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**Static and dynamic connectivity in bed-scale models of faulted and unfaulted turbidites**

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**Abbreviated title:** Connectivity in faulted turbidites

**Abstract:** A range of unfaulted and faulted bed-scale models with sheet-like or lobate bed geometries and faults of comparable sizes to beds have been built and analysed in terms of bed connectivity and fractional permeability assuming permeable sands and impermeable shales and shale smears. A new method has been devised allowing amalgamation ratio to be included explicitly as model input and this property, rather than net:gross ratio, is found to be the dominant control on the connectivity of unfaulted sequences. At the geometrically representative scales considered (horizontal distances of > 1km for beds up to ca. 1m thick and faults up to ca. 5m throw), faulted sequences rarely have lower connectivities than their unfaulted sedimentological equivalents irrespective of whether fault rock properties are included. Models containing stochastically placed shale smears associated with each faulted shale horizon are generally better connected than if deterministic Shale Gouge Ratio cut-offs are applied. Despite the complex interactions between geological input and connectivity of the faulted sequences, the flow properties at representative scales are controlled by three geometrical variables describing connectivity, anisotropy and resolution. If two different faulted or unfaulted systems have identical values of these three variables they will have the same equivalent flow properties. **End of Abstract**
The objective of this work is to investigate controls on the flow characteristics of faulted turbidite reservoirs. Many turbidite reservoirs contain sandstones interbedded with low permeability shales, and in such cases the flow properties of the rock volume depend to a large extent on the connectivity of the sandstone beds. It has been suggested that for situations in which the shales can be assumed impermeable, many flow characteristics can be estimated very rapidly using semi-analytical scaling laws derived from percolation theory (e.g. King 1990; King et al. 2002; see also Stauffer and Aharony 1994; Sahimi 1995 for background discussion on percolation theory and its applications to flow). The important system parameters for this approach are a connectivity measure, which establishes how close the network of sandstone beds is to the percolation threshold; one or more anisotropy terms, which establish flow path tortuosity in different directions; and one or more resolution terms, which establish how closely the scale of interest (e.g. the inter-well spacing in a particular field development plan) approximates to the infinite systems for which results from percolation theory apply strictly. The central tenet to this approach is that the geological details of the system are not relevant per se, but only in as much as they contribute to the three parameters mentioned. In this study we examine faulted and unfaulted bed-scale models of idealised sheet-like turbidite geometries, and discuss geological controls on connectivity as well as whether and how these three more fundamental geometrical terms can be defined in anything other than the most simplistic idealisations of the reservoir geology. A glossary of the terminology used in this paper is given in the appendix.

The first part of the paper concerns connectivity in unfaulted thin-bedded sheet-like or lobate bed geometries (e.g. Fig 1), a correct representation of which is recognised as a significant challenge in turbidite modelling (e.g. Weimer et al. 2000; Browne & Slatt 2002). Despite strong vertical heterogeneity, these systems often appear laterally homogeneous at an outcrop scale owing to the high horizontal to vertical bed anisotropy. A compilation of width to thickness measurements from sheet-like systems (Fig 2a) indicates that turbidite deposits at all hierarchical scales from individual beds to complete systems are typically about 200 times longer than they are thick (+/- a factor of 10). For systems of beds of ca. 1m thickness or less, therefore, bed connectivity is a more significant control on inter-well flow (i.e. flow at length-scales of hundreds of meters to a few kilometres) than is the internal permeability distribution of the beds, provided the latter is small compared to the
permeability contrast between beds and shales. Bed connectivity can be recognised at outcrop as amalgamation surfaces (e.g. Fig 1b), and in this study we make extensive use of the amalgamation ratio (AR; Chapin et al. 1994) as a connectivity measure. Departing slightly from the ambiguous definition of Chapin et al., we define amalgamation ratio as the fraction of sandstone bed bases that are amalgamated with the underlying sandstone bed when measured on a line sample. As noted by Stephen et al. (2001), this is equivalent to the total length (or area) of amalgamation normalised by the total length (or area) of sandstone bed bases in 2D (or 3D), and therefore is analogous to the “connectedness ratio” measured in the process-based fluvial models of Mackey & Bridge (1995) and Karseenberg et al. (2001).

In the second part of the paper we examine the influence of faults on bed connectivity within geometrically representative systems. Previous studies (e.g. Bailey et al. 2002; James et al. 2004) have shown that the influences of fault system characteristics on connectivity are intimately tied to sedimentological characteristics, a recurring issue in our analyses. The focus of the analyses are on faults of comparable sizes to the principal sedimentological length-scales. Fault lengths and maximum fault throws range from a few times smaller to a few times larger than the length and thickness of the beds respectively. As we are principally considering beds thinner than ca. 1m, the faults are sub-seismic with maximum fault throws up to ca. 5m. The effects of fault rock properties are included using Shale Gouge Ratio cut-offs and by explicit stochastic shale smear modelling. In common with the assumption of a binary permeable / impermeable sedimentological system, we assume across-fault sand-on-sand juxtapositions are either permeable in the absence of a shale smear, or impermeable where one or more is present. We therefore do not address permeability decreases caused by cataclastic fault rock.

Much of our modelling has been inspired by the faulted turbidites of the Miocene Mt Messenger Formation, Taranaki, New Zealand (e.g. King et al. 1993; Browne et al. 1996; Browne & Slatt 2002; Childs et al. this volume). The characteristics of both the sediments and faults examined in the modelling are not, however, constrained to those observed in the field area since our aims are more general than a specific understanding of the connectivity within this particular system.
Modelling procedures – unfaulted systems

In the most direct idealisation of a sedimentological system with the kind of geometrical networks investigated by researchers in percolation theory, the net:gross ratio ($NTG$) of the sandstone beds takes the role of the volume fraction of a continuum percolation system (e.g. King 1990; King et al. 2001). A continuum percolation system is conceptually equivalent to an unconditioned object-based model in which beds, sampled from a particular size distribution, are placed at random within the modelling volume. The principal difference is that conventional object based models are discrete (i.e. each bed occupies a particular number of cells in each direction). This discretisation introduces systematic connectivity biases which depend on the number of cells occupied by each bed in each direction and are beyond the scope of this work. A more significant issue is what modifications must be made to the idealisation of a sedimentological system as a continuum percolation model in order that known percolation results can be applied.

Figure 3a shows a vertical cross-section (redrawn from Browne et al. 1996) of thin-bedded sandstones interpreted as compensationally-offset (sensu Mutti & Sonnino 1981) lobe-fringe deposits from the shoreline cliff exposures of the Mt Messenger Formation at Tongapurutu, New Zealand (e.g. Fig 1). The 230m long and 4m thick section is based on correlating sandstone beds between 8 detailed vertical reference stations with close attention paid to the nature of bed contacts. Many of the beds terminate within the mapped section and, despite a relative high $NTG$ (0.65), only 20% of the sandstone beds can be followed across the section, while every shale bed is continuous. Significantly, no amalgamation of sandstone beds is present in this section. Figure 3b shows an example of an unconditioned object-based model where the $NTG$ and sandstone bed size distribution from the natural example (Fig 3a) are reproduced, but the connectivity characteristics are vastly different: 90% of the sandstones but only 75% of the shales can be traced across the section. Similar results are obtained with an object-based shale model, again scaled to the observed bed-size distribution (Fig 3c). The reason for the discrepancies is that the models do not honour the amalgamation of the natural example. If a bed is placed at random within a system which already contains a particular $NTG$, there is a probability equal to this $NTG$ that the base of the bed being placed will overly an existing bed. Therefore an
unconditioned object-based model will have $AR$ approximately equal to $NTG$. Natural systems, however, generally have $AR$ significantly lower than $NTG$ (Fig 2b,c), and hence are less connected than an unconditioned object-based model even if the model has the correct $NTG$ and bed size distribution. Conditioning to produce less connected models has been discussed (e.g. Begg & Williams 1991) but we have been unable to find software allowing us to generate bed-scale models in which both the $NTG$ and $AR$ of the beds can be defined as input. Therefore we have been obliged to devise our own methods for doing this.

In the method devised, a system is generated with an $AR$ equal to the final $NTG$ of the model, and then all cells containing shale are compressed vertically with respect to those containing sand. The resultant (thinner) model has a higher $NTG$ than the initial model, but the same $AR$. This procedure allows the generation of models in which both ratios can be defined as input. The factor by which all shale cells are compressed relative to the sandstone cells is termed the compression factor ($c_f$), and the relationships between this, $AR$ and $NTG$ of the final models is given by $c_f = \frac{1 - NTG^{-1}}{1 - AR^{-1}}$. Our definition means that for a given $NTG$, models with a higher $c_f$ have higher $AR$, and, if $c_f = 1$, the models have the same connectivity characteristics as unconditioned object-based models. Whilst recognising that $c_f$ is a modelling tool rather than a meaningful sedimentological parameter, it is instructive to compare the relationships between $NTG$ and $AR$ associated with particular values of $c_f$ with measurements from different turbidite systems (Fig 3c).

