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Political Campaign Spending Limits

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Abstract
Political campaign spending ceilings are purported to limit the incumbent’s ability to exploit his fundraising advantage. If the challenger does not have superior campaign effectiveness, in contrast to conventional wisdom, we show that the incumbent always benefits from a limit as long as he has an initial voter disposition advantage, however small and regardless of the candidates’ relative fundraising ability. If the challenger has higher campaign spending effectiveness, the effect of limits may be non-monotonic. If the incumbent enjoys a mild initial voter disposition advantage, a moderate limit benefits the challenger. Further restricting the limit favours the incumbent. Stricter limits may lead to the unintended consequence of increased expected spending.

Keywords: Campaign Finance Legislation, Spending Cap, Expenditure Limit, Incumbency Advantage, Efficiency in Fundraising, Effectiveness of Campaign Spending, Initial Voter Disposition, All Pay Auction, Contest, Preferential Treatment Auction.
“[Campaign spending] limits are purported to further three objectives: first, to favour equality, by preventing those with greater means from dominating electoral debate; second, to foster informed citizenship, by ensuring that some positions are not drowned out by others . . . ; third, to enhance public confidence by ensuring equality, a better informed citizenship and fostering the appearance and reality of fairness in the democratic process.”

Supreme Court of Canada
Harper v. Canada (Attorney General), 2004

1. Introduction

Many modern democracies have political campaign spending limits. Walecki (2007) finds that out of 60 democracies studied, 25 have caps on political campaign spending including Canada, the UK, France, Ireland, Israel, Italy, New Zealand and Spain. The Corrupt and Illegal Practices Act of 1883 is characterized by historians as a landmark in the development of democracy in England.¹ The main feature of the act is its introduction of limits on the election expenditures permitted in each constituency. The act was expanded in 2000 under the Political Parties, Elections and Referendums Act. In Canada, spending limits for political parties and candidates were first introduced in 1974 and they were re-regulated in 2003. Spending limits now are considered a cornerstone of Canadian democracy.² Proponents of spending limits suggest that they enhance robust competition in the marketplace of ideas by allowing less established candidates to be heard on an equal footing. Others however argue that incumbents would not have legislated spending caps if limits did not serve them.³ Proponents and opponents of political campaign spending limits base their arguments on one of three sources of asymmetry between the incumbents and challengers; initial voter disposition advantage, efficiency in fundraising and effectiveness in campaign spending.

Opponents of spending limits point to the importance of initial voter disposition advantage. For example in his dissenting opinion in McConnell v. FEC (2003),⁴ Supreme Court Justice Scalia writes: “If all electioneering were evenhandedly prohibited, incumbents would have an enormous advantage. Likewise, if incumbents and challengers are limited to the same quantity of electioneering, incumbents are favored. In other words, any restriction upon a type of campaign speech that is equally available to challengers and incumbents tends to favor incumbents.”

The main argument of proponents of spending limits focuses on incumbents’ higher efficiency in fundraising. In the U.S. Supreme court case McConnell v. FEC (2003), Justice Stevens argues that “[I]ncumbents have pre-existing relationships with corporations and unions, and groups that wish to procure legislative benefits may tend to support the candidate who, as a sitting officeholder, is already in a position to dispense benefits and is statistically likely to retain office. . . . So we do not have a solid theoretical basis for condemning [limits] . . . as a front for incumbent self-protection”
Limits to campaign spending are purported to limit incumbent’s ability to exploit this fundraising advantage.

It is also argued that spending limits affect the balance in the electoral competition due to candidates’ asymmetric effectiveness in campaign spending. Incumbents are already known by the electorate, whereas challengers often need to campaign just to establish name recognition, providing an additional benefit to campaigning. The empirical literature provides vast evidence indicating that challengers employ money more efficiently in terms of turning campaign spending into votes. From this empirical evidence Samuels (2001) argues that limiting campaign spending would therefore benefit incumbents and harm challengers, with potentially deleterious results for democracy. Interestingly, we find that spending limits can benefit the challenger only if the challenger is more effective in campaign spending.

Theoretical literature focuses on these three sources of asymmetry between incumbents and challengers in isolation. In Sahuguet and Perisco (2006) candidates differ only in initial voter disposition and the spending limit is an impediment to the underdog’s ability to overcome his initial disadvantage. Meirowitz (2008) has results on the effect of a spending limit, but only for the case where candidates are asymmetric in fundraising efficiency but are symmetric in all other dimensions. The challenger, who has lower fundraising efficiency benefits from spending limits if voters casts their ballots in favor of the challenger when indifferent. In this paper we adapt the electoral contest model of Meirowitz (2008) to examine spending limits where all three sources of asymmetry may coexist. We find that allowing for more than one source of asymmetry leads to significant changes in the predictions of the model.

Limits are intended to level the playing field in favor of the candidate with lesser resources. However if the challenger does not have superior spending technology Result 1 of the paper shows that this premise does not hold as long as the incumbent has any initial voter-disposition advantage, however small. The limit always helps the incumbent with the initial voter disposition advantage irrespective of who is more effective at fundraising. The challenger must spend more than the incumbent to overcome the incumbent’s head-start advantage. Since the maximum the challenger can spend is given by the limit, the incumbent never needs to spend as much as the limit. This implies that the challenger is effectively constrained by the limit while the incumbent is not.

However, if the challenger can turn campaign spending into votes more effectively, we show that a moderate limit may benefit the challenger when the incumbent has a mild initial voter-disposition advantage. The incumbent must spend more to effectively match his rival’s campaign spending and a mild initial voter disposition advantage does not overwhelm this effect. Hence with moderate limits the incumbent is effectively constrained whereas the challenger is not. Moreover, we find that the effect of a spending ceiling may be non-monotonic. While a moderate limit helps the challenger, further restricting the limit benefits the incumbent.
Another assertion made for campaign spending limits is that spending restrictions can help to restore the efficient use of time of incumbent politicians. Fundraising for a political campaign can distract the politician from his main duties. If stricter limits lead to decreased campaign spending, less time is spent on fundraising and there may be less pay back in terms of political favours to special interest groups. However, we find that, contrary to one of its intentions, the imposition of stricter limits may lead to fiercer competition and increased fundraising.

This paper also makes a small but useful contribution to the auction literature. In the electoral contest the winner takes the seat but both players’ costs of effort are sunk. Hence the contest takes the form of an all-pay auction where one of the contestants has a head-start advantage (initial voter disposition advantage). Hence the contestant with the head-start advantage is subject to “preferential treatment” in the auction literature terminology. Konrad (2002) is the first paper to characterize the equilibrium of a preferential treatment all-pay auction. Konrad (2002) is extended by Meirowitz (2008) to allow for asymmetric marginal costs of bidding and Pastine and Pastine (2009) extend Konrad (2002) to analyze the preferential treatment all-pay auction equilibrium with a cap on the bids. This current paper analyzes a cap in a preferential treatment all-pay auction where contestants may be asymmetric both in the cost of bidding and in the effectiveness of their bids. This is non-trivial because these asymmetries and the preferential treatment interact in the presence of a cap.

Section 2 provides the framework of the model. Section 3 presents the equilibrium of the electoral contest with and without a binding spending cap. Section 4 first summarizes some empirical regularities relevant to the model and then analyzes the implications of the model for parameter values consistent with these empirical regularities. Section 5 discusses the results in the context of the empirical evidence on the effect of spending caps. Section 6 concludes.

2. Framework

We consider a slightly generalized version of the Meirowitz (2008) model. Two candidates indexed by $i \in \{1, 2\}$ run for office. The value of winning the office is the same for both candidates and it is normalized to one. Candidates simultaneously choose their campaign spending levels, $a_i$. The marginal utility cost of raising funds for campaign spending is denoted by $\beta_i > 0$. Candidates differ in the efficiency of raising funds; the lower $\beta_i$, the higher is candidate $i$’s efficiency of fundraising. If $i$ wins his payoff is $1 - \beta_i a_i$. If the opponent wins candidate $i$’s payoff is $-\beta_i a_i$. Note that candidate $i$’s maximum willingness to spend is $1/\beta_i$ if he were certain he would win the election.

A continuum of voters observe the spending levels and cast their ballots based on their initial disposition towards the candidates and the spending of the two campaigns. The outcome of the election is determined by simple majority. Each voter’s initial disposition for Candidate 1 over
Candidate 2 is i.i.d. $t_i \sim U[-\alpha_2, \alpha_1]$. So if $t_i < 0$ voter $i$ initially prefers Candidate 2. After observing the campaign spending of the two candidates the voter’s utility is:

$$u_i = \begin{cases} 
  t_i + \eta_1 a_1 & \text{if Candidate 1 wins} \\
  \eta_2 a_2 & \text{if Candidate 2 wins} 
\end{cases}$$

(1)

where $\eta_i > 0$ is the campaign spending effectiveness of candidate $i$. As in Meirowitz (2008) campaign spending takes the form of persuasive advertising. In this paper we focus on the persuasive role of campaign spending as in, for example, Dixit and Norman (1978). However campaign spending can also serve other important functions which will not be captured by this model.

