Effect of Generator Flow Control Strategies on the Long Term Dynamics of a Model for Power Systems

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Abstract
Cascading component failure can cause system-wide blackouts in power systems. Complex system analysis of the long term dynamics suggests that power transmission systems operate near a critical point. Here the effect of generator operation in a model for power systems is investigated. Two opposing methods of dispatching generators are compared. These two methods are to minimize and maximize the line flows with respect to their limits across the system. These methods are also compared to an economic dispatch. It is shown that the generator dispatch method used affects the frequency distributions and other statistics of blackouts. Dispatching using the maximization method causes a reduction in the frequencies of all blackouts as measured by the amount of load shed. This reduction is concurrent with an increase in the investment in the transmission system. These observations suggest that economic dispatch, while attempting to increase reliability, actually decreases the robustness of the system.

1. Introduction
Cascading events in power transmission systems can lead to a large number of consumers becoming disconnected. Historical studies of real world data have shown that the frequency of these events scale as a power law with size [1].

It has been hypothesized that these distributions are the result of the power transmission system operating close to a critical point and that there is a nonlinear coupling of the effect of migration and the frequency of blackouts. Therefore seemingly sensible measures to prevent blackouts in power systems may lead to counterintuitive results [2].

Simple models of the electric power transmission system have been developed in order to study and better understand the complex nature of the long term dynamics of power systems. In the model the generators are dispatched economically using a linear programming algorithm under the assumption that the costs of all generators are homogenous. In Watts et al. [3] the effect of relaxing the assumption of homogeneity was investigated. In this paper blackout mitigation using generator dispatch is compared to an economic dispatch where homogeneity is assumed. The extension of the model by Watts et al. is used to implement the flow control strategies. The OPA model in this extension is a DC Optimal Power Flow (DC-OPF) model. The generators are dispatched in a manner that minimizes a quadratic expression of the power injections into the system. OPA stands for Oak Ridge National Laboratory, Power System Engineering Research Centre University of Wisconsin, and the University of Alaska after the institutions that collaborated in its development [4].

It would seem intuitive that a useful measure of the security of a power system after an event is the sum of the total fractional flows squared with respect to the line power flow constraints. A lower value of this measure would indicate a less stressed system overall. This is similar to measures used to investigate the impact of N-1 contingencies on power systems [5]. In simulations of power systems in the OPA model this method of curtailment is implicitly expressed through the use of linear programming or an economic dispatch.

In Section 2 a brief description of the OPA model is given. In Section 3 the proposed generator control strategies are defined and their mathematical implementation is shown. A new method to eliminate lines within the OPA model is introduced in Section 4. In Section 5 the results for the IEEE 118 bus test system are presented and the paper concludes with Section 6.
2. OPA model

The OPA model was developed to model the long term dynamics of an evolving power system. The blackout statistics of the model were shown to replicate those of real world power systems [4, 6, 7]. The model is designed to capture the features of the evolution of a power system in the presence of a small continuous load growth. As the load is increased the system moves towards a less stable point in terms of reliability where cascading outages of lines may occur leading to blackouts. In response to these events the system is upgraded by increasing the flow carrying capacity of lines.

The model consists of slow and fast dynamics which together produce a system that tends towards a critical point and produces the power law distributions in the frequency of blackouts of varying sizes. The model is based on the DC load flow assumption. An in-depth discussion of the model can be found in [6]. In this work the model has been adapted to eliminate problems associated with the use of large numbers in the linear programming phase for implementation in Matlab [8], see Appendix.