With the exception of the Mt Messenger Formation data which were collected as part of this study, only a few data have been found from the literature from individual turbidite systems. It appears, however, that different systems may be roughly represented by different values of $c_f$, with the Mt. Messenger system being particularly poorly amalgamated ($c_f$ ca. 0.03) compared to others (e.g. the Angel Fm where $c_f$ appears to be ca. 0.7 based on data from Evans et al. 2003). There is also a hint that different environments may also to be characterised by different degrees of amalgamation (Fig 3b), with fan fringe environments appearing to be less amalgamated than proximal fan environments for the same $NTG$, a feature reflecting their less connected, and presumably less erosive, nature. There are, however,
insufficient data from individual systems for this trend to be substantiated. Figure 3d shows a 2D model generated using a $c_f$ representative of the Mt Messenger Formation measurements and the same bed-size distribution and $NTG$ as the example Mt Messenger section (Fig 3a). Since the $AR$ of the natural example is now honoured, the model has very similar connectivity characteristics.

**Static and dynamic connectivity in unfaulted models**

A standalone software application has been written to perform the modelling. A model is generated according to a predefined set of sedimentological and fault-related characteristics, a static connectivity analysis is performed and (if requested and if the realisation contains a network of connected beds in the flow direction) the model is submitted to a commercial flow simulator, the results of which are then analysed and reported automatically along with the static analysis. This workflow implies that many models can be processed in batch with no user interaction, allowing many realisations of many thousands of parametrically distinct faulted and unfaulted turbidite bed-scale systems to be modelled. Static properties measured and reported in summary files include the amalgamation ratio and net:gross ratios of the final model, and statistics derived from a connectivity analysis. This analysis identifies all the clusters of connected sandstone beds in the model, whether (and how many) continuous clusters of sandstone are present across the model in each of the 3 directions, and records the fraction of the total sand volume contained in the largest cluster (e.g. Fig 3a). This quantity is termed the fractional mass of the largest cluster ($F_M$).

For convenience we generally use $F_M$ as a measure of static connectivity, and find that $F_M = 0.5$ marks the point at which a model becomes macroscopically connected (i.e. the largest connected cluster spans the width of the model). The percolation threshold of a system is defined technically as the density of objects ($\mu$) at which an infinite cluster is first formed, and this density is referred to as the critical density ($\mu_c$) of the system. If $n$ beds of volume $V_B$ are placed randomly in a model of volume $V_0$, the density is defined as $\mu = nV_B/V_0$. At $\mu_c$, the net:gross ratio and amalgamation ratio are also at their critical values ($NTG_C$ and $AR_C$). Since it is
impossible to generate networks of infinite extent, the percolation threshold of a system is estimated in practice by plotting the fraction of models containing connected clusters that span the modelled volume at different densities. Different curves are constructed from models at different resolutions, and their intersection point marks the threshold. Owing to boundary effects a 0.5 probability of forming a connected cluster does not necessarily mark the position of the threshold, and nor does \( F_M = 0.5 \) (see e.g. Gimel et al. 1999 for a discussion). All percolation thresholds given in this paper have also been corrected for the discretisation bias discussed above.

**Static connectivity**

Figure 4 examines the fractional mass of the largest connected cluster of sandstone beds \( (F_M) \) for models with varying \( NTG \), \( AR \) and bed aspect ratio \( (L_A/L_B) \). When \( F_M \) is compared to \( NTG \) (Fig 4e), three trends (associated with the three different \( c_f \) values used in the models) are evident, with the least amalgamated models \( (c_f = 0.01) \) having very low connectivities even at \( NTG \) of 0.8 (Fig 4e).

When plotted against \( AR \) (Fig 4f) neither the \( L_A/L_B \) nor \( NTG \) has an influence on connectivity, and the only thing that matters in these cases is \( AR \), with the systems achieving \( F_M = 0.5 \) at \( AR \) of ca. 0.28. This is the known 3D percolation threshold of a continuum system of cubes and aligned rectangular prisms (e.g. Baker et al. 2002).

The orientation distribution of rectangular prisms is known to influence strongly the percolation threshold (e.g. Saar and Magna 2002), and Figure 5 charts \( AR_C \) for models in which elongated beds of constant volume are oriented at \( \pm \beta^\circ \) to the average bed orientation. High bed aspect ratio models have significantly lower thresholds. At the extreme of our analysis, randomly oriented (i.e. \( \beta = 90^\circ \)) beds with \( L_A/L_B \) ratios of 20 have \( AR_C \approx 0.07 \) (Fig 5c).

The sandstone beds in these models (Figs 4, 5) are all the same size. Quintanilla (2002) and Consiglio et al. (2003) examined binary systems of small and large circles or spheres, and showed that \( NTG_C \) increases with increasing size variability, reaching a maximum at a particular distribution for systems with equal densities of small and large bodies. However the differences are very small, for
example a system comprising equal densities of spheres of volumes 8 and 1 has
$NTG_C$ less than 1% higher than a system of constant sized spheres. In a natural
system we would expect a continuous range in bed size rather than a binary mix of
small and large beds, and in this situation the variability in $AR_C$ as a function of bed
size distribution is likely to be smaller still. For this reason we have only used models
with constant sized beds in our analyses, although we recognise that the bed size
distribution is likely to have more of an influence on the connectivity of fault systems
than it does on unfaulted ones.

The principal sedimentological controls on the connectivity of the unfaulted
bed-scale models are therefore primarily $AR$, and secondarily, for systems of beds
that are elongate in plan view, the orientation variability of the beds ($\beta$). Known
continuum percolation thresholds (e.g. Baker et al. 2002; Saar & Magna 2002),
expressed as a function of $NTG$, can be simply re-expressed as a function of $AR$ to
provide the thresholds of these more sedimentologically realistic models. This
equivalence between the $NTG_C$ of a random system and $AR_C$ of a system generated
using the compression method is an inevitable consequence of the method. We
discuss the implications of this following a discussion on the dynamic connectivity of
the models.

Dynamic connectivity

The models submitted to the flow simulator are necessarily smaller than those
used in the static analyses (which contain up to 17 million grid-blocks), and examples
of high and low resolution models are shown in Figure 6. Our objective in the
dynamic simulation has been to assess representative flow characteristics of the
models. If, following a static analysis, the model is recognised to be connected in one
horizontal direction, injector wells are placed (in the connected cluster) at one edge of
the model, and producer wells are placed at the opposite edge, imposing a particular
pressure gradient across the model. No-flow boundary conditions are assigned to the
other four edges. Flow modelling has been done in two-phases, but using linear
relative permeability curves and identical properties for the two fluids. This allows
simultaneous assessment of the model permeability as well as giving an indication of
how easily it is drained.
The equivalent horizontal fractional permeability \( F_K \) of a model reported in this work is defined as the permeability measured in the heterogeneous model normalised by the permeability assuming a homogeneous model with \( NTG \) of 1.0 and the same sandstone permeability. As in the static analyses, we aim to report results from 10 flow simulations of each system. Since we are often dealing with poorly connected systems, we often need to generate considerably more than 10 static models of each system to get 10 connected ones. Therefore we attempt up to 1000 realisations of each system, but perform only 10 flow simulations. This implies that we would expect at least one set of simulation results for systems with a greater than 0.1% chance of being connected, and 10 sets of results for systems with a greater than 1% chance. The average \( F_K \) values reported are averages of all realisations generated, not just the connected ones. Hence if one realisation has \( F_K = 0.5 \), but 9 realisations are not connected in the flow direction, the average \( F_K \) reported is 0.05.

Figure 7 shows \( F_K \) results for two suites of high-resolution models containing isotropic beds (in plan view; the thickness to length ratio used in these models is 13.333) which accrue amalgamation as a function of \( NTG \) according to two different \( c_f \) values. As expected, independent trends between \( F_K \) and \( NTG \) are observed (Fig 7a). If \( F_K \) is plotted against \( AR \) (Fig 7b), both sets of models approach \( F_K = 0 \) at the same value of \( AR \) (\( AR_C \approx 0.28 \)), but two trends are still evident. This is because, despite having the same connectivity characteristics at a particular \( AR \), the models with the higher compression factor have a higher \( NTG \), and hence a higher \( F_K \). If \( F_K \) is normalised by \( NTG \) (Fig 7c), both sets of models fall on the same trend.

