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<th>High accuracy Location Estimation of a Mobile Tag using One-way UWB Signalling</th>
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<tr>
<td>Authors(s)</td>
<td>Saad, Mohamed M.; Bleakley, Chris J.; Walsh, Michael; Ye, Tingcong</td>
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<tr>
<td>Publication date</td>
<td>2012-10-03</td>
</tr>
<tr>
<td>Publication information</td>
<td>Ubiquitous Positioning, Indoor Navigation, and Location Based Service (UPINLBS), 2012 [proceedings]</td>
</tr>
<tr>
<td>Conference details</td>
<td>Ubiquitous Positioning Indoor Navigation and Location Based Service (UPINLBS), Helsinki, Finland, October, 2012</td>
</tr>
<tr>
<td>Publisher</td>
<td>IEEE</td>
</tr>
<tr>
<td>Item record/more information</td>
<td><a href="http://hdl.handle.net/10197/3969">http://hdl.handle.net/10197/3969</a></td>
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<tr>
<td>Publisher's statement</td>
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<tr>
<td>Publisher's version (DOI)</td>
<td>10.1109/UPINLBS.2012.6409769</td>
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High Accuracy Location Estimation for a Mobile Tag using One-way UWB Signaling

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Abstract—This paper presents a novel algorithm for determining the 3D location of a Mobile Tag (MT) using wireless Base Stations (BSs) and Ultra Wide Band (UWB) one-way signaling. The algorithm is designed for low power, rapid deployment applications in which the BSs are independent, wireless UWB transceivers located at known positions and the tags are receive-only or transmit-only UWB mobile units. The algorithm utilizes multi-BS time synchronization and hybrid Time Difference of Arrival-Time of Flight (TDoA-ToF) localization to achieve high accuracy tag localization. The algorithm consists of two concurrent tasks. In task I, the BSs exploit periodic inter-BS transmissions to obtain high accuracy multi-BS time synchronization. In task II, a hybrid TDOA-TOF algorithm is used to determine the location of the MT. The algorithm is based on timestamping, or control, of packet Time of Emission (ToE) and estimation of the packet Time of Arrival (ToA). As such, the method is appropriate for use with, but not limited to, IEEE 802.15.4a UWB. In simulations assuming a typical 2 cm standard deviation in ToA estimation error the proposed algorithm was found to provide a RMS error of 2.1 ps and 4.8 × 10⁻⁶ ppm for time offset and crystal clock offset between BSs respectively; and a RMS error in MT location estimation of 2.2 cm; which is 36% better accuracy than the conventional TDoA method.

Index Terms—UWB localization, TDOA, TOF, Algorithms, time synchronization

I. INTRODUCTION

OBTAINING accurate location information is important for many applications, including navigation tools for humans and robots, building mapping, interactive games, resource discovery, asset tracking and location-aware sensor networking [1], [2], [3], [4], [5], [6], [7]. GPS works well outdoors, but fails in indoor environments due to signal attenuation by the building fabric [8]. Development of a robust, reliable and accurate indoor location system is still an open research problem. Many proposals for location systems have been reported in the literature [9], [10], [11], [12]. Of these, UltraWideBand (UWB) Radio Frequency (RF) location systems show particular promise since UWB allows transmission of very short RF pulses, enabling precise time delay estimation and accurate location estimates [13], [14].

Various algorithms for location estimation based on UWB signals have been discussed in the literature [15], [16], [17], [18]. Time delay based algorithms are the most popular technique and are used in the majority of UWB location systems. These algorithms usually calculate the Time of Flight (TOF) or Time Difference of Arrival (TDOA) of UWB pulses. TOF based methods determine the time taken for the RF signal to propagate between a source and receiver. In order to measure TOF, tight time synchronization between the source and receiver is required. Typically such synchronization is not available for mobile units. Hence most UWB TOF based algorithms rely on measurement of Round-Trip Time (RTT) [19], [20]. The accuracy of RTT ranging is limited by the additive variable time delay arising from processing at the secondary device, clock offset between nodes, and the need for two-way communication. TDOA methods utilize measurement of the difference between the Times of Arrival of an UWB signal from a single source at multiple synchronized receivers, or vice versa. Theoretically four synchronized Base Stations (BSs) are sufficient for TDOA-based location estimation, but, in practise, more BSs are used to mitigate for noise in the TDOA measurements [21]. TDOA location estimation algorithms previously described in the literature include a closed-form solution [22], error-minimizing methods [23], [24], and Chan’s method [25]. Algorithms using TDOA do not require time synchronization or two-way communication between the Mobile Tag and BSs, only the BSs need to be synchronized. In most cases, BS synchronization is achieved by means, of wired connection between the BSs. Installing wired connections introduces additional cost and complexity which is especially problematic in applications requiring rapid deployment of the location infrastructure, such as in emergency response applications.

