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Title	Optimal firm wind capacity allocation to power systems with security constraints
Authors(s)	Burke, Daniel J., O'Malley, Mark
Publication date	2009-03
Publication information	Burke, Daniel J., and Mark O'Malley. "Optimal Firm Wind Capacity Allocation to Power Systems with Security Constraints." IEEE, 2009.
Conference details	Paper presented at the 2009 IEEE PES Power Systems Conference & Exhibition, March 15-18 2009, Seattle, Washington
Publisher	IEEE
Item record/more information	http://hdl.handle.net/10197/3316
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Publisher's version (DOI)	10.1109/PSCE.2009.4840126

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Optimal Firm Wind Capacity Allocation to Power Systems With Security Constraints

D. J. Burke, *Student Member, IEEE*, and M. J. O'Malley, *Fellow, IEEE*

Abstract— Many countries have declared future renewable energy penetration targets. Wind power connection to power systems is delayed by limited transmission system capacity as attractive wind sites are often located in weakly designed transmission areas. Optimal use of existing transmission system resources should be made in the allocation of capacity connection permits. The volume of wind power connection applications and their power production statistical inter-dependencies suggest that they should be assessed in a collective probabilistic manner. This paper uses a sequential probabilistic load flow method in tandem with a linear programming computational geometry constraint redundancy approach to optimally allocate wind capacities given the transmission system capacity that is securely available.

Index Terms— linear programming, maximal vector power system planning, power transmission, probabilistic load flow, wind energy.

I. INTRODUCTION

The integration of wind power to electric power systems has been increasing at a significant rate [1]. The abundance of this emissions free and cost-predictable source of energy suggests much higher penetration rates are likely in the near future. It is viewed as a promising solution to the climate change and energy security challenges currently facing the power industry [2]. The most attractive wind resources are often located in areas of low transmission system capacity, distant from the traditional load centres, however. While the cost and lead-times of transmission system upgrades required to accommodate likely wind generation projects are significant, public dissent associated with large-scale infrastructural development in recent times will no doubt further compound such difficulties [3].

Clearly power system planners must be innovative in the identification and implementation of new generation connection requirements [4]. Best use of any existing transmission capacity must be made in the short to medium

term – an optimal allocation of wind capacity connection will facilitate as many wind projects as possible. Given the large numbers of wind connection applications to be processed, and the appreciable degree of stochastic interdependence of geographically distant sites due to common weather patterns, any optimisation approach requires a collective and probabilistic analysis of all sites simultaneously.

Heuristic techniques for individual plant connection feasibility study such as ‘incremental transfer capability’ assessed at assumed onerous base-cases such as ‘winter-day-peak’ or ‘summer-night-valley’ [5] may be secure, but will not lead to optimal use of transmission capacity. This paper uses a probabilistic load flow (PLF) approach to identify the true worst case scenarios, regardless of what time of year and under what operational conditions they occur. This is imperative as a transmission network is a distributed infrastructural resource, and the worst-case-scenarios for wind power related flows could conceivably occur at off-peak conditions, and not necessarily in each line simultaneously. Several analytical and Monte-Carlo sampling (MCS) methods for probabilistic load flow have been published in previous literature [6], [7]. As highlighted in [8], the unique shape of wind power multidimensional probabilistic density functions requires a sampling approach for best accuracy. As discussed by the authors in [9] however, random MCS is a memory-less process, and the only way to accurately account for the inter-temporal trends of wind power in a probabilistic load flow context is with the use of sequential or time-series probabilistic load flow techniques. A sequential analysis is required to correctly model the impact of system wind power production on conventional plant unit commitment parameters such as start-up times, ramp rates, minimum up-times, etc.

This paper determines the optimal wind capacity allocation to specific system buses in order to achieve a defined system wind energy penetration target. In contrast to the integer-programming optimisation techniques required for large conventional plant, the relatively small size of wind turbine capacity with respect to each total wind farm capacity and the use of linearised load flow constraints allow linear programming techniques to be used with little inaccuracy trade-off. An initial approach to achieving this with respect to transmission adequacy was outlined in a previous paper by the authors [9]. To determine the feasibility of firm network connections however, the methodology presented in this paper also accounts for ‘N-1’ security constraints as applied in practice. A probabilistic analysis of power system operation is

This work was conducted in the Electricity Research Centre, University College Dublin, Ireland, which is supported by Airtricity, Bord na Mona, Cylon Controls, the Commission for Energy Regulation, Eirgrid, ESB Networks, ESB Powergen, Siemens and Viridian. The work of D. J. Burke was supported by Sustainable Energy Ireland through a postgraduate research scholarship from the Irish Research Council for Science Engineering and Technology.