Percolation theory predicts a power-law relationship between permeability and the proximity of the system to its percolation threshold with a 3D power-law exponent of 1.6 (e.g. Renard & de Marsily 1997). The proximity of the system to its percolation threshold is measured as a function of the density of the permeable objects (\( \mu \)), and \( NTG \) in a random continuum system is related to \( \mu \) through

\[
NTG = 1 - e^{-\mu} \quad \text{(or conversely } \mu = -\ln(1 - NTG); \text{ Shante & Kirkpatrick 1971)}.
\]

The proximity of a particular system to the percolation threshold is given by \( P = \mu/\mu_c - 1 \) and therefore takes a positive value for connected systems and a negative value for disconnected ones. As we have discussed, \( AR \), rather than \( NTG \), is the important
determinant of connectivity, and so the proximity term should reflect this rather than $NTG$. Hence we calculate $\mu = -\ln(1 - AR)$ and observe the expected power-law between $F_K/NTG$ and the $P$ when the latter is calculated using this revised density expression (Fig 8a). The fit to the expected power-law is very close for $P < \text{ca.} 1.0$ (corresponding to $AR = 0.63$). At $P > 1.0$ the systems are very well connected, and the trend becomes asymptotic to $F_K/NTG = 1$.

In the simulations we used two fluid phases, but with each phase having identical properties and with the phase relative permeabilities adding to unity at all saturations. This allows us to examine in a general way the efficiency with which the models are drained. Figure 8b shows the fraction of resident fluid produced when 1%, 10%, 50% and 90% of the production is the injected fluid (had we modelled a more conventional oil-water system we could refer to these graphs as showing oil recovery factor at different water-cuts). We find that close to the threshold only a very small portion of the total sandstone in the model is associated with the flow path, and recovery of the resident fluid is low even at high “water-cuts”. In more connected systems displacement of the resident fluid becomes gradually more piston-like, and progressively more of the resident fluid is produced.

Finite size effects

The effects of finite model sizes are shown in Figure 9. The plots show fractional permeability vs. proximity to threshold for the high-resolution models discussed above and at lower resolutions (e.g. Fig 6b). The power-law relationship is only robust as the model resolution tends to zero (i.e. as $V_B/V_0 \rightarrow 0$), and lower resolution models diverge from this curve as the threshold is approached. This divergence is inevitable. A greater proportion of lower resolution models become connected below the percolation threshold, and if the probability of forming a connected model is greater than zero, so too must be the average fractional permeability. Lower resolution models are more permeable (on average) than higher resolution models (Fig 9a), but are not necessarily more easily drained (Fig 9b). It is often important to understand the effects of finite size from a practical perspective. For example, if the sizes of the beds are (as will often be the case) significant fractions of the scale of interest, then the single permeability value determined from the
percolation relationship is not appropriate, and instead the probability distribution of
the property of interest should be reported (e.g. King et al. 2002).

**Anisotropic systems.**

As well as departing from the ideal power-law relationship owing to finite size
effects, anisotropy also causes shifts in fractional permeability of a system. The
equation \( F_K = NTG(1-t_0)^{-2} \) considers fractional permeability as a function of a
tortuosity term \( t_o \), and crops up frequently in geometrical treatments of permeability
such as the microscopic Carmen-Kozeny equation (Scheidegger 1974) or the
macroscopic statistical streamline equation (Begg & King 1985). The equation
therefore relates permeability to a geometrically meaningful length-term (which can
be calculated for different anisotropies in 2D or 3D) and can be used to deduce the
fractional permeability of an anisotropic system from that of an isotropic one with the
same connectivity characteristics (e.g. Fig 10).

**Applicability of the unfaulted model results**

In contrast to random continuum network models, natural turbidite systems do
not have \( AR \) approximately equal to \( NTG \) (Fig 2). We have shown that despite this it
is still possible to use known percolation results (which relate \( F_K \) to \( NTG \)) for the
more realistic systems examined. Instead of the power-law between \( K_F \) and \( P \)
calculated using the density term associated with \( NTG \) appropriate for random
continuum systems, \( K_F \) for our models with variable and more realistic relationships
between \( AR \) and \( NTG \) can be expressed as a function of both \( NTG \) and \( P \), with the
latter calculated using a density term expressed as a function of \( AR \). In the following
discussion we generalise these results to provide type curves of horizontal
permeability (\( F_K \)) and of the vertical to horizontal permeability ratio (\( K_V / K_H \))
ratio, and also discuss the general applicability of the models generated using the
compression method.

All models resulting from our bed-placement method are based on an initial
unconditioned object-based model in which \( NTG \) is equal to the target \( AR \), and the
modelling procedure then modifies \( NTG \) leaving the bed connectivity unchanged. It
is therefore inevitable both that our results can be related to known continuum percolation thresholds by simply expressing the thresholds as a function of $AR$ rather than $NTG$, and that they emphasise $AR$ as the pre-eminent control on connectivity. The idea that the $AR$ is an important determinant on flow is not new and Clark & Pickering (1996), while discussing a cross-plot of $AR$ (which they call vertical continuity) and the width to thickness ratio of the beds (i.e. $L_B/L_Z$; which they call lateral continuity) for a variety of deep-water systems, mention that these parameters alone should be sufficient to determine the $K_V/K_H$ ratio of the system. For large systems ($V_B/V_0 \rightarrow 0$) of isotropic beds (i.e. systems with $L_A = L_B$), type curves of $F_K$ and $K_V/K_H$ are shown in Figure 11 for different cases of $L_B/L_Z$. These have been calculated from the results discussed above for 2D and 3D systems. Also shown in Figure 11b is the 2D log-linear relationship between $K_V/K_H$ and $AR$ derived by Stephen et al. (2001), and the clear difference between this and the (2D) curves derived in the present study is a consequence of different sedimentological idealisations of the system. In the systems considered by Stephen et al., sandstone beds are of infinite length, and portions of shale layers are removed to give the final model $AR$. Their models are therefore not percolation systems since horizontal flow is possible in their models at $AR = 0.0$, and vertical flow is possible at all non-zero $AR$ values.

The different $K_V/K_H$ ratio models arising from this study and from that of Stephen et al. (2001) can be reconciled by considering the spatial association between erosion and deposition. In the structure-imitating models generated using the compression method, each erosion surface is associated with the amalgamating bed, and each bed has the potential to be erosive over its entire length. A simple 2D process-imitating model is used to explore these issues further. A volume is filled with sand beds from the bottom to the top with geometrical rules crudely mimicking sedimentological ones. Beds from a predefined size distribution are dropped into the model and are encouraged (but not forced) to fall as close to the bottom as possible (ensuring that towers of beds are not formed). Each bed is encased above and laterally in thin shales. Predefined erosion probabilities, specific to each model, allow individual beds to erode into the underlying sequence over a fraction of the bed thickness (e.g. Figure 12a). As the erosion probability increases, $AR$ increases and
the models become more connected. Unlike in the structure-imitating model, however, $NTG$ and $AR$ do not accumulate along curves of equal $c_f$. However, despite this entirely different generation method, these models reproduce the critical amalgamation ratio of models generated using the compression method (i.e. $AR_C = 0.67$ in 2D).

This result demonstrates that the $AR_C$ values observed in our models have a reality beyond the simple mapping between $NTG$ and $AR$ that is the inevitable consequence of our compression method for generating the models. It does not, however, prove that these values are universally applicable. In a second set of process-imitating models, erosion is modelled independently of deposition. In these examples, after each bed is placed there is a probability (or a probable number) of events that erode off a certain thickness of model over a particular length; this length is reported as a fraction (or multiple) of the length of each bed. These events are not associated with any deposition, and the beds themselves are not erosive. Examples of models where the erosion length is smaller and larger than the bed lengths are shown in Figs 12b,c respectively. We find that $AR_C$ in these models is significantly lower than in the case where erosion events are tied to the deposition of the overlying sandstone bed, even if the erosion events are significantly longer than the beds (Figure 12d). Obviously, $AR_C$ will tend to zero as the erosion length / bed length tends to zero, as it does implicitly in the models of Stephen et al. (2001). The relevance of the two approaches is almost certainly facies dependent. In massive or thick-bedded facies, the lengths of amalgamation surfaces are likely to be very small compared to the lengths of the beds (e.g. Figure 13), and a 3D version of the models of Stephen et al. is more appropriate. By contrast, in thin-bedded facies, consideration of finite bed lengths is crucial, amalgamation is low, erosion and deposition are more likely to be coupled, and our approach is probably more appropriate.

It is clear from these considerations that the characteristic property of a continuum percolation model (i.e. $NTG$) is not necessarily the appropriate parameter for characterising connectivity in natural systems, and also that it cannot simply be replaced by $AR$, since other, more subtle, sedimentological characteristics are also significant. Even for a relatively well characterised unfaulted system there will therefore always be considerable uncertainty about defining an appropriate
percolation threshold, yet this definition is the starting point for estimating permeability, since the proximity of the system to the threshold is the principal measure of connectivity in a percolation theory approach. In the following section we examine faulted versions of models generated using the compression method, recognising that sedimentologically these models explore only the small sub-set of circumstances in which erosion is tied to deposition.