In this work, we consider the problem of determining the 3D position of a Mobile Tag (MT) based on one-way UWB periodic signaling using four or more wireless BSs. The BSs
are assumed to be independent, wireless UWB transceivers located at fixed known positions. The Tags are receive-only or transmit-only UWB mobile units. The algorithm relies on accurate timestamping, or control, of packet Time of Emission (ToE) and accurate estimation of the packet Time of Arrival (ToA). As such, the method is appropriate for use with, but not limited to, IEEE 802.15.4a UWB. For the purposes of clarity, we focus on the transmission control case herein. The algorithm utilizes multi-BS time synchronization and hybrid Time Difference of Arrival-Time of Flight (TDOA-TOF) localization to achieve high accuracy tag localization. The algorithm consists of two concurrent tasks. In task I, a novel technique for acquisition of time offset ($\phi$) and crystal clock offset ($\delta$) based on periodic transmissions and cooperation between BSs is used. In this technique each BS records the Times of Arrival of the periodic signals transmitted from all other BSs. By linear fitting, the relative time offset ($\phi_i$) and crystal clock offset ($\delta_i$) between BSs are estimated based on the known periodic Times of Emission (ToEs) and distances between BSs. This allows accurate time synchronization between BSs. The accuracy of time synchronization between BSs is enhanced by fusing multiple relative measurements between BSs to obtain their time offset ($\phi_i$) and crystal clock offset ($\delta_i$) relative to a single Reference BS (RBS) by means of the Best Linear Unbiased Estimator (BLUE).

In task II, a hybrid TDOA-TOF algorithm is used to determine the location of the MT. Once time synchronization between BSs is acquired, a conventional TDOA algorithm is applied to obtain an initial estimate of the mobile tag’s location. Based on this estimate and the known periodic Times of Emission, the time offset ($\phi_{MT}$) and crystal clock offset ($\delta_{MT}$) between the MT and BSs are estimated. Averaging these estimates over multiple transmissions using linear regression, allows accurate time synchronization between the BSs and the MT. A conventional TOF algorithm is then applied to obtain a final, accurate, estimate of the tag’s 3D location.

The proposed algorithm has the following advantages over previous solutions:

- It acquires time synchronization between the BSs without the need for a wired interconnection or external clock reference, reducing system cost and complexity.
- It requires only one-way communication between the BSs and the mobile tags, reducing power consumption and complexity at the tag in the transmit-only case and enabling privacy aware operation of the tags in the receive-only case.
- It achieves higher location estimation accuracy than conventional TDOA algorithms using wired or wireless BSs.
- It provides high-accuracy time synchronization between the BSs and tag.

The rest of this paper is structured as follows. Section II highlights related work. Section III states the problem of interest. Section IV explains the proposed algorithm. Simulation results are provided in Section V. Section VI concludes the paper.

II. Related work

There are two main aspects of the proposed system: multi-BS clock synchronization and hybrid Time Difference of Arrival-Time of Flight (TDOA-TOF) localization. The following paragraphs consider previous work which relates to each aspect of the system.

