BEAT 'EM OR JOIN 'EM?: EXPORT SUBSIDIES VERSUS INTERNATIONAL RESEARCH JOINT VENTURES IN OLIGOPOLISTIC MARKETS*

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Abstract

This paper compares adversarial with cooperative industrial and trade policies in a dynamic oligopoly game in which a home and foreign firm compete in R&D and output and, because of spillovers, each firm benefits from the other's R&D. When the government can commit to an export subsidy, such a policy raises welfare relative to cooperation, except when R&D is highly effective and spillovers are near-complete. Without commitment, however, subsidisation may yield welfare levels much lower than cooperation and lower even than free trade, though qualifications to the dangers from no commitment are noted.

Keywords: Research and Development; R&D spillovers; Cooperative agreements; RJV's (Research Joint Ventures); Strategic Trade Policy; Export Subsidies; Commitment; Dynamic consistency.

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1. Introduction

Governments everywhere see international success for domestic high-technology firms as a desirable objective and measures to encourage such firms as an important component of economic policy. While this perspective suggests adopting an adversarial approach to promoting "national champions", governments are also aware of the potential benefits to home firms of cooperative arrangements such as research joint ventures with foreign firms. Finally, both domestic governments and international regulatory bodies (such as the EU Commission) are increasingly conscious of the need to formulate and enforce guidelines for pro-competitive behaviour at the international as well as at the national level.

Developing an analytical framework to deal with these issues poses formidable challenges. At the very least, it requires the application of a number of sub-disciplines in economics, drawing on work in the fields of industrial and trade policy, technology policy and competition policy. In this paper we take a small step in this direction by combining the insights from two strands of recent literature, the theory of strategic trade policy, associated especially with Brander and Spencer (1985), and the analysis of R&D cooperation in oligopolistic markets pioneered by d'Aspremont and Jacquemin (1988). In particular, we try to throw light on the relative merits of adversarial and cooperative approaches. From the perspective of domestic welfare, should governments encourage home firms to adopt a "Beat 'Em" or a "Join 'Em" approach to foreign rivals?

The theory of strategic trade policy has shown that intervention in oligopolistic markets can raise domestic welfare if governments use their superior commitment powers to "shift profits" towards home firms (at least when firms engage in quantity competition). While the limitations of the theory's interventionist thrust have been well documented, it remains one of the few strictly economic justifications for policy activism. However, the merits of subsidising exporting firms have to be re-examined in the light of two key considerations in
the recent literature. First, a number of authors have noted that the benefits of strategic intervention may be reduced if the government cannot commit in advance to its policy.\footnote{Aspects of this theme have been explored by Maskin and Newbery (1990), Goldberg (1995), Karp and Perloff (1995), O’Sullivan (1995) and Leary and Neary (1998).} Second, the benefits of adversarial policies need to be reconsidered in the light of the potential gains from cooperation between firms. While the effects of R&D cooperation in the presence of R&D spillovers has been extensively studied in a closed-economy context, its implications in open economies have been little studied as yet.\footnote{Exceptions to the general neglect include Motta (1994), Guffens (1995), Qiu and Tao (1995) and Leary and Neary (1999).}

The objective of this paper is to extend the analysis of strategic trade policy to this richer environment and to compare the effects of export subsidies with those of international cooperation on R&D through the formation of research joint ventures. We do this in a canonical model which is introduced in Section 2. This abstracts from issues of domestic competition policy by assuming a duopolistic market in which a single home firm competes with a foreign rival and all output is exported. We also consider a restricted range of policies, ignoring direct subsidies to R&D and assuming that export subsidies are not provided at all if firms cooperate on R&D.\footnote{R&D subsidies are considered by Spencer and Brander (1983), Muniagurria and Singh (1997) and Leary and Neary (1999), while subsidies to cooperating firms are studied by Qiu and Tao (1995).} Section 3 looks at the incentives firms face to invest in R&D with and without R&D cooperation, while Section 4 looks at the role of export subsidies with and without government commitment. Section 5 compares welfare levels between the four equilibria and Section 6 shows how the comparison is facilitated and the results considerably strengthened when special functional forms are assumed. Section 7 draws conclusions while the Appendix gives the technical details of the model’s solution.

\begin{equation}
\begin{align*}
c &= \min\{c(x, x^*), \ c_s = -\theta, \ c_s^* = -\theta'\} \\
c^* &= \min\{c^*(x^*, x), \ c_s^* = -\theta, \ c_s^{**} = -\theta'\}.
\end{align*}
\end{equation}

Here $\theta$ and $\theta'$ measure the effectiveness of each firm’s R&D in reducing its own costs and $\beta$ and $\beta'$ (which lie between zero and one) measure the extent to which R&D has beneficial spillover effects on the rival firm’s costs. The inverse demand function is given by:

\begin{equation}
p = p(q + q^*), \quad p' = -b, \quad (q + q^*)p'/p' = r,
\end{equation}

where $b$ is the slope (not necessarily constant) of the demand function and $r$ is a measure of the concavity of demand. Summing costs and sales revenue and adding any export subsidy payments received (where $s$ is the per unit subsidy) gives the profits of the home firm:

\begin{equation}
\pi = (p(q + q^*) - c(x, x^*) - s)q - \Gamma(x).
\end{equation}

The foreign firm’s profits $\pi^*$ are determined in the same way, except that it receives no subsidies. Finally, with no home consumption, national welfare is simply profits net of...
subsidy payments (if any):

\[ W = \pi - sq = (p-c)q - \Gamma(x) \]  

(4)

In all the games we consider, the two firms engage in Cournot product-market competition in period 2. Hence, with production costs determined by past decisions on R&D and facing a given home export subsidy (possibly zero), output levels are determined by the first-order conditions:

\[ \pi_q = p - c + s - bq = 0 \]  

(5)

\[ \pi_q^* = p - c^* - bq^* = 0 \]  

(6)

for the home and foreign firm respectively.

We consider four distinct games, which differ in terms of whether or not firms cooperate in their choice of R&D and whether or not the home government offers an export subsidy and can commit to it in advance of firms’ R&D decisions:

(i) Game F: Free trade, with no cooperation on R&D. In this benchmark two-stage game, firms choose R&D levels in period 1 and outputs in period 2.

(ii) Game C: Free trade, with cooperation on R&D. The move order is the same as Game F. The difference is that firms cooperate in their choice of R&D so as to maximise the sum of their joint profits. However, outputs in period 2 are still chosen in a non-cooperative Cournot-Nash manner. This game has been extensively examined in a closed-economy context, stemming from the work of d’Aspremont and Jacquemin (1988).

(iii) Game G: Government commitment to an export subsidy. In this three-stage game the government chooses the subsidy to maximise domestic welfare (4). Crucially, the government sets its subsidy before the firms’ choice of both R&D and exports.