**Connectivity in faulted models**

The sedimentological models generated using the compression method and for which connectivity has been analysed (e.g. Figs. 4, 5) are characterised by differences in $NTG$, $AR$, bed aspect ratio ($L_A/L_B$) and bed orientation dispersion ($\beta$). The static connectivity of these models has been shown to be governed principally by $AR$ and secondarily, for beds with higher values of $L_A/L_B$, by $\beta$. The bed size distribution does not influence significantly the connectivity of the unfaulted models, and $NTG$ has no influence whatsoever. In the remainder of the paper we discuss connectivity in faulted versions of these models. The initial objectives of the modelling were to establish the principal factors controlling flow at an inter-bed scale in faulted sheet-like turbidite systems. To this end, static connectivity was measured in 10 realisations of ca. 25000 parametrically distinct models including variability in the four basic sedimentological parameters (listed above) as well as various fault-related parameters (discussed below). Factor analysis was used to establish sensitivities to connectivity and to connectivity changes as a function of model parameters or, more usefully, dimensionless ratios between sedimentological and fault-related parameters. We do not present these analyses, since their results indicated categorically that there are no principal factors controlling connectivity. All variables examined (including those that are not significant in unfaulted models, for example $NTG$ and the bed size distribution) were found to be potentially significant, with the level of significance a function of the levels of other variables. Instead, following a description of the fault modelling procedures used, we discuss a couple of sets of results illustrating aspects of the connectivity responses, and then return to the central question posed at the beginning of this paper: how can known scaling laws from percolation theory be applied to understanding flow in the models.
Fault system modelling

All faulted models contain systems of randomly positioned faults of identical sizes. The examples shown in Figure 14 are applied to the unfaulted models shown in Fig 5 in which the total model thickness is \(12L_Z\), where \(L_Z\) is the bed thickness. Four different fault orientation models are used; randomly oriented, oriented parallel (\(\pm 10^\circ\)) to the principal orientation of the long axes of the beds, oriented perpendicular (\(\pm 10^\circ\)) to this direction, or oriented parallel and perpendicular to this direction with equal probability (the last of these orientation distribution models is termed “orthogonal” in later discussion). The faults themselves are vertical, with triangular horizontal displacement profiles and no vertical displacement gradients. The fault system, characterised by a particular number of faults \((N_F)\) of length \(L_F\) and maximum throw \(T_F\), is most usefully expressed in terms of dimensionless ratios between fault system variables and sedimentological ones. The beds have areal dimensions of ca. \(L_A L_B\), where the bed aspect ratio \((L_A / L_B)\) ranges from 1.0 to 20.0 (e.g. Fig 5). Since the models have been built so the beds have constant \(L_A L_B\) whatever the ratio \(L_A / L_B\), fault density can be expressed most conveniently as the expected number of faults centres per bed \((\bar{N}_F)\). Fault sizes can be made dimensionless by normalising fault length against the average bed length \((\bar{L}_F = L_F / (L_A L_B)^{0.5})\), and maximum fault throw against bed thickness \((\bar{T}_F = T_F / L_Z)\).

Fault property modelling

Models have been analysed using open faults (i.e. those involving only juxtaposition effects), sealing faults, and two types of fault property predictor. The first property predictor uses Shale Gouge Ratio (SGR, Yielding et al. 1997) cut-offs. In the second method, shale smears are modelled explicitly, using the Probabilistic Shale Smear Factor (PSSF) method defined by Childs et al. (this volume). The methods are illustrated in Figure 15; Fig 15a shows an Allan diagram of a fault extracted from a model with \(\bar{L}_F = 3.6\) and \(\bar{T}_F = 5.0\) (e.g. Fig 14b). Shale layers in the
footwall and hangingwall are shown in pale and dark grey, and sand-on-sand juxtapositions are highlighted in yellow.

SGR is calculated at each corner of each faulted connection, assuming that sands have a clay fraction of 0.0, and shales of 1.0, and SGR cut-offs are applied to determine the state of individual sand-on-sand juxtapositions. We assume that individual fault connections are either entirely sealed or wholly or partially open. If a connection is even partially open, the sandstone cells on either side of the fault are in connection, so recognition of the extent of openness is unnecessary for the connectivity analyses. Therefore the minimum SGR of the corners of the connection is taken as diagnostic of the connection SGR (individual connections will generally have different SGR values at each of their corners) and if the specified SGR cut-off is above this value the connection is open, otherwise it is closed (Fig 15b).

The second sort of fault property modelling applies the ideas of probabilistic shale smears discussed by Childs et al. (this volume) on the basis of observations of faults in the Mt Messenger Formation. In this approach a discrete shale smear is associated with each faulted shale layer. The smear is continuous if the Shale Smear Factor (SSF; Lindsay et al. 1993, given by normalising the fault throw by the shale bed thickness $T_B$) is less than a critical value ($SSF_C$). If SSF exceeds $SSF_C$ the smear is assumed to be discontinuous and of length $T_B(SSF_C - 1)$. This smear is placed on the fault surface with equal probability anywhere between the base of the shale footwall cut-off, and the top of the hangingwall shale cut-off. The result of the process is a distribution of shale smears on the fault surface (e.g. Fig 15c), which combine to define sealed and open sand-on-sand juxtapositions (Fig 15d). Childs et al. (this volume) term a binary distribution of smeared and open portions of faults modelled in this way as a Probabilistic Shale Smear Factor (PSSF).

The implementation of the PSSF method used here follows exactly the 1D definitions of Childs et al. (this volume), but further assumptions are needed to cover a 2D fault surface rather than a 1D trace. For example, shale layers are continuous horizontally over certain distances, and it is reasonable (but by no means necessarily correct) to assume that the shale smear derived from these layers will also be continuous over this distance. Therefore the centre of the smear is placed at an arbitrary fraction of the throw over all grid-cells over which the shale layer does not change in character. This will tend to result in shale smears which have a plunge equal
to the average of the apparent dips of the layering on both sides of the fault. Algorithmic complications arise when the shale changes in character along or across the fault, for example by connecting with another shale at the end of a sandstone bed or by terminating through sandstone amalgamation. We have handled these situations in a fashion that will encourage horizontal continuity of the smears, but since many of the issues associated with this kind of modelling are unresolved geologically (as it is usually only possible to measure shale smear distribution on 1D sections across faults), occasional arbitrary assumptions have been made. Figure 15d shows a realisation of PSSF using $SSF_C = 5.0$ for the example fault. A general decrease in open (non-smeared) sand-on-sand connection towards the higher SGR central region of the fault can be observed, but occasional partially open connections are still present even when SGR exceeds 0.3.

**Connectivity with open faults**

The significance of the dimensionless terms for expressing the fault system parameters can be appreciated via the simplified conceptualisations shown in Fig 16 (similar arguments have been made by James et al. 2004). Figure 16a shows half a fault length, ranging in throw from a maximum $T_F$ at the centre to zero at the tip, and a sandstone bed of length $L_A$ and thickness $L_Z$ offset by the fault, for a case where the fault is considerably longer than the beds. The change in fault displacement over the length of a sandstone bed is given by $dT = 2T_F L_A / L_F$. If the stratigraphy is assumed to be entirely unamalgamated and periodic, the total thickness of each sandstone/shale couplet is $L' = L_Z / NTG$. If the beds are assumed to be stacked vertically, the number of beds juxtaposed against any other bed depends on the precise placement of the sequence on each side of the fault (Fig 16b,c), and, on average, is $2NTG (1 + T_F / L_F)$. This function might therefore be expected (in conjunction with the total number of faults expected per bed) to scale with connectivity. Although this sort of simplified approach goes some way towards understanding the connectivity changes observed in some of the geometrically more simple models which come closest to meeting the assumptions made above (i.e. models with extremely low $AR$, and very high values of $T_F$ and $L_F$), it contains too many simplifying assumptions to be able to explain the majority of our model results.
A general analytical solution, though probably possible, would be extremely complex if non-trivial fault and sedimentological parameters are to be included.

Figure 17 shows a selection of connectivity measurements for systems with open faults, illustrating some of the complexities and interdependencies that are a feature of the model behaviour. The figure charts the connectivity of 1920 different faulted models deriving from 60 different unfaulted models. The unfaulted models (labelled “UF” in Fig 17) are characterised by three different NTG values each for two different c_f values, for models containing beds with two different aspect ratios (L_A / L_B), each with 5 different levels of bed orientation variability (β). The 32 fault systems are two combinations each for $N_F$, $L_F$, and $T_F$, for the four orientation models.

The circumstances in which open faults lower connectivity are comparatively rare. Based on an analysis of a coal-measures sequence, Bailey et al. (2002) found that in low dimensionality systems (i.e. ones containing beds with high aspect ratios) with low net:gross ratios, faults will lower connectivity. The modelling results presented here (Fig 17) suggest that this is only the case for well amalgamated systems (i.e. those with high c_f values, Fig 17c) and $β > ca. 15°$. High $L_A / L_B$ models with lower c_f values (Fig 17d), or lower $β$ (Fig 17c) have larger connectivities when faulted than when unfaulted, as do all the models with lower $L_A / L_B$ ratios (Fig 17a, b). One intriguing feature of these models with $L_A / L_B$ ratios of 20 and $β$ is the virtual reciprocity of each set of results between equivalent NTG at the different c_f values (Fig 17c, d). As mentioned, when these models have a high c_f (and therefore a high $AR$ for a given NTG) faults reduce connectivity, and where they have a low c_f they increase it, but this trend is extremely systematic: each combination of fault characteristics causing connectivity lows in the first case cause connectivity highs in the second.

