Missing Data Analysis Using Multiple Imputation in Relation to Parkinson’s Disease

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ABSTRACT
Missing data is an omnipresent problem in neurological control diseases, such as Parkinson’s Disease. Statistical analyses on the level of Parkinson’s Disease may be not accurate, if no adequate method for handling missing data is applied. In order to determine a useful way to treat missing data on Parkinson’s stage, we propose a multiple imputation method based on the theory of Copulas in the data pre-processing phase of the data mining process. Our goal to use the theory ofCopulas is to estimate the multivariate joint probability distribution without constraints of specific types of marginal distributions of random variables that represent the dimensions of our datasets. To evaluate the proposed approach, we have compared our algorithm with seven state-of-the-art imputation methods such as mean, regression, min, max, K-nearest neighbors, Markov Chain Monte Carlo, Expected Maximization methods, on the basis of six dataset cases containing 5%, 15%, 25%, 35%, 45% and 50% missing data percentages, respectively. The accuracy of each imputation method was evaluated using the Root Mean Square Error (RMSE) formula. Our results indicate that the proposed method outperforms significantly the existing algorithms.

Keywords
Data mining; Data pre-processing; Multi-dimensional Sampling; Copulas; Multiple Imputation; Missing data; Parkinson’s Disease.

1. INTRODUCTION AND RELATED WORK
Missing data is unavoidable in health research, but their potential to undermine the validity of research results has often been neglected in the medical literature. This is due to the fact that statistical methods that can resolve problems arising from missing data have, until recently, not been easily available to medical researchers, especially in Parkinson’s Disease, where any missing data can potentially cause difficulties in analyses and lead to bias and loss of information in epidemiological and clinical research.

Parkinson’s Disease (PD) is the second most common adult-onset neurodegenerative disease after Alzheimer’s [1], and it is expected to affect more than one million people in North America alone [2]. Moreover, these statistics are expected to grow because the population is getting older. PD is a degenerative disorder of the central nervous system and one of the most common movement disorders affecting people older than 60 years. The symptoms of PD include primary motor symptoms (e.g., resting tremor, rigidity, slow movement, speech problems, swallowing difficulty, etc.), and non-motor symptoms (e.g., pain, depression, etc.). The management of PD typically involves a complete dataset for the development of reliable and objective tools for assessing PD. However, the major challenge when studying PD is that the data is multi-dimensional, and may have a missing data in important parameters. There are several methods for handling these issues, described in a rich literature. According to [3][4], there are three possible strategies to deal with missing data. The first one is based on missing data ignoring techniques that simply omit the cases that contain missing data [5]. The second one represents missing data based modeling techniques, that define a model from available data and inferences based on the distribution of the data [6]. These methods assume a multi-normality of continuous outcome variables. The third one represents missing data imputation techniques [7][8]. These techniques are a strategy for completing a missing data in the dataset with a plausible value which is the estimation of the true value of the missing observation. These methods keep the full sample size, which can be advantageous for bias, precision, and accuracy. Different missing data imputation techniques are used, amongst which we can mention: mean [9], regression [10], K-nearest neighbors [11], multiple imputation [5] [7],
In this paper, we will focus our attention on the use of missing data multiple imputation methods in the data preprocessing phase of the data mining process. Multiple imputation is a relatively flexible, general purpose approach to deal with missing data, but it needs to be applied carefully to avoid misleading conclusions.

The main goals of this paper are to handle Parkinson’s Disease by analyzing the missing data, preserve essential characteristics of this dataset by preserving the relationships among the variables, and finally estimate missing values with the most accurate method and the smallest error.

The paper is organized as follows: the basic concepts are presented in Section 2, and Section 3 describes the proposed method. The experimental results are given in Section 4, and finally, Section 5 concludes the paper.

2. BASIC CONCEPTS

This section aims to introduce the basic concepts used in our approach. Our technique is based on probabilistic and sampling models, therefore, one needs to recall some fundamental concepts. These include the notions of a Probability Density Function (PDF), Cumulative Distribution Function (CDF), dependence and rank correlations of multivariate distributions and to present a solution for the difficulties of transformation of the density estimation problem. Sklar’s Theorem [13] showed that there exists a unique m-dimensional Copula $C$ in $[0,1]^m$ with standard uniform marginal distributions $U_1,...,U_m$. [13] states that every distribution function $F$ with margins $F_1,...,F_m$ can be written $\forall (X_1,...,X_m) \in \mathbb{R}^m$ as:

$$F(X_1,...,X_m) = C(F_1(X_1),...,F_m(X_m)).$$

Figure 1 shows an overview of a general approach to use Copulas for the proposed approach, that requires the following steps.

