Source Separation on Seismic Data

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Abstract—This article gives a brief description of the Degenerate Unmixing Estimation Technique (DUET) and applies it in a geophysical setting. Source separation has not been fully addressed by geophysicists and is a crucial first step to locating simultaneous sources, which in turn helps with understanding the dynamics of the sources and their source mechanisms. DUET is applied to synthetic seismic signals. The source separation method works successfully to separate two contemporary explosive sources, and two simultaneous oblique tensile cracks in a 3D structural model of Mt Etna. The method is also applied to field recordings on Mt Etna from 2008. The method separates Long Period events from tremor, Long Period events from Volcano Tectonic events and different sources of tremor from each other.

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I. INTRODUCTION

There are many different types of seismicity in geophysical sciences, and we are mainly interested in the two most frequent ones:

- Rapid stress drop earthquakes caused by brittle material failure. These events are characterized by short transient signals, usually occur in isolation and are identified by clear P-wave (longitudinal body wave) onsets.
- Continuous tremor events, often with emergent onsets. These tremor events can be found in two distinct environments: on volcanoes and in subduction zones.

The origins of the latter events are still poorly understood (and may not be unique) but multiple sources are known to be simultaneously active. Separating these events is a challenge which has not previously been fully addressed by conventional geophysical analysis.

This article treats the issue of multiple sources as a blind source separation problem. The DUET [1] method is used in this article to separate sources. The technique uses a pair of sensors, separated by less than half a wavelength of the signal. This method was designed for the separation of speech in a mixture of several speakers, and the medium in which the sound is traveling is assumed to be air.

In seismology, we deal with two types of body waves (longitudinal (P) and transverse (S)) with different velocities and radiation patterns, as well as with surface waves arising due to the interaction of the body waves wavefield with the free surface. The medium through which the waves propagate is usually much more heterogeneous than air, for which this method was originally developed. Nevertheless, we test the application of the method to seismology, with promising results. Any progress made in the ability to separate seismic signals, especially on volcanoes where many phenomena occur at the same time, is of great importance in gaining a better understanding of the underlying physical processes.

II. THE DUET METHOD

DUET was designed for the blind source separation of speech signals, by taking advantage of the sparsity of speech in the time-frequency representation of sources [2]–[6]. Generally, recordings of speech are sparse in the time-frequency domain, even when two or more people are speaking simultaneously. This is because each person usually has a different pitch (frequency). For example, female speakers would have a higher pitch than male speakers. Thus at most one source (speaker) would be dominant at each time-frequency point in the time-frequency representation of speech recordings.

The technique works well when there is little or no overlap between the two sources in the time-frequency domain. Seismic sources inside volcanoes can also exhibit sparsity in the time-frequency domain, and this property allows DUET to successfully separate seismic signals. The assumptions made and the algorithm are described in the following section.

A. Assumptions

1) Anechoic mixing: Consider a mixture of N source signals, \( s_j(t), j = 1, \ldots, N \), is received at a pair of sensors where only the direct path is present. In this case, without loss of generality, we can absorb the attenuation and delay parameters of the first mixture, \( x_1(t) \), into the definition of the sources. As such, the two anechoic mixtures can be expressed as,

\[
x_1(t) = \sum_{j=1}^{N} s_j(t),
\]

\[
x_2(t) = \sum_{j=1}^{N} a_j s_j(t - \delta_j),
\]

where \( N \) is the number of sources, \( \delta_j \) is the arrival delay between the sensors, and \( a_j \) is a relative attenuation factor corresponding to the ratio of the attenuations of the paths between sources and sensors. The anechoic mixing model is not realistic, in that it does not represent echoes, (that is, multiple paths from each source to each mixture). In spite of this limitation, however, the DUET method, which is based on this model has proven to be quite robust even when applied to echoic mixtures [7].