Most systems in the literature use Two Way Ranging (TWR) to compensate for clock drift by cancelling its effect on both sides [26], [27]. The differential Time-Difference-Of-Arrival (dTDOA) algorithm described in [28], [29] is a one-way localization technique compensating for clock drift. The dTDOA is obtained by subtracting two TDOA measurements so that the unknown start time of transmission and clock-offsets are cancelled. In the case of large clock drifts, the algorithm proposes utilizing two sets of dTDOA for clock drift compensation. Some systems estimate crystal clock offset by exploiting multiple transmissions between nodes [20], [30], [31]. The work in [32] proposes Repetitive Response TWR (RR-TWR) employing two (or more) responses from devices at a predetermined interval. Clock drift error is estimated and linear fitting with maximum likelihood is applied to compensate the error caused by random clock jitter assuming a constant-velocity relative motion between devices. A number of previously reported UWB systems exploit periodic signal transmission to compensate for clock parameters and then perform localization. The system proposed in [33], [34] uses a master node and a number of readers at known locations to locate tags at unknown locations. The master node initially transmits a signal which is received by both readers and tags. Readers transmit signals after a fixed pre-determined time period from the reception of the signal transmitted by the master node. The process is repeated periodically with a known fixed time period. The algorithm utilizes the known time periods and known distances between the master node and the readers to calculate clock drift and offset between the reader and master node clocks. A TDOA method is then used to estimate the tag locations. The system uses one way communication and does not need the readers to be synchronized initially. The work reported in [35] uses a reference transponder and a number of base stations to estimate the location of transponders at unknown locations in a similar way. The basic concept of [35] relies on the Frequency Modulated Continuous-Wave (FMCW) radar principle [36]. Thales Research and Technology (UK) proposed an indoor location system where transceiver BSs exchange massages to obtain time offset and time of flight (TOF) between them. The measured TOF between BSs is used for self-calibration and self-survey of new BSs starting from at least 3 reference BSs at known fixed locations. The location of a passive receive-only mobile tag is estimated using a conventional TDOA method since the time offsets between pairs of BSs are known [37].

A number of attempts have been made to combine the basic TOF, TDOA, and AOA algorithms to form hybrid algorithms. For example, a hybrid Time Difference Of Arrival/Angle Of Arrival (TDOA/AA) technique was presented in [38] for indoor positioning systems. The combined AOA method includes both the azimuth AOA (A-AOA) and elevation AOA
(E-AOA) to minimize the errors due to estimation of multipath parameters and gives the position of the estimated mobile terminals by simultaneously resolving a set of linearized location equations. In [40] a hybrid Time-Difference-Of-Arrival/Angle-Of-Arrival (TDOA/TOA) positioning technique with Kalman filters is utilized to reduce non line of sight (NLOS) Time-Of-Arrival (TOA) errors in indoor UWB environments. To deal with the effects of inaccurate NLOS AOA data, an AOA selection process was included. An adjustable Extended Kalman Filter (EKF) structure was used to process the formulated TDOA and select AOA measurements for mobile positioning.

Filter (EKF) structure was used to process the formulated TDOA and AOA positioning techniques to enhance the accuracy of mobile positioning and tracking. The scheme presented in [40] combines TDOA and AOA positioning techniques to enhance the accuracy of positioning compared to the classical technique (TDOA). In [41] a TOA/TDOA hybrid relative positioning system utilizes two way communication and TDOA to reduce both the number of base stations and the number of communication messages used for positioning. The work in [42] by the authors of this paper proposes a hybrid (AOA-TOF) with Timing Lock algorithm for ultrasonic signals to determine the location of a Mobile Device (MD) relative to a number of beacons with high accuracy. The algorithm utilizes an AOA-based location method to obtain an initial estimate of its own location. Based on this, it estimates the Timing Offsets (ToEs) between the MD clock and the BS transmissions. The average TO and the known periodicities of the beacon signals are then used to obtain a second, more accurate, MD location estimate via a TOF method. Unlike the UWB case, clock drift is negligible in the ultrasonic case, which simplifies the algorithm. Due to the speed of propagation of RF signals, offset and drift between clocks in UWB systems causes significant errors in location estimation. Compensating for clock parameters is a challenging task for most UWB ranging systems.

This paper proposes a novel algorithm for determining the 3D location of a Mobile Tag (MT) using wireless BSs and one-way UWB signaling. The algorithm consists of two concurrent tasks. In task I, the BSs exploit periodic inter-BS transmissions to obtain high accuracy time synchronization between BSs. In task II, a hybrid TDOA-TOF is used to determine the location of the tag. In the following sub-sections the two tasks are discussed in detail.

