<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>Impact of distributed generation capacity on losses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Authors(s)</strong></td>
<td>Keane, Andrew; O'Malley, Mark</td>
</tr>
<tr>
<td><strong>Publication date</strong></td>
<td>2006-06</td>
</tr>
<tr>
<td><strong>Publication information</strong></td>
<td>IEEE Power Engineering Society General Meeting, 2006 [proceedings]</td>
</tr>
<tr>
<td><strong>Conference details</strong></td>
<td>Paper presented at the IEEE PES General Meeting 2006, Montreal, Quebec, Canada, 18-22 June 2006</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>IEEE</td>
</tr>
<tr>
<td><strong>Link to online version</strong></td>
<td><a href="http://dx.doi.org/10.1109/PES.2006.1709443">http://dx.doi.org/10.1109/PES.2006.1709443</a></td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/3310">http://hdl.handle.net/10197/3310</a></td>
</tr>
<tr>
<td><strong>Publisher's statement</strong></td>
<td>Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.</td>
</tr>
<tr>
<td><strong>Publisher's version (DOI)</strong></td>
<td>10.1109/PES.2006.1709443</td>
</tr>
</tbody>
</table>
Impact of Distributed Generation Capacity on Losses

A. Keane, Student Member, IEEE, M. O’Malley, Senior Member, IEEE

Abstract—The introduction of distributed generation (DG) onto distribution networks has a significant effect on losses. This effect cannot be characterised as detrimental or beneficial but is dependent on the allocation of DG on each distribution network section. Here the impact of DG on losses has been modelled, facilitating a unique approach to the allocation of DG. This approach has been implemented and tested on sample sections of distribution network and results are presented showing the optimal allocation of DG which improves the efficiency of energy delivery on the distribution network. The temporal variations of load and generation are simulated, illustrating that the allocation improves the efficiency throughout a year. The effect of different plant mixes is also simulated showing that the efficiency of energy delivery is dependent on the load factor and operating characteristic of the plant.

Index Terms—Losses, Power distribution planning, Linear programming, Dispersed storage and generation.

I. INTRODUCTION

Losses are an important consideration when designing and planning the distribution network. Losses are inevitable on any network, however the amount can vary considerably depending on the design of the network. In the past the distribution network was a purely passive system, used only for the delivery of electricity to the consumer. With the introduction of distributed generation, the network is being utilised in a different way with more variable and bidirectional power flows. The level of losses is closely linked to the power flows, therefore the allocation of DG provides an opportunity to ameliorate losses. Large amounts of distributed generation are being connected to distribution networks. In Ireland at the end of 2004, applications received by the system operators concerned the connection of approximately 2,500MW of wind generation. A significant amount of this capacity is to be distribution connected. This is in addition to 920MW of previously contracted wind farm capacity and approximately 100MW of other forms of DG such as landfill gas (LFG) and hydro. In a country with a peak load of approximately 4,500MW this is an extremely large amount of DG to integrate into the distribution network [1].

Under the EU Directive 2001/77/EC, Ireland should provide 13.2% of its electricity generation from renewable sources by 2010 [2]. The EU Directive for renewable energy penetration is part of a strategy to meet the Kyoto Protocol national targets for reducing greenhouse gas emissions. The vast majority of distributed generation is from renewable sources. Hence, the cost effective integration of DG is crucial to the economical achievement of these renewable energy targets. The placement of generation on a first come first served basis invariably limits the overall capacity of distributed generation, through network sterilisation as shown in [3]. Network sterilisation results when capacity is allocated to the bus/buses that are most sensitive to power injections. Thus, limiting the amount of further generation that can be connected at the other buses. A number of approaches for the placement of DG to minimise losses have been proposed. In [4] the authors propose a methodology which places DG at the optimal place along feeders and within networked systems with respect to losses. In [5] an algorithm is presented which places DG in order to reduce transmission and distribution losses. These papers do not look at maximising the amount of energy from DG on the system, but rather are concerned only with the minimisation of losses. In [6] the authors developed a methodology to optimally allocate DG capacity on the distribution network. The constraints considered were voltage rise, thermal limit, short circuit capacity, short circuit level, energy resource and customer initiatives. The methodology ensured that network sterilisation was avoided and the network capacity maximised. However no account was taken for losses.

