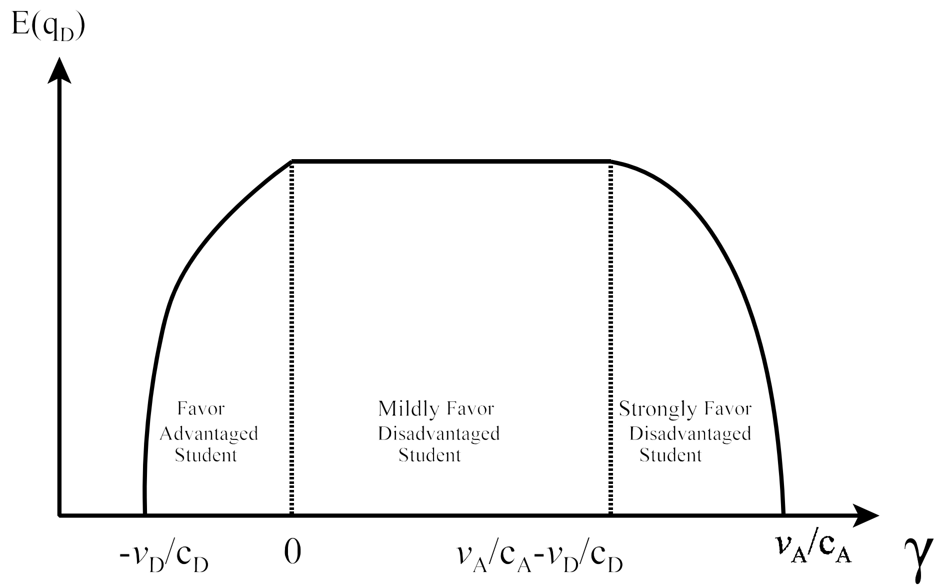

Figure 2:
Expected Achievement of the Student from Disadvantaged Background

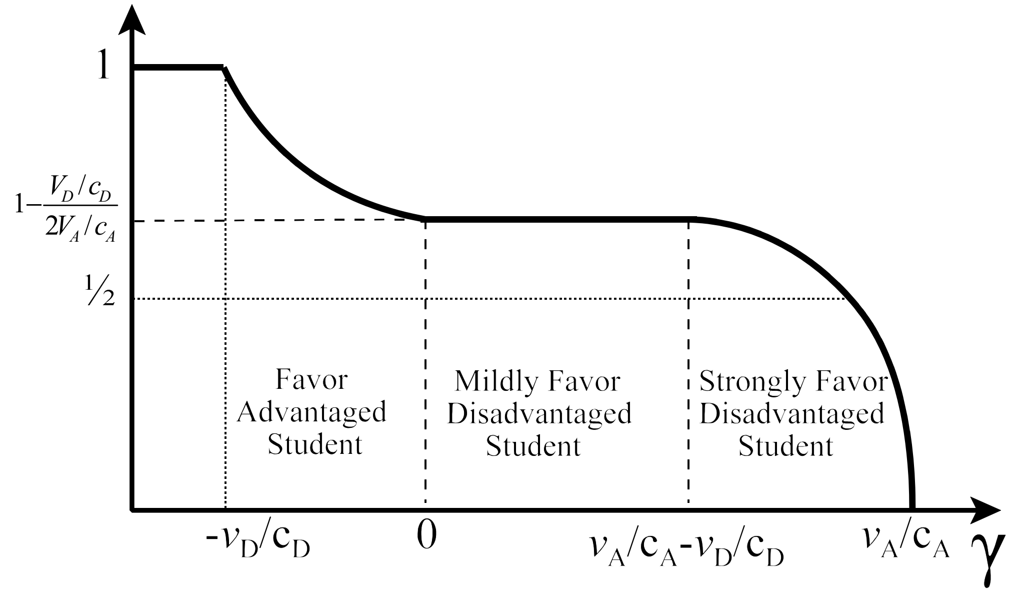

Figure 3:
Probability of Acceptence of the Unfavored Applicant

The admissions policy may favor the applicant from a privileged background (Lemma 2 part a) as is often implemented by giving priority to a list of feeder schools or to applicants who have alumnae parents. In this case, the expected academic achievement of students from both backgrounds deteriorates. The student from the privileged background can guarantee a victory by engaging in an effort level of $\frac{V_D}{c_D}-|\gamma|$, since the applicant from the disadvantaged background would never put in effort that would cost him more than his valuation of the prize, V_D. This leaves the privileged applicant in an advantageous position in the competition both due to his higher valuation to cost ratio and due to the admissions policy that favors him. In equilibrium he has a positive excepted payoff of $\left[V_A-c_A\left(\frac{V_D}{c_D}-|\gamma|\right)\right]$. A stronger preferential treatment tilts the playing field further in his favor. This induces the candidate from the disadvantaged background to become less aggressive. In return, the high-valuation candidate also puts less effort. In equilibrium, the probability of admissions of the high-valuation applicant improves, reducing the diversity of the student body.

If the admissions policy mildly favors the disadvantaged candidate (Lemma 2 part b), the effort distribution of the applicant from the disadvantaged background does not depend on the preferential treatment parameter (see Lemma 1). However it shifts the effort distribution of the high-valuation candidate to the right by γ, the amount of the bonus added to the other candidate's score. Mildly favoring the applicant from a disadvantaged background induces the advantaged student to study more aggressively in order to overcome the preferential treatment. In equilibrium, the probabilities of admissions remain unchanged, since the greater effort of the applicant from the privileged background simply offsets the preferential treatment of the admissions rule. Hence mild preferential treatment has no effect on the diversity of the student body but it widens the score gap.