The authors are with the School of Electrical, Electronic and Mechanical Engineering, University College Dublin, Dublin 4, Rep. of Ireland (e-mail daniel.burke@ee.ucd.ie, mark.omalley@ucd.ie; telephone +353 1 716 1857).

used to define a linear programming model to optimally allocate wind capacity to specific system regions based on both the quality of the wind resource and the transmission system capability in each area. A linear constraint redundancy technique based on the concept of computational geometry maximal vectors is applied to reduce the considerable optimisation dimensionality. Section II of this paper outlines the complete methodology applied in more detail. Section III describes a simple test system to which the algorithm is applied and Section IV presents the results of this process. Section V provides a discussion while Section VI concludes.

II. METHODOLOGY

A. Methodology Overview

As in [9], an initial system-wide wind power output time series is used to perform a year-long unit commitment and dispatch study to determine the true load/wind/conventional generation statistical inter-dependencies. A network DC-load-flow model uses the resulting individual nodal power injection time series' to define network security constraints. An efficient computational geometry pre-processing stage is used to reduce the optimisation constraint dimensionality to a manageable level before a linear programming model determines the optimal firm capacity allocation. Any discrepancy between the initial assumed system wind power output and the trend determined by the linear programming model output capacities (leading to power system imbalance) is resolved through a number of methodology re-iterations. A full sequential probabilistic load flow can be carried out to investigate the resultant system line flows. A flowchart of the methodology applied is given in Fig.1 below.

Fig.1. The optimal capacity allocation methodology flowchart.

B. Determining the System's Stochastic Interdependencies

Any probabilistic load flow approach must correctly account for the true network bus power injection inter-dependencies. Only full sequential unit commitment study with respect to conventional plant characteristics can accomplish this [9]. A network model is not contained within the unit commitment stage and generation re-dispatch is not considered in this paper as it is assumed the firm connection rights of generation already present in the system must be respected. Perfect wind forecasting is assumed, though unit commitment techniques to account for forecast error can be included in practice [10].

Simultaneously recorded nodal load and nominal wind power time series' (both assumed available for study) will inherently contain natural customer demand and weather-dependent power production trends. Each potential wind plant location will have an individual capacity factor and temporal variations in its nominal 1MW time series. The precise determination of individual conventional plant power production time series (which are a function of the total system load and the total system wind production via commitment and dispatch) is initially impossible as the individual wind capacity allocations are the defined outputs of this methodology. Given that the system dispatch is carried out with respect to total system wind power output however, while load flow is based on individual nodal behavior, it may be initially assumed that the system wide wind time series profile thus converges to a smoothened trend based on the general weather patterns over a large geographical area. Therefore the system statistical dependency study can be carried out prior to the load flow and optimization stages, yet the conventional plant power production is still consistent with the individual wind farm behaviour. This is reasonably accurate for a large and dispersed collection of potential wind sites as the stochastic behaviour present in individual time series' is more influenced by the general weather pattern over a wide area as opposed to localized effects. 'Principal component analysis' [11] or 'independent component analysis' [12] can be used to extract and study the common dependency of the wind plant output that causes this overall trend and provide justification for this initial step. In this paper, a simple average of the nominal wind time series' was found to be sufficient, and is used to generate the initial system-wide wind power output trend as in (1) and (2) below. An averaged turbine capacity C_{avg} required to serve a δ_e proportion of load energy can be determined if the average system load $P_{Load-avg}$ and the average capacity factor of all the wind sites cf_{avg} are known. The initial time series for study is determined by scaling the average of the recorded nominal wind farm time series' ts_{avg} by C_{avg} . This will satisfy the annual energy penetration target, while respecting the hour to hour variation exhibited in the individual time series'.