The maximisation of renewable DG capacity alone displaces conventional generation, while the minimisation of losses alone results in more efficient delivery of energy. However, if the minimisation of losses is considered in conjunction with the maximisation of capacity, a different allocation will result, which leads to a larger proportion of the energy produced being delivered to load or exported to the transmission system. This ensures best use of the system and displaces a larger amount of energy from conventional generation. In this paper the impact of DG on losses is modelled, facilitating the calculation of the optimal allocation of generation which maximises the amount of power exported to the transmission system, thus improving the efficiency of energy delivery throughout the year. The optimisation is determined for maximum generation and minimum load as this is the point at which losses are largest for large amounts of DG capacity. The effect of a number of generation plant scenarios are simulated to demonstrate the effectiveness of the allocation over a year and also to illustrate the effect that the load factor and operating characteristics of the DG plant have on losses. In addition, it is shown that even when there is a net import of energy from
the transmission system, the efficiency of energy delivery is enhanced.

In Section II losses are described. In Section III the formulation of an objective function with losses included and the optimisation methodology are outlined. Results are shown in Section IV illustrating the effects that DG can have on losses. The results show that the absolute maximisation of DG capacity does not always lead to the maximisation of energy serving load. Further results are given, illustrating the increased energy delivered from DG when the methodology proposed here is employed. Discussion of the results is in Section V and conclusions are given in Section VI.

II. LOSSES

The transmission of power will always incur a certain amount of electrical losses. Losses can represent a considerable cost, however, similar to any other cost they must be balanced against other costs and objectives and therefore their absolute minimisation may not always be desirable [7]. The integration of large amounts of DG is transforming distribution networks from what were traditionally energy delivery networks to networks that both deliver and harvest energy. A key element to the efficiency of this energy transmission are losses. The losses considered in this paper are the load losses, i.e. the losses which are dependent on the power flows in the system. The effect on losses of increasing power injections can be determined due to the radial structure of the distribution network. Under normal feeding conditions, the flows on any line, and therefore the losses, are dependent on the load and generation downstream of that line, i.e. losses vary monotonically with load and generation. With no generation downstream of a bus, the losses are given by the load downstream. The introduction of generation downstream will change the losses, with the losses initially decreasing until the load at the bus is met and then increasing as the excess power flows back up the line in the opposite direction.

Losses have a quadratic relationship with load and generation. Therefore, to accurately represent them, a piecewise linear approximation is used, with each characteristic divided into segments to aid in their utilisation in a linear programming (LP) formulation. Given that the loss characteristic for each bus is dependent on generation and load, account must be taken for the variability of both generation and load. The relationship between load, generation and losses at each interdependent bus is calculated resulting in loss characteristics for each bus. The loss characteristics are convex, therefore a unique optimal solution exists, which can be determined by application of an LP algorithm.

III. METHODOLOGY

The losses associated with the $i^{th}$ bus are formalised and are shown in Equation (1).

$$P_{Loss_i} = \sum_{j=1}^{N} [P_{DG_i}(1-\eta_{ij}) + P_{LD_i}(1-\rho_{ij})] \quad \text{1}$$

Where $P_{DG_i}$ and $P_{LD_i}$ are the DG capacity and the load at the $i^{th}$ bus respectively, $N$ is the number of buses, $\eta_{ij}$ and $\rho_{ij}$ are the interdependence of losses due to generation and load at the $i^{th}$ bus and generation and load at the $j^{th}$ bus respectively and $P_{Loss_i}$ is the losses due to the $i^{th}$ bus. Without loss of generality, it is assumed that there is one generator connected at each bus. Due to the radial structure, only buses that are connected along the same radial section are interdependent and will therefore have non zero values for $\eta$ and $\rho$. In order to maximise the power delivered, the losses are formalised into the objective function shown in Equation (2).

$$P_{Tx} = \sum_{j=1}^{N} [P_{DG_i}(1-\eta_{ij}) - P_{LD_i}(1-\rho_{ij})] \quad \forall N. \quad \text{2}$$

$P_{Tx}$ (MW) is the amount of generation demanded from or exported to the transmission system. The objective function $P_{Tx}$ (MW) in Equation (2) is maximised subject to the constraints used in the optimal allocation methodology in [6]. These constraints are shown in the Appendix. A linear programming algorithm is employed to maximise the objective function with respect to the constraints given in equations (3), (4), (5), (6), (7) and (8). These constraints limit the DG capacity based on the technical characteristics of the network and the available energy resource and siting restrictions at each bus.