### III. Problem Statement

The goal of this work is to perform accurate 3D location estimation for Mobile Tag (MT) based on one-way UWB periodic signaling using at least four wireless BSs. We assume the following:

- BSs are UWB transceivers that can send and receive RF packets.
- BSs are in direct wireless communication with each other.
- BSs are static and their locations are known.
- Tags are receive-only or transmit-only UWB mobile units.
- The BSs and MT can estimate and record the ToE and/or ToA of the packets.
- The errors in ToE and ToA estimation can be modelled as zero-mean Gaussian distributions with standard deviation \( \sigma \). These error models include the effect of jitter at the transmitter, jitter at the receiver, plus noise, interference and multi-path in the channel.

- The BSs and MT have constant clock offset over short periods of time [43]. Clock offset arises from the crystal cut angle and varies with temperature [44]. Therefore, this assumption is valid over short periods of time (a few seconds) provided that nodes have reached their operating temperature [45].

### IV. Proposed Algorithm

The proposed algorithm utilizes multi-BS time synchronization and hybrid Time Difference of Arrival-Time of Flight (TDOA-TOF) localization to achieve high accuracy tag localization. The algorithm consists of two concurrent tasks. In task I, the BSs exploit periodic inter-BS transmissions to obtain high accuracy time synchronization between BSs. In task II, a hybrid TDOA-TOF is used to determine the location of the tag. In the following sub-sections the two tasks are discussed in detail.

#### A. Multi-BS time synchronization

The multi-BS time synchronization protocol starts with the BSs agreeing on the Reference Base Station (RBS) and the periodic time of transmission \( T_p \). The RBS initiates the process by sending a start-synchronization message then continues sending periodically according to the agreed timing. After an arbitrary random time from receiving the start-synchronization message, each BS starts transmission and sends periodically according to the agreed timing. All BSs continuously listen for packets sent by the other BSs. The data payload of each packet consists of the sending BS’s MAC address or ID, a sequential packet number \( p \) and a timestamp indicating the packet’s ToE estimated at the transmitting BS local time \( t_e(i, p) \). All BSs consider the ToE of their first packet as their local clock reference \( t_e(i, 0) = 0 \). All times measured locally are denoted by superscript \( i \) representing the BS ID.

Considering the packets sent by BS \( i \). The local ToE of packet \( p \) is:

\[
t_e(i, p) = pT_p + \delta_i + \epsilon_m(i, p) \tag{1}
\]

This is the timestamp sent with each packet.

In the case of periodic transmission, \( p \) can be sent as the timestamp, or omitted altogether, since \( T_p \) is known. When this packet is received by a BS \( j \), it records the timestamp and the locally estimated ToA \( t_e(i, p) \).

At the end of signaling, each BS has a sequence of ToE and ToA estimates in the form:

\[
[ t_e(i, 0), t_e(i, 0), (t_e(i, 1), t_e(i, 1)), \ldots (t_e(i, N_p - 1), t_e(i, N_p - 1)) ]
\tag{2}
\]

We refer to this sequence as the packet times of BS \( i \) at BS \( j \).

Due to concurrent transmission by all BSs, similar sequences of packet times are available for all BSs. The actual ToE \( t_e(i, p) \) of packet \( p \) transmitted from BS \( i \) can be expressed as:

\[
t_e(i, p) = \phi_i + pT_p(1 + \delta_i) + \epsilon_m(i, p) \tag{3}
\]
where \( \phi_i \) and \( \delta_i \) are the time offset and crystal clock offset of BS \( i \) and \( e_{cm}(i, p) \) is the ToE error at BS \( i \) for packet \( p \).

The actual ToA of this packet at BS \( j \) is:

\[
t_a(i, p) = \phi_i + pT_p(1 + \delta_i) + e_{cm}(i, p) + D_{ij}/c
\]

(4)

where \( D_{ij} \) is the distance between BS \( i \) and BS \( j \) and \( c \) is the speed of light.

An actual time \( t \) can be converted to the time at any BS \( j (t^j) \) by accounting for the time offset \( \phi_j \) and crystal clock offset \( \delta_j \):

\[
t^j = (t - \phi_j)(1 - \delta_j)
\]

(5)

Substitution gives an expression for the ToA estimated at the slave:

\[
t_d^j(i, p) = (\phi_i + pT_p(1 + \delta_i) + e_{cm}(i, p) + D_{ij}/c - \phi_j)(1 - \delta_j) + e_{ar}(j, p)
\]

(6)

where \( e_{ar}(j, p) \) is the ToA estimation error at BS \( j \) for packet \( p \).