This result is different from Fu (2006) which finds that the probability of acceptance of the candidate from the disadvantaged background improves when subject to mild multiplicative affirmative action. Multiplicative preferential treatment boosts the marginal benefit of effort for the disadvantaged applicant. However, with additive preferential treatment, as in this paper, the bonus points are added irrespective of the performance of the student. Hence the student from the disadvantaged background does not have an additional incentive to put in effort.

Finally, if the admissions policy strongly favors the applicant from a disadvantaged background (Lemma 2 part c), applicant *D* never needs to put in effort greater than $\frac{V_A}{c_A} - |\gamma|$ in order to win the seat in college since the candidate from an advantaged background never puts in effort greater than $\frac{V_A}{c_A}$ given that his valuation of the seat is V_A. This gives applicant *D* an advantage in the competition leaving him a positive expected payoff of $\left[V_D - c_D\left(\frac{V_A}{c_A} - |\gamma|\right)\right]$. A stronger preference tilts the playing field further in favor of applicant *D* and induces less aggressive competition. In equilibrium the probability of admissions of the student from a disadvantaged background goes up.

IV. Student-Body Diversity and Academic Excellence

Below we discuss the optimal choice of γ under two different objective functions. We first analyze the optimal preferential treatment parameter for a purely academic quality oriented college that maximizes the expected academic achievement of the admitted student. We then analyze the optimal choice for a college that aims at full student-body diversification.

Proposition 2: *A preferential treatment admissions rule where* $\hat{\gamma} = \frac{V_A}{c_A} - \frac{V_D}{c_D}$ *maximizes the expected academic quality of the incoming class. This policy increases the expected score of an admitted student from an advantaged background but does not change the expected score of an admitted student from a disadvantaged background. Despite the preferential treatment for the student from the disadvantaged background, it does not improve student body diversity.*

The proof of Proposition 2 is in the Appendix.

Even if the college is indifferent toward diversity and cares solely about the academic quality of the incoming class, the college employs an admissions policy that mildly favors the candidate from the disadvantaged background. To overcome this mild preference, the candidate from the advantaged background puts in more effort which results in higher quality admissions.

Mild preference for disadvantaged students sounds as if it would be beneficial to applicants from underprivileged backgrounds. However with additive preferential treatment such a policy would not help them. In equilibrium, mild preferential treatment leaves student body diversity unchanged and the disadvantaged student is no more likely to get admitted than with an admissions policy that is background blind. But the policy results in a greater inter-group score gap in the admitted class. In expectation, students are therefore more polarized in quality based on their group affiliation. In the job market this is likely to aggravate statistical discrimination.

With a multiplicative affirmative action rule Fu (2006) finds the academic-quality-maximizing choice of admissions policy yields full diversification as a byproduct. Hence there is no tension between academic quality and diversity. This is not the case with an additive preferential treatment policy.

Proposition 3: *The admissions rule where* $\tilde{\gamma}=\left(\frac{V_A}{c_A}\left(\frac{V_A}{c_A}-\frac{V_D}{c_D}\right)\right)^{1/2}>\hat{\gamma}$ *achieves full student-body diversity. This policy induces less effort by the student from a disadvantaged background compared to a group-blind admissions policy. It also leads to a higher score gap than a group-blind admissions policy.*

The proof of Proposition 3 is in the Appendix.

A group-blind admissions policy γ=0, results in a higher percentage of advantaged students being admitted than their representation in the applicant pool. Due to the higher valuation of the prize and or lower cost of effort the student from an advantaged background puts more effort in expectation and has a higher probability of acceptance. In order to achieve a diverse student body the college must implement strong preferential treatment in favor of the disadvantaged student. When $\gamma=\tilde{\gamma}>\hat{\gamma}$ the probability that the disadvantaged student is admitted is ½. Achieving student body diversity through an additive admissions preference comes at a cost. Due to the strong preferential treatment the disadvantaged candidate can relax which leads to a greater score gap between advantaged and disadvantaged students.

With multiplicative affirmative action, the optimal policy that maximizes academic excellence also yields student body diversity (Fu, 2006), however the score gap is increased. Thus the longer term repercussions of either type of preferential treatment admissions policy may be to exacerbate statistical discrimination against students from underprivileged socioeconomic backgrounds. In order to increase diversity without increasing the student score gap or sacrificing academic quality one needs to directly address the underlying causes of the problem.

Proposition 4: *As long as the valuation to cost of effort ratio of the student from the underprivileged background is not too low,* $V_D/c_D>V_A/2c_A$ *, policies that decrease the cost of effort and/or increase the net value of education to the student from the underprivileged background will address all three goals of increasing diversity, increasing academic excellence and decreasing the score gap.*

The proof of Proposition 4 is in the appendix.

The lack of equal representation in college and the inter-group score gap are equilibrium results which arise from the underlying asymmetries in cost of effort and perceived value of college education among students from different backgrounds. Any policy that alleviates these asymmetries makes it a closer contest. This gives students from both backgrounds more incentive to put in effort. Hence policies that attack the source of the problem also lead to an improvement in the academic quality of the incoming class. While with diversity-conscience admissions policies there is a tradeoff between the desirable properties of an incoming class, such a tradeoff does not present itself with policies that help improve the job opportunities and reduce the cost of education to students from underprivileged backgrounds.