The objective function in Equation (2) determines the optimal allocation at a single operating point, in this case maximum generation and minimum load. There is a trade off between the maximisation of capacity and minimisation of losses which is captured by the objective function in Equation (2). The objective function maximises the amount of power exported to the transmission system, i.e. the power output minus the load minus the losses, thus improving the efficiency of energy delivery throughout the year.

The load values used in the optimisation are the minimum values at each bus. When dealing with large amounts of DG capacity, exceeding the load, DG will have its greatest impact on losses when at its maximum and when load is at its minimum. DG will be operated at its maximum as much as possible in order to maximise the generator’s revenue, thus the maximum permissible value is used to calculate the optimal allocation. Some forms of DG, such as wind energy, are intermittent and this will reduce the number of times at which operation at the optimal point occurs. A large amount of intermittent generation will therefore reduce the benefits of the optimal allocation. Indeed if the load factor is very low it may in fact cause the benefits of this approach over other approaches, such as the maximisation of capacity, to be nullified. In addition, the use of a number of different generation types will increase the diversity of the overall generation profile, possibly resulting in reduced operation at maximum overall capacity. These issues are investigated and illustrated in the results in Section IV.

IV. RESULTS

A. 38kV Test System

The test system chosen is a typical section of the Irish 38kV distribution network. Results are given here for a 38/110kV station with 7 buses as shown in Figure 1. The section of
distribution network is modelled in DIgSILENT PowerFactory. Load and generation profiles for a given year were simulated to examine the optimality of the allocation over a year, in particular to determine if the maximisation of $P_T x$ at a key operating point leads to an improvement of energy delivery efficiency. Five generation plant scenarios were selected, each with different characteristics and varying overall load factors. Load and wind generation profiles were obtained from ESB National Grid [8]. The generation profiles for the LFG, biomass and hydro generation along with the other necessary network data was obtained from ESB Networks, the DNO in Ireland [9].

B. Loss Characteristics

Individual loss characteristics for increasing generation are shown in Figure 2. It can be seen that all the buses have the same shaped characteristic, each with a different impact on losses. This figure shows the relative impact of generation at each individual bus on losses with no load present.

Figure 3 shows the loss characteristic of increasing amounts of generation at buses B & C, which are located along the same radial section. It can be seen from the graphs that the relationship between power injections and losses is highly variable and cannot be said to be either beneficial or detrimental. Rather it can be seen that there is a wide range of possible values for losses, in particular it can be seen that the values of $\eta_{ij}$ and $\rho_{ij}$ change depending on the amount of generation and load at all interdependent buses, hence the necessity for a piecewise linear approximation of the characteristics. The total amount of losses on any section of network will be dependent on the allocation of load and generation across all the buses. For example in Figure 3 it can be seen that placing 9MW of generation at bus B with no generation at bus C will result in losses of 0.02MW. Placing 9MW at bus C with none at bus
B incurs losses of 0.47MW. With reference to Figure 1 it can be seen that bus C is at the end of a 20km line, therefore the losses incurred by increasing the flows are much larger than those incurred by bus B which is connected to the transmission station by a 1km line. Similar convex characteristics exist for the interdependence between load and generation at each bus.

These loss characteristics may be used to form the objective function given in Equation (2) which maximises the power exported to the transmission system at a given operating point. The values for $\eta_{ij}$ in Equation (2) are determined from these characteristics and the values for one segment of the linearisation are shown in Table I.

### TABLE I
GENERATION LOSS INTERDEPENDENCIES ($\eta_{ij}$)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.005</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0003</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0.0114</td>
<td>0.0242</td>
<td>0.01926</td>
<td>0.01926</td>
<td>0.01926</td>
<td>0.01926</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>0.025</td>
<td>0.0326</td>
<td>0.0322</td>
<td>0.0322</td>
<td>0.0322</td>
<td>0.0322</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>0.0154</td>
<td>0.0236</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>E</td>
<td>0</td>
<td>0</td>
<td>0.0150</td>
<td>0.0258</td>
<td>0.0258</td>
<td>0.0258</td>
<td>0.0258</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>0.0086</td>
<td>0.0470</td>
<td>0.0470</td>
<td>0.0470</td>
<td>0.0470</td>
</tr>
<tr>
<td>G</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.012</td>
</tr>
</tbody>
</table>