$$C_{avg} = \frac{P_{Load-avg}}{cf_{avg}} \cdot \delta_e \quad (1)$$

$$ts_{syst} = C_{avg} \times ts_{avg} \quad (2)$$

C. DC Load Flow

DC load flow is often used in planning studies as an initial first-pass analysis of the transmission network's active power transport capabilities. Important network operational issues involving steady-state and dynamic voltage concerns, short-circuit levels as well as transient stability behaviour are more suited to detailed in-depth analysis, may not be fully amenable to a manageable optimization process across such a range of many operating points, and are often assessed only once the grid is known to be thermally secure. The algorithm outlined in this paper could be viewed as the best possible wind capacity allocation from which to carry out such analyses, considering all possible solutions [13]. The DC load flow model is linear, and thus linear constraints can be formulated to represent network security criteria in the optimization stage. With a designated reference bus, DC load flow assumes that the line power flow solution f_j in each line j is a linear combination of the power injections, P_i , at every other bus i , [14], as in (3) below. A different set of DC load flow coefficients α_{ij} will exist for each network configuration scenario. In this paper, the reference bus also ensures system load balance for the first few methodology iterations until the total wind time series model error is removed.

$$f_j = \sum_i \alpha_{ij} \times P_i \quad (3)$$

D. Linear Programming Optimization

Most power system regulatory bodies have goals to integrate a fixed amount of wind energy by a future static year timeline [15]. Certain wind sites have better energy-producing characteristics than others, as defined by their capacity factor. A prudent way to achieve energy penetration goals while maximizing the transmission system capability would be to thus allocate the wind capacity on the basis of each wind site's capacity factor. If the most attractive wind capacity factor sites are chosen then less overall wind turbine capacity will be required in the system to achieve the same energy penetration goal. In this paper no transmission expansion is assumed possible (if the optimal firm wind capacity allocation problem is infeasible, then a lower energy penetration must be initially accepted until new transmission is built) and no losses are modeled as DC load flow is used. The optimization cost function is therefore defined by minimising the sum of the individual MW capacity allocations (which are thus the optimization variables) i.e. the total system wind turbine installed capacity as in (6) below. The total installed wind capacity must satisfy the defined energy penetration goal over the whole year, thus the inclusion of the simple linear equality constraint of the optimization variables scaled by the capacity factors of the respective sites as in (7). Without inclusion of network flow constraints, the unconstrained optimum would be to thus locate all wind capacity in the best capacity factor site.

As wind power is a non-dispatchable resource, then grid capability across all possible 'N-G-1' security-limited operating conditions in the year of study should be assessed in a prudent wind generation connection feasibility analysis. In a

practical power system this implies a very large number of possible study scenarios, reaching in the limit the value as expressed in (4) below for T years of data analysis (considering every possible generation and line contingency at each hour with respect to each other line's capacity).

$$T \times 8760 \times N \times \binom{N-1}{N-2} \times \binom{G}{G-1} \quad (4)$$

Line contingencies will change the DC load flow coefficients used, while generation contingencies will change the bus power injection values. The loss of generation and the contribution of primary operating reserve can be used to define network security requirements for all possible generation contingencies (wind plant contingencies can only be modeled for subsequent iterations as the individual wind plant capacities are initially unknown). The localized impact of any possible branch contingencies would suggest that not all line/branch contingency scenarios need to be modeled when analyzing the capability of any particular other line in the network however. In this paper the probabilistic spread of the partial load flow solution without wind in each line for each other possible line contingency was used as a simple practical method to select which line contingencies should be included as constraints for each line under analysis. A more rigorous contingency selection method could use the network's 'line-outage distribution factors' or other methods [14] to initially screen branch contingencies before probabilistic study.

The DC load flow coefficient vectors and the bus power injections are used define the line flow constraints in the PLF optimisation stage. The bus injections of conventional plant and customer load at the individual network buses are known from the unit commitment output and the input model data stages respectively. Thus the net contribution from both of these sources to the power flow in each line j can be evaluated as numerical partial load flow solution value fp_j for each hourly security scenario s based on the relevant DC load flow coefficients. The collective net contribution of the k wind farms to the power flow as a forward or backward flow value in each hourly security scenario s is as of yet unknown. The optimization MW capacity allocation variables C_k when scaled by the nominal 1 MW time series values ts_k and the relevant DC load flow coefficients will determine this. To ensure that the wind capacity allocations do not overload any of the network line capacities LC_j in either the forward or backward flow directions, the inequality constraint of (5) is included for each hourly security scenario (active power limits considered only).

$$\sum_s \sum_j [-LC_j \leq \sum_{i,k} \alpha_{ijs} \times ts_{ks} \times C_k + fp_{js} \leq LC_j] \quad (5)$$