In the original model the system was dispatched for each iteration through the use of a linear programming algorithm. In Watts et al. [3] the model was updated so as to include the heterogeneity of costs of generation in the dispatch. This was done by introducing quadratic programming into the model. Here this adaption is used, however instead of using the objective function to dispatch according to costs of generation it is used to dispatch the generation to minimize the flows across the system as a mitigation strategy before and after a contingency. Conversely the flow strategy of maximizing the flows across the system will also be investigated. The objective function was given as,

\[ \frac{1}{2} \hat{p}^\top O \hat{p} + \bar{n} \hat{p} \]  

where \( \hat{p} \) is the vector of power injections into the system. \( \bar{n} \) is a vector consisting of entries that have a high number when they correspond to those of load nodes. The high number for the load nodes ensures that shedding load is very costly. The matrix \( O \) is discussed in Section 3.

The constraints on minimizing the objective function are the usual power system constraints;

\[ \sum_{i=1}^{n} p_i = 0 \]  

\[ l_i \leq p_i \leq 0 \quad i \in L \]  

\[ g_i \geq p_i \geq 0 \quad i \in G \]  

\[ |f_i| < F_i \]  

where \( G \) is the set of generator nodes and \( L \) is the set of load nodes. \( F_i \) is the capacity limit of line \( i \). \( p_i \) is the load/generation injected into the system at node \( i \) and \( f_i \) is the flow on line \( i \). \( l_i \) and \( g_i \) are the load demand and the generator capacities at node \( i \) respectively. Equation (2) is the power balance constraint, equation (3) and (4) ensure that the load at a node is no more than the demand and that the generation is not greater than the capacity limits at that site. Equation (5) is the capacity constraint on the lines.

At the start of each 'day', each line has an initial probability of failure, \( q_0 \). Once the system is dispatched the flow in the lines are checked against their thermal limits. If

\[ M_i = \frac{f_i}{F_i} > \text{thres} \]  

the line has a failure probability, \( q_i \). \( M_i \) is called the fractional line flow.

When the system reaches a point where there are no more line failures the percentage of the load lost is recorded. If a blackout does not occur in that day the line flow limits are reduced by,

\[ F_i^{k+1} = \frac{F_i^k}{\mu_k} \]  

where \( F_i^{k+1} \) is the capacity limit in line \( i \) at time \( k \) and \( \mu_k \) is a constant greater than 1. This differs slightly from that of the original OPA model and is implemented in this manner to eliminate problems with large numbers in the optimization, see Appendix. However if a blackout occurred, all lines that were at some point during the cascade above the threshold have their limits increased. This is achieved by multiplying the overload lines capacity limit by a constant,

\[ F_i^{k+1} = \partial_i F_i^k \]  

where \( F_i^{k+1} \) is defined the same as in equation (7) and \( \partial_i \) is the line upgrade factor which is greater than 1.
3. Flow control strategies

In this paper we compare two opposing methods of flow control. The first method is to minimize the sum of the square of the fractional flows, $M_i$, across the system. The fractional flow on a line may be considered as an indicator of the stress on that element of the system. Minimizing the fractional flows across the system could be viewed as a strategy that is attempting to minimize the total stress of the system. We also study the opposite strategy, that of maximizing the fractional flows across the system. This maximization would cause some lines to be heavily loaded and would seem to represent a highly stressed system. In equation form we wish to minimize (or maximize) a system state variable, $s$, such that,

$$s = \sum_{i=1}^{m} M_i^2$$  \hspace{1cm} (9)

where $m$ is the number of lines comprising the transmission system. $s$ can be considered the system stress. For comparison we also look at the case of economic dispatch through linear programming.

The line flows are given as a vector,

$$\tilde{f} = (f_1, f_2, ..., f_m)^T$$  \hspace{1cm} (10)

This is given by the formula,

$$\tilde{f} = -\frac{1}{2} AR\tilde{p}$$  \hspace{1cm} (11)

where $A$ is the weighted adjacency matrix and $R$ is a matrix of effect inductances between nodes in the system, this is discussed in more detail in Section 4. The fractional line flows are given by,

$$f_i = F_i - p_i$$  \hspace{1cm} (12)

where $F$ is the diagonal matrix of line flow limits.