- **Step 1: Empirical Copula**

To evaluate the suitability of a selected Copula with estimated parameter and to avoid the introduction of any assumptions on the distribution $F_i(X_i)$, one can utilize an empirical $CDF$ of a marginal $F_i(X_i)$, to transform m samples of $X$ into m samples of $U$. An empirical Copula is useful for examining the dependence structure of multivariate random vectors. Formally, an empirical Copula is given by the following equation [14]:

$$C_{ij} = \frac{1}{m} \sum_{k=1}^{m} I(v_{ij} \leq v_{ij}), i = 1,...,n; \ j = 1,...,m.$$  \hspace{1cm} (3)

where the function $I_{(arg)}$ is the indicator function, which equals 1 if $arg$ is true and 0 otherwise. Here, $m$ is used to keep the empirical $CDF$ less than 1, where $m$ is the number of observations.

- **Step 2: Theoretical Copula**

In the literature, various Copula families have been proposed. As the most frequently used we can cite the following: Gaussian, Student $\chi$, and the Archimedean Copulas. In this paper, we will focus on the Copula that results from a standard multivariate Gaussian Copula. The difference between the Gaussian Copula and the joint normal $CDF$ is that the Gaussian Copula allows to have different marginal $CDF$ types from the joint distribution [15]. However, in probability theory and statistics, the multivariate normal distribution is a generalization of the one-dimensional normal distribution. The Gaussian Copula is defined as follows [12]:
$C(\Phi(x_1), ..., \Phi(x_m)) = \frac{1}{|\Sigma|^\frac{1}{2}} \exp(-\frac{1}{2} X^T (\Sigma^{-1} - I) X).$  

(4)

where $\Phi(x_i)$ is the CDF standard Gaussian distribution of $f_i(x_i)$, i.e., $X_i \sim N(0, 1)$, and $\Sigma$ is the correlation matrix. The resulting Copula $C(u_1, ..., u_m)$ is called Gaussian Copula. The density associated with $C(u_1, ..., u_m)$ is obtained with the following equation:

$c(u_1, ..., u_m) = \frac{1}{|\Sigma|^\frac{1}{2}} \exp\left[-\frac{1}{2} \xi (\Sigma^{-1} - I) \xi\right]$.  

(5)

where $u_i = \Phi(x_i)$, and $\xi = (\Phi^{-1}(u_1), ..., \Phi^{-1}(u_m))^T$.

- Step 3: Generate a sample based on the theoretical Copula

To illustrate the problem of invertible transformations of $m$-dimensional continuous random variables $X_1, ..., X_m$ according to their $CDF$, into $m$ independently uniformly-distributed variables $U_1 = F_1(X_1), U_2 = F_2(X_2), ..., U_m = F_m(X_m)$ [12], let $f(x_1, x_2, ..., x_m)$ be the probability density function of $X_1, ..., X_m$, and let $c(u_1, u_2, ..., u_m)$ be the joint probability density function of $U_1, U_2, ..., U_m$. In general, the estimation of the probability density function $f(x_1, x_2, ..., x_m)$ can provide a nonparametric form (unknown families of distributions).

In this case, we estimate the probability density function $c(u_1, u_2, ..., u_m)$ of $U_1, U_2, ..., U_m$ instead of that of $X_1, ..., X_m$ to simplify the density estimation problem, and then simulate to achieve the random samples $X_1, ..., X_m$ by using the inverse transformations $X_i = F_i^{-1}(U_i)$ [12].

### 2.2 Dependence and Rank Correlation

Since the Copula of a multivariate distribution describes its dependence structure, it might be appropriate to use measures of dependence which are Copula-based. The Pearson correlation measures the relationship $\Sigma$ given by the equation $6$.

$$\Sigma_{ij} = \frac{\text{cov}(X_i, X_j)}{\sigma_{X_i} \sigma_{X_j}}$$  

(6)

where $\text{cov}(X_i, X_j)$ is the covariance of $X_i$, $i = 1, ..., n$ and $X_j$, $j = 1, ..., m$, and $\sigma_{X_i}, \sigma_{X_j}$ are the standard deviations of $X_i$ and $X_j$.

