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ASSESSMENT DYNAMIC RATIO FOR TRAFFIC LOADING ON HIGHWAY BRIDGES

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ABSTRACT

The determination of characteristic bridge load effect is a complex problem. Usually, statistical extrapolation of simulated static load effects is used to derive a lifetime characteristic static load effect. However, when a vehicle crosses a bridge, dynamic interaction occurs which often causes a greater total load effect. This total load effect is related to the static load effect through a dynamic amplification factor (DAF). Specifications often recommend a conservative level for DAF, based on bridge length, number of lanes, and type of load effect only. Therefore significant improvements in the accuracy of this calculation are possible if a DAF, specific to the considered bridge, is applied. In this paper, the authors develop a novel method that considers site-specific bridge and traffic load conditions and allows for the reduced probability of both high static loading and high dynamic interaction occurring simultaneously. This approach utilises multivariate extreme value theory, in conjunction with static simulations and finite element vehicle-bridge dynamic interaction models. It is found that the dynamic allowance for the sample bridge and traffic considered, is significantly less than recommended by bridge codes. This finding can have significant implications for the assessment of existing bridge stock.

KEYWORDS

Bridge Loading, Traffic, Statistics, Dynamics, Simulation, Probabilistic, Modelling.

1. INTRODUCTION

Often in engineering it is not the outcome of a single stochastic process that is of interest, but the combination of several processes. It is well known that truck crossing events cause the truck and bridge to interact dynamically. Consideration of both the static and dynamic components of loading will yield the total load effect experienced by the bridge.

For static traffic loading, many authors have described methods of calculating characteristic values, as reviewed in detail by Caprani [1]. Of particular note is the work of Nowak [2]-[4], Bailey [5], Cooper [6]-[7] and Crespo-Minguillón and Casas [8]. While other approaches may be possible, most recent work uses Extreme Value Theory to extrapolate and determine the characteristic value of load effect.

The dynamic amplification factor (DAF) for a traffic loading scenario represents the ratio of the total load effect, ε_T , to the static load effect, ε_S , imparted to the bridge by that traffic scenario. Consequently, many authors [9]-[10] define it as:

$$\text{DAF} = \frac{\varepsilon_T}{\varepsilon_S} \quad (1)$$

High levels of dynamics can be expected when there is some form of resonance between traffic loading and a bridge. This can arise from a frequency matching between bridge natural frequency and the time between heavy axles arriving at a critical point. Bumps are also significant and vehicle and axle frequencies can lead to a coincidence of effects. The Eurocode traffic load model [11] has been developed from the simulation of static traffic actions and is shown for 2-lane bridges in Figure 1(a). The dynamic amplification factors of Figure 1(b) are applied to the 1000-year characteristic static load effects and are included in the load model of Figure 1(a). Since these specified DAF values are generalised, a significant level of conservatism may be incorporated. The allowance for dynamics specified by the Eurocode fails to recognize the reduced probability of two extremes (static and dynamic aspects) occurring simultaneously.

(a)

(b)

Figure 1: Eurocode Load Model: (a) Static Loading (the α factors reflect traffic on national networks or different classes of road); (b) Allowance for Dynamic Interaction (DAF – Dynamic Amplification Factor).

Multivariate extreme value theory is the statistical tool that is used to analyse critical combinations of several processes. This theory allows for the respective probabilities of occurrence of the variables as well as any relationship between them. In this paper, this theory is introduced to the bridge loading problem in order to incorporate the dynamic interaction of the bridge and trucks into an extreme value analysis for total load effect.

Numerous studies have identified that the potential for a large DAF reduces when the static traffic loading increases [12]-[15]. There have also been numerous field tests of dynamic interaction carried out [15]-[18]. A number of these studies have identified that higher values of DAF tend to occur for single-vehicle events than for multiple-vehicle bridge loading events [12]-[14]. Since many extreme traffic load events for short- to medium-span bridges typically consist of the meeting of two or more heavy vehicles ([1] and [19]), it is logical to expect that a reduced allowance for dynamics is applicable for characteristic maximum loading.

Leaving aside consideration of individual loading events, for the correct assessment of bridge traffic loading, it is the characteristic value of static load effect, $\hat{\varepsilon}_s$, and the characteristic value of total load effect, $\hat{\varepsilon}_T$, that are of interest. Consequently, it is the ratio between these two values that represents the correct allowance for vehicle-bridge dynamic interaction to be applied. In this paper, this ratio is termed the Assessment Dynamic Ratio (ADR). The aim of this paper is to show how a site-specific ADR may be obtained. This value of ADR can then be applied to the results of static simulations to determine a more accurate characteristic total load effect for a prescribed service life and the bridge of interest than found in general recommendations. The calculation of ADR is illustrated for a bridge and traffic sample.