(iv) Game S: Subsidisation without commitment. In this game the government also provides an export subsidy to maximise (4) but, unlike Game G, it cannot commit to the subsidy level before firms choose their R&D levels. In this three-stage game, firms first choose their R&D levels, then the government chooses its subsidy, and finally firms choose their output levels. This game has been considered by Karp and Perloff (1995), O’Sullivan (1995) and Grossman and Maggi (1997).

Notwithstanding the differences between the four games we consider, they share the property of subgame perfection. Thus at each stage in each of the four games, agents take into account the effects of their current decisions on the future decisions of all other agents.

3. Strategic Investment with and without Cooperation

We consider first the two games where the home government does not intervene. Our analysis follows Leahy and Neary (1997), who consider a general multi-firm model of a closed economy, allowing for both quantity and price competition. Specialising to the case of Cournot duopoly permits a significant strengthening of the results. For convenience, we concentrate in the text on the case where the two firms are identical. Allowing for asymmetries has little qualitative effect on the results but necessitates additional notation which is detailed in Section A.1 of the Appendix.

In choosing its optimal level of R&D, each firm takes account of the direct cost-reducing effect and also of the strategic effect of R&D on its rival’s output in the second stage. Thus, in Game F (the benchmark free trade game with no cooperation on R&D) the home firm’s first-order condition for R&D is:

\[ \frac{d\pi}{dx} = \pi_q \times \pi_q^* \frac{dq^*}{dx} = 0. \]  

(7)

Here the direct or non-strategic effect of R&D, \( \pi_q \), equals \( 8q - \Gamma' \); i.e., the gain from a
reduction in the firm’s production costs less the direct cost of the R&D itself. When this effect is zero, R&D is at its efficient level from the home country’s perspective. Hence there is over- or under-investment in R&D depending on whether the second term on the right-hand side is positive or negative. Since the home firm unambiguously gains from a fall in foreign output ($\pi_r = -b q < 0$), the sign of the second term depends on whether additional home R&D raises or lowers foreign output. Following Leahy and Neary (1997), we show in the Appendix, Section A.1, that this in turn depends on the following expression:

$$ b \frac{dq^*}{ds} = - \frac{\bar{\beta} - \beta}{1 + \bar{\beta}} \theta, \quad 0 < \bar{\beta} = \frac{\bar{\beta}}{\beta} \leq 1, \tag{8} $$

where the threshold parameter $\bar{\beta}$ is positive provided foreign output is a strategic substitute for domestic output. (Strategic substitutability, equivalent to downward-sloping reaction functions, is the normal configuration in Cournot competition, implying $r > -2$. We assume it holds henceforth.) Also we see that $\bar{\beta}$ reduces to $\frac{1}{2}$ when demand is linear ($r=0$); and that it is greater than or less than $\frac{1}{2}$ depending on whether demand is concave ($r > 0$) or convex. Pulling together the different components of the first-order condition (7) we obtain:

$$ \frac{d\pi}{ds} = \mu^q - \Gamma' = 0, \quad \text{where:} \quad \mu^q = \left[ 1 + \frac{\bar{\beta} - \beta}{1 + \bar{\beta}} \right] \theta. \tag{9} $$

$\mu^q$ is the marginal return to R&D per unit output in free trade. Equations (8) and (9) show that, when spillovers are low ($\beta < \bar{\beta}$), higher home R&D lowers foreign output and so the home firm has an incentive to over-invest in R&D ($\mu^q > \theta$). Conversely, when spillovers are high ($\beta > \bar{\beta}$), higher home R&D raises foreign output and so the home firm has an incentive to under-invest in R&D ($\mu^q < \theta$). Of course, the foreign firm faces identical incentives. Subject to technical qualifications which can be relegated to a footnote, we may conclude that, in symmetric equilibria, both firms invest less the higher the spillover parameter $\beta$ and invest more the greater the concavity of demand.

Figure 1 illustrates how the strategic effect of investment in R&D depends on the size of the spillover parameter. The curves HH and FF, with intersection at A, represent the home and foreign output reaction functions respectively in the absence of any R&D. The curve $H'H'$ represents the home reaction function when R&D is at its efficient level (where $\pi_r = 0$). Point B (with the same level of foreign output as A) therefore gives the home firm’s output level when it does not invest strategically in R&D. If spillovers are zero, the foreign reaction function is unaffected by home R&D and so the actual equilibrium would be at C. Anticipating this fall in $q'$, the home firm therefore has an incentive to increase its investment beyond the non-strategic level underlying $H'H'$. By contrast, if spillovers are complete, then from symmetry, the foreign reaction function shifts out to $F'F'$ by exactly as much as the home one, and the new equilibrium would be at D. Anticipating this rise in $q'$, which will lower its profits, the home firm therefore has an incentive to reduce its investment below the non-strategic level underlying $H'H'$. By continuity, the conclusion is clear. Provided the foreign reaction function FF is downward-sloping (i.e., provided home output is a strategic substitute for foreign output), there is some threshold level of spillovers, between zero and

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4 Spencer and Brander (1983) drew attention to this effect in the case of zero spillovers. d’Aspremont and Jacquemin (1988) considered spillovers in a linear model and showed that in that case the threshold value of the spillover parameter is $\frac{1}{2}$. 

5 The direct comparison in the text is exact with no further qualifications when the parameters $\theta$, $\beta$ and $\beta$ (i.e., $r$) are constants. More generally, the values of the two marginal returns to R&D may differ between the two equilibria to be compared. Sufficient conditions to justify this comparison are provided in Leahy and Neary (1997), Section II.B. It is shown there that, in symmetric equilibria, output and R&D are higher with strategic behaviour than without if: (i) equilibrium of each type is unique; (ii) $\mu^q > \theta$, where both parameters are evaluated at the same equilibrium (either the strategic or the non-strategic equilibrium); and (iii) the profit functions corresponding to the other equilibrium exhibit the Scade (1989) stability condition when these functions are evaluated at all points along the locus of oligopoly equilibria in $(q, l)$ space between the two equilibria.
zero and one, at which the strategic effect on investment in R&D is exactly zero.

The incentives to invest in R&D are very different in Game C when cooperation by firms leads each of them to choose its R&D so as to maximise their joint profits. The first-order condition for the home firm's choice of R&D is now:

$$\frac{d(\pi + \pi^*)}{dx} = \left\{ \pi_f + \pi_q, \frac{dq}{dx} \right\} + \left\{ \pi_f^* + \pi_q^*, \frac{dq}{dx} \right\} = 0.$$  (10)

This takes account of both direct and strategic effects on foreign as well as home profits.