If the valuation to cost of effort ratio of the student from the underprivileged background is too low, $V_D / c_D < V_A / 2c_A$, only two of the three goals can be reached with social policies that help reduce the cost of effort and increase the net value of education for students from disadvantaged backgrounds. These policies always increase the quality of the incoming class and the diversity of the student body but the student score gap may get worse before it improves. However note that this is just an artifact of the score gap measure as it is traditionally calculated, as the absolute difference between the scores. Take a situation where V_D is very low to begin with. Then the disadvantaged student will have very low effort and so the advantaged student does not need to put a in high effort either. Therefore in absolute terms the score gap will be small, because the scores themselves will be low. As social policies such as scholarships improve the net value of education to students from disadvantaged backgrounds, the competition will be more intense and the scores of both students will be higher. Thus for these very asymmetric cases the score gap will increase, measured in the traditional manner as the difference in the scores of students. However the relative student score gap, measured as the ratio of scores, always decreases as a result of these positive policies.

There are a wide variety of policies which directly address the underlying asymmetry between students from different backgrounds. If applicants from disadvantaged backgrounds have higher costs of effort due to lack of access to effective schools or tutorials, these can be provided.[12] Mentor programs may be useful where applicants from disadvantaged backgrounds have low valuation for college education due to lack of social networks or lack of business community approved mannerisms. If disadvantaged applicants have low valuations due to the high monetary cost of selective colleges, this can be addressed directly through scholarships. If the problem is arising due to statistical discrimination in the post-college job market one should be particularly concerned about preferential admissions policies which are likely to increase the student score gap.

V. Conclusion

Students from a disadvantaged background may have a lower valuation of a degree in college and/or a higher cost of an efficiency unit of effort compared to students from an advantaged background. With a group-blind admissions policy, we find that for the same level of effort a student from an underprivileged background has a lower chance of being admitted than a student from a privileged background. The student body is not diversified and the score gap leaves the door open for long-term statistical discrimination in the job market against students from a disadvantaged background.

Public policy that directly addresses the underlying cause of the problem through scholarships, loans, free SAT tutorials, or higher quality of schooling in disadvantaged areas can provide improvement in academic quality, the score gap and diversity in college. Whereas preferential treatment policies are cheap and easy to implement, all three goals are not mutually inclusive.

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[12] In UK following the January 2009 report from Independent Commission on Social Mobility (www.socialmobilitycommission.org) the government launched New Opportunities White Paper. The plan includes a golden handshake to the most effective teachers in order to encourage them to work at schools in socio-economically disadvantaged areas as well as a new guarantee for high potential young people from low income backgrounds to get the help they need to get to university.

Furthermore, if preferential treatment policies are used they should be carefully designed so that they increase the marginal benefit of effort for disadvantaged applicants. If they fail to do so they may give an incentive for the disadvantaged student to put in less effort. Contrary to one of its intended consequences affirmative action can widen the performance gap further, possibly leading to more severe statistical discrimination against disadvantaged students.

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APPENDIX:

Claims 1 through 5 are used in the proof of Lemma 1.

CLAIM 1: *When* $\gamma \in \left(-\frac{V_D}{c_D}, \frac{V_A}{c_A}\right)$ *there is no equilibrium in pure-strategies and neither applicant will put any probability mass point on any level of effort greater than zero.*

PROOF: Neither applicant will put any probability mass point on any effort greater than zero. Define a range E_u as (0,x) where x is an arbitrary number greater than V_N/c_N. Define e_u' as the suprimum of E_u. Suppose the lowest mass point of applicant *u* in E_u is given by $e_u^* \in E_u$. Then applicant *f* would not put any probability at $e_f = e_u^* - |\gamma|$ as a slight increase in his effort would result in a discrete increase in the probability of winning. As there is no probability of $e_f = e_u^* - |\gamma|$, applicant *u* could lower his effort slightly without changing his probability of winning. A similar argument rules out mass points for applicant *f* on $e_j \in (0, e_j')$. Applicant *u* would not put any probability at $e_u = e_f^* + |\gamma|$. Both players' engaging in zero effort cannot be sustained as a pure-strategy equilibrium either, since the best response to e_u=0 would be effort slightly higher than $|\gamma|$. □

CLAIM 2: *Applicant u will put zero probability on* $e_u \in (0, |\gamma|]$.

PROOF: If applicant *u* contemplates $e_u \in (0, |\gamma|)$ zero effort will win with the same probability as he must exceed his rival's effort by at least $|\gamma|$ in order to win. If $e_u = |\gamma|$ then he can win only if the other exerts zero effort, in which case there is an even chance of winning. So if that effort level gave him a nonnegative payoff he could double his chances of winning by a slight increase in his effort. And if it gave him a negative payoff he could get a zero payoff by dropping his effort to zero.