Effects of fault orientation also appear generally to be a function of c_f, with low c_f models (Fig 17b, d) showing greater sensitivity. This is not always the case however; for example the central panel of Fig 17c with $β = 15°$ is highly sensitive to fault orientation despite a $c_f = 1.0$. A final illustration of the importance of different sensitivities owing to the settings of particular variables is evident in the central panel of Fig 17c. In this case the difference between models with $β = 0°$ and $β = 15°$ is
paramount on connectivity. However, despite similar total connectivity ranges, other sedimentological models (e.g. the central panels of Fig 17a and 17d) show a much more gradual increase in connectivity with increasing $\beta$.

Figure 17 shows only a sub-set of the models analysed, but illustrates the main conclusion: all variables (sedimentological and fault-related) examined are potentially significant controls on the connectivity (and hence flow characteristics) of the models. In certain instances some variables are not important, but generalisations require so many caveats as to render them incomprehensible. As we show below, this behaviour persists as fault properties are also considered.

### Connectivity including fault properties

Figure 18 shows results for the same fault systems as Fig 17 but in this case the 3 panels represent different sedimentological models (Sedimentologies 1, 2 and 3 shown at the bottom of Fig 18), and the different coloured curves show results using different SGR cut-offs (Fig 18a) or critical SSF values (Fig 18b). In most cases the faults provide fairly well connected models with open faults, and fairly poorly connected ones with sealing faults, but the transitional cases, however, behave very differently. Looking first at the SGR cases (Fig 18a) most changes in connectivity occurs at low SGR cut-off values for ‘Sedimentology 1’, fairly regularly across the range of cut-offs for ‘Sedimentology 2’ or at high SGR cut-off values for ‘Sedimentology 3’. Intuitively, perhaps, the dependence between connectivity and SGR cut-off should be associated with a dependence on $NTG$, however no systematic trends have been identified accommodating all the ranges of other sedimentological and fault-related variables examined. The SSF cases (Fig 18b) are very different. ‘Sedimentology 1’ shows only a weak dependence on $SSF_C$ with most models approximately midway between the connectivities observed with open and sealing faults. ‘Sedimentology 2’ shows very little change in connectivity from the open case whatever SSFc is used (these models do, however show a systematic increase in the numbers of small, isolated clusters as $SSF_C$ increases). ‘Sedimentology 3’ shows very similar connectivity to the SGR models at reciprocal cut-offs.

These examples again highlight the extreme complexity of connectivity response to geometrical details of the fault and sediment definitions, but also indicate that extremely different results arise depending on how the fault rock properties are
modelled. We have already mentioned that only particular combinations of conditions allow models containing open faults to have lower connectivity than an unfaulted model. We also find that extremely severe fault property cut-offs are generally needed (i.e. low $SGR_C$ or high $SSF_C$ values). Figure 19a summarises results for about 5000 models that include variability in all fault-related and sedimentological properties considered with the exception of $\beta$. These results show a net loss in connectivity relative to the unfaulted case in only about 10% of models in which all connections with $SGR > 0.1$ are sealed. Bed orientation dispersion is a pre-requisite for open faults to lower connectivity (Fig 17), so had this variable also been included in the models shown, connectivity would be lost at higher SGR cut-offs.

The situation with the probabilistic smears is even more extreme (Fig 19b). Even when $SSF_C$ as high as 10 is considered, most models are only slightly more poorly connected than when fault properties are ignored altogether (i.e. the behaviour of “Sedimentology 2”, Fig 18b, is not unusual). Faults characterised by overlapping discontinuous shale smears, therefore, are less detrimental to large-scale connectivity than might be thought. As Childs et al. (this volume) discuss, once the throw on a fault is larger than the critical smear length ($SSF_C$), there is always a non-zero probability that any position on the fault is not covered by any smears. This is seen in Figure 15c, where small holes in the shale smear coverage exist right up to the centre of the fault. Although rare, these holes in most of our models are frequent enough to ensure bed-scale connectivities that are not substantially lower than those obtained if fault rocks are ignored altogether. The algorithmic similarity between SGR and the reciprocal of SSF is therefore not manifest by equivalent connectivities when the fault smears are modelled deterministically using SGR cut-offs and stochastically using explicit shale smears.

The fraction of the total connectivity change possible as a function of a fault property case is given by the change from the open-fault case normalised by the difference between the open-fault and sealing fault cases. No over-riding trends are observed when this change using an SGR cut-off is compared to the change using the reciprocal critical SSF value (Fig 19c). More sealing fault rock property cut-off values ($SGR_C = 1/SSF_C = 0.1$) show more extreme differences, but even where some combinations of sedimentology and fault system give similar results
(\(SGR_C = 1/SSF_C = 0.5\)), others can still show no changes in connectivity from the unfaulted case when \(SSF_C\) is used despite a large change for the reciprocal \(SGR_C\).

Figure 19d shows the subset of models using the largest faults considered (i.e. high \(L_F\) and \(T_F\)). In these cases cut-off specific trends emerge marking the upper limit of the clouds of data from Fig 19c. A high \(L_F\) implies a small change in fault displacement over the length of a bed, so the differences in plunge (parallel to the fault plane) between the smears and sand beds are lowest. Holes in the smear-covering are therefore more likely to be aligned parallel to the sand-on-sand juxtapositions so a single hole is likely to open up fewer juxtapositions. A high \(T_F\) implies that several shales smears are potentially present at any position on the fault which results in a more predictable PSSF distribution (Childs et al. this volume) in which holes are more likely to occur in regions of lower SGR. Connectivity across faults that are very large (both vertically and laterally) in relation to the beds may therefore be similar if explicit smears are modelled probabilistically or if fault rocks are modelled deterministically using an SGR cut-off, however the appropriate cut-off is certainly not the reciprocal of \(SSF_C\). Most of our models contain relatively small faults, and in these cases there is little association between the responses using SGR cut-offs or the PSSF method. Faults that are larger in relation to the beds are likely to have a more predictable, and more detrimental, influence on across-fault connectivity.

**Dynamic connectivity**

Given the complexities observed in the static connectivity analyses, it is no surprise that the dynamic connectivity results (permeability, drainability) also show complex responses to combinations of fault-related and sedimentological model characteristics. These properties have the additional complication of being directional. Figure 20, for example, shows fractional permeability of isotropic beds faulted by various isotropic and anisotropic fault systems. Reinforcing the static model results, we find that all systems containing open faults (Fig 20a) are more permeable than their unfaulted counterparts, and the same is true of most models with \(SGR_C \geq 0.1\) (Fig 20b).

Rather than consider these results in terms of geological model characteristics, we have analysed them within the context of percolation theory. As discussed at the
start of this paper, one of the underlying implications of the theory is that
c connectivity, anisotropy and resolution are what control flow. These geometrical
properties derive from the geological characteristics of the system, but are not unique
to any particular set of characteristics, and vastly different geological systems could
have identical connectivity, anisotropy and resolution. Percolation theory suggests
that these will all have the same flow properties.

We have shown (Figs 7, 8) that the fractional permeability of high resolution
unfaulted models with different relationships between $AR$ and $NTG$ can be
expressed as a function of the proximity of the system to the percolation threshold
($P$). For unfaulted isotropic systems (i.e. ones in which $L_A/L_B = 1.0$ and hence in
which $\beta$ is irrelevant) generated using the compression method, $P$ depends only on
$AR$ and occurs at $AR_C = 0.28$. A high resolution faulted isotropic system with the
same value of $P$ should therefore have the same fractional permeability. An
immediate problem is that the percolation threshold of the faulted model is unknown,
and potentially varies as a function of all sedimentological and fault-related variables
discussed. It is impossible, therefore, to determine $P$ as a function of a known system
property (unlike the unfaulted models where it could be determined from $AR$).

One of the more basic equations in percolation theory is $B = \mu V_{ex}$, which
relates the average number of connections per object ($B$), to the density ($\mu$) and the
excluded volume ($V_{ex}$) of the objects. The density term $\mu$ has already been
discussed. The excluded volume is defined as the volume surrounding the centre of an
object within which the centre of any other object connected to it must lie. The critical
average number of connections per bed for aligned cuboids ($B_C$) is 2.59 (e.g. Baker
et al. 2002), and, like $\mu$ and $\mu_c$, $B$ and $B_C$ can be used to establish the proximity of
a system to the threshold. An advantage of using $B$, however, is that it is a product of
the sedimentological and fault-related characteristics of a single model, and like $F_M$,
can therefore be measured in each realisation. Figure 21 compares $F_M$ with $B$ for
isotropic faulted models containing differences in $c_f$ and in the full range of fault-
related variables considered. Irrespective of the geological details of the system, they
all lie on the same trend, intersecting $F_M = 0.5$ close to the expected threshold at $B_C$
$= 2.59$. 