Similar calculations to those already given for the no-cooperation game show that this can be written as:

$$\frac{d(\pi + \pi^*)}{dx} = \mu^C q - \Gamma, \quad \text{where:} \quad \mu^C = (1 + \beta) \frac{2 \bar{\beta}}{1 + \bar{\beta}}.$$  (11)

Now, the marginal return to R&D and hence the level of investment in symmetric equilibria is increasing in the spillover parameter $\beta$. However, it is not true that cooperation fully internalises the externality arising from R&D spillovers. Because of strategic behaviour the firm may either under- or over-invest in R&D from a national point of view. Indeed, the final strategic term in (10), $\pi_q dq/dx$, is always negative and for low spillovers it dominates, ensuring that the firm under-invests in R&D. To see this explicitly, we may rewrite $\mu^C$ in a form which brings out its symmetry with $\mu^F$ in (9):

$$\mu^C = \begin{cases} (1 - \frac{\bar{\beta}}{1 + \beta}) \theta & \text{where:} \quad \bar{\beta} = \frac{1 - \bar{\beta}}{2\bar{\beta}}. \end{cases}$$  (12)

This shows that under-investment ($\mu^C < \theta$) is more likely when spillovers are low and demand is convex (i.e., $r < 0$, implying $\bar{\beta} < \frac{1}{4}$, which in turn implies that $\beta > \frac{1}{4}$, since $\bar{\beta}$ and $\beta$ always lie on opposite sides of $\frac{1}{4}$).

Will cooperation lead to more or less investment than non-cooperation? To answer this, we need only compare the values of the two marginal return to R&D parameters, $\mu^F$ and $\mu^C$, subject to the same sort of technical qualifications given in footnote 5. The difference between them is:

$$\mu^C - \mu^F = \frac{1 - 2\bar{\beta}}{1 + \beta} \left( \beta - \frac{1}{2\bar{\beta}} \right) 0.$$  (13)

Thus cooperation leads to more R&D if the spillover parameter exceeds a new threshold, $1/(1 + 2\bar{\beta})$. This lies on the opposite side of $\frac{1}{4}$ from $\bar{\beta}$, coincides with it when demand is linear, and ranges from $\frac{1}{4}$ (when demand is so concave that $\bar{\beta}$ approaches one) to one (when demand is as convex as is possible, given strategic substitutability, so $\bar{\beta} = 0$).

These results are illustrated in Figure 2, which shows how $\mu^F$ and $\mu^C$ vary as functions of $\beta$ for different values of the demand concavity parameter $r$. Higher spillovers lead to more R&D when firms cooperate but less when they do not; and R&D is higher in both regimes when demand is concave (as shown by the solid lines) than when demand is linear (dashed lines) or convex (dotted lines).

The final question to be addressed is whether cooperation raises welfare. Since we have assumed, following the conventions of the strategic trade policy literature, that all output is exported, and since both Games F and C are free-trade equilibria, welfare and home

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6 From a world point of view, equation (11) shows that the cooperative always under-invests in R&D: because of the strategic effects in (10), $\mu^C$ is less than $(1 + \beta)\theta$, which is the marginal world social return to domestic R&D. This is a special case of Proposition 2 in Leible and Neary (1997).

7 Of course, for all but linear or iso-elastic demand functions, the degree of concavity will change as $\beta$ varies. (For convenience $\theta$ is normalised to equal unity in the Figure.)
profits coincide.\footnote{If instead we take a world welfare perspective, then the closed-economy results of Lealh and Neary (1997) apply. In particular, with low spillovers, the reduction in output brought about by R&D cooperation reduces consumer surplus to such an extent that total welfare (profits plus consumer surplus) falls.} Moreover, since the equilibria are symmetric, it follows that home profits must be at least as great with cooperation as without, since the aggregate profits of the two firms are maximised in the former case. Cooperation must lead to higher profits unless the two equilibria coincide, which occurs when the spillover parameter $\beta$ equals the threshold given in equation (13).

4. Export Subsidies with and without Government Commitment

We turn next to equilibria with government intervention. In Game $G$, where the government can commit to its export subsidy prior to decisions on R&D, the behaviour of firms is the same as in the non-cooperative free-trade game of the last section. Anticipating this behaviour, the government in the first stage chooses the export subsidy to maximise welfare, given by equation (4). To see the implications of this, totally differentiate that equation:

$$dW = -s dq - b q dq^* + \left( 6q - \Gamma \right) dx + \beta q dx^*.$$ \hspace{1cm} (14)

The first two terms on the right-hand side are fairly standard. The first reflects the deadweight loss from increased exports when a subsidy is in place, while the second reflects the standard rent-shifting gain in welfare, as the foreign firm is pushed down its reaction function. The other two terms are less familiar.\footnote{Our discussion of the remaining two terms follows Neary and Lealh (1997).} The term in $dx$ can also be written, from (9), as $(\theta - \mu^f) q dx$, showing that it reflects the divergence between marginal social and private returns to R&D. When these differ, there is a motive for subsidisation to offset in part the inefficient investment which the home firm carries out for purely strategic reasons. Of course, this term can be either positive or negative: when spillovers are low, the home firm over-invests strategically so an offsetting tax is warranted, and conversely when spillovers are high. Finally, the fourth term reflects what we may call inter-temporal rent-shifting, as opposed to the conventional intra-temporal kind represented by the second term. When spillovers are strictly positive, home profits and welfare are directly affected by foreign R&D. If we make the plausible assumption that foreign R&D depends negatively on $s$, then this term tends to encourage an export tax: government commitment to an export tax raises foreign R&D and so (because of spillovers) brings about a rise in profits that the home firm cannot credibly attain by itself.

While interpreting the terms in (14) is insightful, not much more can be said about their net impact on the sign of the optimal subsidy at this level of generality. We can of course set the equation to zero to obtain an expression for the optimal subsidy:

$$s = \frac{1}{\beta} \left( - \frac{dq}{dx} \right)^{-1} \left\{ - \frac{b q}{ds} \frac{dq}{dx} + \frac{\beta - \beta^* \beta q}{1 + \beta} \frac{dx}{ds} + \beta \theta q \frac{dx^*}{ds} \right\}.$$

This throws some light on the determinants of $s$. However, its interpretation is hampered by the fact that, in general, we cannot even be sure that the various derivatives take their expected signs. (We would expect an increase in $s$ to raise $x$ and $q$ and to lower $x'$ and $q'$.)\footnote{The principal technical difficulty is that $\mu^f$ and the corresponding marginal return for the foreign firm are in general functions of all the endogenous variables, $x, x', q, q'$ and $s$. In the linear-quadratic case considered in Section 5, these two parameters are constant.} The one thing we can be sure of is that the ability of the government to commit to an export subsidy must increase welfare relative to the free-trade level.