Re-arranging gives:

\[
t_d^j(i, p) = pT_p(1 + \delta_i)(1 - \delta_j) + (\phi_i - \phi_j + e_{cm}(i, p) + D_{ij}/c)(1 - \delta_j) + e_{ar}(j, p)
\]

(7)

By inspection, it is clear that \( t_d^j(p) \) is linear in \( p \) with gradient:

\[
G_{ij} = T_p(1 + \delta_i)(1 - \delta_j)
\]

(8)

and constant offset:

\[
O_{ij} = (\phi_i - \phi_j + e_{cm}(i, p) + D_{ij}/c)(1 - \delta_j) + e_{ar}(j, p)
\]

(9)

where \( e_{cm}(i, p) \) and \( e_{ar}(j, p) \) are random errors.

We can consider the packet times as noisy observations of this linear function. Estimates for the gradient \( \hat{G}_{ij} \) and the constant offset \( \hat{O}_{ij} \) of this function can then be obtained by linear regression [46]. Provided that the errors have a zero mean Gaussian distribution, the accuracy of the estimated \( \hat{G}_{ij} \) and \( \hat{O}_{ij} \) will improve as the number of observations, \( N_p \), increases.

Hence, for sufficiently large \( N_p \), \( \hat{O}_{ij} \) will tend to approximately \( (\phi_i - \phi_j + D_{ij}/c) \) and \( \hat{G}_{ij} \) to approximately \( T_p(1 + \delta_i)(1 - \delta_j) \).

Since \( D_{ij}/c \) and \( T_p \) are known, the relative time offset \( \hat{\phi}_{ij} \) and crystal clock offset \( \hat{\delta}_{ij} \) can be calculated from:

\[
\hat{\phi}_{ij} = \hat{O}_{ij} - D_{ij}/c
\]

(10)

\[
\hat{\delta}_{ij} = \hat{G}_{ij}/T_p - 1
\]

(11)

A similar argument can be made for all pairs of BSs. Redundant estimates of relative time offset \( \hat{\phi}_{ij} \) and crystal clock offset \( \hat{\delta}_{ij} \) are obtained for all possible combinations between BSs.

Let us assume that the RBS has zero time offset \( (\phi_{RBS} = 0) \) and crystal clock offset \( (\delta_{RBS} = 0) \). A graph can be constructed where each vertex is associated with a BS and its estimated \( \hat{\phi}_i \) and \( \hat{\delta}_i \) and each edge is associated with the observed relative time offset \( \phi_{ij} \) and crystal clock offset \( \delta_{ij} \) between BSs. Feeding the redundant estimates to a Best Linear Unbiased Estimator (BLUE) [47] provides more accurate estimates of the time offset \( \hat{\phi}_i \) and crystal clock offset \( \hat{\delta}_i \) of all BSs relative to the RBS. The BLUE algorithm obtains an optimal solution such that the Mean square Error (MSE) is minimized.

This Multi-BS time synchronization protocol runs continuously with an appropriate value of \( N_p \) to ensure accurate time synchronization between BSs all the time.

B. Hybrid Time Difference of Arrival-Time of Flight (TDOA-TOF) Localization

This is a novel method for high accuracy MT localization based on hybrid TDOA-TOF. The algorithm consists of four steps: (1) Signaling; (2) TDoA-based MT location estimation; (3) MT Timing Lock; and (4) TOF-based location estimation. A block diagram of the algorithm is presented in Figure 1.

1) Signaling: It has been stated before that the MT can be transmit-only or receive-only. For the sake of simplicity in the following discussion a receive-only MT is considered. All BSs are assumed to send UWB packets periodically with time period \( T_p \). The data payload of each packet consists of the sending BS’s MAC address or ID, a sequential packet number \( p \) and a timestamp indicating the packet ToE estimated in the transmitting BS’s local time frame \( t'_e(i, p) \). The MT records the ToAs of transmitted packets from all BSs.