2) W-Disjoint Orthogonality: We call two functions \( s_j(t) \) and \( s_k(t) \) W-disjoint orthogonal if, for a given windowing function \( W(t) \), the supports of the windowed Fourier transforms of \( s_j(t) \) and \( s_k(t) \) are disjoint. That is, at each point in the windowed Fourier transform, at most one source (function \( s_j(t) \) or \( s_k(t) \)) is dominant. The windowed Fourier transform

\[
W(t) \sum_{k=1}^{N} a_k s_k(t - \delta_k) W(t - \delta_k)
\]
of $s_j(t)$ is defined,
\[ \hat{s}_j(\tau, \omega) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(t-\tau) s_j(t)e^{-i\omega t} dt. \] (3)
So the W-disjoint orthogonality assumption can be stated as:
\[ \hat{s}_j(\tau, \omega) \hat{s}_k(\tau, \omega) = 0, \forall \tau, \omega, \forall j \neq k. \] (4)
This assumption is the mathematical idealization of the condition, that it is likely that every time-frequency point in the mixture with significant energy, is dominated by the contribution of one source. Note that, if $W(t) \equiv 1$, $\hat{s}_1(\tau, \omega)$ becomes the Fourier transform of $s_1(t)$, which we will denote $\hat{s}_1(\omega)$. In this case, W-disjoint orthogonality can be expressed as:
\[ \hat{s}_j(\omega) \hat{s}_k(\omega) = 0, \forall j \neq k, \forall \omega, \] (5)
which we call disjoint orthogonality.
W-disjoint orthogonality is crucial to DUET, because it allows for the separation of a mixture into its component sources using a binary mask. Consider the mask which is the indicator function for the support of $\hat{s}_j$:
\[ M_j(\tau, \omega) := \begin{cases} 1 & \text{if } \hat{s}_j(\tau, \omega) \neq 0 \\ 0 & \text{otherwise} \end{cases} \] (6)
$M_j$ separates $\hat{s}_j$ from the mixture via
\[ \hat{x}_j(\tau, \omega) = M_j(\tau, \omega) \hat{x}_1(\tau, \omega), \forall \tau, \omega \] (7)
where $\hat{x}_1(\tau, \omega)$ is the time-frequency representation of $x_1$. As such, if we could determine the masks which are the indicator functions for each source, we can separate the sources by partitioning. The question is, how do we determine the masks? As we will see shortly, the answer involves labeling each time-frequency point with delay differences that explain the time-frequency phase between the two mixtures, and these delays cluster into groups, one group for each source.

B. Algorithm
The assumptions of anechoic mixing allows us to rewrite the mixing Equations (1) and (2) in the time-frequency domain as,
\[ \begin{bmatrix} \hat{x}_1(\tau, \omega) \\ \hat{x}_2(\tau, \omega) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 \\ a_1 e^{-i\omega \delta_1} & a_2 e^{-i\omega \delta_2} & \cdots & a_N e^{-i\omega \delta_N} \end{bmatrix} \begin{bmatrix} \hat{s}_1(\tau, \omega) \\ \hat{s}_2(\tau, \omega) \\ \vdots \\ \hat{s}_N(\tau, \omega) \end{bmatrix}. \] (8)
If we include the further assumption of W-disjoint orthogonality, at most one source is active at every $(\tau, \omega)$, and the mixing process can be described for each $(\tau, \omega)$ as:
\[ \begin{bmatrix} \hat{x}_1(\tau, \omega) \\ \hat{x}_2(\tau, \omega) \end{bmatrix} = \begin{bmatrix} 1 \\ a_j e^{-i\omega \delta_j} \end{bmatrix} \begin{bmatrix} \hat{s}_1(\tau, \omega) \\ \hat{s}_j(\tau, \omega) \end{bmatrix} \] (9)
for a given $j$. In the above equation, $j$ depends on $(\tau, \omega)$, in that $j$ is the index of the source active at $(\tau, \omega)$. The main observation that DUET utilizes, is that the ratio of the time-frequency representations of the mixtures does not depend on the source components, but only on the mixing parameters associated with the active source component.
\[ \forall (\tau, \omega) \in \Omega_1, \frac{\hat{x}_2(\tau, \omega)}{\hat{x}_1(\tau, \omega)} = a_j e^{-i\omega \delta_j}. \] (10)
Here, $\Omega := ((\tau, \omega) : \hat{s}_j(\tau, \omega) \neq 0$. In this paper, we only use the relative delay estimates and ignore the relative attenuation estimates. This is done because the amplitudes recorded by sensors are sensitive to the instrument response, thus the relative attenuation estimates are not as reliable and accurate as the relative time delay estimates. The mixing parameters (the relative delay estimate in this case) associated with each time-frequency point can be calculated:
\[ \delta(\tau, \omega) = \frac{1}{\omega} \arctan \left( \frac{\hat{x}_2(\tau, \omega)}{\hat{x}_1(\tau, \omega)} \right). \] (11)
The steps taken to separate the sources using DUET are:

1) Construct the discrete Short-Time Fourier Transform as defined in Equation (3) of the signal at each sensor. In this paper, the window used is a Hamming window.
2) Take the ratio of the mixtures to extract the local delay estimate. A delay estimate will result for each time-frequency point.
3) Generate the histogram of these delays, with each delay bin weighted with the energy of the corresponding time-frequency point.
4) Find the peak delay for each source. The peak delays are found using a weighted k-means clustering variant of the k-means clustering technique, for peak tracking.

The k-means algorithm is a classic iterative technique employed in data clustering problems. The algorithm efficiently partitions the points of a data matrix into k clusters. It achieves this by minimizing (in a least mean squares sense) the sum of distances from each point to its nearest cluster center. Each iteration starts by reassigning points to their nearest cluster center. Each cluster center is then recalculated as the mean of all points which have been assigned to it. This process is repeated until the cluster centers converge according to the chosen criterion. The use of a histogram space has proved to be very powerful in DUET. Rather than searching for peaks in the entire delay space, the data is placed into a bounded histogram with a finite number of bins. A weighted version of the k-means algorithm is performed on the histogram. The weighted histogram is updated with the powers of the corresponding time-frequency points for each frame of data in the STFT of the signal. The histogram bin centers are passed to k-means and each point is weighted by the height of that histogram bin.

5) For each peak found, i.e. for each source, a binary time-frequency mask is then created, based on how close the delay at each time-frequency point is to the peak delays found.
6) This mask can then be applied to each of the mixtures recorded at each sensor to get the time-frequency representation of the separated source recorded at each sensor.
7) The separated source signals are then transformed, using the inverse Short-Time Fourier Transform, back to the time domain.

The flowchart of the separation algorithm is shown in Figure 1. In the figure, the loop incrementing $\text{TF}_{\text{index}}$ is the loop going through all the $(\tau, \omega)$ pairs.

In this section, the assumptions made and the DUET algorithm were described. The following section describes the results of this method on synthetic data and real data.
In this section, the results from source separation are outlined. DUET works well in separating sources that are sparse in the time-frequency domain [1]. Seismological data are not always W-Disjoint orthogonal in the time-frequency domain. As will be seen in Section III-B, Long Period and Volcano Tectonic events overlap at low frequencies, as do Long Period events and tremor. Unlike Long Period events and tremor though, Volcano Tectonic events and Long Period events hardly ever occur simultaneously, thus satisfying the W-Disjoint orthogonality assumption. This paper focuses on the ability of DUET to separate seismic sources, which do not follow the assumption of an acoustic wave traveling in a homogenous medium, and are not W-Disjoint orthogonal. The synthetic tests are run to verify that DUET can separate sources despite the presence of transverse S-waves in addition to longitudinal P-waves (which is what DUET was originally developed for with acoustic waves). The method is then applied to field recordings.

Both the synthetic data and the real data used are from Mt Etna. Mt Etna is an active 3330m high stratovolcano located on the East coast of Sicily, Italy. An eruptive period began on 10th May 2008 and stopped on July 7th 2009. Before the source separation method was applied to the field recordings taken at Mt Etna in June 2008, it was tested on synthetic data. The results of these tests are described in Section III-A. DUET was then applied to field recordings to separate Long Period events from Volcano Tectonic events, Long Period events from tremor and multiple tremor sources. The results of these experiments are outlined in Section III-B.