2. BRIDGE AND TRAFFIC MODELS

The Bridge

The Mura River Bridge in Slovenia is used as the sample site for the method proposed in this paper. The general arrangement of the bridge is shown in Figure 2. It is a two-lane, bi-directional, 32 m simply-supported bridge span which forms part of a multi-span structure. It consists of 5 longitudinal pre-stressed concrete beams, a reinforced concrete slab, and 5 transverse diaphragm beams.

Figure 2: Mura River Bridge, Slovenia: general arrangement.

To determine a total load effect, a three-dimensional finite element model was used. To maximize confidence in the results from the finite element model, the dynamic behaviour of this model has been calibrated against site-measured responses. This calibration was carried out for single and two-truck meeting events using different combinations of 2-axle and 3-axle vehicles [10].

The finite element model, shown in Figure 3, was also used to determine the influence lines for each of the longitudinal girders. These influence lines, as well as polynomial fits to them, are shown in Figure 4. The influence lines, thus modelled, are used to determine the static bridge response to traffic. It can be seen that the asymmetry of the beams is reflected in the influence lines and that the polynomial fits are mostly indistinguishable from the finite element influence lines.

Figure 3: Finite element model of the bridge.

Figure 4: Finite element influence lines for the Mura River Bridge, Slovenia.

The Traffic

One week of Weigh-In-Motion (WIM) data was taken from the A6 motorway near Auxerre, France. The site has 4 lanes of traffic (2 in each direction) but only the traffic recorded in the slow lanes was used and this results in conservative loading. In total, 17 756 and 18 617 trucks were measured in the north and south slow lanes respectively, giving an average daily truck flow of 6744 trucks. The recorded WIM data was

analysed for the statistical distributions of the traffic characteristics of the site for each lane as described in Table 1. More information on the modelling of the traffic characteristics can be found in Caprani [1], OBrien and Caprani [21] and Grave [22]. It is acknowledged that one week is insufficient to accurately capture the key characteristics of traffic at a site but this data, while typical of Western Europe, is only being used to illustrate the procedure.

Table 1: Statistical models for site traffic characteristics.

Traffic Characteristic	Statistical Model
Gross Vehicle Weight (GVW)	Tri-modal Normal distribution
Axle spacings	Uni- or bi-modal Normal distributions, as appropriate
Axle weights for 2- & 3-axle trucks	Tri- or bi-modal Normal distributions, as appropriate
Axle weights for 4- & 5-axle trucks	Expressed as a percentage of GVW for the first and second axles and for the remaining tandem group. In each case, the percentage is modelled as a Normal distribution
Composition	Measured percentage of 2-, 3-, 4- and 5-axle trucks
Speed	Normal distribution - considered independent of truck type and uncorrelated with GVW
Flow rates	For each hour of the day, the average flow rate (ignoring weekend days) was used for all the days available
Headway	Modelled with a number of distributions dependent on flow and gap, as described in OBrien & Caprani [21]

3. SIMULATIONS

Static Simulations

Monte Carlo simulation is used to generate 10 years of bi-directional, free-flowing traffic data. It is considered that there are 250 working days per year (allowing for weekends and holidays) and each such year is broken into 10 periods of 25 working days each. The 10 years of simulated traffic is passed over the influence surface for mid-span bending moment in Beam 1, yielding 10 years of static bending moments. As

a basis for further analysis, the 100 events corresponding to 25-day-maximum static load effect are retained. This is done to minimize the number of events that are to be dynamically analysed, as well as providing a shorter ‘extrapolation distance’.

Of the 100 25-day-maximum events, 20 are found to be 1-truck events, 77 to be 2-truck events and 3 are 3-truck events. The influence surface for Beam 1 is asymmetrical. Consequently, trucks in Lane 1 dominate, with trucks in Lane 2 making a considerably lesser contribution. The 25-day-maximum events are mainly comprised of heavy trucks in Lane 1, and trucks with lower GVW in Lane 2. Figure 5 illustrates some examples of the 25-day-maximum events; the prevalence of heavy trucks in Lane 1 (top lane) is evident.

Figure 5: Examples of 25-day-maximum events – GVW is noted on each truck in tonnes and Lane 1 is uppermost: (a) 1-truck; (b) 2-truck; (c) 3-truck.