Considering the packets sent by BS \( i \) the local ToE of packet \( p \) is:

\[
t'_e(i, p) = pT_p
\]

(12)

This is the timestamp sent with each packet.
When this packet is received by the MT, it records the timestamp and the locally estimated ToA $t_{\text{a}}^\text{MT}(i, p)$. At the end of signaling, the MT has a sequence of all BSs ToE and ToA estimates in the form:

$$\{(t_e^i(0), t_{\text{a}}^\text{MT}(i, 0)), (t_e^i(1), t_{\text{a}}^\text{MT}(i, 1)), \ldots (t_e^i(i, N_p-1), t_{\text{a}}^\text{MT}(i, N_p-1))\}$$ (13)

We refer to this sequence as the packet times of BS $i$ at MT.

2) TDoA-based MT location estimation: The actual ToE and ToA estimates in the form:

$$t_e(i, p) = \phi_i + pT_p(1 + \delta_i) + e_{\text{em}}(i, p) + r_i(p)/c$$ (14)

where $\phi_i$ and $\delta_i$ are the time offset and crystal clock offset of BS $i$ relative to the RBS and $e_{\text{em}}(i, p)$ is the ToE error at BS $i$ for packet $p$.

The actual ToA of this packet at the MT is:

$$t_a(i, p) = \phi_i + pT_p(1 + \delta_i) + e_{\text{em}}(i, p) + r_i(p)/c$$ (15)

where $r_i(p)$ is the range between BS $i$ and the MT at the time of receiving packet $p$ and $c$ is the speed of light.

This ToA can be converted to MT’s local time by accounting for the time offset $\phi_{\text{MT}}$ and crystal clock offset $\delta_{\text{MT}}$:

$$t_a^\text{MT}(i, p) = (\phi_i + pT_p(1 + \delta_i) + e_{\text{em}}(i, p) + r_i(p)/c - \phi_{\text{MT}})(1 - \delta_{\text{MT}}) + e_{\text{as}}(p)$$ (16)

where $\phi_{\text{MT}}$ and $\delta_{\text{MT}}$ are the time offset and crystal clock offset of the MT relative to the RBS and $e_{\text{as}}(p)$ is the ToA estimation error at the MT for packet $p$.

Provided that the maximum value of $(\phi_i + pT_p\delta_i)$ is short enough relative to the speed of the MT such that the MT can be assumed to be static, TDoA equations can be formed. Substituting values from the packet times sequence from step 1 and estimates of time offset $\phi_i$ and crystal clock offset $\delta_i$ from the Multi-BS time synchronization protocol for four or more BSs, the TDoA equations can be solved. An initial estimate of the MT location is obtained by applying any conventional TDoA algorithm [22], [23], [24], [25]. Thereafter, the estimated MT location $(x'(p), y'(p), z'(p))$ and the known location of each BS $i$ $[x_i, y_i, z_i]$ is used to calculate an estimate for the range between BS $i$ and MT at the time of receiving packet $p$, as follows:

$$\hat{r}_i(p) = \sqrt{(X_i - x'(p))^2 + (Y_i - y'(p))^2 + (Z_i - z'(p))^2}$$ (17)

3) MT Timing Lock: Re-arranging Eq (16) gives:

$$t_a^\text{MT}(i, p) = pT_p(1 + \delta_i)(1 - \delta_{\text{MT}}) + (\phi_i - \phi_{\text{MT}} + e_{\text{em}}(i, p) + r_i(p)/c)(1 - \delta_{\text{MT}}) + e_{\text{as}}(p)$$ (18)

Approximating for small values of $(\phi_i - \phi_{\text{MT}} + e_{\text{em}}(i, p) + r_i(p)/c)$ and $\delta_{\text{MT}}$ gives:

$$t_a^\text{MT}(i, p) = pT_p(1 + \delta_i)(1 - \delta_{\text{MT}}) + (\phi_i - \phi_{\text{MT}} + e_{\text{em}}(i, p) + r_i(p)/c) + e_{\text{as}}(p)$$ (19)

Subtracting the estimated values of $\hat{r}_i(p)/c$ arising from step 2 from $t_a^\text{MT}(i, p)$ gives an estimate of the ToE of packet $p$ from BS $i$ with reference to the MT local time:

$$t_e^\text{MT}(i, p) = pT_p(1 + \delta_i)(1 - \delta_{\text{MT}}) + (\phi_i - \phi_{\text{MT}} + e_{\text{em}}(i, p))(1 - \delta_{\text{MT}}) + e_{\text{as}}(p) + e_{\text{rs}}(p)$$ (20)

where $e_{\text{rs}}(p)$ is the error in estimating the range between BS $i$ and MT at time of receiving packet $p$.

