Abstract—Wind power generation is the fastest growing renewable technology worldwide with large on- and off-shore wind farms being connected to the transmission networks. A significant share of it, however, is still being deployed at distribution levels. While distributed wind generation presents traditionally passive distribution networks with well-established technical challenges, reactive support needed by high penetrations of such new generation capacity might also have an impact on the weak areas of the transmission grid. In this work, a multi-period AC optimal power flow-based technique is proposed to find power factor and substation settings that minimise the transmission reactive support required by variable distributed generation while also considering N-1 contingencies. A section of Irish distribution network is analysed. Results show the significant benefits that a passive approach such as the use of optimal power factor and substation settings can achieve.

Index Terms—Distributed generation, optimal power flow, distribution networks, N-1 contingencies.

I. INTRODUCTION

In the last decade, environmental and fuel security concerns have been translated into targets set by governments to diversify their energy mixes. In 2007, European leaders signed up to an EU-wide target where 20% of their overall energy needs have to be sourced from renewable energy sources by 2020. Thus, incentives combined with technology advances have positioned wind power as the fastest growing renewable technology connecting to both transmission and distribution networks. However, while high penetration levels of wind power generation connected at distribution levels are yet to be seen, such volumes may affect the interactions between the distribution and transmission networks. With distributed generation (DG) typically operating at fixed power factors, they might always or partly draw reactive power depending on the (statutory) generation requirements (e.g., [11]). The aggregated effect of such absorption combined with the (weak) VAr support capability of the local transmission grid might result in operation difficulties [2, 3].

This work explores the use of optimal settings for the power factor of DG units as well as the substation on-load tap changer (OLTC) in order to minimise the reactive power imports from the regional transmission system while maintaining thermal and voltage constraints within limits under normal and contingency operations. AC Optimal Power Flow (OPF) is adapted to cater for this and also to take into account the variability of both demand and wind power generation through a multi-period approach based on demand/generation scenarios.

This paper is structured as follows: Section II presents the mathematical formulation of the problem, considering N-1 contingencies and the optimal operation of DG units and the substation OLTC. In section III, the methodology is applied to a part of a typical Irish distribution network. Section IV concludes the work.

II. PROBLEM FORMULATION

The well-known AC Optimal Power Flow [4], formulated as a non-linear programming (NLP) problem, can be adapted to have different objectives and constraints according to the study being carried out. Here, the AC OPF formulation, where thermal and voltage constraints are typically taken account of, is tailored to cater for the variability of both demand and generation (multiple periods or demand/generation scenarios, see Appendix A) [5], as well as for N-1 contingencies (e.g., loss of a line) [6]. The latter, also known as Security Constrained OPF (SCOPF), ensures that no limits are exceeded even during contingencies that might occur in the system. In practical terms, multiple network topologies are simultaneously analysed.

The objective function of this tailored AC OPF will be the minimisation of the total reactive power import (VArh) through the Grid Supply Point (GSP). The multi-periodicity, in terms of demand/generation scenarios and multiple topologies due to N-1 contingencies, is achieved by providing each...
scenario, \( m \), and topology, \( k \), with a different set of power flow variables whereas a unique, \( \text{inter-period} \) set of variables for the power angles and the voltage at the substation is used throughout the analysis. This is shown schematically in Fig. 1.

![Multi-period interdependency](image)

For a given DG installed capacity, the objective function is calculated by considering the main network topology (normal operation), \( k_{\text{MAIN}} \), and multiplying the GSP reactive power draw of each period \( m \) by its corresponding duration, \( \tau_m \), as follows:

\[
\text{min} \sum_{b \in B} g_{\text{GSP},b_{\text{MAIN}},b} \tau_m \quad (1)
\]

subject to the typical AC OPF constraints across all periods \((M)\) and topologies \((K)\), i.e., \( \forall m \in M \) and \( \forall k \in K \). Voltages at bus \( b \) (set of buses) are constrained by max/min levels

\[
V_{b}^\text{min} \leq V_{b,m,k} \leq V_{b}^\text{max} \quad \forall b \in B \quad (2)
\]

Power flows at each end of lines and transformers, \( l \) (\( L \), set of lines):

\[
\left(f^{(1,2),l}_{1,m,k} + f^{(1,2),l}_{2,m,k}\right) = \left(f^{+}\right)^2 \quad \forall l \in L \quad (3)
\]

where \( f^{(1,2),l}_{1,m,k} \) and \( f^{(1,2),l}_{2,m,k} \) are the active and reactive power injections at each end of the branch (denoted 1 and 2) and \( f^{+} \) is the apparent power flow limit on the branch. Active and reactive power injections into each end of the lines are governed by Kirchhoff’s voltage law:

\[
f^{(1,2),l}_{1,m,k} = f^{KVL}_{1,l,1}(V_{m,k},\delta_{m,k}) \quad \forall l \in L \quad (4)
\]

where \( f^{KVL}_{1,l,1}(V_{m,k},\delta_{m,k}) \) and \( f^{KVL}_{1,l,2}(V_{m,k},\delta_{m,k}) \) are standard Kirchhoff voltage law expressions. With tap changing transformers (OLTCs) and voltage regulators, the appropriate terms in (4) for the voltage at the start bus of the line must be divided by the tap ratio \( t_{m,k} \) which is limited to the range

\[
t_{m,k} \leq t_{m,k} \leq t_{m,k}^\text{max} \quad \forall b \in B \quad (5)
\]