Dynamic Simulations

The 100 25-day-maximum loading events obtained from the static simulations are next analysed to determine the total load effect they impart to the bridge. To do this, a finite element vehicle-bridge interaction model, developed by González [23], is used along with the finite element bridge model described earlier. The first mode of vibration of the bridge is longitudinal with a frequency of 3.56 Hz, while the second is a torsional mode with a frequency of 4.38 Hz. Mode shapes three and four occur at frequencies of 13.36 Hz and 13.88 Hz respectively. 3% modal damping is assumed in the simulations.

The trucks are modelled using rigid bodies supported by suspension and tyre systems. The trailer and tractor masses in the trucks are modelled as point masses distributed throughout the frame by rigid elements, and the suspensions and tyres are individually modelled as spring dashpot systems. The 5-axle and 4-axle vehicle models allow for articulation between the tractor and trailer. The 3-axle and 2-axle vehicle models are rigid bodied. The mechanical characteristics (suspension and tyre properties) of the rigid 2-axle and 3-axle configuration truck models are based on parameters given by Lutzenberger and Baumgärtner [24]. The front axles of these rigid vehicles have a suspension stiffness of 213 kN/m and a tyre stiffness of 876 kN/m. Other axle/s have suspension and tyre stiffness of 806 kN/m and 1750 kN/m respectively. The mechanical properties of the articulated truck models are based on values proposed by Kirkegaard et al [25], and are kept constant throughout. The suspension stiffness of the articulated vehicles has been assumed to be 300 kN/m for the second axle and 1800 kN/m for all other axles. A value of 1000 kN/m has been adopted for the stiffness of the tyres in the first axle and 2000 kN/m for all other tyres. Damping for tyre and suspension systems vary between 5 and 80 kNs/m. Mass moments of inertia are adjusted according to tractor and trailer masses, leading to vehicle frequencies that change depending on vehicle configuration, axle spacings and weights; typically between 1 and 2.5 Hz for the body bounce, roll and pitch modes of vibration, and between 8 and 16 Hz for axle hop and roll modes of vibration. It is noted that variations in truck mechanical properties may influence the simulation results.

One independent road profile is used for each wheel path that is specified as constant for all events. The four road profiles (two for each lane) are generated using power spectral density functions for 'very good' conditions, similar to those found in new pavement profiles. A minimum approach length of 100 m is included for all simulations and the profile is continued after the bridge for both lanes. The roughness values (IRI) for 150 m lengths of each of the generated four profiles are 2.72 and 2.79 m/km for left and right wheel paths respectively in lane 1, and 2.75 m/km for both wheel paths in lane 2. It is appreciated that a profile of poorer quality will inevitably lead to an increase in the variability of the dynamic results. However the assessment here is aimed to evaluate a site-specific profile within the accepted highway standard IRI for well maintained pavements. A comprehensive description of the vehicle and bridge models utilised, as

well as the equations governing vehicle-bridge dynamic interaction are given by González et al [26].

The end result of these bridge-truck interaction simulations is a population of 100 25-day-extreme loading events for which both total and static load effects are known. The Eurocode uses a built-in DAF value of 1.17 in the traffic load model proposed for the selected bridge (Figure 1(b)). The combination of the bridge, vehicle dynamic properties and relatively smooth profile will clearly lead to a lower dynamic amplification factor than the Eurocode value. The sections that follow propose a rational approach towards the quantification of this site-specific dynamic allowance for a prescribed return period.

4. STATISTICAL INVESTIGATION OF RESULTS

Preliminary Results

A scatter plot of the 100 total and static load effect values is shown in Figure 6(a). There is strong correlation between static and total load effect – as may be expected since the static component constitutes the greater part of the total load effect. Each point on this graph represents the bivariate data corresponding to a particular loading event. Using equation (1), scatter plots of DAF against total and static load effect are shown in Figure 6(b) and (c) and it can be seen that a positive correlation exists between total load effect and DAF. Counter-intuitively, there is little correlation between the static load effect and DAF. However, this is explained by the fact that only the critical loading events are considered whilst it is the non-critical events that exhibit high DAFs. Hence the negative correlation in the overall population of loading events is concealed.

(a)

(b)

(c)

Figure 6: Scatter plots of maximum-per-month static and corresponding total load effect and DAF: (a) Maximum-per-month static and corresponding total load effect; (b) DAF against static load effect; (c) DAF against total load effect.

Multivariate extreme value analysis

It is usual in multivariate extreme value analyses to adopt the component-wise maxima approach [27]–[28]. In the present study the components are the maximum static effect

and the maximum total effect. Strictly then, 25-day-maximum total and static load effects should have been noted independently of each other and their associated loading events. However, it is not thought that there is much error introduced by using the 25-day-maximum static load effects and the associated total load effect value for the particular loading event. This is because the modelling procedure is more sensitive to the higher extremes than the lower ones.