The double-sided inequality as in (5) can be represented as two single-sided inequalities by simple algebraic rearrangement as in (9) and (10). The linear programming load flow inequality constraints thus comprise of a $2s^*(k+1)$ matrix consisting of k optimization variable constraint coefficient

columns and one numerical constant column. The optimization stage is represented in the formal mathematical programming model of (6), (7), (8), (9) and (10) below.

$$Cost = Min(\sum_k C_k) \quad (6)$$

$$C_{avg} \cdot cf_{avg} = \sum_k C_k \cdot cf_k \quad (7)$$

$$C_k \geq 0 \quad (8)$$

$$\sum_s \sum_j [\sum_{i,k} \alpha_{ijs} \times ts_{ks} \times C_k - (LC_j - fp_{js})] \leq 0 \quad (9)$$

$$\sum_s \sum_j [\sum_{i,k} -\alpha_{ijs} \times ts_{ks} \times C_k - (LC_j + fp_{js})] \leq 0 \quad (10)$$

E. Linear Programming Constraint Redundancy

Optimization algorithm computational requirements are quite sensitive to both the number of variables and constraints in the mathematical model [16]. As (4) would suggest, the total number of constraints could be intractable in a reasonably large power system. While the contingency selection methods of Section II-D reduce the dimensionality somewhat, an intelligent approach should also seek to take advantage of the similarity and possible redundancy of many of the operational scenarios across the representative year of analysis. A simple method involving discretised multi-dimensional clustering was applied by the authors to the network adequacy problem in [9] (i.e. ignoring line contingencies). This would be inappropriate for the large number of network security scenarios to be considered - a more formal approach to optimisation model dimension reduction is proposed here. Constraint redundancy occurs when an inequality constraint such as line C in Fig.2 below does not intersect the feasible space as defined by the lines A, B and the variable axes. It will have no influence on the optimal constrained value of the cost function, and thus the optimal variable values. As will be seen, a great many of the time-series security inequality constraints from (9) and (10) do not intersect the feasible space, i.e. they are made redundant by other more extreme constraints.

Fig.2. Linear program constraint redundancy – no feasible space intersection.

A general and formal mathematical treatment of linear programming redundancy is given in [17]. Dula in [17] discusses the dual representation of linear constraints as points,

with the ‘frame’ or ‘extreme point’ set of the ‘convex hull’ of the points in the dual domain representing the set of non-redundant constraints in the primal representation. Linear programming redundancy and the field of computational geometry [18] are closely linked. As the convex hull of any multidimensional dataset defines every point within its volume, it is the minimal complete representation of the multidimensional dataset [18], and all points interior to the hull are thus redundant. Dula et al in [19] describes an efficient computational method to determine the frame of the convex hull using successively larger linear programs.

The presence of (8) in the mathematical program of this paper allows an intuitively more simple interpretation of linear constraint redundancy issue however. Consider two arbitrary linear constraints as in (11) and (12) below. As C_1, C_2 are ≥ 0 by design in this problem, then if all the coefficients and constant of (11) (i.e. $a_1, a_2, const_a$ etc) are at least greater than or equal to their equivalents in (12), then (12), (corresponding to line type C in Fig.2) will intuitively be a less extreme constraint than (11) and is thus redundant.

$$a_1 \cdot C_1 + a_2 \cdot C_2 + \dots + const_a \leq 0 \quad (11)$$

$$b_1 \cdot C_1 + b_2 \cdot C_2 + \dots + const_b \leq 0 \quad (12)$$

The geometric dual of a line (and thus a constraint) is a point [22], and each constraint such as (11) and (12) can be thus uniquely represented with the points $p_a(a_1, a_2, const_a)$, and $p_b(b_1, b_2, const_b)$. The complete set of inequality constraints in (9) and (10) can therefore be equally defined in this manner as a multi-dimensional point dataset. The problem of identifying the non-redundant constraints in a linear program of this format is analogous to defining the ‘maximal vector’ subset of the geometric dual dataset. A data-point is a maximal vector of a dataset if it is not dominated by any other data-point – this concept was first introduced in [20] and [21]. p_a dominates p_b if $b_1 < a_1$, $b_2 < a_2$, and $const_b < const_a$ etc. The maximal vector problem corresponds to other computational geometry problems such as the ‘Pareto set’ problem and the ‘skyline’ problem [22]. A graphical representation of the maximal vector subset of a point dataset is given in Fig.3 below.