Kirchhoff’s current law describes the active and reactive nodal power balance, \( \forall b \in B \):

\[
\sum_{b \in B} p_{b,m,k} + d^{\text{gen}}_m \eta_m = \sum_{x \in G} p_{x,b} \theta_x + \sum_{m \in \Phi} q_{x,m,k} \eta_m \quad (6)
\]

\[
\text{Here } (p,q)_{b,m,k}^L \text{ are the total power injections onto lines at } b, \text{ and } d^{\text{gen}}_m \text{ are the peak active or reactive demands at the same bus. In period } m, \eta_m \text{ is the demand level relative to peak and } \theta_x \text{ is the generation level relative to nominal capacity as dictated by the intermittent (renewable) resource in that period. If required, the reactive power line injections (6) can be adapted to include shunt capacitance.}
\]

The distribution network has external connections at the GSP substation and might also have interconnectors. Here it is assumed that both can export power so the import/export constraints at the GSP or interconnector \( x \), (set of external sources), are:

\[
p_m^\text{L} \leq p_x \leq p_m^\text{L} \quad \forall x \in X \quad (7)
\]

The GSP is taken as the reference bus \( b_{\text{GSP}} \), with the voltage angle set at zero, i.e., \( \delta_{b_{\text{GSP}}} = 0 \).

A. Optimal Operation of DG units

The considered DG units will have their nominal active power capacities constant, i.e., \( p_m = p_m^\text{const} \). However, while they will typically operate at constant power factors (e.g., \( \cos(\phi_g) = 0.95 \), inductive/capacitive), here the power angle of each generator, \( \phi_g \), will become a variable making it possible to evaluate the optimal power factor that minimises the reactive power drawn over time (1). In practice DG will be required to operate within a certain range of power factors \( (\phi_g^\text{min} - \phi_g^\text{max}) \); the following constraint applies:

\[
\phi_g^\text{min} \leq \phi_g \leq \phi_g^\text{max} \quad (8)
\]

B. Optimal Operation of the OLTC

Through the use of OLTCs, DNOs set the target voltage at the secondary side of the substations to values that are able to cope with the variability of demand in order to keep voltage profiles within statutory limits. This practice, while satisfactory, does not necessarily help minimising (1). Here, the voltage at the substation secondary, \( V_{h} \), is considered as a variable whose final value is constant across all periods and network topologies.

\[
V_{h,m,k} = V_{h} \quad (9)
\]

III. CASE STUDY

In this section the AC OPF-based technique is applied to a section of the Irish distribution network. Firstly the minimum demand-maximum generation scenario is analysed. Then, the variability of both the demand and generation is incorporated in the study. The method was coded in the AIMMS

A. 5-bus Irish Network

The one-line diagram of a typical rural section of the Irish 38kV distribution network is shown in Fig. 2. Corresponding line data in p.u. (S_pu=100MVAr) is included in the Appendix B (Table III). The feeders are supplied by one 31.5MVA 110/38kV transformer. GSP voltage is assumed to be nominal. In the original configuration (no DG), the OLTC at the substation has a target voltage of 1.078pu (41kV) at the busbar. Voltage limits are taken to be ±10% of nominal. The maximum demand of the network is 15.12MW.

In order to investigate whether a diverse number of sources and locations will require different power factor settings, five wind power plants are connected to the network: a 5MW unit each at buses A to C and 3MW generators at buses D and E. Each is capable of providing 0.90 inductive/capacitive power factor. Thus, the total installed capacity accounts for 21MW, exceeding the total peak demand by almost 40%. In a business as usual scenario all these generators are assumed to be operated at 0.95 inductive power factor (i.e., absorbing reactive power) to limit voltage rise.

The main network topology, k_main, is the one with line A-S open, as presented in Fig. 2. The N-1 contingencies considered are the outages of lines Tx-A and Tx-S, as shown in Fig. 3. Half-hourly data for demand and two wind generation profiles (one for each feeder) are broken into a series of bins: 10 ranges for demand (e.g., [0,10%], (10%,20%],...) and 11 ranges for each generation profile (e.g., [0]), (0,10%), (10%,20%),...) are used. The process is illustrated in Appendix A.

![Fig. 2. 38kV network during maximum load conditions. Five wind power sites are considered. Two different wind areas are adopted for the two feeders.](image)

![Fig. 3. N-1 topologies considered. Dashed lines indicate outages.](image)

B. Min Demand – Max Generation

Typically, when optimising the size or the operation of distributed generators the worst case scenario is considered. To illustrate this, the optimal settings for the power factor of the generators and the target voltage at the busbar are investigated considering only the minimum demand/maximum generation scenario, in which voltage rise will become binding. Fig. 4 shows the reactive power (MVAr) imports or exports resulting in each of the cases analysed. Table I presents the corresponding optimal settings.