The situation is very different in Game $S$, when the government cannot commit to its subsidy level until after firms choose their R&D. It therefore chooses the export subsidy to...
set to zero equation (14) without the R&D terms. This leads to the standard static rent-shifting subsidy first derived by Brander and Spencer (1985):

\[ s(x, x^*) = -bq \frac{dq^*}{dq}, \]  

(16)

where \( \frac{dq}{dq} \), the slope of the foreign firm’s static reaction function, is negative provided foreign output is a strategic substitute for domestic output. We write the subsidy as a function of the two R&D levels, because they are chosen prior to it. In the first stage of the game, both firms anticipate this dependence and, in a subgame perfect equilibrium, take it into account in choosing their optimal R&D levels. Thus the first-order condition for home R&D is now:

\[ \frac{dx}{dx^*} = \pi_x^* + \frac{\pi_y^* dq^*}{dx} + \pi_y^* \frac{dx}{dx} = 0. \]  

(17)

A higher subsidy must raise profits \( (\pi_x = q > 0) \), so the impact of the additional final term on the marginal profitability of R&D hinges on the effect of R&D on the magnitude of the optimal subsidy, \( \frac{ds}{dx} \). Leahy and Neary (1998) derive a necessary and sufficient condition for this term to be positive and show that this is the normal case.\[^{11}\] Assuming for concreteness that this is the case, the home firm over-invests further in R&D with a view to obtaining a higher subsidy. It does so both because of the direct effect of the third term in (17) but also because, as shown in the Appendix, a positive value for \( \frac{ds}{dx} \) raises the effectiveness of home R&D in lowering foreign output (i.e., it makes \( \frac{dq}{dx} \) more likely to be negative). Finally, the foreign firm faces a similar incentive. Its first-order condition is:

\[ \frac{dx^*}{dx^*} = \pi_x^* + \frac{\pi_y^* dq^*}{dx^*} = 0. \]  

(18)

Foreign profits are not affected directly by changes in the home subsidy. However, the magnitude of \( \frac{dq}{dx^*} \) depends on the responsiveness of the subsidy to foreign R&D, \( \frac{ds}{dx^*} \). Just as we would expect higher home R&D to mandate a higher subsidy, so we would expect higher foreign R&D to mandate a lower subsidy. This in turn makes it more likely that \( \frac{dq}{dx^*} \) will be negative and so encourages over-investment by the foreign firm too. (See Appendix A.1.)

What is the effect on welfare of these additional incentives to engage in R&D which both firms face as a result of the endogeneity of the subsidy? In general, we can be sure that in this model the government’s inability to commit cannot raise welfare relative to the commitment game and will lower it if R&D has any effect on costs. We can say something further by inspection of (14). The third term (substituting from (17) for the marginal return to R&D in this game) shows that extra investment by the home firm lowers welfare except when spillovers are high. As for extra investment by the foreign firm, it has a direct effect tending to raise welfare, provided there are positive spillovers. However, it also has an indirect effect on the location of the foreign firm’s period-2 reaction function. This effect tends to increase foreign output and is not offset by the subsidy (which, from (16), only neutralises the effects of foreign output changes along a given foreign reaction function). Except for high spillovers this effect is likely to dominate, so reducing home welfare further.

5. Comparing Welfare Across Equilibria

The results of the two previous sections for the levels of welfare in the four games may be summarised as follows:

\[^{11}\] Over-strong sufficient conditions for \( \frac{ds}{dx} \) to be positive are that both the demand curve and the foreign reaction function are non-convex. These conditions are satisfied if demand is linear, as in Section 6 below.
\[ W^C \geq W^F < W^G \geq W^S. \]  
\( (19) \)

From Section 3, cooperation cannot lower welfare relative to free trade and will raise it except when the spillover parameter \( \beta \) equals the threshold value given in equation (13). From Section 4, commitment to an optimal export subsidy always raises welfare relative to free trade; and it raises welfare relative to an optimal subsidy without commitment except when R&D is ineffective (when the commitment issue is irrelevant).

These results are interesting but they leave open two crucial questions. First, when will intervention without commitment raise welfare relative to free trade? Clearly, if R&D is ineffective, there is no cost to the government’s inability to commit and so \( W^F \) exceeds \( W^G \). However, when R&D is effective then, as Karp and Perloff (1995), O’Sullivan (1995) and Grossman and Maggi (1997) have shown, over-investment by the home firm with a view to manipulating the export subsidy can lower welfare relative to free trade.

The second question which the inequalities in (19) leave unanswered, is the one posed in the title to this paper. When will optimal intervention raise welfare relative to international R&D cooperation? This in turn implies two sub-questions, since intervention may be with or without commitment. We can infer from (19) that intervention with commitment dominates cooperation when \( \beta \) equals the threshold given in equation (13). Moreover, intervention without commitment dominates cooperation when R&D is ineffective.

To go beyond these very weak sufficient conditions we must assume particular functional forms for the demand and cost functions.

6. The Linear-Quadratic Case

In order to throw further light on the questions discussed in the last section, we simplify the model by assuming that the behavioural functions take special forms.\(^{12}\) Specifically, we assume that the demand function is linear:

\[ p(q,q^*) = a - b(q + q^*). \]  
\( (20) \)

We also assume that each firm’s marginal production cost function is linear in its own and its rival’s R&D:

\[ c(x, x^*) = c_0 - \theta (x + \beta x^*) \quad \text{and} \quad c^*(x^*, x) = c_0 - \theta (x^* + \beta x) \]  
\( (21) \)

for the home and the foreign firm respectively. Finally, we assume that the R&D cost functions are quadratic in R&D:

\[ \Gamma(x) = \gamma x^2/2 \quad \text{and} \quad \Gamma^*(x^*) = \gamma (x^*)^2/2. \]  
\( (22) \)

(All the parameters \( a, b, c_0, \theta, \beta \) and \( \gamma \) are constant.) The detailed solutions under these specifications are given in the Appendix. Probably the most insightful way to explore their implications is by simulating the levels of welfare in the four games. Fortunately, comparisons between the four games with symmetry depend on only two parameters: the degree of spillovers \( \beta \) and a new parameter \( \eta \), defined as: \( \eta = \theta / \beta \gamma \). Following Lealehy and Neary (1999), this can be interpreted as the \textit{relative effectiveness of R&D}, since \( \theta \) gives the reduction in unit production costs per unit R&D, \( \gamma \) is the marginal cost of R&D and \( b \) (the slope of the demand function) is a measure of the size of the market. Figures 3 to 8 compare the levels of welfare in each of the four games as functions of \( \eta \) and \( \beta \). In Figures 3, 4, 5 and 7, which illustrate the levels of welfare in the four equilibria, units are chosen so that

\(^{12}\) The effects of asymmetries between firms in similar models are considered in Neary (1994), Karp and Perloff (1995) and Lealehy and Neary (1999).
welfare in free trade is unity when $\eta$ is zero.\textsuperscript{13}

Figure 3 shows how welfare in free trade varies with $\eta$ and $\beta$. (The exact expression is given by equation (34) in the Appendix.) Welfare falls with $\eta$ in the absence of spillovers, as additional strategic over-production benefits consumers at the expense of firms.\textsuperscript{14} However, welfare rises with $\eta$ for moderate or high spillovers and (provided $\eta$ is strictly positive) always rises with $\beta$. (Though recall from Figure 2 that with higher spillovers firms engage in less R&D.) This may be compared with the level of welfare when firms cooperate on R&D, shown in Figure 4. (The exact expression is given in equation (36) in the Appendix.) Now welfare rises with $\eta$ even with no spillovers. (Of course, in this case, cooperation has no cost-saving implication and is purely a strategic device which avoids over-investment and so leads to less R&D.) Welfare rises even more rapidly when both parameters attain moderate or high levels, reaching a value of 9.0 (truncated from the figure to facilitate presentation) when both $\eta$ and $\beta$ are unity.