Through relatively aggressive campaign spending a voter can be persuaded to vote for the candidate he did not initially prefer. Voter $i$ casts his vote for Candidate 1 if $t_i + \eta_1 a_1 > \eta_2 a_2$. If the inequality is reversed he votes for Candidate 2. If he is indifferent he flips a coin. Since $t_i \sim U[-\alpha_2, \alpha_1]$, the median voter has $t_i = (\alpha_1 - \alpha_2)/2 = \alpha$. Label candidates as 1 and 2 such that the majority of voters have an initial predisposition for Candidate 1, $a_1 > a_2$ so $\alpha > 0$. Candidate 1 wins the election if he can capture the vote of the median voter, i.e. if $\alpha + \eta_1 a_1 > \eta_2 a_2$. Hence the parameter $\alpha$ gives the initial voter disposition advantage enjoyed by Candidate 1. In case of equality, $\alpha + \eta_1 a_1 = \eta_2 a_2$, Candidate 1 wins with probability 1/2. Following the terminology in Siegel (2009) refer to $\alpha + \eta_1 a_1$ as the “score” of Candidate 1, and $\eta_2 a_2$ as the “score” of Candidate 2. Let $k$ denote the level of the campaign spending ceiling, so a candidate’s spending cannot exceed $k$.

This framework is identical to the electoral contest model in Meirowitz (2008) except for the explicit inclusion of the degree of effectiveness of campaign effort. Meirowitz (2008) sets $\eta_1=\eta_2=1$. Without campaign spending restrictions $\beta$ can capture an aggregate technology of fundraising and campaign spending effectiveness. This is because in Meirowitz (2008) $a_i$ takes the interpretation of spending measured in efficiency units. However spending caps limit the face value of spending, not the efficiency units. Therefore in our framework $a_i$ is the monetary value of campaign spending and the parameters $\beta_i$ and $\eta_i$ represent two different sources of asymmetry. With spending limits, it will be shown that the effects of these two sources of asymmetry are distinctly different from each other.
3. Equilibrium

If Candidate 1’s initial voter disposition advantage is severe, \( \alpha \geq \eta_2 / \beta_2 \), with or without spending limits the unique equilibrium is in pure strategies. Candidate 2 cannot overcome the head-start advantage of the rival, even if Candidate 2 were to spend his maximum willingness to spend, \( 1 / \beta_2 \), and Candidate 1 spent nothing. Hence neither candidate exerts any effort. Likewise with a sufficiently restrictive spending limit, \( k < \alpha / \eta_2 \), the unique equilibrium is in pure strategies where there is no competition. If \( \eta_2 k < \alpha \), Candidate 2 cannot overcome the voters’ initial disposition with a spending equal to \( k \), even if Candidate 1 exerts no effort.

Define a “binding cap” as a cap which is lower than the maximum of the upper bounds of the no-cap equilibrium spending supports. A “more restrictive cap” refers to a smaller \( k \) when the cap is binding.

For each candidate define \( \bar{a}_i \) as the maximum amount \( i \) would be able or willing to spend if he knew he would win for sure:

\[
\bar{a}_i \equiv \min(k, 1 / \beta_i)
\] (2)

The function \( M_i(a_j) \) gives the amount candidate \( i \) must spend to effectively match the score of his rival when the rival spends \( a_j \):

\[
M_1(a_2) = (\eta_2 a_2 - \alpha) / \eta_1
\]
\[
M_2(a_1) = (\eta_1 a_1 + \alpha) / \eta_2
\] (3)

By construction these functions are inverses of each other. Candidate 1 is referred to as the “strong” candidate if his maximum willingness or ability to spend meets or exceeds what he needs to spend to effectively match the score of his rival if the rival were to spend his maximum willingness or ability to spend, \( \bar{a}_1 \geq M_1(\bar{a}_2) \). Otherwise Candidate 2 will be referred to as the strong candidate. The strong candidate’s rival will be referred to as the “weak” candidate. Whenever \( \bar{a}_i > M_i(\bar{a}_j) \) candidate \( i \) has the option of guaranteeing victory and a positive payoff with spending just above \( M_i(\bar{a}_j) \) if he desires.

Below we describe the equilibrium for the non-trivial cases where there is competition in equilibrium, i.e. where voter’s initial disposition is not too strong, \( \alpha < \eta_2 / \beta_2 \), and the cap is not too restrictive, \( k > \alpha / \eta_2 \). In the interest of space we ignore the non-generic special case \( k = \alpha / (\eta_2 - \eta_1) \) where \( M_i(k) = k \).

The victor captures the political seat but both candidates’ costs of effort are sunk. Hence the electoral contest has the form of an all-pay auction. With and without a cap, the unique equilibrium is in mixed-strategies. If Candidate 1 were to spend \( \alpha' \), the optimal response of Candidate 2 would be to spend enough to beat the rival’s score or to spend zero if that spending yields a negative
payoff or is not possible due to the spending cap. In either case $a'$ would not be the best response of Candidate 1.

### 3.1. Characterization of the Equilibrium

The two propositions below characterize the equilibrium of the electoral contest. The first proposition describes the equilibrium for parameter values where Candidate 1 is the strong candidate, $\bar{a}_1 \geq M_1(\bar{a}_2)$. The second proposition describes the equilibrium for parameter values where Candidate 2 is the strong candidate, $\bar{a}_1 < M_1(\bar{a}_2)$. The propositions are preceded by Lemma 1 and Lemma 2 that give the parameter values where Proposition 1 and 2 apply without a binding cap and with a binding cap, respectively. To reduce unnecessary notation, throughout the paper define the interval $[b,c]$ as $[0,c]$ whenever $b<0$ and as the empty set whenever $b>c$ or $c<0$. Similarly define open intervals.

**Lemma 1: Non-binding cap:**

(a) if $\alpha \in \left(0, \frac{b_2}{\beta_2} - \frac{b_1}{\beta_1}\right)$, then $k > \max\left(\frac{1}{\beta_1}, M_2\left(\frac{1}{\beta_2}\right)\right)$ is not binding and $\bar{a}_1 < M_1(\bar{a}_2)$ without a binding cap.

(b) if $\alpha \in \left(\frac{b_2}{\beta_2} - \frac{b_1}{\beta_1}, \frac{b_2}{\beta_2}\right)$, then $k > \max\left(M_1\left(\frac{1}{\beta_1}\right), \frac{1}{\beta_2}\right)$ is not binding and $\bar{a}_1 \geq M_1(\bar{a}_2)$ without a binding cap.

**Proof:** Appendix.

Without a binding cap if each candidate were to spend their maximum willingness to spend, Candidate 2 would win if $\alpha \in \left(0, \frac{b_2}{\beta_2} - \frac{b_1}{\beta_1}\right)$ and Candidate 1 would win if $\alpha \in \left(\frac{b_2}{\beta_2} - \frac{b_1}{\beta_1}, \frac{b_2}{\beta_2}\right)$. In case (a), Candidate 2 is the strong candidate and in case (b) Candidate 1 is the strong candidate.

A cap greater than the supremum of the no-cap equilibrium supports of both candidates, $k > \max(a^\text{sup}_1, a^\text{sup}_2)$ is not binding. The supremum of the no-cap equilibrium support of the weak candidate $j$ is given by $1/\beta_j$. If it were lower than $1/\beta_j$, the strong candidate would never spend more than just enough to exceed the supremum score of the weak candidate. The weak candidate would then have an incentive to deviate by increasing his upper bound to guarantee a win with a positive expected value. Since the supremum of the equilibrium support of the weak candidate $j$ is given by $1/\beta_j$, the strong candidate has the upper bound of $M_j\left(\frac{1}{\beta_j}\right)$.

**Lemma 2: With a binding cap:**

(a) if $k(\eta_2 - \eta_1) < \alpha$ then $\bar{a}_1 \geq M_1(\bar{a}_2)$.

(b) if $k(\eta_2 - \eta_1) > \alpha$ then $\bar{a}_1 \geq M_1(\bar{a}_2)$.

**Proof:** Appendix.
At an intuitive level, Candidate 1 is strong in the competition if the binding cap is very restrictive. In case (a) there is not much scope for Candidate 2 to overcome Candidate 1’s initial voter disposition advantage. Case (b) exists only if \( \eta_2 > \eta_1 \). When Candidate 2 is more efficient in spending, with a higher spending limit Candidate 1’s initial voter disposition advantage becomes relatively less important. Because Candidate 2 is more efficient in spending, at the upper end of their equilibrium distributions Candidate 1 must outspend him in order to win. Hence the spending cap will effectively only constrain Candidate 1. If the cap is not too restrictive Candidate 2 is able to use his greater efficiency in spending to gain the strong position in the competition.

With a binding cap asymmetries in the effectiveness in campaign spending and in the efficiency in fundraising have qualitatively distinct effects on the equilibrium of the contest. In Meirowitz (2008) without a cap, \( \beta_i / \eta_i \) is captured by a single parameter, and takes the interpretation of the cost of raising funds for one effective unit of spending. However when there is a cap, the legislation puts a ceiling on the dollar value of spending. While fundraising efficiency determines the maximum willingness to spend, the variation in the effectiveness in campaign spending determines the identity of the strong candidate when there is a binding cap.