The business as usual (BAU) case, where 0.95 inductive power factor is adopted by all the generators and the original target voltage at the bus is maintained, is the one requiring the largest amount of reactive power from the transmission grid. In fact, the reactive support needed by the network in this case is almost five times that without any generation (2.2MVAr, 40% of peak load). Solely considering the optimisation of independent power factors (pf case) the proposed technique was able to find a better (although mostly inductive) settings that resulted in much less reactive power being imported. With more flexibility, i.e., incorporating the OLTC target voltage (pf+V case), the generators are able to make the network a net MVAr exporter. It is also noticeable that when the N-1 contingencies are considered, voltage restrictions make the results less optimistic (particularly for the pf case).

![Fig. 4. Minimum Demand–Maximum Generation Scenario: MVAr imports or exports resulting from the cases considering optimal settings for the power factors (pf) and the target voltage at the OLTC (+V), as well as N-1 constraints.](image)

**Table I**

<table>
<thead>
<tr>
<th>DG unit</th>
<th>Optimal Power Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>only ( k_{\text{MAIN}} )</td>
</tr>
<tr>
<td></td>
<td>pf</td>
</tr>
<tr>
<td>A</td>
<td>0.900 (c)</td>
</tr>
<tr>
<td>B</td>
<td>0.997 (l)</td>
</tr>
<tr>
<td>C</td>
<td>0.945 (l)</td>
</tr>
<tr>
<td>D</td>
<td>0.963 (l)</td>
</tr>
<tr>
<td>E</td>
<td>0.987 (l)</td>
</tr>
</tbody>
</table>

C. Variable Demand and Generation

While the previous analysis illustrates the ability of the proposed methodology to find the optimal settings that minimise the reactive support needed by DG units, the findings are not applicable given the inherent variability of demand. Renewable sources of generation such as wind power certainly make the analysis even more complex. Fig. 5 shows the net annual GVarh imports or exports resulting in each of the cases analysed considering the variability of demand and (wind power) generation. Table II presents the corresponding optimal settings.
Without DG units the network has annual reactive power imports of 36.9GVArh. This figure goes up to 60.8GVArh when all the wind farms operate at 0.95 inductive power factor (BAU case, Fig. 5). If the optimal, independent, power factor settings are applied on their own (pf), the net annual imports decrease by 40% relative to the BAU case for normal network operation ($k_{\text{MAIN}}$), and by 30% with N-1 contingencies being considered. It can also be noted that, in both cases, only generator A resulted in a capacitive power factor. In fact, the resulting new power factors are mainly driven by voltage (rise) constraints produced by the reverse active power flows. Hence, while it is possible for generator A (electrically close to the substation) to have a capacitive behaviour, the other DG units have to remain with inductive power factors, albeit typically improved from the business as usual case..

Incorporating the target voltage at the OLTC as a variable into the formulation (pf+V case) allows, as expected, the required reactive power support from the GSP to decrease further. With an intact network ($k_{\text{MAIN}}$), net annual reactive exports of 3.8GVArh are possible. However, including N-1 contingencies this figure can ‘only’ be reduced to 10.5GVArh imported, i.e., less than a quarter of the BAU requirements. In terms of the resulting power factors, the lower values for the target voltage at busbar, make capacitive behaviour viable (although the security constrained case requires generator C to be inductive).

V. APPENDIX

A. Demand/generation scenarios

The time-varying characteristics of demand and (wind) generation over a given time period are considered by using aggregated output/demand scenarios instead of the full output/demand time. Thus, time-series data (e.g., hourly) is reduced to a set of demand/generation scenarios where their joint probability defines the number of coincident hours (i.e., time duration) over the year. Fig. 6 presents an example of how these multiple demand/generation scenarios are considered. This procedure is extended in the similar manner to cater for more than one resource or generation profile [5].

B. Line and Transformer Data

Table III presents the line and transformer data for the 38kV 5-bus network (Fig. 2).
### TABLE III
LINE AND TRANSFORMER PARAMETERS FOR THE 38kV 5-BUS NETWORK

<table>
<thead>
<tr>
<th>Line</th>
<th>R</th>
<th>X</th>
<th>Smax</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSP - Tx</td>
<td>0.2500</td>
<td>0.3150</td>
<td>0.1975</td>
</tr>
<tr>
<td>Tx - A</td>
<td>0.0296</td>
<td>0.0863</td>
<td>0.3817</td>
</tr>
<tr>
<td>A - B</td>
<td>0.5841</td>
<td>0.6244</td>
<td>0.1975</td>
</tr>
<tr>
<td>B - C</td>
<td>0.3875</td>
<td>0.4072</td>
<td>0.1975</td>
</tr>
<tr>
<td>Tx - S</td>
<td>0.0669</td>
<td>0.0800</td>
<td>0.3817</td>
</tr>
<tr>
<td>S - D</td>
<td>1.0591</td>
<td>1.1130</td>
<td>0.1975</td>
</tr>
<tr>
<td>D - E</td>
<td>0.1550</td>
<td>0.1629</td>
<td>0.1975</td>
</tr>
</tbody>
</table>

### REFERENCES


### BIOGRAPHIES

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