We know from Section 3 that cooperation always raises welfare, except when $\beta$ attains its threshold level $\bar{\beta}$, which with linear demands equals $\bar{\beta}/2$. Comparison of Figures 3 and 4 shows in addition that the gains from cooperation are increasing in $\eta$ and in the gap between $\beta$ and $\bar{\beta}$. (It is shown in the Appendix, equation (37), that the ratio $W^C/W^F$ is symmetric in $\beta$ around the $\beta=\bar{\beta}$ threshold.) Recall however, that the reason for the superiority of cooperation is very different in the low- and high-spillover cases. For low spillovers, it allows the firms to reduce their R&D and avoid strategic over-investment; whereas for high spillovers it has the more natural effect of increasing investment, though still not to a level which is efficient from a world perspective.

Turning next to the two cases where the government intervenes to provide an export subsidy (G and S), a complication arises. If R&D is even moderately effective, the effect of the subsidy may be to drive the foreign firm from the market. For parameter values at which this happens (intermediate or high values of $\eta$ and low values of $\beta$) we assume that the home government offers a subsidy just sufficient to drive foreign profits to zero. In Game G, the government commits in advance of R&D decisions to the entry-preventing subsidy. Moreover, for some parameter values an optimal rent-shifting equilibrium may exist, in which the foreign firm earns strictly positive profits, and yet entry prevention yields higher welfare. The reason is that, with entry prevention, home investment in R&D is always at the efficient level (defined by $\theta q=1$), since, if the foreign firm does not enter, the home firm has no incentive to over-invest for strategic reasons.

The outcome of these considerations (the detailed calculations for which are given in the Appendix) is the welfare function for Game G shown in Figure 5. It is clearly the upper envelope of two single-peaked functions. That covering the larger part of the parameter space corresponds to the rent-shifting equilibrium. It starts at 1.125 (=9/8) when there is no R&D and increases relatively gently in both $\eta$ and $\beta$ thereafter. The other function corresponds to the entry-prevention equilibrium. It applies when R&D is highly effective (so subsidisation causes the home firm to expand a lot and squeeze out foreign output) and spillovers are relatively low (so the foreign firm does not benefit from home R&D). It is clearly strongly increasing in $\eta$ and decreasing in $\beta$.

We know from general principles that the welfare level attainable when the government commits to a subsidy must exceed that in free trade. However, comparing Figures 3 and 5, it is striking that the gains from an optimal rent-shifting subsidy are

\textsuperscript{13} From equation (34) in the Appendix, this implies the normalisation $(a-c)^2=9b$. Copies of the GAUSS program used to calculate the equilibria are available on request.

\textsuperscript{14} This result was also noted in Leahy and Neary (1996), footnote 6.
relatively modest: 12.5% in the static case when η is zero, as we have seen, and even less when both η and β are high. By contrast, the gains from entry prevention (when η is high and β is low) are considerable. Figure 6 shows that entry prevention also leads to large gains relative to cooperation. However, with rent shifting the gains are much smaller and when spillovers are high and R&D very effective rent-shifting is inferior to cooperation.

This figure provides one answer to the question posed in the title of the paper. Commitment dominates cooperation for most parameter values, though only modestly except when it enables the home government to deter entry by the foreign firm.

Of course, the assumption that the government can commit in advance to its export-subsidy prejudices the comparison against cooperation. The story is very different when the government cannot commit to its subsidy, as illustrated in Figure 8. Once again, there is both a rent-shifting and an entry-prevention regime. However, since the government moves second, it is effectively the home firm which determines which regime prevails. In the entry-prevention case, the home firm chooses a level of investment which is just sufficient to lead to a subsidy that will drive the foreign firm’s profits to zero. As in Game G, the rent-shifting regime prevails over most of the parameter space. Welfare is lower the more effective is R&D, though it falls less rapidly for higher spillovers. This contrasts sharply with the entry-prevention regime, which as in Game G is the dominant policy when R&D is highly effective and spillovers are low. Welfare now increases rapidly in η and declines rapidly in β. Note that there is a wide range of parameters (high η and intermediate to high β) for which welfare is negative, often highly so. (Of course, home profits are always positive.) Comparing Figure 7 with Figures 3 and 4, the rent-shifting equilibrium in Game S yields higher welfare than either free trade or cooperation only for low values of η, although if entry prevention occurs Game S may yield considerably higher welfare.

The last issue we address diagrammatically is the relative size of the subsidies in the G and S equilibria. These are illustrated in Figures 8 and 9 respectively. The relatively flat portion of Figure 8 corresponds to the optimal rent-shifting subsidy with government commitment. Although the vertical scale masks its variation, it is increasing in η for low β and decreasing in η for high β; while for positive η it is a U-shaped function of β. These patterns reflect the complicated interaction of the different motives for subsidisation discussed in connection with equation (15) (and illustrated for the linear-quadratic case in equation (40)). However, all this variation is dwarfed by the contrast with, first, the entry-prevention subsidy in Game G, which is typically much larger and is strongly decreasing in both η and β; and, second, the optimal subsidy in both regimes of Game S illustrated in Figure 9. (Once again, some extremely high values of the optimal subsidy have been truncated to facilitate viewing the figure.) For all positive values of η the optimal subsidy without commitment is considerably greater than the corresponding subsidy with commitment. This suggests that a simple device to avoid the welfare losses from non-commitment would be to place a ceiling on the subsidy rate or on the total amount of subsidy payments.

---

15 This does not mean that, for parameter values where entry prevention dominates, the home firm is actually a Stackelberg leader, although just as in the static Brander-Spencer game the outcome is the one it would choose if it were. In the first stage of the game, the entry-preventing level of investment by the home firm and a zero level by the foreign firm are best responses to each other.

16 With no spillovers, the no-commitment case dominates free trade for low and high values of η but not for intermediate values: a U-shape also found by Grossman and Maggi (1997) but for different reasons. In their model, only the home firm invests in R&D and when η is high the extra subsidy-induced investment has a low social cost. In our symmetric model, the U-shape reflects the higher welfare attainable with entry prevention when η is high.
7. Conclusion

This paper has compared adversarial with cooperative industrial and trade policies in a dynamic oligopoly game in which a home and a foreign firm compete in R&D and output and, because of spillovers, each firm benefits from the other's R&D. We have shown that the relative merits of assisting domestic firms versus facilitating international research joint ventures depend on three key features: the effectiveness of R&D in reducing costs, the extent of spillovers between firms, and the degree to which the government can commit to its export subsidy policy in advance of firms' investment decisions.