Note that with a binding cap, only the weak candidate will be effectively restricted by the cap and his rival will be able to use that to capture the strong position in the contest. If the weak candidate \( j \) is effectively restricted by \( k \), then the strong candidate \( i \) does not spend more than \( M_i(k) < k \). The reversed inequality, \( M_i(k) > k \), would be a contradiction of \( j \) being the weak candidate.

**Proposition 1.** For \( k > \alpha / \eta_2 \) and \( \alpha \in (0, \beta_2 / \eta_2) \): if \( \bar{a}_1 \geq M_1(\bar{a}_2) \) then the equilibrium is characterized by unique cumulative density functions \( F_1(a_1) \) and \( F_2(a_2) \) for Candidates 1 and 2’s campaign spending respectively:

\[
F_1(a_1) = \begin{cases} \frac{\beta_1}{\eta_1} (\alpha + \eta_1 a_1) & \text{for } a_1 \in [0, M_1(\bar{a}_2)] \\ 1 & \text{for } a_1 \in (M_1(\bar{a}_2), \infty) \end{cases}
\]

\[
F_2(a_2) = \begin{cases} 1 - \frac{\beta_1}{\eta_1} (\eta_2 \bar{a}_2 - \alpha) & \text{for } a_2 \in [0, \alpha / \eta_2] \\ 1 - \frac{\beta_1}{\eta_1} \eta_2 \bar{a}_2 + \frac{\beta_1}{\eta_1} \frac{\alpha}{\eta_2} a_2 & \text{for } a_2 \in \left( \frac{\alpha}{\eta_2}, \bar{a}_2 \right] \\ 1 & \text{for } a_2 \in (\bar{a}_2, \infty) \end{cases}
\]

(a) Expected values: \( EV_1 = 1 - \frac{\beta_1}{\eta_1} (\eta_2 \bar{a}_2 - \alpha) \geq 0 \), with equality only if \( \bar{a}_1 = M_1(\bar{a}_2) \), and \( EV_2 = 0 \).

(b) Expected spending: \( E(a_1) = \frac{\beta_1 \eta_2}{2 \eta_1} [(\eta_2 \bar{a}_2 - \alpha) / \eta_1]^2 + (1 - \beta_2 \bar{a}_2)(\eta_2 \bar{a}_2 - \alpha) / \eta_1 \) and \( E(a_2) = \frac{\beta_1 \eta_2}{2 \eta_1} \left[ \bar{a}_2^2 - \left( \frac{\alpha}{\eta_2} \right)^2 \right] \).
(c) Probability that Candidate 2 wins: \[ \text{prob}_2 = \frac{1}{2} \frac{\beta_2 \beta_1}{\eta_1} \left[ \bar{a}_2^2 - \left( \frac{\alpha}{\eta_2} \right)^2 \right]. \]

**Proof:** Appendix.

Figure 1. Equilibrium Distributions with a Binding Cap when Candidate 1 is Stronger

The equilibrium distribution functions with a binding cap are graphed in Figure 1, hence \( \bar{a}_2 = k \). In equilibrium, contestants are indifferent between all spending levels in the support of their equilibrium strategies given the equilibrium distributions of their rival. Since \( \bar{a}_1 \geq M_1(\bar{a}_2) \), Candidate 1 is willing and able to exceed the maximum score Candidate 2 is willing and able to reach. The strong candidate (Candidate 1) never exceeds \( M_1(k)^+ \) since the weak candidate (Candidate 2) is restricted by \( k \). The strong candidate has a probability mass at \( M_1(k)^+ \). Candidate 2 never spends in the range \((0, \alpha/\eta_2)\), since he needs to spend at least \( \alpha/\eta_2 \) in order to overcome the initial voter disposition advantage of Candidate 1. Both the weak and the strong candidates have a probability mass at zero campaign spending. The probability masses at zero are increasing in \( \alpha \). The higher Candidate 1’s initial disposition advantage the greater is the chance that Candidate 2 is passive in his campaign effort which allows Candidate 1 to remain passive in campaign spending with positive probability.

**Proposition 2.** For \( k > \alpha/\eta_2 \) and \( \alpha \in (0, \frac{\alpha}{\eta_2}) \): if \( \bar{a}_1 < M_1(\bar{a}_2) \) then the equilibrium is characterized by unique cumulative density functions \( F_1(a_1) \) and \( F_2(a_2) \) for Candidates 1 and 2’s campaign spending respectively:

\[
F_1(a_1) = \begin{cases} 
1 - \frac{\beta_2}{\eta_2} \eta_1 \bar{a}_1 + \frac{\beta_1}{\eta_1} a_1 & \text{for } a_1 \in [0, \bar{a}_1] \\
1 & \text{for } a_2 \in (\bar{a}_1, \infty)
\end{cases}
\]

\[
F_2(a_2) = \begin{cases} 
0 & \text{for } a_2 \in [0, \alpha/\eta_2] \\
\frac{\beta_2}{\eta_2} (\eta_2 a_2 - \alpha) & \text{for } a_2 \in \left( \frac{\alpha}{\eta_2}, M_2(\bar{a}_1) \right] \\
1 & \text{for } a_2 \in (M_2(\bar{a}_1), \infty)
\end{cases}
\]
(a) Expected values: \( EV_1 = 0, \text{ and } EV_2 = 1 - \frac{\beta_1}{\eta_1} (\eta_1 \bar{a}_1 + \alpha) > 0. \)

(b) Expected spending: 
\[
E(a_1) = \frac{\eta_1 \beta_1 \bar{a}_1^2}{2 \eta_2} \quad \text{and} \quad E(a_2) = \frac{\beta_2 \bar{a}_2}{2 \eta_2} (\eta_2 \bar{a}_2 + 2 \alpha) + \frac{L}{\eta_2} (1 - \beta_1 \bar{a}_1) (\eta_1 \bar{a}_1 + \alpha).
\]

(c) Probability that Candidate 2 wins: 
\[
\text{prob}_2 = 1 - \frac{1}{2} \frac{\beta_1 \beta_2 \eta_2 \bar{a}_2^2}{\eta_1}.
\]

**Proof:** Appendix.

![Figure 2. Equilibrium Distributions with a Binding Cap when Candidate 2 is Stronger](image)

The equilibrium distributions with a binding cap are graphed in Figure 2. Candidate 2 is the strong candidate since \( \bar{a}_1 < M_1 (\bar{a}_2) \). Given that Candidate 1 has a head-start advantage, this case is only possible if Candidate 2 is more efficient in campaign spending and/or better at fundraising.

In this case the probability that Candidate 2 wins does not depend on the degree of initial voter disposition towards Candidate 1 (see Proposition 2(c)). If \( \alpha \) increases, Candidate 2 simply becomes more aggressive to overcome his rival’s greater head start while Candidate 1’s equilibrium distribution remains the same. Hence Candidate 2’s probability of victory remains unchanged. This is not the case for parameter values where Proposition 1 applies. The stronger position of Candidate 1 allows him to compete away all of 2’s expected gains. Thus if \( \alpha \) increases, Candidate 2 cannot become more aggressive to compensate. This allows Candidate 1 to reduce his campaigning and still have a higher probability of winning than he did with a lower \( \alpha \).

### 3.2. Additional Features of the Equilibrium

The model implies that it may be difficult to empirically establish whether an existing spending limit is binding. Natural intuition would suggest that there would be a large number of political candidates who spend the maximum permissible amount if the spending cap were binding. For instance, Evans (2007) reports that campaign spending limits are seldom binding based on the fact that candidates so rarely spend at the limit. However, a significant feature of the equilibrium is
that although the cap is binding and alters both candidates’ behavior, neither candidate has a probability mass at the cap. Hence according to the model, these empirical observations do not necessarily indicate that the limits are not binding.

While politicians spend a lot of time and effort trying to raise money to fund their political campaigns, academic research finds only weak evidence that campaign spending affects vote shares at the national level. Using data on U.S. House 633 elections from 1972 to 1990, Levitt (1994) cannot identify statistically significant effects of incumbent nor challenger campaign spending on electoral outcomes. The model implies that in equilibrium there is positive probability that Candidate 2 spends more than Candidate 1 but not by enough to overcome the voters’ initial preferences. Hence the model is consistent with the weak empirical evidence of the effect of campaign spending on election outcomes.

4. Results

In this section we discuss the equilibrium implications of the model for the set of parameter values that are consistent with empirical regularities. We label Candidate 1 as the incumbent since incumbents often enjoy the initial voter disposition advantage either due to reputation they build over their term in office or due to their campaign activities in prior elections. As we present the results the spending limit will be deemed to “benefit” a particular candidate if the limit increases his probability of winning and his expected value from the contest.