C. Optimal Generation Allocation

Table II shows a comparison between two generation allocations with the load fixed. The first allocation (Max. Capacity) employs the maximisation of capacity methodology from [6], with the second allocation (Max. $P_{TX}$) determined by the approach proposed here. It can be seen that when losses are not considered, the total generation allocation is slightly higher. However with losses considered, the slightly smaller allocation of 34.92MW results in more efficient delivery of energy. The difference in the total allocation is only 0.11MW, but there is a significant difference in the allocation of the generation between the buses, indicating that for a relatively small reduction in the total DG capacity, an allocation exists which by taking account of losses and capacity, results in more efficient delivery of energy. The amount of power exported to the transmission system is given by $P_{TX}$. It can be seen that the reduction in losses results in a larger export of power from a slightly smaller amount of generation capacity at maximum generation output.

### TABLE II
GENERATION ALLOCATIONS

<table>
<thead>
<tr>
<th>Bus</th>
<th>Load (MW)</th>
<th>Max. Capacity</th>
<th>Max. $P_{TX}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.25</td>
<td>1.92</td>
<td>0.87</td>
</tr>
<tr>
<td>B</td>
<td>1.00</td>
<td>0.00</td>
<td>1.6</td>
</tr>
<tr>
<td>C</td>
<td>0.75</td>
<td>1.00</td>
<td>6.00</td>
</tr>
<tr>
<td>D</td>
<td>2.50</td>
<td>10.00</td>
<td>5.70</td>
</tr>
<tr>
<td>E</td>
<td>0.50</td>
<td>6.16</td>
<td>5.61</td>
</tr>
<tr>
<td>F</td>
<td>3.00</td>
<td>8.74</td>
<td>6.00</td>
</tr>
<tr>
<td>G</td>
<td>2.00</td>
<td>7.21</td>
<td>9.1</td>
</tr>
<tr>
<td>Total</td>
<td>10.00</td>
<td>35.03</td>
<td>34.88</td>
</tr>
</tbody>
</table>

$P_{TX}$

The allocations shown in Table II are optimised for operation at maximum generation and minimum load. The load and generation vary throughout the day, causing the optimal allocation to thereby change at every instant. Hence, in terms of energy, it is impossible to define a single allocation of generation as the optimal, however, the load will follow a predictable profile and generation profiles are available, which allow an assessment of the effect of the generation allocation on losses over a year.

D. Generation Plant Scenarios

The allocation determined by the algorithm results in a larger export to the transmission system ($P_{TX}$) at times of high generation and low load. The frequency of this scenario is dependent on the load factors and generation profiles of each of the generator types. However, due to the quadratic nature of losses, the maximisation of the exported power at this key time when losses are at their maximum, should result in an overall net reduction in the amount of energy imported from the transmission system, even when using intermittent generation. To demonstrate this, the five plant scenarios shown below were simulated over a year. The load flow analysis is calculated for fifteen minute intervals over the year. The sending voltage at the transmission station is set to a typical value. The generator and load power factors are each set to average values within their permissible range.

1) No Generation
2) All Wind generation
3) Wind & LFG generation
4) Wind, LFG & Biomass generation
5) Biomass, LFG & Hydro generation

Each scenario has a different mix of generation types, however the generation allocated to each bus in each scenario matches those shown above in Table II. In particular, scenario
3 places 5MW of LFG generation at bus D, with the remaining generation being made up of wind generation. Scenario 4 places 5MW of biomass generation at bus F in addition to 5MW of LFG at bus D, with the remainder once again made up by wind generation. Finally in scenario 5, 5MW of hydro generation are placed at bus F, with 5MW of LFG once again at bus D and the remaining capacity is made up by biomass.

Figure 4 shows the losses and power imported and exported over a typical day for plant scenario 4. The import/export characteristic is shown to illustrate the link between the level of generation output and the savings in losses. This improvement in losses can be seen, with a larger reduction in losses evident when the generation output is high, corresponding to the right hand side of the graph when power is exported. The impact of the optimal allocation can be seen by the reduced losses at this time over the maximum capacity allocation. It can be seen on the left hand portion of the graph that when the generation output is lower, the losses are lower as would be expected and more significantly, the saving in losses between the two allocations is reduced.