Concerning the effects of R&D cooperation, we drew on the results of Leahy and Neary (1997) to show that it raises profits and hence (ignoring home consumption) raises domestic welfare if spillovers are either relatively high or relatively low. However, the reasons it does so differ between the two cases. When spillovers are low, an individual firm has an incentive to over-invest in order to give itself a strategic advantage against its rival in subsequent product-market competition. Cooperation over-internalises this externality, serving in effect as an anti-competitive device to restrict R&D and output. A supra-national body concerned with encouraging competition at the international rather than merely the domestic level should not view cooperation as desirable in this case. By contrast, when spillovers are high, each individual firm faces an incentive to under-invest, since otherwise its foreign rival will enjoy the benefits of its cost-reducing investments. In this case, cooperation leads to over-investment from a national point of view, to the benefit of foreign consumers. At some intermediate level of spillovers these two incentives offset each other and there is no social or private gain from R&D cooperation.

Provided the government is able to commit to an export subsidy before decisions on R&D are taken, and so avoid strategic manipulation by firms, such a policy typically yields modest welfare gains relative to R&D cooperation. Two exceptions to this generalisation (highlighted in Figure 6) were noted. First, if R&D is highly effective and spillovers are near-complete, cooperation on R&D is sure to raise welfare considerably more than subsidisation. Second, if R&D is highly effective but spillovers are relatively low, welfare may be maximised by a policy which subsidises exports sufficiently to prevent the foreign firm from entering the market. In such cases, the welfare gains from exploiting a monopoly position where R&D greatly reduces costs are considerably greater than the benefits from cooperation. Of course, such a policy would be vulnerable to pressures from international regulatory bodies and to retaliation by foreign governments. Nevertheless the scale of the potential welfare gains draws attention to the incentives which governments face to engage in such predatory policies. The answer to the question posed in the title of this paper therefore seems to be that "Beat 'Em" is mildly better than "Join 'Em" but, if you can get away with it, "Kill 'Em" is best of all.

A key determinant of the implications of our results for policy making is the magnitude of the spillover parameter. Empirical evidence, reviewed by Griliches (1992), tends to find that between 20% and 40% of the cost savings from R&D expenditures cannot be appropriated by the firms which carry out the investments. This would suggest that relatively low values for the key parameter \( \beta \) in our results are appropriate, implying that adversarial strategies are likely to be preferable to cooperative ones. On the other hand, many authors have suggested that the degree of spillovers should be viewed as a choice variable. Kamien, Muller and Zang (1992) go further and argue that cooperative synergies will always ensure that spillovers are complete (\( \beta = 1 \)) when cooperation occurs. (They reserve the term "R&D cartel" for the type of cooperation we have considered, where the size of \( \beta \) is unaffected by the decision to cooperate.) Which of these arguments dominates...
presumably depends on the particular industry to which the model is applied. However, in all cases the caveat noted in Leahey and Neary (1997) should be borne in mind: cooperation always raises profits relative to non-cooperation, so the relevant issue for policy is whether cooperation should be prohibited rather than whether it should be encouraged.

The final issue on which our results throw light is the likelihood and size of welfare losses from strategic trade policy in cases where the government cannot commit to its subsidy in advance of firms' investment decisions. In such circumstances, both firms have incentives to alter their investment levels, typically upwards, so as to influence the magnitude of the export subsidy. Such strategic investments are socially wasteful, of course, and even for moderate levels of R&D effectiveness, the resulting welfare losses may more than offset the gains to strategic trade policy, leading to welfare levels lower than would be attainable had the government committed to free trade. (The possibility of such immiserizing intervention was independently noted by Karp and Perloff (1995), O'Sullivan (1995) and Grossman and Maggi (1997).) Our simulations show that the welfare losses resulting from an inability to commit to future policies may indeed be substantial. However, they also point to two qualifications to this argument. First, if R&D is relatively effective and spillovers are low, the home firm may be able to engage in a level of investment which prevents the foreign firm from entering the market. The surprising feature of this outcome is not that the existence of a strategic trade policy programme leads to a monopoly position for the home firm even though both firms are ex ante identical. Rather it is that the monopoly profits which accrue to the home firm when R&D is highly effective may raise welfare above the level it would have reached even with R&D cooperation. Second, we noted that, whether the foreign firm enters or not, the subsidy level which emerges in equilibrium is many times greater than that which would be provided if there was no R&D and/or if the government could commit in advance. This suggests that capping the level of subsidy payments may be a simple but effective way of avoiding the capture of subsidy programmes by domestic firms.

Appendix

A.1 Strategic Investment with and without Cooperation

To calculate the strategic effects on R&D choice in equations (7) and (10), we need to solve the period-2 output game. Totally differentiating the first-order conditions for output, (5) and (6):

\[
\begin{bmatrix}
\pi_{w} & \pi_{w}' \\
\pi_{f} & \pi_{f}'
\end{bmatrix}
\begin{bmatrix}
d_q \\
d_q'
\end{bmatrix}
= 
- \begin{bmatrix}
\theta & \beta \theta' \\
\beta & \beta \theta'
\end{bmatrix}
\begin{bmatrix}
x \\
x'
\end{bmatrix}
= 
\begin{bmatrix}
\theta & \beta \theta' \\
\beta & \beta \theta'
\end{bmatrix}
\begin{bmatrix}
\pi_{w} & \pi_{w}' \\
\pi_{f} & \pi_{f}'
\end{bmatrix}
\begin{bmatrix}
d_q \\
d_q'
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
d_x.
\]

(23)

The diagonal elements in the coefficient matrix are negative from the home and foreign second-order conditions and the determinant must be positive for stability:

\[
\Delta = \pi_{w}(x - \pi_{w}) - \pi_{w}(x - \pi_{w}) > 0.
\]

(24)

Solving (23) with \(x\) fixed and defining \(\psi = \partial x / \partial z\) yields:

\[
\begin{bmatrix}
d_q \\
d_q'
\end{bmatrix}
= 
\begin{bmatrix}
\beta \theta \pi_{w} - \beta \theta' \pi_{w} \\
\beta \theta \pi_{f} - \beta \theta' \pi_{f}
\end{bmatrix}
\begin{bmatrix}
\pi_{w} & \pi_{w}' \\
\pi_{f} & \pi_{f}'
\end{bmatrix}
\begin{bmatrix}
d_q \\
d_q'
\end{bmatrix}
\begin{bmatrix}
\theta & \beta \theta' \\
\beta & \beta \theta'
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
d_x.
\]

(25)

These can be simplified by calculating the profit function derivatives explicitly. (For example, \(x_{w} = -b(2 + \sigma r), \Delta = b \theta(3 + r), \) etc., where \(\sigma\) is the market share of the home firm.) Setting \(\psi\) equal to zero the expression for \(d q / d x\) can be simplified to:

\[
b \frac{d q^*}{d x} = \frac{-\bar{\beta} \pi_{w} - \beta \pi_{w}^*}{\bar{\theta} + \beta}, \quad \bar{\beta} = \frac{\pi_{w}^*}{\pi_{w}} = \frac{1 + \sigma r}{2 + \sigma r},
\]

(26)

where \(\bar{\theta} = \theta / \theta'\) and \(\sigma\) is the foreign market share. The denominator is positive from the
home firm’s second-order condition and the numerator is positive provided foreign output is a strategic substitute for domestic output. Imposing symmetry (\( \theta=1 \), \( \beta'=\beta \), \( \sigma=\sigma'=\frac{1}{2} \)) ensures that \( \tilde{\beta} \) is less than one and takes the special form given in equation (8).