Result 1: If \( \eta_1 \geq \eta_2 \), then imposing a binding spending cap or making an existing cap more restrictive benefits the incumbent as long as the incumbent has any initial voter-disposition advantage, however small. This result holds regardless of the candidates’ relative fundraising abilities.

Proof: Appendix.

Without a cap on spending either Proposition 1 or 2 may apply depending on the candidates’ relative efficiency in fundraising. However with a binding cap if \( \eta_1 \geq \eta_2 \), Candidate 1 is stronger so Proposition 1 applies regardless of relative fundraising efficiency. Since the incumbent is more efficient in converting spending into votes and he as an initial voter disposition advantage, the challenger must spend more than the incumbent in order to effectively match his score, \( M_2(a_1) > a_1 \). With or without a cap the supremum of Candidate 2’s spending will be higher than the supremum of the incumbent’s. Hence the cap will effectively only restrict the challenger. The incumbent does not need to spend up to the cap in order to match the maximum permissible score of his rival, \( M_1(k) < k \). The incumbent is effectively unrestricted. Note that the matching functions
depend only on the initial voter disposition advantage and the spending efficiency of the two candidates, not on their fundraising abilities.

This advantage allows the incumbent to capture a positive expected value from the contest equal to \( 1 - \beta_1 M_1(k) \) (see Proposition 1(a)). Hence as the cap becomes more restrictive the challenger becomes more constrained which is to the advantage of the incumbent who is popular \( a \) priori. This decreases the overall aggressiveness of the challenger, which in turn induces less aggressive spending from the incumbent, leading to decreased expected aggregate spending. With a more restrictive cap (lower \( k \)), the incumbent's probability of winning goes up and expected total campaign spending goes down. The cap always helps the candidate with the head-start advantage.

This result is different from Proposition 6 in Meirowitz (2008) where there is no head-start advantage. Meirowitz (2008) finds that the cap may benefit the challenger when the candidates only differ in efficiency of fundraising as long as the voters resolve ties in favor of the challenger. Without a head-start advantage, candidates tie when they both spend at the limit. This is not the case with \( a > 0 \). We find that the candidate who happens to be the \( a \) priori popular candidate always benefits from the cap because the cap only effectively restricts his rival.

The main argument of the proponents of spending limits is that caps put candidates with lesser means on an equal footing. The main argument of opponents is that caps limit challengers ability to overcome incumbents’ head start with the voters. Result 1 shows that if \( \eta_1 \geq \eta_2 \), the argument of the opponents of caps always trumps the main argument in favor of caps. No matter how dramatic the difference in fundraising abilities, the cap always benefits the candidate with a head-start advantage, no matter how small that advantage may be. Since often the incumbent is the candidate with a head-start advantage the primary argument in favor of caps needs qualifications at best.

To capture a more complete picture of the effect of a spending cap, in what follows, we allow for \( \eta_1 < \eta_2 \). There is vast empirical evidence indicating that challengers tend to have higher effectiveness of campaign spending.\(^{13}\) Incumbents are already known by the electorate. However challengers often must still establish name recognition. This provides an additional benefit to campaigning for challengers.

In addition to their initial voter disposition advantage, incumbents also tend to have higher efficiency in fundraising since they are in a position to deliver political favors to donors. Hence we discuss the equilibrium implications for \( \beta_2 > \beta_1 \).\(^{14}\) Note that this captures the source of asymmetry that forms the main argument made in favor of spending caps.

**Result 2:** For \( \beta_1 < \beta_2 \) and \( \eta_1 < \eta_2 \), if the incumbent has a large initial voter disposition advantage, \( a \in \left( \frac{\eta_1 - \eta_2}{\beta_1}, \frac{\eta_2}{\beta_2} \right) \), a more restrictive campaign spending cap always benefits the incumbent and always reduces expected spending.

**Proof:** Appendix.
In the range of $\alpha$ specified in Result 2, Proposition 1 applies with and without a spending cap. The large initial voter disposition advantage overwhelms the advantage the challenger enjoys due to his higher effectiveness in spending. The imposition of a binding cap first effectively restricts Candidate 2 before it restricts Candidate 1 since the supremum of Candidate 2’s no-cap equilibrium distribution is higher, $\frac{1}{\beta_2} > M_1\left(\frac{1}{\beta_1}\right)$. With a binding cap, the expected spending of Candidate 2 decreases. The cap also drives down the expected spending of Candidate 1 since he never needs to spend more than $M_1(k) < k$ to guarantee a victory. This leads to an increase in the expected value of the contest to the incumbent. The more restrictive the ceiling, the higher the expected payoff and the probability of winning for the incumbent. Even though the challenger is more efficient in spending, since he starts out so far behind (because $\alpha$ is so large) he must outspend his rival in order to catch up. The effect of a campaign spending cap is to limit his ability to do so.

Result 3 below shows that the identity of the candidate who captures the strong position depends on the level of the cap if the incumbent has a mild initial voter disposition advantage, $\alpha \in \left(\frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1}, \frac{\eta_2}{\beta_1}\right)$. When the cap is not binding the incumbent is strong. However with a moderate binding cap the challenger captures the strong position. And with a very restrictive cap, the initial voter disposition advantage overwhelms the effectiveness in spending advantage of the challenger and the incumbent captures the strong position.

**Result 3:** For $\beta_1 < \beta_2$ and $\eta_1 < \eta_2$, if the incumbent has a moderate initial voter disposition advantage, $\alpha \in \left(\frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1}, \frac{\eta_2}{\beta_1}\right)$, a moderate cap $k \in \left(\frac{\alpha}{\eta_2 - \eta_1}, \frac{1}{\eta_1} \left(\frac{\eta_2}{\beta_1} - \alpha\right)\right)$ benefits the challenger. But a very restrictive cap $k < \frac{\alpha}{\eta_2 - \eta_1}$ benefits the incumbent. The effect of a spending limit on expected spending is non-monotonic as well.

**Proof:** Appendix.

To prove Result 3 the appendix establishes that: (a) the introduction of a barely binding spending limit leads to a jump down in the probability that the incumbent wins and in the expected value of the contest to the incumbent. A barely binding cap also results in a jump up in expected total campaign spending. (b) As long as the limit is moderate; $k \in \left(\frac{\alpha}{\eta_2 - \eta_1}, \frac{1}{\eta_1} \left(\frac{\eta_2}{\beta_1} - \alpha\right)\right)$, a more restrictive limit causes a decrease in the probability that the incumbent wins while increasing the expected value of the contest to the challenger. A more restrictive cap decreases expected total spending. (c) Reducing the limit from $\left(\frac{\alpha}{\eta_2 - \eta_1}\right)^+ \to \left(\frac{\alpha}{\eta_2 - \eta_1}\right)^-$ leads to an increase in the probability that the incumbent wins as well as improving the expected value to the incumbent. The reduction in the cap leads to an increase in the expected campaign spending of the incumbent. Further reductions in the spending limit decrease both candidates’ expected spending while increasing the incumbent’s expected value and probability of winning.
For the parameter values in Result 3, Figures 3-5 graph the expected payoffs, the expected campaign spending and the probability that the challenger wins as a function of the level of the cap.

The identity of the strong candidate depends on the level of the cap. When the cap is not binding, the incumbent is the strong candidate and Proposition 1 describes the equilibrium. When the cap is binding at a moderate level, the playing field is tilted in favor of the challenger who is more effective in campaign spending and the equilibrium is characterized in Proposition 2. With a tight spending ceiling, it is the incumbent who captures the strong position and the equilibrium is once again given by Proposition 1 (see Lemma 2).

When the ceiling becomes barely binding, the limit first hits the incumbent before it hits the challenger. The supremum of the no-cap equilibrium support of the incumbent is higher than of the challenger, $M_1 \left( \frac{1}{\beta_2} \right) > \frac{1}{\beta_2}$, since the incumbent needs to be aggressive in order to overcome the effectiveness of the campaign spending of the challenger. Imposition of the cap restricts the incum-
bent, but does not effectively restrict the challenger. Since the incumbent is restricted by the cap, the challenger can guarantee a win by exceeding the supremum score of the incumbent. This drives the expected payoff of the incumbent down to zero (Figure 3), and yields a positive expected payoff for the challenger. The imposition of the ceiling changes the identity of the strong candidate. The cap tilts the playing field in favor of the challenger which makes the challenger more aggressive in campaign spending; resulting in a jump up in the expected spending of the challenger (Figure 4). This results in a jump up in the probability that the challenger wins (Figure 5). The imposition of the spending cap benefits the challenger.

![Figure 5. Probability Challenger Wins](image.png)

A further decrease in the spending cap leads to a decline in expected spending of both candidates and the expected payoff of the challenger goes up, as long as the cap is moderate. At $k < \frac{a}{\eta_2 - \eta_1}$ however, the overall advantage in the contest switches over to the incumbent once again. Even though the challenger is more effective in spending, the cap is too small for the challenger to catch up with rival’s initial voter disposition advantage. Just below $\frac{a}{\eta_2 - \eta_1}$ the identity of the strong candidate switches from the challenger to the incumbent, the playing field tilts in favor of the incumbent which induces more aggressive campaign spending of the incumbent.