Fig.3. Maximal vector subset of a 2-D dataset.

The maximal vector subset could be found using a simple search and comparison approach by progressively cycling each point through the set and removing any dominated points.

Those still remaining at the end of the search process would be the non-dominated maximal vector set. The maximal vector set will thus tend to consist of constraints with generally larger than average coefficients. The large amount of load flow security constraints in a practical sized power system problem would make this naïve search very time-consuming however. A much more efficient method to find the maximal vectors (particularly in high dimensional datasets i.e. constraint sets with many optimization variables) is to intelligently preprocess the point dataset with an ordering scheme, as discussed in [23]. Godfrey et al in [23] used a data vector coefficient logarithmic sum to order the dataset points prior to the search and comparison process. Data-points with a group of relatively large coefficients will also generally have a larger than average entropy sum of their coefficients, and if these are swept through the constraint matrix first of all (i.e. in order of their sum value) then they will initially remove proportionally more constraints than a purely naïve search. In contrast this paper uses the product of the ‘rank’ (i.e. its ordered position with respect to the other constraint coefficients for that optimization variable (a term unrelated to linear algebra matrix rank)) of each constraint coefficient to determine the ordered position of the data-point p as in (13). This is a superior preprocessing scheme as coefficient ranks are a more natural measure of each data-point’s dominance in the overall dataset.

$$order_p \propto \prod_{d=1}^{k+1} rank_{p_d} \quad (13)$$

Constraints are swept through the total constraint set in order of this rank product, creating an efficient preprocessing scheme with no associated simplification error. The constraints with the highest coefficient rank products will quickly remove a large number of redundant constraints, but progress will slow down as constraints with lower overall rank product are tested for dominance. However the total time to carry out the optimization is a sum of the preprocessing and optimization solver times, so thus it is wise to have a stopping criterion in the preprocessing stage, i.e. when the remaining number of constraints is less than a defined number that the LP solver can efficiently handle.

III. TEST-SYSTEM

Similar to [9], the test system used for the illustration of this methodology is a modified version of the IEEE 14-bus test system [24], as depicted in Fig.4. It was desired to integrate sufficient wind capacity to satisfy a 15% (i.e. $\delta_e = 0.15$) wind energy penetration target, while respecting ‘N-G-1’ security limitations in the transmission system. The network topology of [24] was chosen while recorded load and wind time series data, and existing conventional plant data were taken from the Irish power system. Details of the conventional plant, the maximum load values and the wind farm capacity factors at each bus, as well as the assumed line flow capacity limits, are contained in the Appendix section. The parameters of the conventional plant unit commitment characteristics assumed are based on those available from [25]. The parameters and

ratios of the conventional generation plants are representative of the mixture of base-load, mid-merit and peaking plant found on a typical power system. The total conventional generation capacity was 2256.1 MW. Wind generation connection to buses 2, 3, 5, 9, 12, 13, and 14 was assumed possible.

The recorded year-long load time series’ (of hourly resolution) were arbitrarily scaled to suit the test system, while preserving the statistical quality of typical load patterns. The peak system load for the test system was 1861 MW. The average load was 1025 MW. The assumed line capacities were chosen based on the zero-wind penetration scenario PLF results. It was assumed, for system inertial and frequency stability reasons, [26], that an arbitrary minimum level of 450 MW of load had to be served by conventional plant at all times (the value chosen to ensure that at least 2 conventional plants were online at any time). This lead to the occasional curtailment of wind for system security reasons at particularly low load levels, however the low frequency of occurrence of this event was observed to have a negligible impact on the desired 15% energy penetration level. The recorded wind time series were normalized to 1 MW capacity to be scaled by the outputs of the subsequent optimal wind allocation process. The recorded wind time series were found to have varying capacity factors. High capacity factor time series are located on buses 3, 5, 9, and 12. Lower capacity factor wind farms are placed on buses 2, 13, and 14.

Fig. 4. The modified IEEE 14-bus test system schematic.