For one flow control strategy the state variable $s$ is to be minimized,

$$\text{min}(M^T M)$$  \hspace{1cm} (13)

using equations (11) and (12) we get,

$$M^T M = \frac{1}{2} p^T R A F^{-2} AR \tilde{p}$$  \hspace{1cm} (14)

therefore the matrix $O_{\text{min}}$ is given by the equation,

$$O_{\text{min}} = \frac{1}{2} R A F^{-2} A$$  \hspace{1cm} (15)

For the case of maximizing the fractional flows we simply set,

$$O_{\text{max}} = -O_{\text{min}}$$  \hspace{1cm} (16)

In the case of the maximizing the flows the optimization problem becomes concave and an optimal solution may not be found. If this occurs then the solution proposed by the economic dispatch is used. The use of the economic dispatch in cases where an optimal solution is not found quickly will tend to reduce the effect that the maximization method would have on the statistics of the blackout time series.

4. Elimination of lines during a cascade

In the original OPA model line outages were achieved using an approximation. The susceptances of the lines were decreased by a large number and the capacity limit of the line was also reduced by a large number. Using this method the objective function discussed above would include these lines as part of the minimization. In order to avoid this, a more exact method of line deletion was implemented.

The DC load flow equation was reformalized as,

$$\tilde{f} = -\frac{1}{2} AR\tilde{p}$$  \hspace{1cm} (17)

where $A$ is the adjacency matrix of the system and $R$ is a matrix such that,

$$-\frac{1}{2} R Y = I + \tilde{e} \tilde{w}$$  \hspace{1cm} (18)

where $Y$ is the usual admittance matrix of the system, $\tilde{e}$ is a column vector with all entries equal to 1, $\tilde{w}$ is a row vector of “terminal weight numbers” [9]. The entries in $R$ have the physical interpretation of the effective inductance between nodes of the network. Using equation (18) and the DC load flow assumptions we can derive the DC load flow equation presented in Section 3.

After a line outage, where the failed line was connected between nodes $r$ and $s$, the matrix $R$ is updated using the equation,

$$r_{ij}^{\text{new}} = r_{ij}^{\text{old}} + \left(1 - \frac{1}{4} y_y r_{rs}^2 \right) \frac{1 - y_y r_{rs}^2}{1 - y_y^{\text{old}} r_{rs}^{\text{old}}}$$  \hspace{1cm} (19)

A proof of this, for infinite resistance networks, is given in Csetri et al. [10]. The row in the matrix $A$ corresponding to the eliminated line is also deleted.

In the case of islanding of the network it is found that the denominator of equation (19) goes to zero. If this occurs, all entries in the $R$ matrix, for which the numerator are non-zero, are still connected and therefore their value is left unchanged. For entries for which the numerator is zero the associated nodes are now in distinct islands and therefore these entries are set to a constant large number, $\infty$. The new equality constraints are determined from the matrix, $R$. A matrix $E$ is obtained by setting,
The rank of $E$ is equal to the number of islands. The new equality constraint can be given as,

$$E_r \vec{p} = \vec{0} \quad \text{(21)}$$

where $E_r$ is the set of independent rows of $E$.

5. Results

The method was applied to the IEEE 118 bus system. To examine the sole effect of generator dispatch on the distribution of blackouts the maximum load shed at each individual node was taken for the results of a linear dispatch. This ensures that any variations in the flows on the lines are the result of differences in the dispatch of generation.

The vector $\vec{n}$ was taken to contain a value of 1 for generator nodes and 1000 for load nodes. $\text{thres, } q_0$ and $q_1$ are set to 0.9, 0.001 and 0.1 respectively. The line upgrade factor, $\partial_j$, was taken as 1.007 times the previously value. $r_\infty$ was chosen to be equal to 1000.