Kendall’s rank correlation (also known as Kendall’s coefficient of concordance) is a non-parametric test that measures the strength of dependence between two random samples $X_p, X_p'$ of $n$ observations with $i = 1, ..., n$. The notion of concordance can be defined by the following equation:

$$\tau = P[(X_p - X_p')(X_p' - X_p) > 0] - P[(X_p - X_p')(X_p' - X_p) < 0].$$  

(7)

For the Gaussian Copula, Kendall’s $\tau$ can be calculated as follows:

$$\tau = \frac{2}{\pi} \arcsin \Sigma_{X_i X_j}.$$  

(8)

### 3. PROPOSED IMPUTATION APPROACH

In this section, we propose a new multiple imputation approach based on the theory of Copulas to address the problem of multivariate missing data. A Copula provides a suitable model of dependencies to compare with well-known multivariate data distributions in order to better distinguish the relationship between the data.

The main goals of this paper are to: (a) handle Parkinson’s Disease by analyzing the missing data in the multivariate case, (b) preserve the essential characteristics of the data by preserving the relationships among the variables, (c) model multivariate random variables without imposing constraints to specific types of marginal distributions of data, (e) provide a valid statistical inference, (f) provide highest accuracy (g) estimate missing values with the most effective method.

The main steps of the proposed imputation approach can be described by the following algorithm:

**Algorithm 1 Proposed algorithm**

**Input:** Incomplete dataset $X_{(n_1,m)} = \{X_1, X_2, ..., X_m\}$

**Output:** Complete dataset $\hat{X}$.

1. Generate a Sample $W_{(n_2,m)}$ based on a theoretical Copula.
2. Determine $V$ which is the subset of rows that contain missing data $W'_j$.
3. Determine $Index$ which contains the index of subset of no missing values set in the original data.
4. Determine $R$ the subset of rows in the theoretical sample $W_{(n_2,m)}$ that verify $w'_j = v_j$ by comparing $(V, W, Index, \xi)$, where $\xi$ is a relative error.
5. Complete the dataset $\hat{X}$.

Here $n_1$ is the number of rows of the dataset $X$, $n_2$ is the number of rows of the dataset $W$. For more clarification, we consider an example taken from Parkinson’s Disease dataset in the following table. We suppose that the dataset contains a missing data in the biomedical test ($T_1$) denoted by “ ? ”.

<table>
<thead>
<tr>
<th>Test</th>
<th>T1</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
<th>T5</th>
<th>T6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient 1</td>
<td>?</td>
<td>0.021</td>
<td>0.221</td>
<td>?</td>
<td>0.277</td>
<td>0.301</td>
</tr>
<tr>
<td>Patient 2</td>
<td>0.925</td>
<td>?</td>
<td>0.507</td>
<td>0.511</td>
<td>0.602</td>
<td>?</td>
</tr>
<tr>
<td>Patient 3</td>
<td>1.024</td>
<td>1.185</td>
<td>?</td>
<td>1.182</td>
<td>1.504</td>
<td>1.504</td>
</tr>
</tbody>
</table>

In this example (Table 2), we want to estimate the missing data $T3$ and $T5$ for patient 4. The entries of the row in question are: 1.406, 1.823, “ ? ”, 4.385, “ ? ”, 4.640.

To impute the missing data “ ? ”, we generate the appropriate Copula having the same parameters as the empirical sample.

By applying the step 4 of the algorithm 1, we obtain the Table 3 which is a collection of rows of a theoretical sample $W_{\{\xi\}}$ obtained by comparing the known values: 1.406, 1.823, 4.385, 4.640 of patient 4. The red boxes represent the estimated values. For each column of Table 3, we compute the mean, and we impute the results obtained in the PD dataset according to the index of each values.

4. EXPERIMENTAL RESULTS

Our experiments were performed on Parkinson’s Disease real-world datasets taken from the machine learning repository [16], which are from the Healthcare database.

Parkinson’s Disease.

The dataset was created by Athanasios Tsanas and Max Little of the University of Oxford, in collaboration with 10 medical centers in the US and Intel Corporation who developed the telemonitoring device to record the speech signals. This dataset was accumulated employing the Intel AHTD which is a telemonitoring system designed to facilitate remote and internet-enabled measurements of a variety of PD-related motor impairment symptoms. The dataset is collected at the 42 patient’s home, with early-stage Parkinson’s disease recruited to a six-month trial of a telemonitoring device for remote symptom progression monitoring transmitted over the Internet, and processed appropriately in the clinic to predict the Unified Parkinson Disease Rating Scale score (UPDRS). Columns in this dataset represent 16 biomedical UPDRS scores. The information about these attributes includes: clinician’s motor UPDRS score (Jitter(%), Jitter(Abs), Jitter:RAP, Jitter:PPQ5, Jitter:DDP); several measures of variation in fundamental frequency (Shimmer, Shimmer(dB), Shimmer:APQ3, Shimmer:APQ5, Shimmer:APQ11, Shimmer:DDA); several measures of variation in amplitude (NHR,HNR); two measures of ratio of noise to tonal components in the voice (RPDE); a nonlinear dynamical complexity measure (DFA); signal fractal scaling exponent (PPE). Each row corresponds to one of 5875 voice recordings from these individuals.