□

CLAIM 3: *If* $\gamma \in \left(-\frac{V_D}{c_D}, 0\right) \cup \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_D}{c_D}\right)$ *applicant u has an infimum effort level of zero. The expected value of the game to applicant u is zero.*

PROOF: Bidder *u*'s infimum effort must be less than V_u/c_u since there can be no probability mass at V_u/c_u by Claim 1. Suppose that applicant *u* has an infimum effort of $e_u^{\inf} \in \left(|\gamma|, \frac{V_u}{c_u}\right)$. Then applicant *f* would never choose $0 < e_f \le e_u^{\inf} - |\gamma|$. If he did he would have positive effort and would lose for sure, since the probability of applicant *u* choosing exactly $e_u^{\inf}$ is zero in this range by Claim1. Therefore applicant *u* could lower his infimum effort without changing the probability of winning. Suppose that $e_u^{\inf} = |\gamma|$ where applicant *u* was mixing in the open interval above $|\gamma|$ but not at $|\gamma|$, by Claim 2. Then applicant *f* would never choose zero effort as this gives zero payoff and he can win for sure with effort of $\frac{V_u}{c_u} - |\gamma| + \varepsilon$ yielding a positive payoff. Take an effort level of $e_u = e_u^{\inf} + \varepsilon = |\gamma| + \varepsilon$, the probability that applicant *u* wins with this effort is $\int_{|\gamma|^+}^{|\gamma|+\varepsilon} \mathrm{f}_f(x - |\gamma|)dx$. Since applicant *f* has no mass points on (0, ε] by Claim 1, this probability is close to zero for small ε, yielding a negative expected payoff for applicant *u*. Hence applicant *u*'s infimum effort cannot be |γ|. $e_i^{\inf} \in (0, |\gamma|)$ is not possible by Claim 2. Therefore zero effort is in the support of the mixed strategy of applicant *u*. At this effort level he loses for sure, so the expected value of the game for applicant *u* must be zero. □

CLAIM 4: *If* $\gamma \in \left(-\frac{V_D}{c_D}, 0\right) \cup \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_D}{c_D}\right)$ *applicant u has a suprimum effort of* V_u/c_u*. Applicant f has a suprimum effort of* $\frac{V_u}{c_u} - |\gamma|$*. The expected value of the game to applicant f is* $V_f - c_f\left(\frac{V_u}{c_u} - |\gamma|\right) > 0$.

PROOF: Suppose that applicant *u* had a suprimum effort of $e_u^{\sup} < V_u/c_u$. In that case applicant *f* would not set $e_f > \max[0, e_u^{\sup} - |\gamma|]$ as he can win for sure with that effort since either $e_u^{\sup} < |\gamma|$ or by Claim 1 the probability of applicant *u* choosing exactly $e_u^{\sup}$ is zero. Therefore applicant *u* could win for sure with $e_u = e_u^{\sup} + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 3. Suppose that applicant *f* had a suprimum effort of $e_f^{\sup} < V_u - |\gamma|$. Then applicant *u* could win for sure with $e_u = e_f^{\sup} + |\gamma| + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 3. Applicant *f* can win for sure with effort of $\frac{V_u}{c_u} - |\gamma|$ since by Claim1 the probability of applicant *u* choosing exactly $\frac{V_u}{c_u}$ is zero. Since $\frac{V_u}{c_u} - |\gamma|$ is in the support of his mixed strategy and he wins for sure with that effort, the expected payoff for applicant *f* is $V_f - c_f\left(\frac{V_u}{c_u} - |\gamma|\right)$. □

CLAIM 4: *If* $\gamma \in \left[0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right]$ *the advantaged applicant has an infimum effort of γ. The disadvantaged applicant has an infimum effort of zero. The expected value of the game to the disadvantaged applicant is zero.*

PROOF: Applicant *A* can win for sure with an effort of $\frac{V_D}{c_D} + \gamma$ yielding a payoff of $V_A - c_A\left(\frac{V_D}{c_D} + \gamma\right) \geq 0$. He would not choose zero effort since he would lose for sure. $e_A^{\inf} \in (0, \gamma)$ is not possible by Claim 2. $e_A^{\inf} = \frac{V_D}{c_D} + \gamma$ would be a pure strategy with a pure strategy best reply by applicant *D*, but there cannot be a pure-strategy Nash equilibrium by Claim 1. Suppose that applicant *A* has an infimum effort of $e_A^{\inf} \in \left(\gamma, \frac{V_D}{c_D} + \gamma\right)$. Then applicant *D* would never choose $0 < e_D \leq e_A^{\inf} - \gamma$. If she did she would be paying a positive amount and would lose for sure, since by Claim 1, the probability of applicant *A* choosing exactly $e_A^{\inf}$ is zero in this range. Therefore, applicant *A* could lower his infimum effort without changing the probability of winning. Suppose applicant *D* had an infimum effort level of $e_D^{\inf} \in \left(0, \frac{V_D}{c_D}\right]$ then applicant *A* would never choose $e_A \le e_D^{\inf} + \gamma$. If he did, he would lose for sure yielding a negative payoff. Since by Claim 1 the probability of applicant *D* choosing exactly $\frac{V_D}{c_D}$ is zero and he can always guarantee a positive payoff of $V_A - c_A\left(\frac{V_D}{c_D} + \gamma\right) \ge 0$ with effort of $\frac{V_D}{c_D} + \gamma$. But then applicant *D* would prefer zero effort to $e_D^{\inf}$. Therefore a effort of zero is in the support of the mixed strategy of applicant *D*. At this effort she loses for sure, so the expected value of the strategy for applicant *D* must be zero. □

CLAIM 5: *If* $\gamma \in \left[0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right]$ *the advantaged applicant has a suprimum effort of* $\frac{V_D}{c_D} + \gamma$. *The disadvantaged applicant has a suprimum effort of* V_D / c_D. *The expected value of the game to the advantaged applicant is* $V_A - c_A\left(\frac{V_D}{c_D} + \gamma\right) \ge 0$.