In addition to being controlled by proximity to the percolation threshold, the flow properties of isotropic models are functions of the model resolution (Fig 9). A fault that entirely offsets the bed breaks it into two objects, each considered separately when calculating $B$. Since the number of objects has thus increased as a consequence of the fault, the average object volume ($V_B$) must decrease, resulting in a net increase in the resolution of the models (i.e. a decrease in $V_B/V_0$). Figure 22 charts the fractional permeability and drainability of ca. 200 isotropic faulted models (including, but not limited to, those shown in Fig 20a,b) as a function of their proximity to percolation calculated from measured values of $B$. The data have been separated into two classes dependent on the resolution of the faulted beds, and these are compared in Fig 22 to results for unfaulted isotropic systems at the same resolutions (Fig 9). The drainability data are noisy in both the faulted and unfaulted cases (Figs 9b, 22b), however both dynamic properties fall approximately within the same ranges as the unfaulted cases, supporting the initial notion that proximity to the percolation threshold and resolution are the only significant controls on the flow characteristics in these isotropic models.

**Discussion and Conclusions**

This study has been directed towards understanding controls on inter-bed flow in faulted sheet-like or lobate turbidite systems. Conceptually, there are geometrical parallels between interbedded sandstone/shale sequences and random percolation systems. King et al. (1990, 2002) have argued that, since the flow characteristics of percolation systems can be formalised mathematically (e.g. King et al. 1999a, b; Andrade et al. 2000; Lopez et al. 2003), exploiting these parallels provides a useful method for rapidly establishing likely behaviour. In this paper we have been concerned with establishing the strength of the parallels and, where they are present, with defining measures that allow the geological system to be expressed within the same mathematical framework as the percolation system.

In common with a percolation approach, we have focused exclusively on representative, stationary volumes containing beds and faults from a single size distribution (usually constant sized). These conditions may seldom be met in a natural system since volumes of at least 7-10 bed lengths are required for a system to be
considered representative, yet gradual property trends or abrupt sedimentological transitions are often present over such length scales. The presence of different types of objects within a single volume can also modify significantly the system connectivity (e.g. a few strongly erosive channels can connect up an otherwise disconnected system of poorly amalgamated sheets). Such systems are beyond the scope of the present study.

The simplest link between sedimentological and percolation systems is through the net:gross ratio. In a continuum percolation model the net:gross and amalgamation ratios are equal, but in natural systems the latter is often considerably lower than the former, resulting in systems with less sandstone connectivity at the same net:gross ratio. A modelling scheme we refer to as the compression method allows models to be built with more natural associations between the two ratios, and the connectivities of models generated using this procedure have identical dependencies with respect to amalgamation ratio as random continuum models do with respect to net:gross ratio. In such systems, therefore, the principal control on connectivity is the amalgamation ratio with the orientation dispersion of elongate beds an important secondary control. The bed size distribution is not significant.

A series of simple 2D models have been developed to address sedimentological implications of the compression method. The compression method implies that erosion is directly linked to deposition and, when it occurs, has the potential to cover the entire length of a bed. If erosion events are not associated with the deposition of an overlying bed, the systems become connected at lower amalgamation ratios which depend on the ratio between the length of the erosion events and the length of the beds. It is also likely that the bed and erosion size distributions may be significant in these cases. Hence although the connectivity of unfaulted models generated using the compression method can be reconciled with known results, natural systems are more complex with essentially unknown percolation thresholds dependent on properties that are never included in traditional continuum percolation studies (e.g. Baker et al. 2002, Consiglio et al. 2003, Saar & Magna 2002).

A series of models have investigated connectivity in systems characterised by different sedimentological and fault characteristics. All models have been built using the compression method, and sedimentary variables considered include the net:gross and amalgamation ratios of the systems, and the aspect ratios and orientation.
variability of the beds. Fault-system variables include fault frequency, length, maximum throw and orientation. Sandstone connectivity has been calculated using open faults, sealing faults and using two methods for predicting fault properties. In the first method across-fault juxtapositions that exceed a specified SGR cut-off are deemed sealing. In the second more innovative method, shale smears are modelled stochastically as a function of SSF cut-offs. All the beds and faults in any particular model are the same size, and the faults range from a few times smaller to a few times larger than the individual beds. We find all the fault system variables we have modelled to be influential on the connectivity of the sequences, with the particular variables being more or less influential depending on the values of other fault-related or sedimentological variables. The connectivity of faulted turbidite beds is therefore extremely complex and is controlled by interactions of different variables.

Some general conclusions can be made. Open faults are rarely capable of reducing the connectivity of the system. Systems modelled with deterministic SGR cut-offs are less connected than systems modelled with stochastic smears with the reciprocal SSF cut-off. The two methods become more equivalent for systems with higher fault throw to bed thickness and fault length to bed length ratios. In general, low SGR (<0.1) cut-offs or high SSF cut-offs (>>10) are required before a faulted system becomes less connected than its unfaulted counterpart. However particular systems may however lose connectivity at less extreme cut-offs.

It is important to appreciate that these conclusions apply to the geometrical configurations we have examined, i.e. intra-bed connectivity in representative volumes in which the faults are of comparable sizes to the beds. Our modelling results (and others, e.g. James et al. 2004) indicate a general decrease in across-fault connectivity with a decrease in the fault displacement gradient normalised by bed size. Therefore faults that are much larger than the beds are likely to be more detrimental to over-all connectivity than faults in the systems examined, given the same shale smear modelling criteria.

For inter-well distances of up to 1 km, the thickness of beds we are considering are < ca. 1m (Fig 2a), and the faults have maximum throws of up to a few meters. Inter-well connectivity in sedimentological systems characterised by thicker units may be controlled by the continuity of individual beds or bed-sets rather than by a representative network of them. If one of these individual units is wholly or partially offset by a fault, the connectivity at the length-scale of interest (< ca. 1 km) will be
lowered, despite the fact that connectivity within a representative network (i.e. > ca. 5 km) with the same fault to bed size ratios may increase. This apparent duality of connectivity behaviour is a consequence of the volume of interest being most appropriately represented by single objects as opposed to a representative network of them.

Our flow simulation results support the notion that geological details of a representative system, irrespective of whether it is faulted or not, are important only inasmuch as they contribute to the eventual connectivity, anisotropy and resolution of the connected network of beds, and that these three terms dictate the flow behaviour. For models with simple associations between erosion and deposition, the required connectivity term (the proximity of the system to the percolation threshold) can be established as a function of amalgamation ratio. In the presence of faults an analogous connectivity term can be expressed as a function of the average number of connections per bed \( B \). This, however, cannot be valid in all cases. The ideas underlying percolation theory rely on the notion of random distributions of objects, and in such cases aggregate system properties, such as the number of connections per bed, will show only a limited variability normally distributed about a mean value. It is possible to conceive of a system in which both the faults and the beds individually meet these conditions but which combine to produce a network that does not. For example a very poorly amalgamated system of beds might contain a sparse network of short faults. If these faults have high enough maximum throws they may produce a network in which \( B \) exceeds the critical value, but which is not macroscopically connected since the faults are spaced too far apart for the regions of high connectivity to be mutually connected. In this case the distribution of connections per bed will be distinctly bimodal, and the condition of spatial homogeneity required by the theory is violated. These conditions are likely to be fairly extreme, and our results show that in many cases the flow characteristics of a faulted system can be established using the three basic system measures.

As a general conclusion we consider that, despite superficial similarities between interbedded sandstone / shale sequences and random percolation systems, at present it is impossible even in the absence of faults to establish \textit{a priori} the proximity of a natural system to its percolation threshold, since the dependencies on the threshold as a function of natural associations between erosion and deposition are not known. This does not mean that a percolation approach to understanding flow
characteristics is not of potential value. Provided a modelling scheme is used which allows inclusion of the geological factors most significant on connectivity, static modelling could be used to estimate the requisite connectivity term as a function of connections per bed. Alternatively, in an approach that would not rely on static modelling, the connectivity and anisotropy terms associated with flow in a particular reservoir might be estimated at a particular resolution through inversion of available flow data (e.g. well tests). Likely flow characteristics on a field-wide basis could then be explored using these terms at a revised resolution.

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Appendix - Glossary of terminology

**$NTG$** Net: Gross Ratio (dimensionless). The fractional volume of a sequence occupied by sandstone.

**$AR$** Amalgamation Ratio (dimensionless). The fraction of sandstone bed bases eroded into an underlying sandstone bed when measured on a vertical sample-line.