Additional empirical evidence can be used to eliminate some sets of parameter values. Without spending limits incumbents tend to have a higher probability of victory. In the 2008 U.S. House elections, 94 percent of incumbents who chose to run for election were reelected. The average reelection rate in U.S. House election cycles from 1964 to 2008 is 93 percent. The same figure for the U.S. Senate is 81 percent. In Canada, prior to the introduction of spending limits, on average 79 percent of incumbents who re-ran were elected. Examining the probability of victory of the incumbent without spending limits, note that only a subset of $a$ in the range specified in Result 3 may be empirically relevant. If $a \in \left( \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1}, \left( \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \right)^{1/2} \right)$ without cap the incumbent is strong and $EV_1 > 0$ however the challenger has a higher probability of victory. This theoretical prediction is not consistent with empirical observations. Only if $a \in \left( \left( \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \right)^{1/2}, \frac{\eta_2-\eta_1}{\beta_2} \right)$ without cap the incumbent is strong and and has a higher probability of victory (Figures 3-5 apply

14
for this range of $\alpha$). However also note that this range exists only if $\frac{\beta_2 - \beta_1}{\bar{\beta}_1} > \frac{\eta_2 - \eta_1}{\eta_1}$, i.e. if the fundraising advantage of the incumbent is significant in comparison to the effective spending advantage of the challenger.

With these parameter values and a moderate binding cap, the probability that the challenger wins exceeds $1/2$. Whether this theoretical prediction is empirically relevant or not depends on the interpretation of the source of asymmetry in spending effectiveness. The only source of advantage for the challenger that can help him to catch up with the incumbent’s head-start advantage is superior campaign spending effectiveness. The challenger may have higher spending effectiveness just because to him campaign spending has the additional benefit of establishing name recognition while the incumbent already enjoys name recognition. Under this interpretation of the source of the challenger’s spending efficiency it is not believable that he would have a greater than $1/2$ chance of victory. At best he would be able to catch up to the incumbent. Once he establishes the same level of name recognition his efficiency of spending would be the same as well. Hence the blame for this counterintuitive prediction goes to the linear technology of spending effectiveness. In order to capture this interpretation of asymmetry in campaign spending, one would need to specify a technology with decreasing returns.\textsuperscript{16}

However under other interpretations for the source of the asymmetry in spending effectiveness a greater than $1/2$ chance of winning for the challenger is more plausible. For example, the incumbent has already established impressions in voters’ minds over the years he has been in office whereas the challenger is a blank page. Changing people’s minds may be harder than creating a first impression.\textsuperscript{17} With this interpretation for the asymmetry in $\eta$, it may be possible that the challenger captures the strong position with a higher chance of winning if the headstart advantage is not too severe.

Finally, if the incumbent’s initial voter disposition advantage is small, $\alpha \in (0, \frac{\eta_2 - \eta_1}{\bar{\beta}_1})$, without a spending limit the spending effectiveness advantage of the challenger overwhelms the head-start advantage and the efficiency in fundraising advantage of the incumbent (see Lemma 1). In this case, without a spending limit, the challenger has a higher probability of winning. This violates the empirical regularity that incumbents tend to have a higher probability of victory than challengers when there is no spending limit.
5. Relation to Empirical Evidence

In countries with spending limits the debate on political campaign spending limit legislation is revived repeatedly prior to each election. Even in the U.S. where mandatory spending limits were struck down by the Supreme Court, the effect of expenditure limits is of interest. Public financing of political campaigns in the U.S. has taken a new lease on life with the movement loosely grouped under the banners “Clean Elections” or “Fair Elections.” Candidates who choose to participate the Clean Elections initiative avail of public funds but are subject to spending ceilings. If the Californian Voluntary Campaign Spending Limit Legislation is ratified via referendum in 2010, more than a quarter of the U.S. population will be living in states with voluntary caps on campaign expenditures. In this section we discuss the effects of spending limits and compare the implications of the model with findings in the empirical literature.

5.1. Do limits help level the playing field?

Candidates may differ in their resources due to varying degrees of access to fundraising. The main stated aim of campaign spending limit legislation is to create a political system where candidates with lesser resources can compete on an equal footing. However our model does not lend support to this argument as long as the challenger is not more effective in campaign spending; Irrespective of the identity of the candidate with more efficient fundraising technology, spending limits benefit the incumbent with the initial voter disposition advantage (Result 1).

But if the challenger is more effective in campaign spending, the limit may help the challenger as long as the limit is not too restrictive. In the absence of a limit, if the initial voter disposition advantage of the incumbent is mild, the incumbent may need and be able to spend more than the challenger in order to overcome challenger’s effective spending. The imposition of a barely binding limit then first restricts the incumbent while not effectively restricting the challenger (Result 3).

Employing data on Canadian Federal elections, Milligan and Rekkas (2008) show that smaller limits tend to lead to closer elections. This is encouraging, however our model suggests that transfer of policy recommendations from one political environment to the next may be problematic. The effect of limits depends on the political institutional framework. In parliamentary systems with party discipline (such as in Canada, Brazil etc.) often incumbents do not enjoy the same degree of name recognition as U.S. Senators, see Samuels (2001). Hence while in Canada limits may help challengers, in the U.S. they might benefit incumbents.
5.1.1. Probability of Victory

Milligan and Rekkas (2008) find that campaign spending limits do not affect the probability of victory in any meaningful magnitude. This empirical finding is not inconsistent with our model since the model suggests that in some races the spending limit will give a boost to the incumbent while in others it can strengthen the electoral prospects of the challenger (Results1-3) because the degrees of asymmetries between the candidates tend to vary across different races.

5.1.2. Large versus Small Parties

A policy report by Phillips (2007) on strengthening democracy commissioned by former British Prime Minister Tony Blair suggests that "[l]owering the national expenditure limit for campaigning may help small and new parties to compete with the two principal established parties.” The model may help shed some light to this discussion. If the larger party can be assumed to enjoy a head-start advantage in initial voter disposition, the smaller party may need to spend more in order to win the election. In contrast to the claim above, Results 1 and 2 suggest that a cap on campaign expenditure may in fact benefit the larger party rather than the smaller party. Furthermore Result 3 suggests that even if the existing cap benefits smaller parties, it does not necessarily follow that further lowering the limit will have the same effect.

5.2. Do limits reduce the time and effort spent for fundraising?

Running for elections is an expensive endeavor. The need to raise funds may take time away from other duties and raises the concern that legislative outcomes may be driven by money. If limits can help reduce expected campaign spending, they may help to improve the quality of democracy. Gross, Goidel and Shields (2002) find that in the U.S. from 1978 to 1997 in Gubernatorial elections, the existence of voluntary spending caps accompanied by public funding reduced the expected spending of both the incumbent and the challenger. Palda and Palda (1985) employs cross sectional data from 95 constituencies of Ontario in the 1979 Canadian Federal Elections. The study finds that a $1 increase in the limit leads to a $0.58 increase in candidate expenditure.

While empirical evidence seems to be in favor of limits due to a reduction in campaign spending, the implications of the model call for caution. The effect of a limit on expected spending is shown to be non-monotonic (Figure 4) when the initial voter disposition advantage of the incumbent is mild. A very strict limit can invite more aggressive spending of the incumbent compared to a higher ceiling. When the limit levels the playing field it can lead to fiercer competition. Contrary to one of the intended consequences of spending limits, this may yield increased effort for fundraising and opens the door wider for monied interest’s influence in policy making.
6. Conclusion

The model makes a number of simplifying assumptions on voter behavior as well as on the information structure of the contestants. However as long as one candidate as a higher supremum spending in equilibrium (either in pure or mixed strategies), he would be the first to be restricted by the introduction of a barely binding cap whereas the rival would not be effectively constrained. Hence the rival would benefit from the imposition of a barely-binding cap. The identity of the candidate with higher no-cap equilibrium spending support necessarily depends on the relative effectiveness of candidates’ spending and the degree of initial voter disposition advantage, as in this model.

Appendix

Here we consider cases where there is competition in equilibrium: \( \alpha < \eta_2 / \beta_2 \) and \( k > \alpha / \eta_2 \). We also omit the non-generic case where \( k = M(k) \leq \min(1 / \beta_1, 1 / \beta_2) \). Let the notation \( S \) and \( W \) refer to the strong and weak candidates respectively.

Claim 1. Candidate \( W \) does not put a probability mass point on any level of spending greater than zero. Candidate \( S \) does not put a probability mass point on any level of spending \( a_S \in (0, M_S(\bar{a}_W)) \). There is no equilibrium in pure strategies.

Proof: Suppose that \( W \)‘s lowest mass point in \( (0, \bar{a}_W) \) is \( a' \). \( M_S(a') > \bar{a}_S \) conflicts with the definition of the weak candidate. If \( M_S(a') < \bar{a}_S \) or if \( M_S(a') = \bar{a}_S \) and \( k > 1 / \beta_S \) then \( S \) would not spend \( M_S(a') \) or in the open interval below it as a slight increase in spending to just above \( M_S(a') \) would result in a discrete increase in his probability of winning. Therefore \( W \) could decrease spending slightly from \( a' \) with no decrease in his probability of winning.