The two allocations in Table II were used with each scenario. Using the load and generation profiles, a profile of the losses and the net energy import or export ($E_{Tx}$) over a year is determined for the network shown in Figure 1. Tables III and IV show the results of these simulations. The results shown here are for generators and loads with power factors of 0.95 and 0.96 respectively. In all the plant scenarios simulated the maximisation of $P_{Tx}$ reduced the amount of losses over the maximum capacity allocation and either reduced the net import from the transmission system or increased the net export to the transmission system as illustrated by the net change in energy imported or exported ($\Delta E_{Tx}$) in Table IV.

Plant scenario 2 is all wind generation, with a total load factor (LF) of 0.36. Wind generation is intermittent by nature but wind turbines are designed to run at their maximum capacity and will do so whenever possible. The benefits of the optimal allocation are generally reduced for plant scenarios with lower load factors as can be seen from the results in Tables III and IV. The net amount of energy demanded from the transmission system over the year is reduced by 191.47MWh. Plant scenario 3 is a combination of wind and LFG generation. The LFG plant has a individual load factor of 0.76, which results in an increased overall load factor of 0.42. The impact of the LFG plant with its different output profile and higher load factor can be seen. The increased energy production means that overall the generation is operating closer to its maximum capacity for a greater proportion of the time. This is reflected in the results in Table IV, where the optimal allocation reduces the net energy import from the transmission system by 263.93MWh.

In scenario 4, the biomass generation employed has a individual LF of 0.86, further increasing the overall load factor to 0.49. In this case, the reduction in the import from the transmission system is 432.25MWh. Finally in scenario 5, the combination of biomass, LFG and hydro generation which has an individual load factor of 0.35, results in an overall load factor of 0.77. In this case there is a net export of energy from the distribution network onto the transmission system. The optimal allocation is effective once more with an extra 455.33MWh exported to the transmission system. It can be seen from Table IV that in general a higher overall load factor yields a greater saving. However, it can also be seen from scenario 5 that the addition of 5MW of hydro which rarely operates at its maximum capacity, causes the expected savings as a result of the high load factor to be diminished. This shows that in addition to the load factor, the operating characteristic of the generation will affect the optimality of the allocation. The network parameters, such as power factors and sending voltage, vary throughout the year and affect losses, however, they will not affect the relative difference between each allocation.

### V. Discussion

The new approach demonstrated here illustrates the impact of DG on losses. With regard to emissions targets, the allocation of DG will have a significant impact on the feasibility.
of meeting these targets economically. It has been shown that maximising capacity with no regard for losses does not maximise the potential benefits of DG. The approach given here, maximises the amount of active power delivered from DG at a key operating point. It has been shown that by maximising power at this peak condition, the energy demanded by the distribution system is reduced over a year. Reactive power flows also have a significant effect on losses. Each generator has to operate within the range of acceptable power factors as set by the DNO; these power factors will affect the losses, but it is the placement of DG capacity that is optimised rather the operation of DG.

In this paper, DG capacity is allocated on the basis of firm connection agreements i.e. the generators are not dispatched or curtailed, which has traditionally been the case in Ireland. A limited form of non-firm access is now permitted on the Irish system, however, it will not be used to minimise losses but rather to permit a further penetration of DG, which is a higher priority [10]. In Ireland, generic loss factors are currently used to take account of any perceived saving in losses resulting from distributed generators [11]. However rather than a generic value dependent on the voltage level, a more accurate value could now be assigned on a site specific basis to individual generators using the analysis shown here, providing a price signal rewarding generators for improving losses and penalising them for any increase in losses they cause.