To calculate the first-order condition for R&D in Game C, we also need the expression for \( \frac{dq}{dx} \) from (25). Substituting in (10) with \( \psi \) equal to zero gives:

\[
\mu^c = \mu^f - \frac{1 + 2\tilde{\beta}'}{1 + \tilde{\beta}'} \left( \tilde{\beta} - \frac{\tilde{\theta}}{1 + 2\tilde{\beta}'} \right) \frac{\theta' q'}{q}, \quad \text{where:} \quad \tilde{\beta}' = \frac{\pi_{q'} \pi_{\psi}}{\pi_{\psi} \pi_{q}}, \quad (27)
\]

Imposing symmetry (so \( \tilde{\beta}'=\tilde{\beta}, \tilde{\theta}=\theta \) and \( q'=q \)) gives equation (11) in the text.

In Game S, these calculations may be repeated with a non-zero value for \( \psi \). This shows that the expression for \( \frac{dq^*}{dx} \), equation (8), becomes:

\[
\frac{dq^*}{dx} = -\frac{\tilde{\beta}(\theta+\psi)-\beta' \theta}{\tilde{\theta} \tilde{\beta}}. \quad (28)
\]

This shows that, other things equal, a positive value for \( \psi \) tends to reduce \( \frac{dq}{dx} \) and so to increase the incentive for the home firm to over-invest. A similar series of derivations shows that the same is true for the foreign firm, assuming that \( \frac{\partial v}{\partial x} \) is negative.

A.2 Product-Market Competition with Linear Demands

With linear demands given by (20), the first-order conditions for output, equations (5) and (6), simplify to:

\[
\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} bq \\ bq^* \end{bmatrix} = \begin{bmatrix} a-c^* \\ a-c \end{bmatrix} \quad (29)
\]

Solving for \( q \) and \( q^* \) and substituting from the cost functions (21) gives:

\[
3b \begin{bmatrix} q \\ q^* \end{bmatrix} = \begin{bmatrix} A + (2-\beta)bx - (1-2\beta)bx^* + 2s \\ A + (2-\beta)bx^* - (1-2\beta)bx - s \end{bmatrix}, \quad (30)
\]

where \( A=a-c_0 \). These equations apply in all four equilibria. In symmetric free-trade equilibria (Games F and C) they simplify to:

\[
3bq = A + (1-\beta)bx. \quad (31)
\]

A.3 Non-Cooperative Choice of R&D in the Linear-Quadratic Case

Calculating \( \frac{dq}{dx} \) from (30) and substituting into the R&D first-order condition (7) gives equation (9) specialised to the linear-quadratic case:

\[
\frac{2}{\beta} (2-\beta) \theta q = \gamma x. \quad (32)
\]

With symmetry, \( q=q' \) and \( x=x' \), so the full solution for Game F can be found from (31) and (32). The level of welfare can then be obtained by substituting from the first-order condition for output (5) into the welfare function (4):

\[
W = \pi = bq^2 - \gamma x^2/2. \quad (33)
\]

Calculating this explicitly for the free-trade case gives:

\[
W^F = \frac{9-2(2-\beta)^2 \eta}{9-2(1+\beta)(2-\beta) \eta} \frac{A^2}{b}. \quad (34)
\]

Since welfare and profits are identical, this expression cannot be negative if both firms are to enter the market, which requires that \( \eta \leq \frac{9}{2(2-\beta)^2} \). If this constraint is to bind for all admissible values of \( \beta \), then \( \eta \) cannot exceed 1.125 (-9/8). Within this range, \( W^F \) is increasing in \( \beta \) for \( \eta>0 \) and increasing in \( \eta \) if and only if \( 2\beta>2(1+\beta)(2-\beta) \eta \). Normalised by its zero-R&D value (\( A^2/b \), \( W^F \) equals 0.36 (-9/25) when \( \eta=1 \) and \( \beta=0 \).
and equals 2.52 (=63/25) when \( \eta = 1 \) and \( \beta = 1 \).

### A.4 Cooperative Choice of R&D in the Linear-Quadratic Case

When the two firms cooperate on their choice of R&D, the first-order condition for R&D, equation (10), becomes:

\[
0 q + \frac{1 - 3 \beta}{3} (q - 2q^*) = \gamma x. \tag{35}
\]

Imposing symmetry and combining with the first-order condition for output (5) and the expression for welfare (33), allows us to calculate welfare explicitly in this case too:

\[
W^C = \frac{1}{9 - 2(1 - \beta)^2 \eta} \frac{A^2}{b}. \tag{36}
\]

As before, \( \eta \) cannot exceed 1.125 for \( W^C \) to be non-negative at all admissible values of \( \beta \). \( W^C \) is increasing in both \( \eta \) and \( \beta \). Normalised by its zero-R&D value \( (A^2/b) \), \( W^C \) equals 1.29 (=9/7) when \( \eta = 1 \) and \( \beta = 0 \) and equals 9.0 when \( \eta = 1 \) and \( \beta = 1 \). Finally, the ratio of \( W^C \) to \( W^F \) may be calculated from (34) and (36) as:

\[
\frac{W^C}{W^F} = 1 \cdot \frac{18(1 - \beta)^2 \eta}{[9 - 2(1 + \beta)^2 \eta][9 - 2(1 - \beta)^2 \eta]}, \tag{37}
\]

which equals unity when \( \beta = \frac{1}{2} \) and elsewhere is greater than unity, increasing in \( \eta \) and symmetric around the \( \beta = \frac{1}{2} \) threshold.