The only remaining possibility is that \( M_S(a') = \bar{a}_S = k \). If \( a' < \bar{a}_W < k \) then \( M_W(\bar{a}_S) < \bar{a}_W \) a contradiction of the definition of \( S \). Suppose that \( a' = \bar{a}_W = 1 / \beta_W < k \). \( S \) will put no mass in the open interval below \( k \) as moving it up to \( k \) results in a discrete increase in the probability of winning. If \( S \) put no probability mass at \( k \) then \( W \) could reduce spending from \( a' \) with no loss. If \( S \) did put mass at \( k \) then \( W \) would not win with certainty at \( a' = 1 / \beta_W \) and so it would result in a negative expected payoff. \( a' = \bar{a}_W = k \) results in the non-generic case where \( k = M_k(k) \) and is therefore omitted.

The symmetric argument establishes that \( S \) can have no mass point on \( a' \in (0, M_S(\bar{a}_W)) \).

Candidate \( W \) cannot have a mass point on any positive level of spending. If in a pure-strategy equilibrium \( W \) had zero spending. \( S \)’s optimal response would be \( a'_S = \max(0, M_S(0) + \varepsilon) \). However in this case \( W \) would prefer \( M_W(a'_S) + \varepsilon > 0 \). Both of these levels of spending are affordable and possible since \( \alpha > \eta_2 / \beta_2 \) and \( k > \alpha / \eta_2 \). Hence there is no equilibrium in pure strategies. \( \square \)

Claim 2. Candidate \( 2 \) puts zero probability on \( a_2 \in (0, \alpha / \eta_2] \).

Proof: Candidate 2 will not choose spending of \( a_2 \in (0, \alpha / \eta_2] \) as zero spending wins with the same probability. Candidate 2 can win with \( a_2 = \alpha / \eta_2 \) only if \( a_1 = 0 \). Either this chance is small enough that his expected value is negative, in which case he would prefer zero spending, or a slight increase in his spending would result in a discrete increase in his probability of winning. \( \square \)
Claim 3. Candidate $W$ has an infimum spending level of zero, $a_W^{\inf} = 0$ and $EV_W = 0$.

Proof: Suppose $a_W^{\inf} > 0$. If $M_\bar{S}(a_W^{\inf})>0$ then Candidate 1 would never choose $a_S \in (0, M_\bar{S}(a_W^{\inf}))$ as $S$ would be putting in positive spending and would lose for sure since the probability of $W$ having spending of $a_W^{\inf}$ is zero by Claim 1. Claim 1 also implies that the probability of $S$ choosing exactly $M_\bar{S}(a_W^{\inf})$ is zero, therefore $W$ could lower his spending without changing his probability of winning. If $M_\bar{S}(a_W^{\inf})<0$ then spending of zero would give $W$ the same probability of winning as the conjectured $a_W^{\inf} > 0$, a contradiction. If $M_\bar{S}(a_W^{\inf})=0$ then $S=I$ and $a_W^{\inf} = \alpha / \eta_2$. By Claim 2 the probability of 2 having spending of $\alpha / \eta_2$ is zero. Hence if $a_W^{\inf} = \alpha / \eta_2$ Candidate 2 is mixing in the open interval above $\alpha / \eta_2$. In this case 1 would put no probability at zero spending by the above argument and by Claim 1 Candidate 1 has no mass point on $(0, \epsilon)$. Hence 2’s probability of winning with spending of $M_2(\epsilon) = (\alpha / \eta_2)^+$ is approximately zero for small $\epsilon$. So with spending $M_2(\epsilon)$ Candidate 2 is putting in positive spending for a negligible probability of winning, hence $a_W^{\inf} \neq \alpha / \eta_2$. $a_W^{\inf} \in (0, \alpha / \eta_2)$ is not possible by Claim 2 so 2’s infimum spending must be zero. So in all cases $a_W^{\inf} = 0$.

Zero is in the support of W’s mixed strategy. If $W=2$ then he loses with certainty with that spending so $EV_W = 0$. If $W=1$ then, from the definition of $W$, $\bar{a}_1 < M_1(\bar{a}_2)$, so 2 will not choose zero spending as he would lose for sure and he can guarantee victory and a positive payoff with spending of $M_2(\bar{a}_1)$. By Claim 2 there is zero probability of $a_2 \in (0, \alpha / \eta_2]$ so 1 has zero probability of winning with zero spending. Since $a_W^{\inf} = 0$ and $W=1$, $EV_W = 0$. \(\square\)

Claim 4. Candidate $W$ has a supremum spending of $a_W^{\sup} = \bar{a}_W$ while Candidate $S$ has a supremum spending of $a_S^{\sup} = M_S(\bar{a}_W)$ and $EV_S = 1 - \beta_S M_S(\bar{a}_W) \geq 0$, with equality only if $\bar{a}_1 = M_1(\bar{a}_2)$.

Proof: $a_W^{\sup} = 0$ is not possible by Claim 1. If $a_W^{\sup} \in (0, \bar{a}_W)$ then $S$ would never set $a_S > \max(0, M_S(a_W^{\sup}))$ since $S$ can win for sure with that spending as the probability of W choosing $a_W^{\sup}$ is zero by Claim 1. Therefore $W$ could win for sure with spending $a_W^{\inf} + \epsilon$ yielding a positive payoff for small enough $\epsilon$, a contradiction of Claim 3. Likewise $a_S^{\sup} < M_S(a_W^{\sup})$ allows candidate $W$ an opportunity to guarantee a positive payoff and hence contradicts Claim 3. Candidate S can win for sure with spending of $M_S(a_W^{\sup})$ so $a_S^{\sup} = M_S(\bar{a}_W)$ and $EV_S = 1 - \beta_S M_S(\bar{a}_W)$. By the definition of $a_S$ this is strictly greater than zero whenever $\bar{a}_S > M_S(\bar{a}_W)$ and equal to zero if $\bar{a}_S = M_S(\bar{a}_W)$. \(\square\)

Claim 5. For Candidate 1 spending levels almost everywhere on $a_1 \in (0, a_1^{\sup})$ and for Candidate 2 spending levels almost everywhere on $a_2 \in (\alpha / \eta_2, a_2^{\sup})$ must have positive probability.

Proof: Suppose there were an interval $(t, v)$ in $(\alpha / \eta_2, a_2^{\sup})$ where Candidate 2 had zero probability of spending in $(t, v)$. Then 1 would have zero probability of spending on $(M_1(t), M_1(v))$ since he could lower his spending to $M_1(t)$ and have the same chance of winning by Claim 1. But in this case 2 would never have spending of $v + \epsilon$ as he could lower his spending to $t$, saving $v + \epsilon - t$ in spending and losing only $F_1(M_1(v + \epsilon)) - F_1(M_1(v))$ in probability of winning. By Claim 1 this loss in probability is negligible for small $\epsilon$. So if there were an interval of zero probability it must go all the way up to $a_2^{\sup}$, which contradicts Claim 4. A symmetric argument rules out ranges of zero probability for Candidate 1 on $(0, a_1^{\sup})$. \(\square\)

Proof of Lemma 1

From Claim 4, $a_W^{\sup} = \bar{a}_W$ and $a_S^{\sup} = M_S(\bar{a}_W)$. With any non-binding cap $\bar{a}_W = 1 / \beta_W$ hence a non-binding cap requires $k > \max(1 / \beta_W, M_S(1 / \beta_W))$. S=2 requires $\bar{a}_1 < M_1(\bar{a}_2)$ which implies $M_2(\bar{a}_1) < \bar{a}_2$. So from equation (2), $M_2(1 / \beta_1) < 1 / \beta_2$. Equation (3) yields $\alpha < \frac{\eta_1}{\beta_2} - \frac{\eta_2}{\beta_1}$ which
completes part (a). $S=1$ requires $\bar{a}_1 \geq M_1(\bar{a}_2)$ so from (2), $1/\beta_1 \geq M_1(1/\beta_2)$. Equation (3) yields $\alpha \geq \frac{\beta_2}{\beta_1} - \frac{\eta_1}{\beta_1}$ which completes part (b). □

Proof of Lemma 2

From Claim 4, $a_{S}^{\sup} = M_S(\bar{a}_W)$ and $a_w^{\sup} = \bar{a}_W = \min(1/\beta_w, k)$. Suppose $a_{S}^{\sup} = \bar{a}_W = k$, hence $a_{S}^{\sup} = M_S(k)$. This must be less than $k$ by the definition of $S$. Thus for $S=2$ in equilibrium, $M_2(k) < k$ which yields $(\eta_2 - \eta_1)k > \alpha$ which gives part (b). For $S=1$ in equilibrium $M_1(k) < k$, noting that we are not considering the non-generic special case where $M_1(k) = k$, which yields $(\eta_2 - \eta_1)k < \alpha$ which gives the result in part (a) as long as $k > 1/\beta_w$.