IV. RESULTS

The unit commitment stage was carried out using the commercial software tool Plexos, version 4.907 [27]. The optimal scheduling task for one year of 24 hour segments using the ‘rounded relaxation’ method took approximately 13 minutes for the test system described above, implemented on a 3.6GHz Pentium dual-core driven, 4 GB RAM enabled Dell Optiplex GX620 desktop PC. The MATLAB [28] software environment was used to implement the constraint redundancy preprocessing stage, trimming the original $\sim 15 \times 10^6$ possible line flow constraints to a user-defined stopping criterion of 2.5×10^6 in approximately 14 minutes. When the constraint

redundancy analysis was allowed to generate the minimal set of non-dominated maximal vectors only $\sim 30 \times 10^3$ constraints remained, though the computation time tradeoff of 56.4 minutes was substantial. A graph of the increasing order of rank-product values can be seen in Fig.5. As illustrated, relatively few constraints will dominate most of the scenario sets, and therein lies the efficiency of the ordered rank-product redundancy preprocessing step.

Fig. 5. A graph of ordered rank-product values.

The LP optimization solver employed was the MATLAB ‘linprog’ ‘medium-scale’ interior point algorithm. This found the optimal solution to the reduced problem in approximately 19.3 seconds (the minimal set of $\sim 30 \times 10^3$ constraints was solved in 0.45 seconds in comparison). Once the initial output capacities were determined the entire process was re-iterated a number of times until the capacity allocation settled to an optimal, secure, and balanced solution. A table of capacity allocations to each wind farm bus in each of the iterations is included below in Table I. The inconsistency of the total system wind power output trend before and after each of the iterations is shown in Fig. 6 below. As this would correspond to an erroneous reference/slack bus power injection to balance the power system, the reiteration is carried out to reduce it to a negligible level in the converged capacity allocation. With a larger wind farm data set than 7 possible farms in a realistic-sized power system, the initial smoothened wind output model assumption would be more accurate and the presence of any balancing error with respect to the total system behaviour would likely be less significant. The probabilistic load flow of the converged capacity allocation values was subsequently investigated. As the edge of the line flow distribution of line from bus 4 to bus 5 under contingency in branch from bus 5 to bus 6 corresponds to its capacity of 530 MW, as illustrated in Fig.7, it is known that this line is a binding constraint on the wind capacity allocation process.

TABLE I
OPTIMAL WIND CAPACITY ALLOCATION (MW)

Iteration Number	Wind Farm Bus Number						
	2	3	5	9	12	13	14
1	0	151.8	128	142.9	56.8	0	0
2	0	162.1	166.7	142.9	0	0	0
3	0	175.7	153.6	142.9	0	0	0
4	0	173.7	155.5	142.9	0	0	0

Fig. 6. Histogram of power imbalance between iterations – converges to negligible value between iterations 3-4.

Fig. 7. Probabilistic load flow distribution for branch 4-5 under contingency in branch 5-6.

V. DISCUSSION

The algorithm presented in this paper allocates wind capacity with respect to the quality of the wind resource and the capability of the transmission network in each area. It is based on representative time series data of 1 year’s length. As the algorithm is related to the identification of worst case scenarios at the edge of line flow distributions, then it is possible that 1 year of wind data may be statistically unreliable. As the importance of sequential probabilistic load flow was emphasised in [9], sequential wind data-growing techniques such as in [29] could be used to give more consistent results. Any wind data growing technique for network flow studies should be able to generate separate time series’ preserving the correct cross correlative trends between the individual wind sites (for load flow) as well as the auto correlative trend of the total system wind power output (for system unit commitment).

The LP constraint redundancy algorithm applied here efficiently reduced the original security constraint set by a user-specified criterion of approximately 85%. Though the test system applied here was of a small size, a methodology based on efficient pre-processing such as this is applicable in larger more realistic power systems also. Similar or more attractive dimensionality reduction should be readily achievable in larger

power systems, and parallelized pre-processing algorithms could conceivably be used for larger initial constraint sets. A wide range of preprocessing algorithms are available in the computational geometry and optimization literature, and the most attractive method can be applied given the nature of each problem. MATLAB was the software environment chosen in this paper for ease of code development given the simple nature of the example, but a more efficient programming language such as C/C++ or others could be applied in practical-scale problems to drastically reduce the computation time required for preprocessing. Commercial optimization programs such as CPLEX report that they can handle ‘millions’ of constraints, and also use preprocessing steps to increase solver efficiency [30]. However the constraint set for this type of problem in even a reasonably-sized power system could easily run to the order of billions, and therefore some prudent user preprocessing such as in this paper would likely be required even before using such advanced commercially available solvers for the reduced problem.