PROOF: Suppose that applicant *A* had a suprimum effort of $e_A^{\sup} < \frac{V_D}{c_D} + \gamma$. Then applicant *D* would never set $e_D > \max[0, e_A^{\sup} - \gamma]$ as he can win for sure with that effort since either $e_A^{\sup} < \gamma$ or by Claim 1 the probability of applicant *A* choosing exactly $e_A^{\sup}$ is zero. Therefore applicant *D* could win for sure with $e_D = e_A^{\sup} - \gamma + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 4. Suppose that applicant *D* had a suprimum effort of $e_D^{\sup} < \frac{V_D}{c_D}$. Then applicant *A* would never set $e_A > e_D^{\sup} + \gamma$ since he could win for sure with $e_A = e_D^{\sup} + \gamma$. Therefore applicant *D* could win for sure with $e_D = e_D^{\sup} + \varepsilon$ yielding a payoff greater than zero for small enough ε, a contradiction of Claim 4. By Claim 1, applicant *A* will win for sure with effort of $\frac{V_D}{c_D} + \gamma$, and it is in the support of his mixed strategy, so the expected value of the game to the advantaged applicant is $V_A - c_A e_A^{\sup} = V_A - c_A\left(\frac{V_D}{c_D} - \gamma\right) > 0$. □

CLAIM 5: *For the advantaged applicant, effort levels almost everywhere on* $e_A \in [e_A', e_A'']$ *and for the disadvantaged applicant, effort levels almost everywhere on* $e_D \in [b_D', b_D'']$, *must have positive probability, where*

$$\forall \quad \gamma \in \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_A}{c_A}\right) \qquad e_A' = \gamma,\ e_A'' = \frac{V_A}{c_A} \qquad \textit{and} \qquad e_D' = 0,\ e_D'' = \frac{V_A}{c_A} - \gamma$$

$$\forall \quad \gamma \in \left(0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right) \qquad e_A' = \gamma,\ e_A'' = \frac{V_D}{c_D} + \gamma \qquad \textit{and} \qquad e_D' = 0,\ e_D'' = \frac{V_D}{c_D}$$

$$\forall \quad \gamma \in \left(-\frac{V_D}{c_D}, 0\right) \qquad e_A' = 0,\ e_A'' = \frac{V_D}{c_D} + \gamma \qquad \textit{and} \qquad e_D' = -\gamma,\ e_D'' = \frac{V_D}{c_D}$$

PROOF: Suppose there were an interval of effort levels (t, s) in [e_A', e_A''] on which applicant *A* had zero probability. Then applicant *D* would place zero probability on effort levels in (t-γ, s-γ) since she could lower her effort to t-γ and have the same chance of winning. But in this case applicant A would never effort of $s+\varepsilon$ as he could lower his effort to t, saving $s+\varepsilon-t$ in effort and losing only $\mathrm{F}_D(s+\varepsilon-\gamma)-\mathrm{F}_D(t-\gamma)$ in probability. By Claim 1 the loss in probability is negligible for small ε. So if there is an interval of zero probability it must go up to the top of the range, which depending on the level of γ, contradicts either Claim 4 or 5. A symmetric argument rules out ranges of zero probability for applicant D in $e_D \in [e'_D, e''_D]$. □

PROOF OF LEMMA 1: ***Characterization of the Equilibrium.***

(i) $\gamma \in \left(-\frac{V_D}{c_D}, 0\right) \cup \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_D}{c_D}\right)$. Claims 1, 2, 3, 4 and 5 show that applicant u must be indifferent among effort levels almost everywhere in $\{0\} \cup \left(|\gamma|, \frac{V_u}{c_u}\right]$ and applicant f is indifferent among all effort levels almost everywhere in $\left[0, \frac{V_u}{c_u} - |\gamma|\right]$. The expected value of the game to applicant u is zero by Claim 3. When he puts forth effort $e \in \left(|\gamma|, \frac{V_u}{c_u}\right]$ applicant u wins the place with value V_u only with the probability that applicant f puts forth effort less than e-|γ|. Hence, $V_u\,\mathrm{F}_f(e-|\gamma|)-c_u e = 0$. So, $F_f(e) = \left[\frac{c_u}{V_u}(e+|\gamma|)\right] \forall e \in \left[0, \frac{V_u}{c_u} - |\gamma|\right]$, and applicant f has a probability mass equal to $\frac{c_u|\gamma|}{V_u}$ at zero. The expected value of the game to applicant f is $\left[V_f - c_f\left(\frac{V_u}{c_u} - |\gamma|\right)\right]$ by Claim 4. When he puts forth effort $e \in \left[0, \frac{V_u}{c_u} - |\gamma|\right]$ applicant f wins the place with value V_j only with the probability that applicant u does not exceed applicant f's effort by more than |γ|: So indifference implies $V_f F_u(e+|\gamma|) - c_f e = V_f - c_f\left[\frac{V_u}{c_u} - |\gamma|\right]$. Hence $F_u(e) = \frac{V_f - \frac{c_f}{c_u}V_u + c_f e}{V_f}$ $\forall e \in \left(|\gamma|, \frac{V_u}{c_u}\right]$, and applicant u has a probability mass equal to $\frac{V_f - \frac{c_f}{c_u}V_u + c_f|\gamma|}{V_f}$ at zero effort. And he puts zero probability on $(0,|\gamma|]$ by Claim 2.