If $k > 1/\beta_w$ then for the cap to be binding $a_{S}^{\sup} = M_S(1/\beta_w) = k \leq 1/\beta_S$. From the definition of $S$, $\bar{a}_S \geq M_S(\bar{a}_W)$ with equality only if $S=1$. Hence the only possibility for a binding cap with $k > 1/\beta_w$ is $k = M_1(1/\beta_2)$, in which case $\bar{a}_1 = M_1(\bar{a}_2)$. Therefore $k = \left(\frac{\eta_1}{\beta_1} - \alpha\right) / \eta_1 > \frac{1}{\beta_1}$ which implies $\alpha < (\eta_2 - \eta_1) \frac{1}{\beta_2} < (\eta_2 - \eta_1)k$ which completes the proof of part (a). □

Proof of Proposition 1

Claims 2-5 demonstrate that in equilibrium 2 is indifferent among all spending levels almost everywhere on $(0, 1] \cup (\alpha/\eta_2, \bar{a}_2]$ and 1 is indifferent among spending levels almost everywhere on $[0, M_1(\bar{a}_2)]$.

$E_{V_2} = 0$ by Claim 3. On $a_2 \in (\alpha/\eta_2, \bar{a}_2)$ Candidate 2 wins with probability $F_1(M_1(a_2))$ as there is zero probability that $a_1 = M_1(a_2)$ by Claim 1. So indifference of 2 in that range implies $F_1(M_1(a_2)) - \beta_2 a_2 = 0$. This yields $F_1(a_1) = \beta_2(a + \eta_1 a_1) / \eta_2 \forall a_1 \in [0, M_1(\bar{a}_2)]$.

Hence 1 has a probability mass of $\alpha \beta_2 / \eta_2$ at zero and a mass in the open interval above $M_1(\bar{a}_2)$ of $1 - \beta_2 \bar{a}_2$. Note that this mass is zero if the cap is not binding. $E_{V_1} = 1 - \beta_1 M_1(\bar{a}_2) \geq 0$ by Claim 4. On $a_1 \in (0, M_1(\bar{a}_2))$ candidate 1 wins with probability $F_2(M_2(a_1))$ as there is zero probability that $a_2 = M_2(a_1)$ by Claim 1. So the indifference of 1 in that range implies $F_2(M_2(a_1)) - \beta_1 a_1 = 1 - \beta_1 M_1(\bar{a}_2)$. This yields $F_2(a_2) = 1 + \beta_1 \eta_2 (a_2 - \bar{a}_2) / \eta_1 \forall a_2 \in (\alpha/\eta_2, \bar{a}_2]$. 2 has a probability mass of $1 - \beta_1 \eta_2 \bar{a}_2 / \eta_1$ at zero spending. By Claim 2 Candidate 2 puts zero probability on $(0, \alpha/\eta_2)$.

Part (a) Expected Value: Given by Claim 3 and Claim 4.

Part (b) Expected Spending: On $a_2 \in (\alpha/\eta_2, \bar{a}_2]$ the p.d.f. of 2’s spending is $f_2(a_2) = \beta_1 \eta_2 / \eta_1$. Hence his expected spending is $E(a_2) = \int_{\alpha/\eta_2}^{\bar{a}_2} f_2(x) x \, dx$. On $a_1 \in (0, M_1(\bar{a}_2)]$ the p.d.f. of 1’s spending is $f_1(a_1) = \beta_2 \eta_1 / \eta_2$ and 1 has a probability mass in the open interval above $M_1(\bar{a}_2)$. Hence his expected spending is $E(a_1) = \int_{0}^{M_1(\bar{a}_2)} f_1(x) x \, dx + \int_{M_1(\bar{a}_2)}^{1} [1 - F_1(M_1(\bar{a}_2)) - M_1(\bar{a}_2)] \, dx$.

Part (c) Probability Candidate 2 wins: In equilibrium there is zero probability of ties where $a_2 = M_1(a_2)$ by Claim 1 so the probability that 2 wins is given by $P_{22} = \int_{0}^{\bar{a}_2} F_1(M_1(x)) f_2(x) \, dx$. □

Proof of Proposition 2

Claims 2-5 demonstrate that in equilibrium 2 is indifferent among all spending levels almost everywhere on $(0, \alpha/\eta_2, M_2(\bar{a}_1)]$ and 1 is indifferent among spending levels almost everywhere on $[0, \bar{a}_1]$. $E_{V_1} = 0$ by Claim 3. On $a_1 \in [0, \bar{a}_1]$ Candidate 1 wins with probability $F_2(M_2(a_1))$ as there is zero probability that $a_2 = M_2(a_1)$ by Claim 1. So indifference of 1 in that range implies $F_2(M_2(a_1)) - \beta_1 a_1 = 0$. This yields $F_2(a_2) = \beta_1 (\eta_2 a_2 - \alpha) / \eta_1 \forall a_2 \in (\alpha/\eta_2, M_2(\bar{a}_1)]$. Hence 2 has no probability on $(0, \alpha/\eta_2]$ and a probability mass in the open interval above $M_2(\bar{a}_1)$ of $1 - \beta_1 \bar{a}_1$. Note that this mass is zero if the cap is not binding. $E_{V_2} = 1 - \beta_2 M_2(\bar{a}_1) \geq 0$ by Claim 4. On $a_2 \in (\alpha/\eta_2, M_2(\bar{a}_1))$ Candidate 2 wins with probability $F_1(M_1(a_2))$ as there is zero probability that $a_1 = M_1(a_2)$ by Claim 1. So the indifference of 2 in that range implies $F_1(M_1(a_2)) - \beta_2 a_2 = 1 - \beta_2 M_2(\bar{a}_1)$. This yields $F_1(a_1) = 1 + \beta_1 \eta_2 (a_1 - \bar{a}_1) \forall a_1 \in [0, \bar{a}_1]$. 1 has a probability mass of $1 - \beta_2 \eta_1 \bar{a}_1 / \eta_2$ at zero spending.

Part (a) Expected Value: Given by Claim 3 and Claim 4.
Part (b) Expected Spending: On \( a_2 \in (\alpha / \eta_2, M_2 (\bar{a}_1)) \) the p.d.f. of \( 2 \)'s spending is 
\[ f_2(a_2) = \beta_1 \eta_2 / \eta_1 \] and he has a probability mass in the open interval above \( M_2 (\bar{a}_1) \). Hence his expected spending is 
\[ E(a_2) = \int_{\bar{a}_1}^{M_2 (\bar{a}_1)} f_2(x) \, dx + [1 - F_2 (M_2 (\bar{a}_1))] M_2 (\bar{a}_1). \] On \( a_1 \in (0, \bar{a}_1] \) the p.d.f. of \( 1 \)'s spending is 
\[ f_1(a_1) = \beta_2 \eta_1 / \eta_2. \] Hence his expected spending is 
\[ E(a_1) = \int_{0}^{\bar{a}_1} f_1(x) \, dx. \] □

Proof of Result 1
Since \( \eta_2 < \eta_1 \), by Lemma 2 with a binding cap Proposition 2 applies and hence \( \delta E_{\bar{a}_1} / \delta k < 0 \) and \( \delta \text{prob}_i / \delta k > 0 \). By Lemma 1, if \( \alpha \in \left( \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1}, \frac{\eta_2}{\beta_2} \right) \) then Proposition 1 applies without a cap as well and hence Proposition 1’s subsections also yield the result for the initial imposition of the cap.

If \( \alpha < \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \), which can occur if the challenger has a fundraising advantage, then by Lemma 1 without a cap Proposition 2 applies and hence the imposition of a cap switches the equilibrium from Proposition 2 to Proposition 1. Nevertheless, from the subsections of the propositions, without a cap \( \text{EV}_1 = 0 \) while with a binding cap \( \text{EV}_1 \geq 0 \). Without a binding cap \( \bar{a}_1 = 1 / \beta_1 \) and hence \( \text{prob}_1 = 1 - \frac{1}{2} \bar{a}_1 \eta_1 / \eta_2 \beta_1 \) while with a binding cap \( \bar{a}_2 = k \) so \( \text{prob}_1 = 1 - \frac{1}{2} \bar{a}_2 \eta_1 / \eta_2 \beta_1 \left( k^2 - \left( \frac{\alpha}{\eta_1} \right)^2 \right) \). Since \( \eta_2 < \eta_1, M_2 (1 / \beta_1) \geq 1 / \beta_1 \) so from Lemma 1 a barely binding cap has \( k = M_2 (1 / \beta_1) \). From that and the fact that \( \alpha < \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \) the incumbent’s probability of victory is higher with a barely binding cap than with no cap on campaign spending. □

Proof of Result 2
Since \( \beta_1 < \beta_2, \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} > \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \) so \( M_1 (1 / \beta_2) < 1 / \beta_2 \). From Lemma 1 part (b) a binding cap has \( k \leq 1 / \beta_2 \). Hence for any binding cap \( \alpha > (\eta_2 - \eta_1) k \) so Lemma 2 implies that Proposition 1 applies. In the absence of a binding cap \( \bar{a}_1 = 1 / \beta_1 \) and \( \bar{a}_2 = 1 / \beta_2 \). Therefore \( \alpha > \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \) implies \( \bar{a}_1 > M_1 (\bar{a}_2) \) and hence Proposition 1 applies without a cap as well. The subsections of Proposition 1 yield the results. □

Proof of Result 3
In the absence of a binding cap \( \bar{a}_1 = 1 / \beta_1 \) and \( \bar{a}_2 = 1 / \beta_2 \). Therefore \( \alpha \geq \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \) implies \( \bar{a}_1 \geq M_1 (\bar{a}_2) \) and hence Proposition 1 applies without a cap.