The algorithm presented here is designed to allocate capacity allocation of wind power under security constraints in the power system. However the proportion of time that a wind farm is near maximum output level is relatively small compared to a base-load or mid-merit conventional generation plant. A post-optimal analysis of the linear program applied in Section II suggested that only a handful of the original $\sim 15 \times 10^6$ security constraint scenarios were actually binding on the optimal output solution. Thus the wind plant operation in a few hours during the year (under the assumption that a line or generation contingency could occur, the probability of which is already small) determined the overall capacity allocated. Given the low capacity credit nature of wind power [31], it may be more prudent in transmission access policy to define the problem and methods used to solve it as the integration wind energy instead of wind capacity to networks by considering the occasional curtailment of a wind farm if required during specific onerous events. More wind capacity could be connected to the same network if the transmission system were viewed as a wind energy harvesting device as opposed to a wind capacity facilitation system.

The methodology described in this paper is somewhat related to the ‘composite’ system planning concepts of the traditional vertically integrated utility (VIU), where generation and transmission were planned in tandem. In contrast, most modern power systems now operate based on a ‘de-regulated’ market-oriented paradigm. However the large volumes of wind power plants seeking to connect and their stochastic power production interdependence, and the present delays in transmission expansion implementation may require a return to integrated system planning methods to some extent. Regulatory policy could be one way to ensure that any development undertaken by individual market participants is aligned to the overall objective of optimal evolution of the power system for all parties concerned [32].

VI. CONCLUSION

This paper presents a methodology to optimally allocate wind capacity in a collective manner to power transmission system networks considering both the quality of the wind resource and the real-power transmission system capability. This paper provides a framework to cater for the probabilistic nature of power transmission system operation with respect to security criteria with high wind penetration, and defines the optimal basis for detailed further study of other critical system operation parameters such as voltage stability etc. The methodology combines a sequential probabilistic load flow approach and a computational geometry based linear programming constraint redundancy preprocessing method to achieve this. Results are presented for a simple 14 bus test system, but the efficiency of the method suggests that it should be easily applicable to much larger power systems also. Post-optimal analysis suggests that future methods should have wind energy integration at the kernel of their approach, as power system planning worst-case analyses associated with capacity integration may be inefficient for wind.

VII. APPENDIX

TABLE II
MAXIMUM BUS LOAD VALUES

Bus Number	Maximum Load (MW)
2	100
3	150
4	200
6	400
9	250
10	250
11	100
12	290
13	250
14	100

TABLE III
WIND FARM TIME SERIES CAPACITY FACTORS

Wind Farm Capacity Factor	Wind Farm Bus Number						
	2	3	5	9	12	13	14
	0.24	0.33	0.34	0.31	0.29	0.22	0.25

TABLE IV
CONVENTIONAL PLANT INFORMATION

Bus Number	Plant Description	
	Capacity (MW)	Fuel
1	2*286	Coal
2	1*400	Gas (CCGT)
3	7*95	Gas (OCGT)
4	1* 90, 1*117.6	Peat
10	3*109.5	Oil
13	1*83	CHP

TABLE V
TRANSMISSION LINE ACTIVE POWER CAPACITY LIMITS

From Bus	To Bus	Capacity (MW)
1	2	800
1	5	800
2	3	550
2	4	480
2	5	500
3	4	600
4	5	530
4	7	785
4	9	460
5	6	850
6	11	500
6	12	300
6	13	340
7	8	100
7	9	760
9	10	570
9	14	460
10	11	580
12	13	300
13	14	350

VIII. ACKNOWLEDGMENT

The authors thank Mr. Mark Norton, Mr. Paul Smith and others with the Irish TSO Eirgrid, for their timely co-operation and assistance in the supply of time series data.

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X. BIOGRAPHIES

Daniel Burke (M 2007) graduated from University College Cork, Ireland with a BEEE in Electrical and Electronic Engineering in 2006. He is currently conducting research pursuant to a PhD in power systems at the Electricity Research Centre in University College Dublin, Dublin Ireland. He is a postgraduate student member of the IEEE.

Mark O'Malley (F'07) received B.E. and Ph. D. degrees from University College Dublin in 1983 and 1987, respectively. He is the professor of Electrical Engineering in University College Dublin and is director of the Electricity Research Centre with research interests in power systems, grid integration of renewable energy, control theory and biomedical engineering. He is a fellow of the IEEE.