**$c_f$** Compression Factor. Dimensionless modelling parameter which relates $AR$ and $NTG$ through the expression $c_f = \left[1 - NTG^{-1}\right]/\left[1 - AR^{-1}\right]$. 

**$L_A, L_B, L_Z$** The length, width and maximum thickness of individual sandstone beds (meters). The ratio $L_A/L_B$ is the bed aspect ratio referred to in the text (unless the vertical aspect ratio; $L_B/L_Z$, is mentioned explicitly).

**$T_B$** The thickness of a shale bed (meters).

**$\beta$** Bed orientation dispersion (degrees). Individual sandstone beds are oriented in any model at $\pm 90^\circ$ to the principal direction of bed alignment.

**$V_B/V_0$** Bed volume / Modelling volume. This ratio is a convenient measure of the resolution of unfaulted models since in individual models all sandstone beds are the same size.

**$\mu$** Bed density parameter. In a random continuum system containing $n$ beds of constant size, this density term is given by $\mu = nV_B/V_0$. In such a system $\mu$ is related to $NTG$ through the expression $NTG = 1 - e^{-\mu}$.
\( B \)  
Bed connectivity parameter. The average number of beds connected to any particular bed in a model.

\( NTG_C, AR_C, \mu_C, B_C \)  
The values of \( NTG, AR, \mu \) and \( B \) at the percolation threshold of the system. The percolation threshold marks the density at which a large (technically infinite) volume first contains a cluster of objects that connects all the edges of the system (i.e. an infinitely large cluster).

\( P \)  
The proximity of a system to its percolation threshold. It is given by \( P = \frac{\mu}{\mu_C} - 1 \) or by \( P = \frac{B}{B_C} - 1 \). \( P \) takes positive values for macroscopically connected systems and negative ones for disconnected systems.

\( F_M \)  
The Fractional Mass of the largest cluster of connected sandstone beds in a model. This is the most common measure of static connectivity used in this paper and is defined as the volume of sandstone beds contained in the largest cluster normalised by the total volume of sandstone beds (see Figure 3e).

\( F_K \)  
Fractional Horizontal Permeability (dimensionless). Defined in this paper as the directional permeability normalised by the permeability of a homogeneous model with \( NTG = 1 \) and the same sandstone permeability.

\( K_V / K_H \)  
The ratio between the vertical and horizontal permeability. Note that \( K_V / K_H \) in this paper is only discussed with reference to systems with isotropic horizontal permeability (i.e. in which \( L_A = L_B \)).

\( N_F \)  
The number of faults in a model. For the models described in this paper, all faults in a particular model are the same size.
$\bar{N}_F$  
Dimensionless fault frequency parameter expressed as the average number of fault centres (i.e. points of maximum throw) contained in each sandstone bed.

$L_F, T_F$  
The length and maximum throw of a fault (meters).

$L_F, T_F$  
Dimensionless measures of fault size normalised by bed size. They are given by $\bar{L}_F = L_F / (L_A L_B)^{0.5}$ and $\bar{T}_F = T_F / L_Z$.

$SGR$  
Shale Gouge Ratio. The fraction of shale that has passed a particular point on a fault. In this work the $V_{\text{shale}}$ of sandstone and shale beds are taken as 0.0 and 1.0 respectively.

$SGR_C$  
Shale Gouge Ratio Cut-off. If every point within an individual sand-on-sand juxtaposition has $SGR$ values exceeding $SGR_C$ then a continuous shale smear is assumed to be present within this juxtaposition.

$SSF$  
Shale Smear Factor. This is defined at a particular location on a fault surface, and for a particular shale bed, as the ratio between the local fault throw and the thickness of the shale bed.

$SSF_C$  
Critical Shale Smear Factor. Shale beds with $SSF$ exceeding this value are assumed to be disconnected from the source shale beds on both the footwall and hangingwall sides of the fault. Shale beds with lower $SSF$ values are assumed to form continuous smears between the source layers in the hangingwall and footwall.

$PSSF$  
Probabilistic Shale Smear Factor. A binary fault surface property marking the presence of one or more probabilistically placed shale smears.
References


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Figure captions

Figure 1. Uninterpreted and interpreted photographs of thin bedded turbidites from the Mount Messenger Formation at Tongaporutu beach, New Zealand. Note that the line drawings show sandstones in yellow and shales in grey, while in the photos the sandstones are reddish-brown and the shales pale grey. (a) Section showing small sandstone beds encased in shale. (b) Section showing abundant amalgamation of sandstone beds. (c) Close-up of the shale break (bed i) to the left of the rucksack in (b). Note also bed ii, which although continuous at the scale of this photo is eroded at a larger scale.

Figure 2. (a) Fan body width ($L_B$) to thickness ($L_Z$) measures for modern and ancient systems, compiled from published data. (b-c) Net:gross ratio ($NTG$) vs. Amalgamation ratio ($AR$) separated on the basis of environment (b) and turbidite system (c; this only shows systems for which 4 or more data are available). The lines in (c) represent the trends observed in the end member systems represented by the Angel and Mt Messenger Formations. The data in (a) are from Al Ja’a’idi (2000), Badalini et al. (2000), Browne et al. (1996), Bruhn & Walker (1994), Carr & Gardner (2000), Chapin et al. (1994), Clark & Gardner (2000), DeVay et al. (2000), DeVille Wickens & Bouma (2000), Elliott (2000), Evans et al. (2003), Jennette et al. (2000), Kleverlaan (1994), Lyons (1994), Lüthi (1981), Mahaffie (1994), Nilsen & Abbate (1985), Nilsen (1984), Nilsen (1990), Parea & Ricci-Lucchi (1975), Pickering et al. (1989), Ricci-Lucchi (1978), Ricci-Lucchi & Pignone (1978) and Stow & Johansson, (2000). With the exception of the new Mt Messenger Formation measurements, the data in (b) and (c) derive from measurements or logs presented by Amy et al. (2000), Booth et al. (2003), Browne et al. (1996), Evans et al. (2003), Haughton (1994), Johnson et al. (2001), Mattern (2002), Rozman (2000), Satur et al. (2000), Sinclair (1994), Talling (2001) and Tomasso (2001).

Figure 3. (a) Bed correlation along a ca. 230m long section of thin-bedded lobe fringe facies from Tongaporutu beach, redrawn from Browne et al. (1996). (b) Sandstone beds with approximately the same size distribution and net:gross ratio ($NTG$) placed randomly in a shale background. (c) Shale beds with approximately the same size distribution and $NTG$ placed randomly in a sandstone background. (d) Sandstone beds with approximately the same size distribution, amalgamation ratio ($AR$) and
NTG placed in a shale background using the compression method. (e) 3D model with constant sized circular beds at $NTG = 0.7$, $AR = 0.3$. Shales are shown in grey, and the largest cluster of mutually connected sandstone bed is coloured purple. The volume of this cluster, normalised by the total volume of sandstone, defines the Fractional Mass ($F_M$) of the model.

Figure 4. (a – c). Example realisations of systems generated with net:gross ratio ($NTG$) of 0.6 and amalgamation ratio ($AR$) of 0.26 for beds with different aspect ratios ($L_A / L_B$). (d) Fifteen cases of $AR$ and $NTG$ have been modelled for each of the three aspect ratios. (e) Fractional Mass of the largest cluster of beds ($F_M$) vs. $NTG$. (f) $F_M$ vs. $AR$. Error bars reflect the variability observed in 10 realisations and the insert shows the different symbols and colours used.

Figure 5. (a – b) Example realisations with different bed aspect ratios ($L_A / L_B$) and orientation dispersions ($\beta$). In each case net:gross ratio ($NTG$) is 0.6 and amalgamation ratio ($AR$) is 0.17. (c) Critical amalgamation ratio ($AR_C$) as a function of $L_A / L_B$ and $\beta$.

Figure 6. Examples of high (a) and low (b) resolution unfaulted simulation models. Model resolution is defined as the ratio between the volume of a bed of the model ($V_B / V_0$) and is 0.000125 in (a) and 0.00625 in (b).

Figure 7. (a) Horizontal Fractional permeability ($F_K$) vs. net:gross ratio ($NTG$) for high resolution models ($V_B / V_0 = 0.000125$) built with different compression factors (red: $c_f = 0.1$. Black: $c_f = 1.0$). (b) $F_K$ vs. amalgamation ratio ($AR$), and (c) $F_K / NTG$ vs. $AR$. Colours as in (a).

Figure 8. (a) Power-law relationship between fractional horizontal permeability normalised by net:gross ratio ($F_K / NTG$) and the proximity of the system to the percolation threshold ($P$), where $P$ is measured as a function of amalgamation ratio, for the models shown in Figure 7. (b) Drained volume (i.e. the fraction of the connected volume occupied by the injected fluid) vs. $P$, when the indicated percentage of the production is the injected fluid.
Figure 9. (a) Fractional horizontal permeability normalised by net:gross ratio ($F_K / NTG$), against the proximity of the system to the percolation threshold ($P$) for models with different resolutions ($V_B / V_0$). The solid line shows the power-law relationship observed in the high resolution models (Fig 8a). (b) As (a) but showing the drained volume when 10% of the production is the injected fluid.