### A.5 Government Commitment to a Subsidy in the Linear-Quadratic Case

With government intervention the equilibrium is not symmetric, so we need to distinguish \( x' \) from \( x \) and \( q' \) from \( q \). When the government can commit to a subsidy we need to set equation (14) to zero and solve for the optimal subsidy. The first step is to use the two first-order conditions for R&D (equation (32) for the home firm and the corresponding equation for the foreign firm) to eliminate \( dx \) and \( dx' \) from (14). This yields:

\[
dW = - \left[ s + \frac{2}{3}(1 - 2\beta)(2 - \beta) \eta \right] dq - \left[ 1 - \frac{2}{3}(1 - 2\beta) \eta \right] bq dq^*. \tag{38}
\]

The next step is to calculate the slope of the foreign firm's generalised output reaction function, which shows, from the perspective of the home government, how the foreign firm's output responds to changes in home output, taking account of the foreign firm's anticipatory adjustment in R&D. For given levels of R&D the familiar static reaction function is the second equation in (29). Using the two first-order conditions for R&D once again to eliminate \( x \) and \( x' \), we can solve for the generalised reaction function:

\[
2 \left[ 1 - \frac{2}{3}(2 - \beta) \eta \right] q^* = \frac{A}{b} - \frac{1 - \frac{2}{3}(2 - \beta) \eta}{q}. \tag{39}
\]

The slope simplifies to \( -\beta \) when R&D is ineffective \( (\eta = 0) \). Substituting for \( dq'/dq \) from (39) into (38) and equating to zero, we can solve for the optimal subsidy:

\[
s = \frac{1}{2} \left[ \frac{1 - \frac{2}{3}(2 - \beta) \eta}{1 - \frac{2}{3}(2 - \beta) \eta} - \frac{2}{3}(1 - 2\beta)(2 - \beta) \eta \right] bq. \tag{40}
\]

The first term in parentheses represents the rent-shifting motive for subsidisation (both inter- and intra-temporal), and is always positive; while the second represents the consequence of offsetting the strategic over- or under-investment by the home firm, and is negative if and only if \( \beta \) is less than \( \frac{1}{2} \). Along with the four first-order conditions, we now have five equations in the five unknowns, \( q, q', x, x' \) and \( s \). While explicit solution is not insightful, it is straightforward to calculate the results and substitute them into the expression for welfare, which in the presence of a subsidy is not (33) but rather:

\[
W = (bq - s) q - \gamma x^2/2. \tag{41}
\]
A.6 Subsidisation without Government Commitment in the Linear-Quadratic Case

With linear demands the static optimal subsidy (16) becomes simply: \( s = bq_2 / 2. \)

Substituting this into the first-order conditions for output (30) allows us to calculate the key derivatives needed to assess the firms' choice of R&D:

\[
\frac{dx}{dx} = \frac{2 - 2\beta}{4} \sigma; \quad \frac{dq}{dx} = -\frac{2 - 3\beta}{4\beta} \sigma; \quad \frac{dq^*}{dx^*} = -\frac{1 - 2\beta}{2\beta}. \tag{42}
\]

Using these results in the home and foreign first-order conditions for R&D, (17) and (18), we obtain:

\[
(2 - \beta)q = \gamma x, \tag{43}
\]

\[
\frac{1}{2} (2 - 2\beta) q^* = \gamma x^*. \tag{44}
\]

Once again, we have five equations in the five unknowns, \( q, q^*, x, x^* \) and \( s \). It is most convenient to reduce the system to two equations in \( q \) and \( q^* \), which may be solved for:

\[
\Delta \begin{bmatrix} q \\ q^* \end{bmatrix} = \begin{bmatrix} 2 - (1 - \beta)(3 - 2\beta) \eta \\ 1 - 2(2 - \beta)(1 - \beta) \eta \end{bmatrix} \frac{\Delta}{\sigma}, \tag{45}
\]

where:

\[
\Delta = (2 - (2 - \beta) \eta) \{ 4 - \frac{1}{2} (3 - 2\beta)^2 \eta \} - \frac{1}{2} (1 - 2\beta)(3 - 2\beta)(2 - 2\beta) \eta^2. \tag{46}
\]

The solution for \( q \) can be used to calculate welfare, since from (41) and (43):

\[
W^S = [1 - (2 - \beta)^2 \eta]^2 q^2. \tag{47}
\]

Since firms move first in this game, we also need to check that profits are positive. Home profits may be calculated in a similar manner to welfare as:

\[
\pi^S = (1 - \frac{1}{2} (2 - \beta)^2 \eta) bq^2. \tag{48}
\]

This is positive throughout the relevant parameter space. As for foreign profits, they are proportional to foreign output. Hence, from (45), we see that the foreign firm is driven out of the market for relatively low values of \( \eta \) when spillovers are low; as low as \( \eta = 0.25 \) when \( \beta = 0 \). Moreover, because the home firm over-invests so much in this game, there are some parameter values at which welfare is negative even though the foreign firm is still profitable. (Home welfare falls to zero at the same value of \( \eta \) (0.25) as foreign output when spillovers are zero but at lower values when spillovers are positive.)

A.7 The Entry-Preventing Subsidy in Game G

If foreign R&D and output are zero, the first-order conditions for home and foreign output (29) simplify to:

\[
2bq = A + \beta x + s \quad \text{and} \quad bq = A + \beta 0x. \tag{49}
\]

To these must be added the home first-order condition for R&D which, since there is no foreign output to be strategically manipulated (\( dq'/dx = 0 \)) is simply the efficient condition \( \beta q = \gamma x \). These three equations may be solved to give simple expressions for home output, the subsidy and welfare:

\[
\tilde{q}^0 = \frac{A}{1 - \beta \eta} b, \quad \tilde{q}^0 = \frac{1 - (1 - \beta) A}{1 - \beta \eta} b \quad \text{and} \quad \tilde{W}^0 = \frac{(1 - 2\beta) A^2}{2(1 - \beta \eta)^2 b}. \tag{50}
\]

Which of the entry prevention or rent-shifting regimes prevails depends on whether the level of welfare \( \tilde{W}^0 \) is greater or less than that in the rent-shifting Game G of Section A.5 above.

A.8 Entry-Preventing Investment in Game S

If the foreign firm is to be just deterred from entering, the first-order conditions for
output given in (49) continue to apply. In addition, the subsidy must still obey the static rule $s = bq/2$. These three equations can be solved for the entry-preventing level of investment (which is independent of $\gamma$) and the implied levels of output, profits and welfare:

$$
\begin{align*}
\bar{q}^* &= \frac{1}{2-3\beta} \frac{A}{\theta}, \\
\bar{q}^* &= \frac{2(1-\beta) A}{2-3\beta} b, \\
\bar{q}^* &= \frac{8(1-\beta)^2 \eta - 1}{2(2-3\beta)^2} \frac{A^2}{\eta} b\quad\text{and}\quad \bar{w}^* = \frac{4(1-\beta)^2 \eta - 1}{2(2-3\beta)^2} \frac{A^2}{\eta} b
\end{align*}
$$

(51)

In this case, which of the entry-prevention or rent-shifting regimes prevails depends on whether the level of profits $\bar{q}^*$ is greater or less than that in the rent-shifting Game S of Section A.6 above. The requirements that both output and profits must be positive mean that entry prevention cannot occur unless $\beta < 2/3$ and $\eta > 1/(1-\beta)^2$.

References


**Figure 1: Strategic Effect of Investment in R&D with and without Spillovers**
Figure 2: Marginal Return to R&D with and without Cooperation

Figure 3: Welfare in Free Trade

Figure 4: Welfare with Cooperation on R&D

Figure 5: Welfare with Government Commitment

Figure 6: Welfare with Commitment Relative to Cooperation
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