(ii) $\gamma \in \left[0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right]$. In this case f=D and u=A. Claims 1, 2, 4, 5, and 5 show that applicant A is indifferent between effort levels almost everywhere in $\left(\gamma, \frac{V_D}{c_D} + \gamma\right]$ and applicant D is indifferent between effort levels almost everywhere in $\left[0, \frac{V_D}{c_D}\right]$. The expected value of the game to applicant A is $\left[V_A - c_A\left(\frac{V_D}{c_D} + |\gamma|\right)\right]$, by Claim 5. When he exerts effort $e \in \left(|\gamma|, \frac{V_D}{c_D} + |\gamma|\right]$ applicant A wins the place with value V_A only if applicant D exerts effort less than e-γ. Therefore $V_A F_D(e-|\gamma|) - c_A e = V_A - c_A\left(\frac{V_D}{c_D} + |\gamma|\right)$. So, $F_D(e) = \frac{V_A - \frac{c_A}{c_D}V_D + c_A e}{V_A}$ $\forall e \in \left[0, \frac{V_D}{c_D}\right]$. Applicant D has a probability mass of $\left(1 - \frac{V_D/c_D}{V_A/c_A}\right)$ at zero. The expected value of the game to applicant D is zero by Claim 4. When he exerts effort $e \in \left[0, \frac{V_D}{c_D}\right]$ applicant D wins the place with value V_D only if applicant A exerts effort less than $e+\gamma$. So, $V_D\ F_A(e+\gamma)-c_D\ e=0$. Therefore $F_A(e)=\frac{c_D}{V_D}(e-|\gamma|)\ \forall e\in\left(|\gamma|,\frac{V_D}{c_D}+|\gamma|\right]$. Applicant A puts zero probability on $(0,\gamma]$ by Claim 2.

In each of the three ranges of γ the equilibrium distribution functions are continuous in γ. They are also continuous at the joins between ranges. For the lower join this can be seen by taking the left limit of the equilibrium distribution functions as $\gamma\to 0$, which are the same as the distributions at $\gamma=0$. Likewise the continuity at the upper join between ranges can be seen by taking the right limit of the distributions as $\gamma\to\frac{V_A}{c_A}-\frac{V_D}{c_D}$ which are the same as the distributions when $\gamma=\frac{V_A}{c_A}-\frac{V_D}{c_D}$.

□

PROOF OF LEMMA 2:

Start with part B. If $\gamma\in\left[0,\frac{V_A}{c_A}-\frac{V_D}{c_D}\right]$, then from Lemma 1 the expected academic achievement of the disadvantaged (favored) applicant is given by

$$\int_0^{V_f/c_f} x\,\mathrm{f}_f(x)dx=\int_0^{V_f/c_f}\frac{c_u x}{V_u}dx=\frac{c_u}{2V_u}\left(\frac{V_f}{c_f}\right)^2 \tag{1}$$

which does not depend on γ and hence proves part b (*i*). The diversity of the student body is given by the probability that the unfavored applicant is admitted

$$\begin{aligned}\mathrm{Prob}_u &= \int_{|\gamma|^+}^{|\gamma|+V_f/c_f}\mathrm{f}_u(x)\,\mathrm{F}_f(x-|\gamma|)dx=\\ &\int_{|\gamma|^+}^{|\gamma|+V_f/c_f}\frac{c_f}{V_f}\left(\frac{V_u-(c_u/c_f)V_f+c_u(x-|\gamma|)}{V_u}\right)dx=1-\frac{V_f/c_f}{2V_u/c_u}\end{aligned} \tag{2}$$

which does not depend on γ which proves part b (*ii*). In this range of γ, the expected academic achievement of the advantaged (unfavored) applicant is given by

$$\int_{|\gamma|^+}^{|\gamma|+V_f/c_f} x\,\mathrm{f}_u(x)dx=\int_{|\gamma|^+}^{|\gamma|+V_f/c_f}\frac{c_f x}{V_f}dx=\frac{V_f}{2c_f}+|\gamma| \tag{3}$$

which is increasing in γ. Comparing (1) and (3) at $\gamma=0$ the expected score of the advantaged applicant is greater than the expected score of the disadvantaged applicant since in this range the favored applicant is from the disadvantaged background so $V_u/c_u>V_f/c_f$, which proves part b (*iv*). Higher γ increases the expected score of the advantaged applicant, equation (3), without changing the score of the disadvantaged applicant, equation (1), which proves part b (*iii*).

If $\gamma \in \left(-\frac{V_D}{c_D}, 0\right) \cup \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_A}{c_A}\right)$ then from Lemma 1the expected academic achievement of the favored applicant is given by:

$$\int_0^{\frac{V_u}{c_u}-|\gamma|} x\,\mathrm{f}_f(x)dx = \int_0^{\frac{V_u}{c_u}-|\gamma|} \frac{xc_u}{V_u}dx = \frac{c_u}{2V_u}\left(\tfrac{V_u}{c_u} - |\gamma|\right)^2 \tag{4}$$

while the expected academic achievement of the unfavored applicant is given by:

$$\int_{|\gamma|^+}^{\frac{V_u}{c_u}} x\,\mathrm{f}_u(x)dx = \int_{|\gamma|^+}^{\frac{V_u}{c_u}} \frac{c_f x}{V_f}dx = \frac{c_f}{2V_f}\left[\left(\tfrac{V_u}{c_u}\right)^2 - |\gamma|^2\right] \tag{5}$$

Both of these expressions are strictly decreasing in $|\gamma|$ which proves part a (*i*). From Lemma 1, in this range of γ the probability that the unfavored applicant is admitted is given by:

$$\mathrm{Prob_u} = \int_{|\gamma|^+}^{V_u/c_u} \mathrm{F}_f(x-|\gamma|)\,\mathrm{f}_u(x)dx = \int_{|\gamma|^+}^{V_u/c_u} \left(\frac{c_u x}{V_u}\right)\frac{c_f}{V_f}dx = \frac{c_u c_f}{2V_u V_f}\left[\left(\frac{V_u}{c_u}\right)^2 - |\gamma|^2\right] \tag{6}$$

In the range $\gamma \in \left(-\frac{V_D}{c_D}, 0\right)$ as equation (6) is strictly decreasing in $|\gamma|$. Moreover

$$\lim_{\gamma \to 0} \mathrm{Prob_u} = \frac{V_u / c_u}{2V_f / c_f} = \frac{V_D / c_D}{2V_A / c_A} < \frac{1}{2} \tag{7}$$

therefore increasing $|\gamma|$ decreases the diversity of the student body and proves part a (*ii*).