By inspection, it is clear that $t_a^\text{MT}(i, p)$ is linear in $p$ with gradient:

$$G_i = T_p(1 + \delta_i)(1 - \delta_{\text{MT}})$$ (21)

and constant offset:

$$O_i = (\phi_i - \phi_{\text{MT}} + e_{\text{em}}(i, p) + e_{\text{rs}}(p)) + e_{\text{as}}(p)$$ (22)

where $e_{\text{em}}(i, p)$, $e_{\text{rs}}(p)$ and $e_{\text{as}}(j, p)$ are random errors.

Estimates for the gradient $G_i$ and the constant offset $O_i$ of this function can then be obtained by linear regression of the $r_a^\text{MT}(i, p)$ observations [46].

Provided that the errors have zero mean Gaussian distribution, the accuracy of the estimated $G_i$ and $O_i$ will improve as the number of observations, $N_p$, increases. Hence, for sufficiently large $N_p$, $O_i$ will tend to approximately $(\phi_i - \phi_{\text{MT}})$ and $G_i$ to approximately $T_p(1 + \delta_i)(1 - \delta_{\text{MT}})$.

Since $\phi_i$ and $\delta_i$ is continuously estimated using the multi-BS time synchronization protocol described earlier. The MT time offset $\phi_{\text{MT}}$ and crystal clock offset $\delta_{\text{MT}}$ relative to RBS can be calculated from:

$$\phi_{\text{MT}} = \hat{G}_i - \phi_i$$ (23)

$$\delta_{\text{MT}} = \hat{G}_i/T_p(1 + \delta_i) - 1$$ (24)

A similar argument can be made for all BSs. Redundant estimates of $\phi_{\text{MT}}$ and $\delta_{\text{MT}}$ are obtained and averaged to get a final refined estimate of MT time offset $\phi_{\text{MT}}$ and crystal clock offset $\delta_{\text{MT}}$ relative to RBS.

4) TOF-Based Location Estimation: The MT time offset $\phi_{\text{MT}}$ and crystal clock offset $\delta_{\text{MT}}$ estimates obtained in step 3 and the BSs time offset $\phi_i$ and crystal clock offset $\delta_i$ estimates arising from the multi-BS time synchronization protocol are substituted in Eq (20) to calculate an estimate of the packet ToE reference to the MT local time. The TOF of the signal can then be calculated as the difference between this predicted ToE $(t_e^\text{MT}(i, p))$ and the ToA estimate obtained in step 1 $(t_a^\text{MT}(i, p))$.

This estimated TOF can then be used to determine the range between the MT and BSs based on the speed of light ($c$). Overall, the range from the MT to BS $i$ at reception of packet $p$ is given by:

$$\hat{r}_i(p) = (t_a^\text{MT}(i, p) - t_e^\text{MT}(i, p))/c$$ (25)

Finally, these range estimates $\hat{r}_i(p)$ are provided to a conventional multilateration algorithm for calculation of the MT location [48].

The update rate of the MT clock parameters $\phi_{\text{MT}}$ and $\delta_{\text{MT}}$ can be adjusted independently of the update rate of the TOF-
Based location estimates in order to reduce the computational complexity of the system so as to meet real-time constraints. For example, the TOF-Based location update rate (step 4) may be 20 Hz while the TDOA-only location and clock parameter update rate might be 5 Hz. This will increase the time required for clock parameter convergence but, once the clock parameters estimates have converged, accuracy will be unaffected.