(a) The introduction of a barely binding limit. From Lemma 1 part (b) a barely binding cap has \( k = \max (1 / \beta_2, M_1 (1 / \beta_2)) \). Since \( \alpha < (\eta_2 - \eta_1) / \beta_2, k = M_1 (1 / \beta_2) = (\frac{\eta_2}{\beta_2} - \alpha) / \eta_1 \). Therefore \( \bar{a}_1 = k, \bar{a}_2 = 1 / \beta_2 \) and \( \bar{a}_1 < M_1 (\bar{a}_2) \) so with a barely binding cap Proposition 2 applies. Plugging these values into the results in the subsections of Proposition 1 and Proposition 2 and noting that \( \alpha > \frac{\eta_2}{\beta_2} - \frac{\eta_1}{\beta_1} \) proves that the introduction of a barely binding limit leads to a jump down in the probability that the incumbent wins and a jump up in the expected total campaign spending.

(b) Making a moderate limit, \( k \in \left( \frac{\alpha}{\eta_2 - \eta_1}, \frac{1}{\eta_1} \left( \frac{\eta_2}{\beta_2} - \alpha \right) \right), \) more restrictive. From the proof of part (a) and Lemma 2 in this range of \( k \) Proposition 2 applies so its subsections and noting that \( \bar{a}_1 = k \), establishes that in this range of \( k \), making the limit more restrictive decreases the probability that the incumbent wins and decreases expected total spending.

(c) Reducing the limit from \( \left( \frac{\alpha}{\eta_2 - \eta_1} \right)^+ \) to \( \left( \frac{\alpha}{\eta_2 - \eta_1} \right)^- \). From the proof of part (a) and the fact that \( \alpha < (\eta_2 - \eta_1) / \beta_2 \) the cap is binding for \( k = \left( \frac{\alpha}{\eta_2 - \eta_1} \right)^+ \). From Lemma 2 when \( k > \alpha/(\eta_2 - \eta_1) \) Proposition 2 applies and \( \bar{a}_1 = k \) while when \( k < \alpha/(\eta_2 - \eta_1) \) Proposition 1 applies and \( \bar{a}_2 = k \). Taking left limit of the results in the subsections of Proposition 2 and the right limit of the results in the
subsections of Proposition 1 as \( k \to \alpha/(\eta_2 - \eta_1) \) and noting that \( \alpha < (\eta_2 - \eta_1) / \beta_2 \) and \( \beta_1 < \beta_2 \) establishes that reducing the limit from \( \left( \frac{\alpha}{\eta_2 - \eta_1} \right)^+ \) to \( \left( \frac{\alpha}{\eta_2 - \eta_1} \right)^- \) leads to an increase in the probability that the incumbent wins and leads to an increase in the incumbent’s expected campaign spending. For all \( k < \left( \frac{\alpha}{\eta_2 - \eta_1} \right) \), Proposition 1 applies so its subsections show that further decreases in \( k \) will benefit the incumbent. □

Notes

1 See Seymour (1970).

2 See Walecki (2007).

3 There is some indirect support for the argument. See Evans (2007) and Bender (1988) in the Canadian and U.S. context, respectively.

4 In the U.S. the Federal Election Campaign Act (FECA, 1974) provided for ceilings on campaign expenditures in Presidential, Senate and House elections. In 1976 however, the U.S. Supreme Court overruled Section 608(c) of the FECA and deemed expenditure limits to be unconstitutional (Buckley v. Valeo).


6 While this structure lends itself to useful interpretation, any continuous distribution of tastes would yield the same type of competition between candidates. All that is needed is a median voter model where voters can be influenced by campaign spending and one candidate has a potential head-start advantage. Meirowitz (2008) generates the same game form from a different underlying model.

7 Campaign spending may convey information to the electorate, either directly or indirectly through signaling. In Soberman and Sadoulet (2007) campaign advertising provides direct information about the valence of the candidate à la Butters (1977). The left and right wing candidates are symmetric in all respects. Hence there are no implications for incumbency advantage. In Prat (2002) campaign spending provides indirect information to voters and campaign finance restrictions result in less informed voters. Therefore limits hurt incumbents that have high valence and benefit incumbents that have low valence. In multi-candidate races campaign spending may also serve a coordinating function, as in Pastine and Pastine (2002).

8 Incumbents may be able to use resources from their office to campaign for reelection. These resources include mailing privileges, the right to weekly trips to their constituencies, regular publications informing voters on policy actions or plans for the future. These appear in the expense account of the office and are not included in the accounting of campaign expenditure. These privileges effectively imply that there are two different
levels of spending cap for the incumbent and the challenger. But in this paper we will abstract from this issue.

9 This particular case is non-generic in the sense that for any given level of \( k \), if \( \alpha, \eta_1 \) and/or \( \eta_2 \) are drawn from continuous distributions there is zero probability of the case occurring. Nevertheless it is interesting theoretically. In this non-generic case if both players spend the maximum permitted amount they tie and the contest is decided by lottery. Hence the equilibrium mirrors the equilibrium in Che and Gale (1998) and can be solved using their approach. If the cap is binding but not too restrictive the players play mixed strategies with a mass point at the cap and zero probability for a range just below the cap. However if the cap is very restrictive both players use pure strategies of spending the maximum permissible amount and hoping to win the lottery.

10 In the UK 2001 general elections, the Conservative and the Unionist parties each spent 83 percent of the spending limit. The Labour Party spent 71 percent of the limit, see Walecki (2007). Combining all Canadian federal races in the 1997 and 2000 election years, Milligan and Rekkas (2008) report that 89 percent of all candidates and 66 percent of incumbents spent less than 90 percent of their spending limit.

11 In Levitt (1994) alternative specifications of spending give statistically significant results for challenger spending. A one percent increase in challenger spending yields about one percent increase in vote share. However incumbent spending is insignificant both in levels and in logs.

12 Using Congressional election data from 1978 American National Election Study, Jacobson (1981) reports that 50 percent of the sampled voters recall the incumbent’s name, while only 17 percent of the voters recall the challenger’s name. 40 percent of the voters claim that family or friends had contact with the incumbent. The same figure for the challenger is only 11 percent. In the 1997 and 2000 Canadian General Elections, Milligan and Rekkas (2008) find that incumbents enjoy an 8 percent vote share advantage having controlled for campaign spending.

13 See the discussion in the introduction. Additionally, Erikson and Palfrey (1998) and Samuels (2001) show that the effectiveness of increased incumbent spending declines with seniority, giving further evidence for the hypothesis that the marginal benefit of spending declines with name recognition. However, for Brazil Samuels (2001) finds that incumbents and challengers gain equally from campaign spending. Samuels (2001) attributes this to the Brazilian political and institutional context which allows the incumbent to gain very little name recognition from holding office.

14 Palda (1992) shows that the larger the government wealth in control of the politician and the more power the politician has over the state budget, the more money the politician raises for his campaign. Hall and Wayman (1990) shows that politicians with positions of power in congressional committees are better fundraisers.
15 Data comes from http://www.punditsguide.ca/files/Incumbency_Table1.html and http://www.opensecrets.org/bigpicture/reelect.ph. In Brazil since 1945 in democratic elections 69 percent of incumbents who ran for reelection were victorious, Samuels (2001).

16 We thank Todd Kaplan for bringing this point to our attention.

17 The 2008 primary race between Senators Hillary Clinton and Barack Obama may be a good example of this.

18 The Clean Elections movement has been gaining momentum in recent years putting serious pressure on candidates. Clean Elections have been adopted by Arizona, Connecticut, Maine, Massachusetts, New Mexico, Vermont, North Carolina, and in the cities of Albuquerque, New Mexico and Portland, Oregon. Maine was the first state to pass a Clean Elections Law which went into effect in 2000. In 2008, 85 percent of successful candidates accepted voluntary spending limits. Connecticut is the first state to enact Clean Elections for all state offices. In the first run in 2008, 80 percent of the winners were Clean Elections candidates. In legislative elections, the percentage of incumbents who opted for Clean Elections where available was 51 percent in 2002, 76 percent in 2004 and 82 percent in 2006 (see http://www.commoncause.org).

19 However the limit varied across constituencies and was set based on expected variation in travel cost and mailing expenses. Hence it is hard to interpret this result.

20 See Che and Gale (1998) and Pastine and Pastine (2009) for similar intuition for contribution caps. Stricter contribution caps can lead to an increase in expected contributions.

References