A. Results on synthetic data

The DUET algorithm was applied to two different types of sources (two separate source mechanisms):

1) Explosive sources - these are isotropic sources that generate only P-waves. The separation results on these sources are shown in Section III-A1.

2) Oblique tensile cracks - these are non-isotropic sources that generate both P- and S-waves. The separation results on these sources are shown in Section III-A2.

1) Explosive sources: The 3D structural model for Mt Etna used in this instance, was inelastic, and its physical properties varied spatially. The lower regions of the model were constructed from the tomographic model of [12]. As tomography models are known to be poorly constrained near the Earth’s surface, a low P-wave velocity near surface gradient ranging from 1600m/s at the surface to 3000m/s at 500m depth was added to the model. The top surface of the model was defined using the actual Digital Elevation Model of Mt Etna (the topography model of Mt Etna and the sensor configuration is shown in Figure 2(a) and Figure 2(b). The Elastic Lattice Scheme [13] was used to propagate full wavefield viscoelastic waves through the model.

The source mechanisms were purely isotropic, and were located at depths of 280m and 1640m below the summit. The source time function is a repetitive Ricker wavelet, of duration 80s. A Ricker wavelet is a special type of wavelet often used for modeling purposes and is defined by its dominant frequency. It is characterized by a decaying amplitude spectrum around the dominant frequency. The Ricker wavelet is by definition zero-phase and is used because it is simple to understand and often seems to represent a stylized seismic source. An example of a Ricker wavelet with a dominant frequency of 1Hz is shown in Figure 3(a). One of the sources had a single frequency present, at 1Hz. The second source was W-Disjoint orthogonal to the first source in the time-frequency domain (at most one source was dominant at every time-frequency point), but the frequencies in this source overlapped with those in the first source during some samples (1000-2000) as shown in Figure 3(b). The explosive source generated only P-waves (the elastic wave analogy to acoustic waves, although the shear modulus did play a role in controlling its velocity), however S-waves were generated in the model through P- to S-conversions inside the medium, and on the free surface, respectively. Dispersive surface waves were also present in the recorded signals, due to the interaction of P- and S-waves at the volcano surface. White Gaussian noise was added to the signal at each sensor (The variance of the white gaussian noise was 10% of the r.m.s of the variance of the signal).

Fig. 1. Flowchart of the DUET separation algorithm.
In all the experiments covered in this article, only the vertical component of the seismograms are used. Seismographs record data in the vertical component and two horizontal components (North-South and East-West). Traditionally, geophysicists primarily use the vertical component of the seismogram. The horizontal components of the seismogram are sensitive to tilt caused by changes in temperature and wind, compared to the vertical component. DUET was developed to separate acoustic sources recorded by microphones, which only have one component. In addition, by choosing the vertical component, the horizontal transverse surface waves (Love waves) are neglected, thus reducing the number of non-acoustic waves recorded in the signal.

Figure 4 shows the first 40 seconds (this was the duration of the signal analyzed by DUET) of the mixtures at a pair of sensors, and their time-frequency representations. The sampling frequency was 50 Hz. The histogram of delays calculated is shown in Figure 5(a). Two peaks were found to be present, at -4.6205 and 0.0369, using a weighted k-means clustering variant of the k-means clustering technique. A binary time-frequency mask was then created for each source to separate them in the time-frequency domain (shown in Figure 5(b)). Note that the masks are time-frequency dependent and could not be replaced with fixed band-pass filters. Each of these masks is applied to the time-frequency domain of the separated signals in order to synthesize the separated sources back to the time domain. The separated sources are shown in Figure 6. In the time domain, the blue plot is the source separated by the DUET algorithm, and the red plot is the original individual source at that sensor. The metric used to determine the success of the algorithm mathematically was the signal to interference ratio,

\[ SINR_1 = 10 \log_{10} \left( \frac{M_1(\tau, w)\delta_1(\tau, w)}{M_1(\tau, w)\delta_2(\tau, w)} \right), \]

where \( M_1 \) is the binary mask used to separate source 1 (from Equation (7)) and \( s_i(\tau, w) \) is the time-frequency representation of the \( i^{th} \) source. The sources were separated almost perfectly with a signal to interference ratio of 15dB. There was an overlap in the frequencies of the two sources, between samples 1000 and 1500, which results in a slight increase in the interference between the sources in this range of samples.