5.1. Blackout frequency distributions

The log$_{10}$-log$_{10}$ plot of the distributions of blackouts for three different optimization cases is shown in Figure 1. The amount of load shed during a blackout is a measure of the size of a cascading event in transmission systems. It is observed that there is little if any difference in the distributions between minimizing the flows during dispatch and dispatching the system economically. However the frequencies of blackouts of all sizes are reduced in the case where the system is dispatched to maximize the flows. These results may seem non-intuitive as the system in this case is being dispatched in a manner that attempts to maximize the stress in the system.

Two alternative measures of the size of blackouts are the number of lines failed [2] and the number of overloaded lines during an event. In Figure 2 the log$_{10}$-log$_{10}$ plot of the frequency distributions of the number of line failures during a cascade is given. Again little difference is observed between minimizing flows and a linear or economic dispatch. There is an increase in the number of small and medium sized cascades in the maximization case. This result would seem to contradict those given in Figure 1. The failure of lines during a cascading event weakens the ability of the transmission system to transmit power and therefore it would seem reasonable that an increase in the number of line failures would increase the amount of load shedding that occurs. A resolution to this apparent conflict can be found by examining how the transmission system is upgraded after a cascading blackout. The number of lines upgraded after the cascade is equivalent to the size of the event for this measure. Figure 3 gives the frequency distribution of line upgrades. In the maximization case we see a decrease in the frequency of small upgrade events concurrent with an increase in the frequency of larger events. Overall the increase seen in medium range cascading failures gives an increase in the average number of upgrades per blackout. This increase causes the system to become more robust to blackouts. There is therefore a tradeoff between the increase in line failures and the increase in system robustness due to the maximization of flows in the lines. It is apparent from the results in Figure 1 that for the system tested that the dominant force is the increase in system robustness and that this reduces the frequency of blackouts of all sizes when measured by the amount of load shed.
5.2. Fractional Line Flows

When a statistical equilibrium is reached the average fractional line overloads, $<M_i>$, for each line becomes robust [4]. An example for the economic dispatch case is given in Figure 4. In Dobson et al. [4] the distribution of the fractional flows was attributed to the network topology and distribution of generation and loads. In Figure 5 the cumulative density function, (CDF), of the average fractional lines flows is given for the three differing methods of generation dispatch. It is seen that this distribution also depends on the method in which the generators are dispatched. If we use this averaged fractional line flow we can determine the average system stress. This is given in Table 1. Due to the increase in the rate of upgrades to the system, on average, the strategy of attempting to maximize the fractional flows, leads to a reduction in the average stress of the system over time.

![Log-Log Plot of Line Outage Frequency Distributions](image1)

**Figure 2.** Frequency distribution of line failure events: minimizing flows (crosses), maximizing flows (diamonds) and economic dispatch (circles)

![Log-Log Plot of Frequency Distribution of Line Upgrade Events](image2)

**Figure 3.** Frequency distribution of line upgrade events: minimizing flows (crosses), maximizing flows (diamonds) and economic dispatch (circles)

![Average Fractional Flow](image3)

**Figure 4.** Average line fractional flows: economic dispatch case
Table 1. Average system stress for various cases

<table>
<thead>
<tr>
<th></th>
<th>Minimization Case</th>
<th>Maximation Case</th>
<th>Economic Dispatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{ave}$</td>
<td>54.4517</td>
<td>47.4110</td>
<td>61.0817</td>
</tr>
</tbody>
</table>

5.3. Upgrades per Blackout and Blackout Rates

The average number of days between blackouts, $\alpha$, and the average number of upgrades per blackout, $\beta$, for the various dispatch policies are given in Table 2. A decrease is seen in the maximization case for the average time between blackouts which indicates that the system in this case is more robust to failure. In order to achieve this robustness however the average number of upgrades to the system after a blackout has almost doubled. To see whether this average increase was universal for all blackout sizes the blackouts were binned according to size of load shed and the average number of upgrades for each bin was taken. The results are plotted in Figure 6. It is seen that this average increase is universal for the maximization case.