The results show that the plant mix of distributed generation has a considerable impact on the losses and the energy exported to the transmission system. This further optimisation problem has been detailed in [12]. The results in Tables III and IV show the effectiveness of the allocation, for plant mixes with both high and low load factors. The approach in this paper maximises the power export to the transmission system from DG at a key operating point. Indeed, it is evident from the results that even with a large DG capacity, there will still often be a net import of energy from the transmission system. Hence, the maximisation of power export at one point results in improved efficiency of energy delivery be it export or import from the transmission system. It has been shown that the approach is effective for variable and intermittent forms of generation such as wind power, but that the inclusion of generation that only operates very infrequently at its maximum power factor has a significant impact on the optimal allocation. Previously, losses could only be ameliorated by uprating overhead lines or other equipment. The introduction of DG provides the DNO with a new variable that may be optimised for everyone’s benefit. Distribution networks are no longer exclusively used for the delivery of electricity to consumers and with the penetration of DG set to increase over the coming years, the cost effective integration of DG will be crucial to meeting emissions targets and also to the economical running of power systems across the world.

VI. CONCLUSION

A new approach has been proposed for the allocation of DG on distribution networks. The methodology enables the optimal allocation of large amounts of DG, such as those seen in Ireland. It has been tested on a sample section of distribution network. The impact of DG in radial distribution networks on losses has been modelled, thus allowing an accurate assessment of the impact of DG and load on losses. Losses have been shown to be an important factor, with a significant impact on the amount of energy that reaches the load or transmission system. It has been shown that neither the maximisation of capacity alone nor the minimisation of losses alone is the optimal way to maximise the benefit of DG to society, but rather an objective function which takes account of both losses and capacity. The effectiveness of the approach has been demonstrated for a number of different DG plant scenarios, illustrating that the load factor and operating characteristic of DG plant has a significant impact on the efficiency of energy delivery.

APPENDIX

Thermal Constraint

\[ I_i < I_{i_{\text{Rated}}} \quad i \in N. \]  

(3)

Where \( I_i \) is the current flowing from generator \( i \) to bus \( i \), \( I_{i_{\text{Rated}}} \) is the maximum rated current for the line between each generator and its corresponding bus.

Short Circuit Level

\[ \sum_{j=1}^{N} \delta_{jT_x} P_{DG_{j}} + \alpha_{T_x} \leq SCL_{\text{Rated}}. \]  

(4)

Where \( \delta_{jT_x} \) is the dependency of the SCL at the transmission station to power injections at bus \( j \). \( \alpha_{T_x} \) is the initial SCL at the transmission bus with no generation present.

Short Circuit Ratio

\[ P_{DG_{i}} - 0.1 \cos(\phi) \sum_{j=1}^{N} \delta_{j} P_{DG_{j}} \leq 0.1 \cos(\phi) \alpha_{i} \quad i \in N. \]  

(5)

Where \( \cos(\phi) \) is the power factor at the generator.

Voltage Rise

\[ \sum_{j=1}^{N} (\mu_{ij} P_{DG_{ij}} + \nu_{ij} P_{LD_{ij}}) + \beta_{i} \leq V_{\text{max}} \quad i \in N. \]  

(6)

Where \( \mu_{ij} \) and \( \nu_{ij} \) refer to the dependency of the voltage level at bus \( i \) on power injections and load at bus \( j \) respectively, \( \beta_{i} \) refers to the initial voltage level at the \( i \)th bus with no generation.
Transformer Rating

\[
\sum_{i=1}^{N} (P_{DG_i} - P_{LD_i}) \leq P_{TrafoCap}, \tag{7}
\]

Where \(P_{TrafoCap}\) refers to the rating of the transformer.

Energy Resource & Customer Initiatives

\[
P_{Installed_i} \leq P_{DG_i} \leq P_{Avail_i} \quad i \forall N. \tag{8}
\]

Where \(P_{Avail_i}\) and \(P_{Installed_i}\) are the available energy resource and any existing generation at the \(i^{th}\) bus respectively.

ACKNOWLEDGMENT

The authors gratefully acknowledge I. Codd, D. Phelan and T. Walsh of ESB Networks and their colleagues in the Electricity Research Centre, in particular R. Doherty for their help with this work.

REFERENCES


Andrew Keane (S’04) received a B.E. degree in electrical engineering from University College Dublin in 2003. He is currently conducting research for a Ph. D. degree in the Electricity Research Centre, University College Dublin with research interests in power systems planning, distributed generation and distribution networks.

Mark O’Malley (S’86-M’87-SM’96) received B.E. and Ph. D. degrees from University College Dublin in 1983 and 1987, respectively. He is currently the Professor of Electrical Engineering at University College Dublin and director of the Electricity Research Centre with research interests in power systems, control theory and biomedical engineering.