Multivariate distribution modelling can be divided into two separate parts: modelling the marginal distributions and modelling the dependences through a copula [28]. In this way, appropriate marginal and dependence structures can be developed separately and then combined [29]. When multivariate data is analysed to find extremes, the copula representing the dependence structure also becomes extreme [30] and Pickands [31] provides a general class of extreme-value copulas. Tawn [32] describes the use of this copula for the bivariate extreme value case.

Based on Pickands' general extreme value copula, Stephenson [33] discusses eight different forms of bivariate extreme value distributions that have emerged in the literature. Of these, the Gumbel Bivariate Extreme Value Distribution (BEVD) is found to model the observed dependencies between total and static load effect well:

$$G_{Gu}(x, y) = \exp\left\{-\left(z_1^{1/\alpha} + z_2^{1/\alpha}\right)^\alpha\right\} \quad (2)$$

where $0 < \alpha < 1$ is a parameter reflecting the degree of dependence. Independence is represented by $\alpha = 1$ and complete dependence occurs when $\alpha \rightarrow 0$. The variables z_1 and z_2 are the transformed marginal distribution variables given by [32]. Capéraà et al [34] describe the fitting of the Pickands' dependence function upon which this work is based.

Bivariate Extreme Value Analysis of Load Effect Data

In the analysis that follows, software developed by Stephenson [33] is used in conjunction with bespoke algorithms, written in the *R* language for statistical computing [35]. Stephenson's method for simulating multivariate extreme value random variables is also used [36].

The static and total load effect data is fitted using the Gumbel logistic Bivariate Extreme Value Distribution (BEVD), given in equation (2). The results of the fit can be seen in Figure 7. Figure 7(a) shows a contour plot of the bivariate probability density function while Figure 7(b) illustrates the empirical and fitted dependence structure of the data. It can be seen that the dependence function is modelled quite well - this is the determining

factor in the choice of the Gumbel BEVD for this work and its parameters are given in Table 2.

Figure 7: Results and diagnostic plots of the BEVD fit: (a) Bivariate distribution; (b) Dependency structure.

Table 2: Parameters of fitted 25-day-maximum bivariate extreme value distribution.

Marginal Distributions					
Total Load Effect			Static Load Effect		
μ	σ	ξ	μ	σ	ξ
6.972	0.3851	0.2071	6.756	0.2423	0.1233
Dependence Measures					
BEVD dependence parameter			Tail dependence		
$\alpha = 0.5347$			$\lambda_U = 0.5514$		

Notes:

μ , σ , and ξ are the location, scale, and shape parameters of the Generalized Extreme Value distribution;

λ_U is the tail dependence measure of [29]

Bootstrapping for lifetime load effects

The BEVD fit of Figure 7 represents the distribution of 25-day-maximum events. In order to determine the distribution of characteristic maximum events, and since it is not feasible to raise the BEVD to a power, a bootstrapping approach is used [37]. To do this, 1000 synthetic 25-day-maximum events are simulated from the fitted BEVD model (corresponding to 100 years of 250 days each). Any inaccuracy of the BEVD fit to the data is therefore not taken into account. The component-wise maxima are recorded. Consequently these maxima are not related through an individual loading event. In this way, the maximum total and static load effects from the 1000 bootstrap replications of the bridge lifetime are noted. These points are given in Figure 8 alongside the 25-day-maximum data points.

Figure 8: Maximum-per-month data and bootstrapped lifetime data with fitted bivariate distributions.

The ratio of static characteristic maximum load effect to characteristic total load effect is termed here as the Bridge Lifetime Dynamic Ratio (BLDR). This recognizes that the same loading event is not necessarily responsible for the characteristic maximum total and characteristic maximum static load effects. Figure 9 shows BLDR plotted against the bootstrapped total and static maximum-in-lifetime load effects. It can be seen from Figure 9(b) that there is a negative linear correlation between BLDR and static effect. This is significant: it means that the dynamic increment is falling as more extreme load effects are considered.

Figure 9: Scatter plots (showing linear regression lines) of Bridge Lifetime Dynamic Ratio: (a) Total load effect; (b) Static load effect.

Assessment Dynamic Ratio

A Gumbel BEVD is fitted to the simulated lifetime maxima. The fit is shown in Figure 8, but is not evident due to the number of points. The parameters of the fit are given in

Table 3. It is clear that there is dependence between the static and total maximum lifetime load effect values – even though they are not related through individual loading events – and this must be a result of the dependence in the parent distributions.