2) Oblique Tensile Crack Sources: In this case, an infinite homogenous medium was used, with the sensor configuration equivalent to that during the real seismic experiment on Mt Etna in 2008 [14] (shown in Figure 2(b)). Seismic signals were generated analytically using Equation (4.29) in [15]. The seismic sources were oblique tensile cracks and were located at depths of 280m and 1640m below the summit. Both P-waves and S-waves were generated in the model. In this way, we are
investigating the influence of P- and S-waves on the separation algorithm, while the effects of surface waves, free-surface reflections and the scattering of the wavefield on topography are neglected. The source time function was a repetitive Ricker wavelet of duration 80s.

Figure 7 shows the first 46 seconds of the mixtures at a pair of sensors and their time-frequency domain representations. The sampling frequency was 50 Hz. The histogram of delays calculated is shown in Figure 8. Two peaks were found to be at -1.2893 and -0.0681 samples. The separated sources are shown in Figure 9.

The result presented here is for a sensor pair where the separation results were successful. Unlike the case of explosive sources, the separation is not successful for every pair of sensors for tensile crack sources. This is due to the different radiation pattern and propagation velocities of longitudinal P-waves (acoustic equivalent) and shear elastic S-waves. DUET was developed to separate pure acoustic waves, which is not the case with tensile crack sources. The S-waves have higher energy compared to the case of explosive sources, where P-waves dominate. The wave propagation differs from pure acoustic waves, and Equations (1) and (2) no longer hold true.

The sources were separated with a signal to interference ratio of 8dB. The increase in interference compared to the explosive source case could be due to different radiation patterns and propagation velocities of P- and S-waves, respectively, the wavefield properties which differ from a simple acoustic case.

In this section, the results of source separation using DUET on synthetic data were discussed. Section III-B outlines the results of source separation on field recordings from Mt Etna in 2008.

B. Results on field recordings from Mt Etna in 2008

50 sensors were deployed on Mt Etna in close proximity to the summit during the second half of June 2008 (sampling frequency 100Hz). This data set included:

- Eruptive tremor, most likely caused by
  - lava flow on the East flanks, from flank eruption at about 2800m altitude (containing frequencies in the range 2-5 Hz).
  - degassing from the summit craters

- Long Period (LP) events with frequencies in the range 0.3-5 Hz (with peak frequencies in the range of 0.3-1.5Hz), located below the summit crater. A typical LP event is shown in Figure 10(a).

- Volcano Tectonic (VT) events with frequencies in the range 0.3-20Hz. A typical VT event is shown in Figure 10(b).

Figure 2(b) shows the seismic sensors near the summit of Mt Etna. Three results of separation on these recordings are described in Sections III-B1, III-B2 and III-B3.
1) LP and VT: DUET was used to separate an LP event from a VT event. The input data to the algorithm were from sensors ET08 and ET81 (shown in Figure 2(b)), from 21st June 2008 during an LP event, followed by a VT event.

Before applying DUET, a bandpass filter with corner frequencies of 0.3Hz and 1.5Hz was applied to the signals. The unfiltered mixtures are shown in Figure 11 and the mixtures after the band pass filter are displayed in Figure 12. The peak delays were -12.5 and 2.2 samples. The separated signals at ET08 are displayed in Figure 14.

The LP and VT events have been separated with the tremor present in VT events. The source with delay -12.5 samples corresponds to the VT event and tremor. The source with delay 2.2 samples corresponds to the LP event.

The current method of classification of sources is the application of a bandpass filter in the relevant frequency range (0.3-2 Hz for LP events and 2-20Hz for VT events) and picking these events out by eye. Software has been developed for automatic classification of events, but they are not completely satisfactory. The advantage of using DUET in this case is that the low frequencies of the VT event is not lost due to filtering. The low frequency components of VT events help with understanding the source mechanism of the event.