V. Simulation results

In order to evaluate the performance of the proposed algorithm, simulation results were obtained. Simulations were performed using Matlab to model four transmitters in a 4x4x4 m room. The BSs were located at the corners of the room ceiling. The MT was assumed to move over different locations in a circular track in the room. The period of packet transmission \( T_p \) was set to 10 millisecond which results in a system maximum update rate of 100 Hz. A random crystal clock offset with values between -40 to 40 ppm (parts per million) [49] were assumed for MT and BSs clocks. The ToA estimation error of the received UWB packets was modelled by adding Gaussian noise to the true ToAs with zero mean (Bias) and various values for standard deviation (\( \sigma \)). The packet ToE errors was assumed to be negligible compared to the ToAs for well controled UWB devices [50]. Error-minimizing methods [23], [24] were used for TDOA-based and TOF-based location estimation.

Assuming ToA measurement with zero bias (B=0) and 2 cm standard deviation (\( \sigma = 2 \) cm) which is similar to the UWB line of sight ranging parameters reported in [51] modelled from real experimental data.

Figure 2 and Figure 3 show the BS time offset \( \hat{\phi} \) and crystal clock offset \( \hat{\delta} \) estimation errors for varying numbers of received packets \( N_p \). The error decreases as \( N_p \) increases.

Figure 4 shows the cumulative 3D error in the estimated MT location for 200 different locations obtained using the conventional TDoA method and the proposed method. It can be seen from Figure 4 that the location estimates obtained using the proposed method are more accurate than the location estimates obtained using the conventional TDoA method.

Table I states the RMS error for BS time offset \( \hat{\phi} \) and crystal clock offset \( \hat{\delta} \) obtained using the inter-BS time synchronization protocol for various values of ToA standard deviation.

Table II shows the plane RMS error for the TDOA-only 3D location estimates and the Hybrid TDOA-ToF 3D location estimates and states the percentage improvement for various values of ToA standard deviation. Due to the poor BS geometry for a location point of view the error in the vertical axis is 2-3 times higher than the plane.

The results provided in Tables I and II show the effectiveness of the proposed method in reducing location estimation error. Clearly the proposed method has high immunity to random errors such as ToA measurement.

<table>
<thead>
<tr>
<th>ToA Standard deviation ( \sigma ) (cm)</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time offset ( \hat{\phi} ) RMS error (ps)</td>
<td>0.1</td>
<td>0.7</td>
<td>1.1</td>
<td>2.1</td>
<td>3.3</td>
<td>3.8</td>
<td>4.5</td>
<td>12.4</td>
</tr>
<tr>
<td>Crystal clock offset ( \hat{\delta} ) RMS error (ppm*10^{-6})</td>
<td>0.12</td>
<td>0.20</td>
<td>0.32</td>
<td>0.48</td>
<td>0.65</td>
<td>0.72</td>
<td>1.22</td>
<td>2.32</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ToA Standard deviation ( \sigma ) (cm)</th>
<th>0.1</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method RMS error (cm)</td>
<td>0.11</td>
<td>0.54</td>
<td>1.08</td>
<td>2.18</td>
<td>3.29</td>
<td>4.44</td>
<td>8.14</td>
<td>11.9</td>
</tr>
<tr>
<td>Conventional TDoA RMS error (cm)</td>
<td>0.17</td>
<td>0.83</td>
<td>1.68</td>
<td>3.39</td>
<td>6.01</td>
<td>8.72</td>
<td>13.81</td>
<td>20.59</td>
</tr>
<tr>
<td>Improvement (%)</td>
<td>34%</td>
<td>35%</td>
<td>36%</td>
<td>36%</td>
<td>34%</td>
<td>38%</td>
<td>41%</td>
<td>42%</td>
</tr>
</tbody>
</table>

TABLE II

RMS error of MT location estimates and percentage improvement vs. standard deviation of ToA
applying the hybrid TDoA-ToF method, as compared to a significant improvement in location estimation accuracy when high time synchronization accuracy between BSs and providing improved accuracy of location estimation.

Institutions, cycle 4 (PRTLI4).

This work was conducted as part of the NEMBES project which was funded by the Higher Education Authority (HEA) of Ireland under the Programme for Research in Third Level Institutions, cycle 4 (PRTLI4).

ACKNOWLEDGMENT

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[34] ——, “Non-synchronised time difference of arrival localisation scheme with time drift compensation capability,” *IET communications*, vol. 5, p. 693, 2011.