Table 3: Parameters of 100-year lifetime fitted BEVD distribution.

Marginal Distributions					
Total Load Effect			Static Load Effect		
μ	σ	ξ	μ	σ	ξ
8.386	0.0950	0.2425	7.881	0.1079	0.1566
Dependence Measures					
BEV dependence parameter			Tail dependence		
$\alpha = 0.5513$			$\lambda_U = 0.5346$		

Notes:

μ , σ , and ξ are the location, scale, and shape parameters of the Generalized Extreme Value distribution;

λ_U is the tail dependence measure of [29]

The characteristic static load effect value is commonly found using Monte Carlo simulation and some form of statistical analysis. It is a value that links the characteristic total load effect to the characteristic static load effect that is of interest, not the distribution of BLDRs. The ratio of the characteristic total load effect to the characteristic static load effect is the previously-defined ADR, denoted φ , and is derived from the marginal distributions of the BEVD fit to the lifetime data:

$$\varphi_q = \frac{\hat{\varepsilon}_T}{\hat{\varepsilon}_S} = \frac{G_T^{-1}(q)}{G_S^{-1}(q)} \quad (3)$$

where q is the quantile of interest and $G_S(\cdot)$, $G_T(\cdot)$ are the cumulative distribution functions for the static and total load effects respectively. The characteristic value for bridge load effect is defined in EC 1.3 [11] as being that value which is expected to be exceeded with a probability of 5% in 50 years which gives a return period of approximately 1000 years. Therefore, when $G_S(\cdot)$, $G_T(\cdot)$ are the cumulative distribution functions for maximum-in-100-year-lifetime static and total load effects, $q = 0.9$ and. This value is appropriate to relate characteristic static to total load effect values, and is shown in Figure 10. The ADR for this particular bridge and traffic example has a value of 1.058. This figure is in contrast to the 1 in 100-year DAF value,

which for the data in this study gives 1.167. Thus there is a significant difference between the proposed approach and a more traditional approach. In making these comparisons, it is emphasised that this example has a Class A road profile and that other bridge natural frequencies may result in higher ADR values.

Figure 10: Representation of the ADR for the Eurocode quantile.

By applying the stability postulate [27] to the marginal distributions, plots of ADR against various other design lives by quantile can be derived and are shown in Figure 11; some values are given in Table 4. It is clear that as the design life or quantile increase, the ADR reduces. Caprani [1] re-parameterizes the ADR and derives a numerical distribution of ADR, concluding that for the considered study the return period at which no dynamic allowance is required is of the order 2.6×10^6 years. Thus it is clear that a dynamic allowance will be necessary for this bridge and traffic, even at the extreme lifetime level.

Figure 11: ADR quantile plots for various design lives, showing Eurocode calculation.

Table 4: Derived ADR values for sample design lives and quantiles.

Design Life (yrs)	Quantile	
	0.5	0.9
50	1.0649	1.0597
75	1.0639	1.0584
100	1.0631	1.0575
200	1.0612	1.0554
1000	1.0562	1.0505

5. SUMMARY

In this paper, the current means of allowing for dynamic interaction of bridge and truck(s) is reviewed and shown, for a particular bridge and traffic data, to be conservative. Simulations of static load effect are used to obtain 25-day-maximum loading events, which are then modelled dynamically, using a finite element vehicle-bridge interaction model that will provide the total load effect. It is shown that there is

significant statistical correlation between the static and total load effects, and dependence models are described that allow for this.

Bivariate extreme value analysis is used to model the 25-day-maximum total and static load effects, and allow for their inter-dependence. Parametric bootstrapping is used to generate 100-year extremes from the fitted bivariate distribution. In this manner, a distribution for bridge lifetime dynamic ratio is derived. It is again shown to be bivariate, with a similar dependence structure to the 25-day-maximum events, even though the variables are no longer related through a single event.

It is shown that the required dynamic allowance reduces with increasing load effect and that, for the bridge and traffic studied, it is quite small (5.8%). It is also shown that this allowance decreases slowly with increasing safety level. Whilst the dynamic allowance results presented here are specific to this bridge and traffic, the method can be applied to other bridge and traffic data. It should be noted that, for this bridge, the natural frequency is sufficiently far apart from relevant vehicle frequencies to lead to a small dynamic allowance. Similar small values may not occur for other bridges.

The consideration of loading scenarios that are statically critical for the bridge is significant as these are the load cases that govern the bridge safety. Therefore, this approach to the dynamic allowance problem should be of great value in the modelling and assessment of traffic load effects on existing bridges, and for the prescription of dynamic allowance factors in bridge loading codes.

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