2) LP and tremor: In this case, DUET was used to isolate an LP event from a mixture of the LP event and tremor. The data that the algorithm was applied to were from sensors ET82 and ET83 (shown in Figure 2(b)), from 20th June 2008 during an LP event.

Before applying DUET, a bandpass filter with corner frequencies of 0.3Hz and 10Hz was applied to the signals. The mixtures at the two sensors are shown in Figure 15. The histogram of the delays calculated is shown in Figure 16. The peak delays were -12.1 and 11.2 samples. We also analyzed the
signals associated with the histogram energy located between delays -5 and 7 samples. The signals associated with this area appeared to be echoes of the main events. The separated signals at ET82 are displayed in Figure 17.

It can be seen that the LP event is clearer in the separated source with delay 11.2 samples (the second separated source). The source with delay -12.1 samples appears to be tremor which also contains higher frequencies of the LP event. This leads us to believe that DUET could be used as a tool for identifying LP events and denoising.

3) Tremors: In this instance, DUET was used to separate multiple tremors. This is the most practical application of the algorithm. The input data to the algorithm were from sensors ET08 and ET81 (shown in Figure 2(b)) from 21st June 2008.

Before applying DUET, a bandpass filter with corner frequencies of 0.3Hz and 8Hz was applied to the signals. The mixtures at the two sensors are shown in Figure 18. The histogram of the delays calculated is shown in Figure 19. The peak delays were -9.1 and 8.2 samples. The separated signals at ET08 are displayed in Figure 20.

The two separated sources could be from two different tremor sources, or it could also be a scatterer due to topography. An extension of the separation algorithm could be to localize the separated sources to find the source location, which would help understand the source mechanism causing the tremors. This is the most important potential practical application of this source separation method, and would be very valuable to geophysicists. Separating tremor sources and locating them would lead us to better understand the source mechanisms and the causes of the tremors.

IV. CONCLUSIONS AND FUTURE WORK

In this article, the DUET blind source separation algorithm was applied in a geophysical setting. This algorithm was
that DUET could potentially be a powerful tool for identifying, separating and locating LP events and tremor. This could help geophysicists better understand the source mechanism behind tremors. The natural next step would be to use the DUET and existing array location methods [16], [17], to localize the separated sources.

AUTHORS

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Scott Rickard (Scott.Rickard@ucd.ie) received the S.B. degree in mathematics in 1992, the S.B. degree in computer science and engineering in 1993, and the S.M. degree in electrical engineering and computer science, also in
Fig. 15. (a) Filtered mixtures (0.3Hz-10Hz) of LP and tremor events from ET82 (top) and ET83 (bottom) in the time domain, and (b) Mixtures of LP and tremor events from ET82 (top) and ET83 (bottom) in the Short Time Fourier Transform domain.

Fig. 16. Histogram of delays for the LP event and tremor mixture with peaks at -12.1 and 11.2.

Fig. 17. (a) The Short Time Fourier Transform of the separated signals - tremor (top) and LP (bottom) - at ET82, and (b) Separated sources - tremor (top) and LP (bottom) - in the time domain at ET82.

1993, all from MIT. He received the M.A. and Ph.D. degrees in applied and computational mathematics from Princeton University, Princeton, NJ, in 2000 and 2003, respectively. He is currently an Associate Professor in the School of Electrical, Electronic & Communications Engineering at the University College Dublin, Ireland. His research interests include time-frequency/scale analysis, blind source separation and localization, computational and mathematical finance, time-varying channel models, frequency-hopping and Costas arrays.

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Fig. 18. (a) Filtered mixtures (0.3Hz-8Hz) of tremor events from ET08 (top) and ET81 (bottom) in the time domain, and (b) Mixtures of tremor events from ET08 (top) and ET81 (bottom) in the Short Time Fourier Transform domain.

Fig. 19. Histogram of delays for the multiple tremor mixture with peaks at -9.1 and 8.2.

Fig. 20. (a) The Short Time Fourier Transform of the separated signals at ET08, and (b) Separated sources in the time domain at ET08.