If $\gamma \in \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_A}{c_A}\right)$, the probability that the unfavored applicant is accepted is given by equation (6) where f=D and u=A. Setting this expression equal to ½ and solving for γ yields

$$\tilde{\gamma} = \left[\frac{V_A}{c_A}\left(\frac{V_A}{c_A} - \frac{V_D}{c_D}\right)\right]^{1/2} \tag{8}$$

the level of γ that results in full diversity. Since $V_A/c_A > V_D/c_D$,

$$\frac{V_A}{c_A} - \frac{V_D}{c_D} < \tilde{\gamma} < \frac{V_A}{c_A} \tag{9}$$

which proves part c (*i*). In this range of γ the expected academic achievement of the disadvantaged student is given by (4) which is strictly decreasing in γ. Since from Lemma 1 the equilibrium distribution functions are continuous in γ, so are the expected scores of the applicants, which proves part c (*ii*). □

PROOF OF PROPOSITION 1:

When γ=0 the probability of a D applicant with effort e being admitted is $F_A(e)$ and the probability of a A applicant with effort e being admitted is $F_D(e)$. When γ=0, u=A and f=D so from Lemma 1, for all $0<e<V_D/c_D$, $F_A(e)=c_D\, e/V_D$ while $F_D(e)=\left[V_A-(c_A/c_D)V_D+c_A e\right]/V_A$. $F_A(e)<F_D(e)$ $\forall$ $e\in\left(0,V_D/c_D\right)$. □

PROOF OF PROPOSITION 2:

The expected academic quality of the admitted student (Q) is given by

$$Q=\int_u x\,f_u(x)\,F_f(x-|\gamma|)dx+\int_f x\,f_f(x)\,F_u(x-|\gamma|)dx \tag{10}$$

If $\gamma\in\left[0,\frac{V_A}{c_A}-\frac{V_D}{c_D}\right]$, then from Lemma 1

$$Q=\int_{|\gamma|^+}^{|\gamma|+V_f/c_f} x\frac{c_u c_f}{V_u V_f}\left(\frac{V_u}{c_u}-\frac{V_f}{c_f}+x-|\gamma|\right)dx+\int_0^{V_f/c_f} x^2\frac{c_u c_f}{V_u V_f}dx \tag{11}$$

hence Leibniz' rule yields

$$\frac{\partial Q}{\partial|\gamma|}=\frac{c_u}{2V_u}\left(2\frac{V_u}{c_u}-\frac{V_f}{c_f}\right)=\frac{c_A}{2V_A}\left(2\frac{V_A}{c_A}-\frac{V_D}{c_D}\right)>0 \tag{12}$$

So the academic quality of the admitted student in increasing in γ in this range.

If $\gamma\in\left(-\frac{V_D}{c_D},0\right)\cup\left(\frac{V_A}{c_A}-\frac{V_D}{c_D},\frac{V_A}{c_A}\right)$ then from Lemma 1equation (10) becomes

$$Q=\int_{|\gamma|^+}^{V_u/c_u} x^2\frac{c_f c_u}{V_f V_u}dx+\int_0^{-|\gamma|+V_u/c_u} x\frac{c_f c_u}{V_f V_u}\left(\frac{V_f}{c_f}-\frac{V_u}{c_u}+|\gamma|+x\right)dx \tag{13}$$

hence Leibniz' rule yields

$$\frac{\partial Q}{\partial|\gamma|}=\frac{c_u c_f}{V_u V_f}\left\{-|\gamma|^2+\frac{1}{2}\left(\frac{V_u}{c_u}-|\gamma|\right)\left[\left(\frac{V_u}{c_u}-\frac{V_f}{c_f}-|\gamma|\right)-\frac{V_f}{c_f}\right]\right\}<0 \tag{14}$$

the sign of which can be seen from the fact that if γ<0, $V_u/c_u<V_f/c_f$ and if $\gamma>\frac{V_A}{c_A}-\frac{V_D}{c_D}$ then $\frac{V_u}{c_u}-\frac{V_f}{c_f}>\gamma$. Hence in these ranges of γ the expected academic quality of the admitted student is strictly decreasing in $|\gamma|$. Since by Lemma 1 the equilibrium distribution functions are continuous in γ, so is the expected academic quality of the admitted student. Hence the expected academic quality of the admitted student is maximized at the top of the mildly favor D range of γ, at

$$\hat{\gamma}=\frac{V_A}{c_A}-\frac{V_D}{c_D} \tag{15}$$

If $\gamma \in \left[0, \frac{V_A}{c_A} - \frac{V_D}{c_D}\right]$, then applicant D is favored and from Lemma 1 the expected achievement of an admitted student from a disadvantaged background (Q_D) is given by