6. Discussion

The results above show that attempts to minimize network utility by means of generator dispatch in the hope of mitigating cascading blackouts can lead to the opposite effect in the long term behavior of the OPA model. Dispatching the system in such a manner leads to neglect of possible cascading failure paths and increases the frequency of blackouts of all sizes. It is also shown that dispatching the system through a homogenous economic dispatch is practically the same as attempting to minimize the global stress of the system. As the statistics observed in the time series of blackouts by using these methods are almost equivalent.

The reduction in blackout size, as measured by load shed, when the flows are maximized in the system through generator dispatch comes at the cost of increased investment into the running of the system. Any increase in reliability must be
considered with the cost of system maintenance in order to determine if it is economically viable. The increase in the cost of maintenance needs to be subtracted from the savings due to a decrease in risk of blackouts in order to determine if such a policy is beneficial to all parties. In a very simplistic cost-benefit analysis we assume that the cost of a blackout, \( c \), is directly proportional to its size.

\[
c = aB \quad \text{(22)}
\]

where \( a \) is the cost of a total system blackout and \( B \) is the size of the blackout measured as the fraction of load shed to demand. It is also assumed that the cost of a line upgrade is the same throughout the system and that this cost, \( l \), is a fraction of \( a \) such that,

\[
l = Qa \quad \text{(23)}
\]

The value for \( Q \) at which the reduction in blackout frequencies and sizes matches the increase in investment into the system can be determined by the data obtained through simulation. In the case of the above results it was found that \( Q \) is approximately 0.5% of the total cost of a complete system wide blackout. For values below this, there is an economic incentive to dispatch the system to maximize the line fractional line flows.

The above analysis avoids differences in cost due to differing dispatch as the generators are assumed to all have the same cost curve. In a general system the cost of running the dispatch differently to that of the economic dispatch would have to be included.

The reduction in blackouts with all sizes occurring with an increase in the average upgrades after each blackout also may indicate that a policy of randomly upgrading lines after an event which were not necessarily involved in the blackout may have some benefit to the system. While not examined in this paper an interesting question would be whether the maximization case leads to an optimal allocation of line upgrades for cases when more lines limits are increased than are involved in the blackouts of the economic and minimization dispatch cases.

It should be noted that the results observed above may have some system dependence. In order to determine if this behavior of the model is universal for all system configurations, tests will be completed using differing test systems.

7. Conclusion

It has been observed that the method in which generators are dispatched in the OPA model has an effect on the frequency distribution of blackouts as measured by various metrics. Dispatching the system in a manner that attempts to maximize system stress leads to a reduction in the frequency of blackouts of all sizes as measured by the amount of load shed. Conversely attempting to minimize system stress through generator dispatch does not lead to any noticeable changes in blackout statistics from that of an economic dispatch where costs are assumed to be homogenous.

8. Appendix

8.1. Modification to the slow dynamics of the OPA model

In the original model the load at each node is increased, before each iteration, by an amount, \( \mu \). Therefore at iteration \( k+1 \) the load at node \( i \) is given by,

\[
p_i^{k+1} = \mu_k p_i^k \quad \text{(24)}
\]

where \( p_i^k \) is the load at node \( i \) at time \( k \). The value of \( \mu_k \) was set to give an average ‘yearly’ increase in the load of about 3%. The concept of ‘yearly’ increase comes from the analogy of one iteration being equivalent to a ‘daily’ increase in the system loadings. This increase causes the flows in the lines to approach their limits.

The continual increase in the load results in large values for the load and line limits after a number of iterations. When the model was implemented in Matlab this caused problems for the linear programming algorithm. To prevent the use of such large values the model was changed so that the load remained constant while the line limits themselves were lowered. So at iteration \( k+1 \) the line flow limit for the line labeled, \( i \), is given by equation (7). This modification does not affect the dynamics of the model as the changes in the relative values of the line flows to the line limits are consistent with the original model. In a similar manner the generator limits are also decreased.

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9. References