Simulation Study.

To evaluate our proposed approach, we have implemented and performed extensive simulation experiments with seven imputation methods: mean, regression, min, max, K-nearest neighbors, Markov Chain Monte Carlo, Expected Maximization methods and the proposed approach, Table 5 shows the numerical results obtained for the missing values

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \hat{X}_i)^2}{n}}
\]  

where \( n \) = missing values percentage \times the length of data.

By implementing different imputation techniques: mean, regression, min, max, K-nearest neighbors, Markov Chain Monte Carlo, Expected Maximization methods and the proposed approach, Table 5 shows the numerical results obtained for the missing values

<table>
<thead>
<tr>
<th>Methods</th>
<th>5%</th>
<th>15%</th>
<th>25%</th>
<th>35%</th>
<th>45%</th>
<th>50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PA</td>
<td>0.019</td>
<td>0.021</td>
<td>0.221</td>
<td>0.231</td>
<td>0.277</td>
<td>0.301</td>
</tr>
<tr>
<td>EM</td>
<td>0.925</td>
<td>0.525</td>
<td>0.507</td>
<td>0.511</td>
<td>0.602</td>
<td>0.609</td>
</tr>
<tr>
<td>Regression</td>
<td>1.024</td>
<td>1.185</td>
<td>1.453</td>
<td>1.182</td>
<td>1.504</td>
<td>1.504</td>
</tr>
<tr>
<td>K-nn</td>
<td>0.452</td>
<td>0.495</td>
<td>0.583</td>
<td>0.670</td>
<td>0.703</td>
<td>1.703</td>
</tr>
<tr>
<td>Mean</td>
<td>0.905</td>
<td>1.056</td>
<td>1.023</td>
<td>1.031</td>
<td>1.026</td>
<td>1.982</td>
</tr>
<tr>
<td>Max</td>
<td>0.578</td>
<td>0.734</td>
<td>0.777</td>
<td>1.733</td>
<td>7.601</td>
<td>7.353</td>
</tr>
<tr>
<td>Min</td>
<td>1.206</td>
<td>5.246</td>
<td>5.066</td>
<td>5.051</td>
<td>4.847</td>
<td>4.902</td>
</tr>
</tbody>
</table>

4.0.1 Interpretation of the results

According to Table 5 and Figure 2, we notice that the increase of missing values by 5 % to 50 % causes a decrease in accuracy for different methods.

Max imputation provides the lowest accuracy for most of the databases (15 % to 50 %), but it works well in the case of 5 % of missing data. The minimum error values achieve 0.019 for the Proposed Approach (PA), 0.507 for the EM method, 1.504 for regression imputation, 4.640 for the MCMC method, 1.703 for K-nearest neighbors method, 1.982 for the Mean imputation, 7.353 for the Max, and 4.902 for the Min method. On the other hand, the maximum error values achieve 0.925 for the EM method, 1.024 for regression imputation, 1.406 for the MCMC method, 0.452 for K-nearest neighbors method, 0.905 for the Mean imputation, 0.578 for the Max, and 1.206 for the Min method. We can observe that EM and K-nearest neighbor approaches work well and have better results than Mean, MCMC, Min, and Max methods, respectively. The results obtained by the proposed approach are much better than EM, regression, MCMC, K-nearest neighbor, Mean,
Max, and Min imputation for all datasets containing 5%, 15%, 25%, 35%, 45% and 50% missing data percentages, respectively.

5. CONCLUSION

In this paper, we have proposed a multiple imputation method based on sampling techniques to handle missing data from Parkinson’s Disease. We have evaluated the proposed approach by comparing our algorithm with seven imputation methods such as, mean, regression, min, max, K-nearest neighbors, Markov Chain Monte Carlo, Expected Maximization methods and the proposed approach) on the basis of six dataset cases containing missing data from Parkinson’s Disease dataset.

References