$$Q_D = \frac{1}{1-\text{Prob}_u} \int_f x\, \text{f}_f(x)\, \text{F}_u(x+|\gamma|)dx = \frac{1}{1-\text{Prob}_u} \int_0^{V_f/c_f} \frac{c_u c_f}{V_u V_f} x^2 dx \tag{16}$$

since Prob_u is given by (2) and does not depend on γ, Q_D does not depend on γ. The expected achievement of an admitted student from an advantaged background (Q_A) is given by

$$\begin{aligned} Q_A &= \frac{1}{\text{Prob}_u} \int_u x\, \text{f}_u(x)\, \text{F}_f(x-|\gamma|)dx \\ &= \frac{1}{\text{Prob}_u} \int_{|\gamma|^+}^{|\gamma|+V_f/c_f} \frac{c_f}{V_f} \left[\frac{V_u - (c_u/c_f)V_f - c_u|\gamma| + c_u x}{V_u} \right] x dx \end{aligned} \tag{17}$$

Since Prob_u does not depend on γ, Leibniz' rule yields

$$\frac{\partial Q_A}{\partial |\gamma|} = \frac{c_u c_f}{2V_u V_f \text{ Prob}_u} \left[2\left(\tfrac{V_u}{c_u} - \tfrac{V_f}{c_f} - |\gamma|\right)\tfrac{V_f}{c_f} + \left(\tfrac{V_f}{c_f} + |\gamma|\right)^2 - |\gamma|^2 \right] > 0 \tag{18}$$

which is positive since in this range of γ, $V_u/c_u > V_f/c_f$. The policy $\hat{\gamma}$ does not change student body diversity compared with γ=0 by Lemma 2 part b (*ii*). □

PROOF OF PROPOSITION 3:

The full diversity policy is derived in equation (8). Clearly $\tilde{\gamma} \in \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}, \frac{V_A}{c_A}\right) = \left(\overline{\gamma}, \frac{V_A}{c_A}\right)$. At the full diversity policy the expected effort of by an applicant from a disadvantaged background is lower than with no preferential treatment by Lemma 2 part c (*ii*). The score gap at γ=0 is given by equation (3) minus equation (1):

$$\text{Score Gap}\big|_{\gamma=0} = \frac{V_D}{2c_D} - \frac{c_A}{2V_A}\left(\frac{V_D}{c_D}\right)^2 \tag{19}$$

The score gap at $\gamma = \tilde{\gamma}$ is given by the difference between (5) and (4):

$$\text{Score Gap}\big|_{\gamma=\tilde{\gamma}} = \frac{c_D}{2V_D}\left[\left(\frac{V_A}{c_A}\right)^2 - \tilde{\gamma}^2\right] - \frac{c_A}{2V_A}\left[\frac{V_A}{c_A} - \tilde{\gamma}\right]^2 \tag{20}$$

Multiplying this out and plugging in $\tilde{\gamma}$ from (8) for all squared terms and simplifying yields

$$\text{Score Gap}\big|_{\gamma=\tilde{\gamma}} - \text{Score Gap}\big|_{\gamma=0} = \tilde{\gamma} - \frac{c_A}{2V_A}\left(\frac{V_A}{c_A} - \frac{V_D}{c_D}\right)\left(\frac{V_A}{c_A} + \frac{V_D}{c_D}\right) \tag{21}$$

which is positive as long as

$$\tilde{\gamma} > \frac{c_A}{2V_A}\left(\frac{V_A}{c_A} - \frac{V_D}{c_D}\right)\left(\frac{V_A}{c_A} + \frac{V_D}{c_D}\right) \tag{22}$$

Squaring both sides, plugging in $\tilde{\gamma}$ from (8) and simplifying, the difference in the score gaps is positive as long as

$$4\left(\frac{V_A}{c_A}\right)^3 > \left(\frac{V_A}{c_A} - \frac{V_D}{c_D}\right)\left(\frac{V_A}{c_A} - \frac{V_D}{c_D}\right)^2 \tag{23}$$

which is always true. This can be seen by noticing that the (V_D / c_D) that maximizes the R.H.S. is $\frac{V_D}{c_D} = \frac{1}{3}\frac{V_A}{c_A}$ but that even at this level the expression still holds. □

PROOF OF PROPOSITION 4:

When γ=0, the proportion of disadvantaged applicants who are admitted is given by (2) which is less than ½. Hence $\frac{\partial \text{Prob}_D}{\partial V_D} = \frac{c_A}{2c_D V_A} > 0$ and $\frac{\partial \text{Prob}_D}{\partial c_D} = \frac{-c_A V_D}{2c_D^2 V_A} < 0$ indicate that increasing V_D or decreasing c_D will increase diversity. The academic achievement of the admitted class is given by (11). This simplifies to

$$Q = \frac{V_D}{2c_D} + \frac{c_A}{6V_A}\left(\frac{V_D}{c_D}\right)^2 \tag{24}$$

which is increasing in V_D and decreasing in c_D. The score gap is given by (19) which is shown to be positive in the proof of Lemma 2 part b (*iii*). Therefore

$$\frac{\partial \text{Score Gap}}{\partial V_D} = \frac{c_A}{2c_D V_A}\left(\frac{V_A}{c_A} - 2\frac{V_D}{c_D}\right) \tag{25}$$

and

$$\frac{\partial \text{Score Gap}}{\partial c_D} = \frac{-c_A V_D}{2c_D^2 V_A}\left(\frac{V_A}{c_A} - 2\frac{V_D}{c_D}\right) \tag{26}$$

Hence as long as $V_D / c_D > V_A / 2c_A$ increasing V_D or decreasing c_D will decrease the score gap.

□