Figure 10. 2D simulation results (symbols) for horizontal fractional permeability ($F_K$) vs. the proximity to the percolation threshold ($P$), for randomly distributed squares and rectangles aligned parallel (open symbols) or perpendicular (filled symbols) to the flow direction. The solid lines are the model fits derived from the isotropic case using geometrical arguments. See text for discussion.

Figure 11. Plots of amalgamation ratio ($AR$) vs. (a) fractional horizontal permeability ($F_K$) and (b) $K_V / K_H$ ratio. The plots show 2D results in black and 3D results in red, and the three curves for each case are for beds with horizontal to vertical aspect ratios ($L_B / L_Z$) of 50, 200 and 600 (note that these are indistinguishable in the 2D horizontal permeability case). The 2D result of Stephen et al. (2001) is also indicated on (b). See text for discussion.

Figure 12. (a-c) Example realisations of the 2D process-imitating models close to their percolation thresholds. (a) Erosion is tied to individual beds with an erosion probability of 0.8. (b) Erosion is independent of deposition with an erosion / bed length ratio of 0.2 and an erosion frequency of 4. (c) As (b) but with an erosion / bed length ratio of 4 and an erosion frequency of 0.6. (d) 2D critical amalgamation ratio ($AR_C$) as a function of the erosion / bed length ratio, for the case where erosion and deposition are independent. Note that these thresholds have been derived from much higher resolution models than shown in (a-c). The horizontal line shows the 2D percolation threshold for cases where erosion and deposition are linked ($AR_C = 0.67$). See text for discussion.

Figure 13. Thickly bedded sandstones from the Mount Messenger Formation at Tongaporutu beach, New Zealand. The total cliff height is about 20m high. Shale beds are present but the sands are frequently amalgamated by localised erosion.
Figure 14. Example realisations of some of the fault systems applied to the suite of
models shown in Figure 5. (a) Randomly oriented faults with $L_F = 1.8$, $N_F = 0.5$ and
$T_F = 2.5$; see Appendix and text for definitions. (b) Faults oriented parallel to the
dominant bed orientation with $L_F = 1.8$, $N_F = 1.0$ and $T_F = 5.0$. (c) as (b), but with
$L_F = 3.6$. (d) As (c), but $T_F = 1.4$ and with the faults oriented both parallel and
perpendicular to the preferred bed orientations (i.e. the “orthogonal” orientation
model).

Figure 15. Illustration of the fault property modelling. (a) Allan diagram drawn
looking from the footwall side for a fault of the same scale as those shown in Fig 14b.
Shale layers on the hangingwall are shown in dark grey and on the footwall in paler
grey. Sand-on-sand juxtapositions are yellow. (b) As (a) but with the sand-on-sand
juxtapositions coloured for SGR. (c) Probabilistic shale smears on the fault surface
generated using SSF$_C = 5$. Smear tops are shown as the thicker lines, and smear bases
as thinner lines. (d) Smear realisation superimposed on the across-fault juxtapositions.
Non-smeared sand-on-sand juxtapositions are shown in yellow.

Figure 16. (a) Idealised Allan diagram of a half-fault (pale grey) showing the
locations of a sandstone bed in the hangingwall and footwall and a shale bed in the
hangingwall. For the case illustrated the sandstone bed in the footwall (thicker line)
will be juxtaposed against 2 (b) or 3 (c) sandstone beds in the hangingwall. See text
for discussion.

Figure 17. Fractional Mass of the largest cluster of beds ($F_M$) for different
combinations of fault and sedimentological characteristics. No fault properties are
included. Fault systems are characterised by the values of $L_F$, $T_F$ and $N_F$ at the top
of the diagram (see Appendix and text for definitions) and, within each $T_F$ column,
by the four fault orientation models shown in the blow-up in (a). UF signifies an
unfaulted model. (a) to (d) are for the four basic combinations of bed aspect ratio
($L_A / L_B$) and compression factor ($c_f$) indicated at the right of each graph. The three
different panels of each graph are for the different net:gross ($NTG$) cases indicated,
and the different coloured curves reflect different bed orientation dispersions ($\beta$).
The spots show the mean $F_M$ value for 10 realisations of each system, and the error bars are +/- 1 standard deviation.

Figure 18. Fractional mass of the largest cluster of beds ($F_M$) for different combinations of fault and sedimentological characteristics, using different (a) SGR cut-offs or (b) critical SSF values. The fault systems are the same as in Fig 17, and the characteristics of the three different sedimentological models are indicated below each panel.

Figure 19. Cumulative distributions of the change in the Fractional mass of the largest cluster of beds ($F_M$) from the unfaulted state as a function of (a) SGR cut-off values and (b) critical SSF values for models with aligned beds. The colours in (b) are for models with the reciprocal $SSF_C$ value to the $SGR_C$ values labelled in (a). (c) Fraction of the total possible change in $F_M$ using SGR cut-offs plotted against the same combination of faults and sedimentology using the reciprocal $SSF_C$ value, for $SGR_C = 0.1$ and 0.5. (d) As (c) but showing only the models with high $L_F$ and $T_F$.

See text for discussion.

Figure 20. (a). Fractional permeability for cases with open faults, characterised by the $L_F$, $T_F$ and $N_F$ values indicated at the top of the figure and three fault orientation models (faults oriented parallel or perpendicular to the flow direction or at random). The 6 different sedimentological models are characterised by the net:gross and amalgamation ratios indicated. (b) Fractional permeability for the $NTG = 0.4$, $AR = 0.063$ cases for different SGR cut-offs. Cases for which no results are shown have a probability of <0.001 of being connected in the direction of flow.

Figure 21. Fractional mass of the largest cluster ($F_M$) vs. the average number of connections per body ($B$) measured in faulted isotropic models (i.e. circular beds and randomly oriented faults) with otherwise widely different fault and sedimentological characteristics. The models define a trend with $F_M = 0.5$ at the known percolation threshold for unfaulted models ($B_C = 2.59$) as indicated by the dashed lines.
Figure 22. (a) Fractional permeability / net: gross ratio ($F_K/NTG$) and (b) drained volume when 10% of the production is the injected fluid, plotted against proximity to the percolation threshold deduced from the measured average number of connections per body ($B$), for a wide range of isotropic faulted models. The faulted bodies have average sizes in the $V_B/V_0$ ranges indicated. The lines show the limits of the ranges of results obtained for unfaulted models at the comparable resolutions (see Fig 9).
(a) $L_A/L_B = 9, \beta = 15^\circ$

(b) $L_A/L_B = 5, \beta = 90^\circ$

(c) Critical Amalgamation ratio ($AR_C$) vs. Orientation dispersion ($\beta, ^\circ$)
Model resolution ($V_B/V_0$):

- 0.000125
- 0.003 - 0.0063
- 0.0003 - 0.001
- 0.09

(a) Log (Fractional Permeability / Net: gross) vs. Proximity to threshold (P)

(b) Drained Volume vs. Proximity to threshold (P)
Bed Aspect ratio (Length parallel to flow: across-flow length)

5:1
5:2
1:1
2:5
1:5
(a)

Footwall Shale

Hangingwall Shale

Sand - Sand juxtapositions

(b)

SGR:

- Light yellow: <0.1
- Yellow: >0.1 and <0.2
- Orange: >0.2 and <0.3
- Red: >0.3

(c)

Individual smear Top

Individual smear bases

Probabilistic shale smears

(d)

(a) and (c) combined
The diagram illustrates the fractional mass of the largest cluster ($F_M$) as a function of $\bar{L}_F$, $\bar{N}_F$, and $\bar{T}_F$ for different $U_F$ values. The graph is divided into two parts:

(a) SGR Cut-offs
- $SGR_C$ values are represented by different colors:
  - $1.0$ (purple diamonds)
  - $0.5$ (red circles)
  - $0.3$ (orange triangles)
  - $0.2$ (green squares)
  - $0.1$ (blue diamonds)
  - $0.0$ (black triangles)
- $SSF_C$ values for different sealing conditions:
  - Open (purple diamonds)
  - $2$ (red circles)
  - $3.3$ (orange triangles)
  - $5$ (green squares)
  - $10$ (blue diamonds)
  - Sealing (black triangles)

(b) SSF$_C$ Values
- 'Sedimentology 1': $L_A/L_B = 20$, $\beta = 45^\circ$, NTG = 0.2, AR = 0.024
- 'Sedimentology 2': $L_A/L_B = 20$, $\beta = 45^\circ$, NTG = 0.4, AR = 0.074
- 'Sedimentology 3': $L_A/L_B = 3.2$, $\beta = 45^\circ$, NTG = 0.3, AR = 0.3